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MDCCCLXIX.

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THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the

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thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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Adjudication of the Medals of the Royal Society for the year 1868 by the President and Council.

The Copley Medal to Sir Charles Wheatstone, F.R.S., for his Researches in Acoustics, Optics, Electricity, and Magnetism.

A Royal Medal to Mr. Alfred Russell Wallace, for his Labours in Practical and Theoretical Zoology.

A Royal Medal to the Rev. George Salmon, D.D., F.R.S., for his Researches in Analytical Geometry and the Theory of Surfaces, published in the Philosophical Transactions, the Transactions of the Royal Irish Academy, and the Quarterly Journal of Mathematics.

The Rumford Medal to Dr. Balfour Stewart, F.R.S., for his researches on the qualitative as well as quantitative relation between the emissive and absorptive powers of bodies for heat and light, published originally in the Transactions of the Royal Society of Edinburgh, and the Proceedings of the Royal Society of London, and now made more generally accessible by the publication in 1866 of his Treatise on Heat.

The Bakerian Lecture was delivered by Professor Henry E. Roscoe, F.R.S.; it was entitled "Researches on Vanadium."

The Croonian Lecture was not delivered.

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PHILOSOPHICAL TRANSACTIONS.

I. The Bakerian Lecture.—*Researches on Vanadium.*

By Henry E. Roscoe, *B.A., F.R.S.*

Received November 20,—Read December 19, 1867.

I. INTRODUCTION.

Amongst the physical properties which point out the general relationship and classification of chemical substances, none has so deservedly obtained the confidence of chemists as isomorphism. The vanadium compounds have, however, proved a remarkable and unexplained exception to the conclusions which generally follow from well-ascertained identity of crystalline form.

Rammelsberg*, and afterwards, more completely, Schabus†, pointed out the fact that the mineral vanadinite from several localities (a compound of lead vanadate and lead chloride) is isomorphous with apatite, pyromorphite, and mimetesite, minerals consisting of calcium phosphato-fluoride, lead phosphato-chloride, and lead arsenato-chloride, having the formulæ $3(Ca_3P_2O_8).CaFl_2$; $3(Pb_3P_2O_8).PbCl_2$, and $3(Pb_3As_2O_8).PbCl_2$. The crystalline form of all these minerals is an hexagonal prism, terminated by 6-sided pyramids. The angle P: P (terminal) found by Rammelsberg for vanadinite, and by different observers for the other minerals, is as follows:—

(1) Vanadinite .	142° 30′;	(3) Pyromorphite .	142° 15′;
(2) Apatite . .	142° 20′;	(4) Mimetesite . .	142° 7′;

and the relation of the axes

(1) 1 : 0·727;	(3) 1 : 0·736;
(2) 1 : 0·732;	(4) 1 : 0·739.

So far, indeed, has the isomorphism of these compounds been traced, that in many specimens these minerals have been found to crystallize together in all proportions; and Heddle‡ describes a crystal in his possession the upper half of which consists of vanadinite and the lower half of pyromorphite.

* Pogg. Ann. Bd. xcviii. p. 249 (1856). † Ibid. Bd. c. p. 297.

‡ Greg and Lettsom's British Mineralogy, pp. 409–410 (1858).

We are indebted to the researches of BERZELIUS for almost all we know concerning the chemical relations and composition of vanadium and its compounds. From his celebrated investigation on Vanadium* (1831), the results of which have since been more or less completely confirmed by SCHAFARIK† and CZUDNOWICZ‡, it appears that the formula of vanadic acid is VO_3. Hence it is evident that we have here to do with either a case of dissimilarly constituted substances acting as isomorphous bodies and crystallizing together, or else the conclusions of BERZELIUS are erroneous, and the true formula of vanadic acid is V_2O_5, corresponding to the pentoxides of phosphorus and arsenic. The first of these alternatives has been properly accepted by most chemists as the only present solution of the difficulty, inasmuch as the definite experimental data given by BERZELIUS render the assumption of any other formula but VO_3 for vanadic acid perfectly gratuitous in the absence of experiments proving these data to be erroneous.

BERZELIUS based his conclusions on the following experiments, viz. (1) the constant loss of weight which vanadic acid undergoes on reduction in hydrogen at a red heat; (2) the action of chlorine on this reduced oxide, when a volatile chloride is formed and a residue of vanadic acid remains, which is found to be exactly one-third of the quantity originally taken for reduction in hydrogen. Hence BERZELIUS concludes that the number of atoms of oxygen in the oxide is to that in the acid in the proportion of 1 to 3; so that (assuming the lowest oxide to contain one atom of oxygen) the acid contains three atoms, a result which BERZELIUS finds borne out by its capacity of saturation. The question whether the acid contains one or two atoms of metal BERZELIUS decides in favour of the former view, by finding that no compound corresponding to the alums is formed when vanadic acid is brought together with sulphuric acid and potash. The analyses of the volatile chloride made both by BERZELIUS and SCHAFARIK confirm this conclusion, and place beyond all doubt the fact that, if the atomic weight of vanadium be taken to be 68·5 and O=8, the formula of vanadic acid is VO_3, that of the oxide prepared by reduction VO, and that of the chloride VCl_3.

In the present communication I shall show that, whilst confirming these fundamental results in every particular, I still arrive at a totally different conclusion from BERZELIUS respecting the constitution of vanadic acid, and all the other vanadium compounds; for I shall prove that the true formula of vanadic acid is V_2O_5 (when O=16), and the true atomic weight of vanadium 51·3, inasmuch as the substance supposed by BERZELIUS to be vanadium is not the metal but an oxide with an atomic weight of 67·3, nearly that of BERZELIUS's metal, whilst the supposed terchloride is an oxychloride.

The following are the grounds upon which I base my conclusions; the experimental proofs are contained in the sequel:—

(1) An oxide of vanadium exists which unites with three atoms of oxygen to form vanadic acid. Hence this latter substance must contain more than three atoms of oxygen.

* Pogg. Ann. Bd. xxii. p. 1 (1831).

† Ann. Ch. Pharm. cix. p. 84 (1859).

‡ Pogg. Ann. Bd. cxx. p. 33 (1863).

(2) The following vanadium oxides have been obtained both in the dry and wet way, and their compositions determined:—

*(1) V_2O_2, vanadium dioxide, or divanadyl . . . . = 134·6
(2) V_2O_3, vanadium trioxide (Berzelius's suboxide) = 150·6
(3) V_2O_4, vanadium tetroxide . . . . . = 166·6
(4) V_2O_5, vanadium pentoxide (vanadic acid) . . . = 182·6

(3) The so-called terchloride of vanadium (VCl_3. V=67·3) contains oxygen; it is an oxychloride having the formula $VOCl_3$ (V=51·3); it may be called vanadyl trichloride, or vanadium oxytrichloride, and corresponds to $POCl_3$, phosphorus oxychloride.

(4) Three other solid oxychlorides exist, having the composition
(1) $VOCl_2$, vanadyl dichloride, or vanadium oxydichloride.
(2) VOCl, vanadyl monochloride, or vanadium oxymonochloride.
(3) V_2O_2Cl, divanadyl monochloride.

(5) All the native vanadates are tribasic:—
(*a*) Vanadinite from Zimapan, analyzed by Berzelius (Pogg. Ann. Bd. xxii. p. 63), contains nearly three equivalents of lead oxide to one of vanadium pentoxide.
(*b*) Eusynchnite, analyzed by Czudnowicz (Pogg. Ann. Bd. cxx. p. 27), contains three equivalents of a mixture of lead and zinc oxides to one of vanadium pentoxide.
(*c*) Arœoxene, analyzed by V. Kobell (Journ. für. Chem. Bd. l. p. 496), was found to have a similar composition to eusynchnite.
(*d*) An American vanadinite, analyzed by Smith (Journ. Pr. Chem. Bd. lxvi. p. 433), is a tribasic compound.
(*e*) Dechenite from the Pfalz is, according to Brush (Sill. Am. Journ. series 2, vol. xxxiv. p. 116), identical with Arœoxene, and therefore a tribasic compound.
(*f*) Volborthite, according to Credner's analysis (Pogg. Ann. Bd. lxxiv. p. 546), also contains one of vanadic acid to three of basic oxide.

(6) Vanadic acid fused with sodium carbonate displaces three molecules of carbon dioxide, showing that normal or ortho-sodium vanadate is tribasic, the formula of this salt being Na_3VO_4.

(7) The so-called monovanadates are salts corresponding to the monobasic phosphates, and may be termed metavanadates; thus, $NaVO_3$, NH_4VO_3, $Ba2VO_3$. The so-called bivanadates are anhydro-salts, similar in constitution to the anhydro-salts of chromic and boric acids.

* [I had originally intended to call the first and third oxides vanadium monoxide and dioxide respectively, giving to these substances the simplest empirical formulæ VO=67·3 and VO_2=83·3, as it is as yet uncertain whether the molecular weights of these oxides, in accordance with the law of even atomicities, are multiples of the above, or whether, as in the only similar case, that of the nitrogen oxides, they are represented by the above numbers themselves. Acquiescing in the opinion expressed by several chemists that the former alternative is the most probable, I have now adopted the names as they stand in the text.—Jan. 20, 1868.]

(8) Vanadium nitride has been prepared, which, on analysis, was shown to contain 51·3 parts by weight of vanadium to 14 parts of nitrogen.

All the reactions according to which vanadic acid was supposed (BERZELIUS, RAMMELSBERG, SCHAFARIK, &c.) to contain three atoms of oxygen with an atomic weight V = 67·3 can equally well be explained when V_2O_5 (V = 51·3) is taken to represent the composition of this substance. That this is the case is seen from the following:—

	Berzelius's formulæ. (V = 68·5. O = 8.)	New formulæ. (V = 51·3. O = 16.)
(1)	$VO_3 + H_2 = VO + H_2O_2$	$V_2O_5 + 2H_2 = V_2O_3 + 2(H_2O)$
(2)	$3(VO) + Cl_6 = VO_3 + 2(VCl_3)$	$3(V_2O_3) + 6Cl_2 = V_2O_5 + 4(VOCl_3)$

II. OCCURRENCE AND PREPARATION OF THE VANADIUM COMPOUNDS.

1. The sources of vanadium, although numerous, have up to the present time yielded the compounds of this metal in such extremely small quantities that vanadium salts may still be counted amongst the greatest of chemical rarities; and consequently the satisfactory investigation of their properties has proved so difficult that contradictory statements, concerning even fundamental points, are made by recent experimenters. My attention was directed some time ago to the occurrence of vanadium in some of the copper-bearing beds of the Lower Keuper Sandstone of the Trias, worked at Alderley Edge and Mottram St. Andrews, in Cheshire; and I was fortunate enough to secure a plentiful supply of this rare metal by the purchase of a lime precipitate containing vanadium which had been obtained as a residue in the extraction of cobalt from the cobalt-bed sandstone at Mottram, by the Alderley Edge Copper Mining Company. The geological features of the remarkable deposit of metallic salts which occurs in this Keuper Sandstone have been well described by Mr. HULL*. The horizontal beds of sandstone contain copper as blue and green carbonates, lead both as carbonate and as galena, cobalt as black cobalt-ochre, and iron oxides, all in workable quantities, together with arsenic, silver, manganese as dioxide, and barium as heavy spar. The sandstone is of a soft and uniform texture, and is metalliferous to a depth of at least 60 feet; it is coloured variously from green and brown to black, according to the nature of the covering of metallic ore which surrounds the grains of sand. The following arrangement of the beds is given by Mr. HULL:—

	ft.	in.
1. Yellowish sandstone	4	0
2. Shaly clay with a band of copper-sand at the bottom	2	6
3. Ferruginous sandstone, with large nodules containing carbonate of lead	6	0
4. Cobalt bed, laminated sandstone containing earthy cobalt	4	6
5. White compact sandstone, with carbonate of lead	5	0
6. Iron-stained sandstone, with cobalt, manganese, and iron	12	0

The copper is disseminated throughout the sand in quantities on an average of not

* Geological Magazine, vol. i. p. 65 (1864).

more than from 0·5 to 2·5 per cent.; it is dissolved out by hydrochloric acid, and the metal precipitated from the solution by scrap iron. The lead is contained in small crystals throughout the mass, and is separated from the sand by maceration and washing; some of the rock contains as much as from 30 to 40 per cent. of the ore.

The sandstone from which the vanadium precipitate was prepared possesses a light colour, and contains from 0·1 to 0·3 per cent. of the oxides of cobalt, nickel, and copper disseminated as small black, green, and red specks throughout the mass. After crushing, the metals were dissolved out by hydrochloric acid; bleaching liquor and milk of lime were then added to alkaline reaction; a portion of the copper, together with the whole of the nickel and cobalt, remained in solution, whilst the lead, iron, arsenic (partly derived from the acid used), a little copper, and the vanadium were precipitated. As the Mottram mine is now closed I have been unable to obtain a sample of the above-mentioned sandstone for examination, and therefore I cannot state positively whether the vanadium occurs as vanadinite, although for other reasons I am inclined to think that it does.

The above-mentioned lime precipitate was believed by the manager of the mines to contain about 10 per cent. of copper; but when it was dissolved in acid and the copper thrown down by zinc, the solution still retained a bright blue colour, which I soon recognized as being due to vanadium. A rough analysis of the crude lime precipitate shows that it contains about 2 per cent. of vanadium, together with lead, arsenic, iron, lime, and sulphuric and phosphoric acids.

In order to prepare pure vanadium compounds in quantity from this material, I was glad to avail myself of the kindness of my friends Messrs. Roberts, Dale, and Co., who were good enough to place their works at my disposal for this purpose. Three cwt. of the crude material was dried, and then finely ground with four times its weight of coal, and the mixture well furnaced with closed doors for several days until the greatest part of the arsenic had been driven off. The coal having been thus burnt off, the mass was then ground up with one quarter of its weight of soda-ash, and well roasted in a reverberatory furnace with open doors for two days, to oxidize the vanadium to a soluble vanadate; the mass was next lixiviated, and the solution drawn off from insoluble matters: the liquid was acidified with hydrochloric acid, and sulphurous acid was then passed into the solution to reduce the arsenates, when the remaining arsenic was precipitated by sulphuretted hydrogen.

The deep-blue solution thus obtained was carefully neutralized by ammonia (an excess causes much of the vanadium to pass into solution), the precipitated vanadium oxide washed on cloth filters, oxidized by nitric acid, and evaporated to dryness. The well-dried crude vanadic acid was then boiled out with a saturated solution of ammonium carbonate, which left iron oxide and calcium sulphate, alumina, &c. insoluble, and the filtrate evaporated until the insoluble ammonium vanadate separated out. This crude vanadate was then washed with sal-ammoniac solution to free it from soda-salt, and recrystallized. In order to prepare from this salt pure vanadic acid, it was roasted in the air and the powdery acid obtained was suspended in water, into which ammonia gas was passed; the dissolved

ammonium vanadate was separated by filtration from a residue containing silica, phosphates, &c., and was crystallized by evaporation in platinum; the pentoxide obtained from this salt was free from phosphorus. Another mode, which was adopted in order to obtain perfectly pure vanadium pentoxide, was to prepare the pure oxychloride, and this, on being decomposed by water, yielded the acid as a fine orange-coloured powder. In order to free this powder from any trace of obstinately adhering silica, it was moistened with sulphuric acid and exposed in a platinum vessel for some days to hydrofluoric-acid gas; after expelling the sulphuric acid and on fusion, a mass of large transparent crystals of chemically pure vanadic acid was obtained.

The vanadium minerals appear invariably to contain more or less phosphorus, and this fact in itself serves to establish the close relationship which exists between these two elements. The complete separation of this substance from vanadium is attended with great difficulty: if much phosphorus is contained with the vanadium, the method which has proved most effectual for its removal is to deflagrate the finely divided impure acid with its own weight of sodium in a well-covered wrought-iron crucible, and wash the resulting mixture of vanadium oxides by decantation until the wash-water ceases to give an alkaline reaction; frequently this operation had to be repeated three times before the molybdenum test ceased to indicate phosphorus. In other preparations, in which the phosphorus had been more completely removed by previous operations, the process described above, of repeated crystallization of the ammonium salt, was found capable of furnishing a product in which no trace of yellow precipitate was produced by molybdenum.

The action of even traces of phosphoric acid on vanadic acid is most remarkable; if present in quantities exceeding 1 per cent. of the weight of the vanadium, phosphoric acid altogether prevents the crystallization of the vanadic acid, and the fused mass possesses a glassy fracture, and a black vitreous lustre; this effect is not produced by the presence of either silica, arsenic, or the lower oxides of vanadium. Still more singular is the protective influence which even traces of phosphoric acid exert upon the reducing action of hydrogen on vanadic acid at a red heat. Thus a mixture of pure vanadic acid with 1 per cent. of phosphorus pentoxide weighing 1·5888 grm., on ignition in a current of hydrogen lost only 0·0007 grm.; if no phosphorus had been present it must have lost 0·2784 grm.

III. ATOMIC WEIGHT DETERMINATION OF VANADIUM BY REDUCTION OF VANADIUM PENTOXIDE IN HYDROGEN.

The method originally adopted by Berzelius for the determination of the atomic weight of vanadium, viz. the reduction of vanadic acid in a current of hydrogen, is perfectly reliable, inasmuch as the reduced trioxide (the suboxide of Berzelius) does not undergo any further change when the temperature is raised to a bright-red heat, and the reduction of the pentoxide to trioxide at temperatures much below this is perfectly definite. The number obtained by Berzelius according to this method was V=68·5; the

exact number which the following experiments yield, when calculated on BERZELIUS's hypothesis, is 67·3.

In the following determinations of atomic weights the quantity of powdered pentoxide employed was always more than 5 grms.* It was placed either in a bent tube of hard glass, which when heated in a magnesia bath was found not to lose more than 0·0001 grm. after repeated ignition and cooling, or in a platinum boat placed in a glass tube.

Before reduction the powdered vanadium pentoxide was gently heated in a current of dry air until the weighings, after two successive operations, were found to be constant. After the reduction, the tube was allowed to cool completely in a current of hydrogen, and this gas was then displaced by a stream of dry air. If the reduced oxide be even slightly warm when it is exposed to the air it absorbs oxygen, suddenly glows, and is superficially converted into a blue oxide; if, however, the oxide be cold, it may be exposed to dry air for some time without change of weight, an experiment in which nitrogen was substituted for air giving results identical with those in which air was used.

In the experiments in which the oxide was contained in the glass tube the latter was carefully stoppered, and allowed to remain for half an hour in the balance-case before weighing; when the platinum boat was used it was carefully withdrawn, with the reduced oxide, from the combustion-tube and placed in a stoppered test-tube for weighing. These precautions are absolutely necessary in order to ensure accuracy, owing to the hygroscopic nature of the trioxide.

The hydrogen employed was most carefully purified and dried; all joints and stoppers in the apparatus were well secured by copper wire and paraffin. The gas passed through solutions of silver nitrate, sodium pyrogallate, caustic soda, and sulphuric acid, and a tube filled with metallic copper kept red-hot during the experiment was placed before the final drying-tube to ensure absence of oxygen. In order to ascertain whether atmospheric oxygen diffused into the apparatus, a weighed tube containing phosphorus pentoxide was attached to the further end of the combustion-tube during an experiment, and allowed to remain for four hours whilst hydrogen was passing through the heated tube; at the end of the operation the drying-tube had gained only 0·0002 grm., proving the absence of diffused oxygen. For the purpose of drying the hydrogen, boiled sulphuric acid alone can be used. In many of the preliminary experiments phosphorus pentoxide was used in the last drying-tube, and in all these cases the reducing action of the hydrogen was not complete, the numbers thus obtained for the atomic weight being all too high. After much labour this was found to be entirely caused by small traces of the light particles of the phosphorus pentoxide, which were invariably carried over, in spite of stoppers of cotton-wool, with the air and hydrogen into the vanadic acid,

* In order to ensure accuracy, it is absolutely necessary to employ a larger quantity of material than was taken by BERZELIUS. The largest amount which he used was 2·2585 grms. vanadic acid, and the smallest 0·6499 grm. An error of one milligramme on the first weight will produce a variation of $\pm 0{\cdot}2$ in the atomic weight, whilst a similar error on the smaller quantity would throw out the result by $\pm 0{\cdot}7$. If 5 grms. of substance are operated upon the variation arising from a milligramme error is $\pm 0{\cdot}086$.

the presence of the smallest quantity of phosphorus pentoxide rendering, as has been shown, the complete reduction of the vanadic acid impossible. Thus, for example, in two determinations it was found that

(1) 2·9232 grms. vanadium pentoxide yielded 2·4840 grms. trioxide, or V=65·4.
(2) 4·2826 grms. yielded 3·5649 grms. trioxide, or V=55·4.

On testing the residual oxide for phosphorus, the top layer of trioxide in the boat was found to contain phosphorus pentoxide, whereas the lower layers were free from it. In order to show that the reduction in these cases was incomplete, 3·4886 grms. of the reduced oxide from an experiment of the kind was treated in a tube with dry chlorine; when all the chloride had volatilized, the residual vanadic acid was fused and found to weigh 2·165 grms.; if the oxide had been completely reduced, the residual vanadic acid would have weighed 1·407 grm.

No less than nine determinations of atomic weight were made with every possible precaution, in which the results varied from 52·2 to 65·4, owing solely to the employment of drying-tubes containing phosphoric anhydride. As soon as this source of error was removed and a sulphuric-acid tube substituted, the pentoxide was completely reduced to trioxide and the numbers became constant.

The formula of vanadic acid being V_2O_5, and that of the oxide obtained by reduction V_2O_3, the atomic weight of vanadium is found from the equation

$$x=\frac{8(5b-3a)}{a-b},$$

where $a=$ the weight of vanadic acid taken, and $b=$ the weight of reduced oxide obtained.

Determination No. 1.—The vanadic acid used in this and the following determinations was prepared from ammonium vanadate by roasting; the acid thus obtained was found to contain traces of phosphorus and silica. To free it from these substances it was treated as follows: the powdered acid was first deflagrated with its own weight of metallic sodium in an iron crucible, the reduced oxide completely washed by decantation, treated with hydrochloric acid to free it from iron, and oxidized with nitric acid. This vanadic acid was then reduced in a current of hydrogen and converted into the oxychloride by heating in a current of chlorine; after rectification the chloride was decomposed by water, and the resulting powdery acid dried and afterwards moistened with pure sulphuric acid and exposed in a platinum vessel for ten days to the action of hydrofluoric-acid gas. On fusing and cooling this purified acid, dark red transparent crystals, five to six centimetres in length, were obtained, stretching across the bottom of the basin.

Platinum boat after first heating in air + tube* 19·6287 grms.
" " second " " 19·6287 "

* The glass tube and stopper were frequently reweighed to ascertain if the friction of the stopper produced any appreciable diminution of weight. At the end of the series of determinations they were found to have

Boat, tube, and vanadic acid after first heating in air	1 hour	. . .	27·368 grms.
„ „ „ „ second	„	2 hours . . .	27·3684 „
„ „ „ „ third	„	2 hours . . .	27·3684 „

Hence weight of vanadium pentoxide taken = 7·7397 grms.

After the reduction the weighings were not begun until every trace of water had disappeared; two weighings serve, therefore, to show that a constant weight has been reached.

Boat, tube, and reduced oxide after first heating in hydrogen	5 hours	. .	26·0120 grms.
„ „ „ „ second	„	3½ hours . .	26·0114 grms.

The atomic weight calculated from these numbers, according to the above formula, is V = 51·257.

Determination No. 2.—Platinum boat after first heating in air + tube 19·6258 grms.
„ „ second „ „ 19·6259 grms.

Boat, tube, and vanadic acid after first heating in air	2¼ hours	. .	26·2074 grms.
„ „ „ „ second	„	2½ hours . .	26·2079 grms.

Hence weight of vanadium pentoxide taken = 6·5819 grms.

Boat, tube, and reduced oxide after first heating in hydrogen for			3½ hours .	25·058	grms.
„ „ „ „ second	„	„	3 „ .	25·0569	„
„ „ „ „ third	„	„	3 „ .	25·0560	„
„ „ „ „ fourth	„	„	3 „ .	25·0555	„

The atomic weight calculated from these numbers is V = 51·391

In order to confirm this result, the reduced oxide was roasted in a current of dry air until it again attained a constant weight. The powdery acid obtained was of a very brilliant orange-colour; the following numbers show that the original weight was regained to within 0·0004:—

Platinum boat, tube, and vanadic acid after gently heating in a current of air for 14 hours	26·2075 grms.
Ditto, after heating for 3 hours longer	26·2073 grms.

Determination No. 3.—The pentoxide employed for this determination was prepared by the decomposition of another portion of rectified oxychloride by water. It was free from phosphorus, but was not treated with hydrofluoric acid. A hard glass tube heated in a magnesia bath was used in this experiment.

Tube after first heating and cooling	38·4666 grms.
„ second „	38·4667 „
Tube and vanadic acid after heating 1 hour in dry air .	43·6562 „

Hence the weight of vanadium pentoxide taken = 5·1895 grms.

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lost 0·0004 grm., a weight perfectly appreciable with the balance employed, but without influence on the results. Stas likewise noticed a regular decrease of weight in a stoppered tube owing to the same cause.

Tube and reduced oxide after heating in hydrogen over three BUNSEN'S lamps 42·7689 grms.

First heating in hydrogen gas in combustion-furnace					42·7678 grms.
Second	„	„	„	„	42·7511 „
Third	„	„	„	„	42·7486 „

The atomic weight calculated from these numbers is V=51·485.

Determination No. 4.—The acid used in this experiment was prepared by roasting, and afterwards completely oxidizing with nitric acid, pure ammonium vanadate which had been recrystallized four times, and had been prepared from acid obtained by the decomposition of the rectified oxychloride in water. The acid was free from phosphorus and silica, having been exposed, moistened with pure sulphuric acid*, for several days to the action of hydrofluoric-acid gas in a platinum vessel. Platinum boat and tube (constant weight) 25·2630 grms.

Boat, tube, and vanadic acid after			first heating in air		30·3109 grms.
„	„	„	second	„	30·3074 „
„	„	„	third	„	30·3083 „
„	„	„	fourth	„	30·3080 „

Hence the weight of vanadium pentoxide taken =5·0450 grms.

Boat, tube, and reduced oxide after			first heating with hydrogen			29·4290 grms.
„	„	„	second	„	„	29·4252 „
„	„	„	third	„	„	29·4244 „
„	„	„	fourth	„	„	29·4244 „

The atomic weight calculated from these numbers is V=51·353.

On gently roasting the reduced oxide in a current of air, the constant weight 30·3074 grms. was attained; this is within 0·0006 grm. of the original weight of boat, tube, and vanadic acid taken.

The mean of these four determinations is 51·371; hence we may assume the atomic weight of vanadium, as obtained from the reduction of pentoxide to trioxide, to be 51·4 as probably true to within ±0·07.

Determination	No. 1.	51·257,	difference from mean		−0·114
„	No. 2.	51·391	„	„	+0·020
„	No. 3.	51·485	„	„	+0·114
„	No. 4.	51·353	„	„	−0·018
Mean . . .		51·371	Mean error		±0·066

BERZELIUS'S mean number, calculated for V_2O_5, is found to be 52·55 with a mean error of 0·12, whilst the subsequent estimations of CZUDNOWICZ serve only as an approximate control of BERZELIUS'S number, giving a mean of 55·35 with a mean error of 2·33.

* A special experiment proved that every trace of sulphuric acid is easily expelled from vanadium pentoxide on ignition.

The fact that these older determinations all give a higher number than the above experiments, shows either that the acid was not fully oxidized, or, more probably, that the oxide was not completely reduced. It has already been stated (1) that the presence of the slightest trace of phosphorus prevents the complete reduction of the pentoxide to trioxide in hydrogen, (2) that all the naturally occurring vanadium ores contain phosphorus which can only be detected, when present in small traces, by molybdic acid, a test unknown in BERZELIUS's time, and (3) that the complete separation of phosphorus from vanadium is attended with great difficulty. Hence we may fairly conclude that the difference of 1·15 in the atomic weight between the above experiments and those of BERZELIUS is due to the presence of small traces of phosphorus in the vanadic acid used by the great Swede.

IV. THE VANADIUM OXIDES.

The oxides of vanadium of which BERZELIUS determined the composition were three in number:—

1. The suboxide, or lowest oxide, obtained by reducing vanadic acid in hydrogen. This oxide BERZELIUS supposed to contain one atom of oxygen, and gave to it the formula $VO=76·5$.

2. Vanadic oxide, to which BERZELIUS gave the formula $VO_2=84·5$, founding his view upon the analysis of a hydrated sulphate as well as on that of the precipitated oxide dried *in vacuo*.

3. Vanadic acid, $VO_3=92·5$, was shown to contain three times as much oxygen as the suboxide.

BERZELIUS describes several other intermediate oxides, but he did not isolate or analyze any of them.

All these three oxides exist and possess in the main the properties which BERZELIUS assigns to them; they all, however, contain for every one atom of BERZELIUS's vanadium (68·5) one additional atom of oxygen (O=16), with the existence of which he was unacquainted. Besides these oxides, a still lower one has been found containing one atom less oxygen than BERZELIUS's suboxide, and therefore having the atomic weight of BERZELIUS's metal; this oxide acts as a radical; it may be termed vanadyl, VO, and may be supposed to exist in the higher oxides.

Thus we have (V=51·3):—

1. Vanadium dioxide, or vanadyl, V_2O_2.
2. Vanadium trioxide (BERZELIUS's suboxide), V_2O_3, or V_2O_2+O.
3. Vanadium tetroxide (BERZELIUS's vanadic oxide), V_2O_4, or $V_2O_2+O_2$.
4. Vanadium pentoxide (vanadic acid), V_2O_5, or $V_2O_2+O_3$.

1. *Vanadium Dioxide (or Vanadyl)*, $V_2O_2=134·6$.—In its power of uniting with oxygen vanadium surpasses even uranium, as observed by PELIGOT*. Like uranium, the

* Ann. de Ch. et de Phys. 3 sér. tomes v. & xii.

metal vanadium can be separated from its last atom of oxygen with such extreme difficulty, and the lowest oxide is found to enter as a radical into so many compounds, that to this oxide the name vanadyl (VO) may appropriately be given.

Vanadium dioxide can be obtained in the dry state, as a grey powder possessing a metallic lustre, by passing the vapour of vanadyl trichloride ($VOCl_3$), mixed with excess of hydrogen, through a combustion-tube containing red-hot carbon. Lower solid oxychlorides (see page 25) are found to be deposited in the further portions of the tube, whilst vanadium dioxide remains as a bright grey metallic powder mixed with the charcoal. On strongly igniting this mixture in a current of hydrogen, every trace of chlorine is removed, and a pure mixture of charcoal and dioxide remains.

Vanadium dioxide possesses a grey metallic lustre, dissolves in acids without evolution of hydrogen, and yields a lavender-coloured solution which bleaches strongly. Vanadium dioxide is insoluble in water.

Preparation 1. *Analysis* 1.—0·8650 grm. of a mixture of vanadium dioxide and carbon, prepared as described, and freed from chlorine by heating in hydrogen, was treated with hydrochloric acid: the carbon, collected on a tared filter and dried at 120° C., weighed 0·6126 grm.; the filtrate, evaporated to dryness with nitric acid, the residue fused and weighed, yielded 0·3605 grm. of vanadium pentoxide.

Analysis 2.—Another portion of the same substance free from chlorine was burned in a platinum boat in a current of oxygen. The carbon dioxide formed was weighed, and the residual vanadium estimated as pentoxide: 0·0996 grm. substance yielded 0·2482 grm. CO_2, and 0·0416 grm. V_2O_5. Therefore the percentage of vanadium dioxide contained in the grey powder is as follows:—No. 1, 105·3; No. 2, 96·1.

Vanadium Dioxide in solution.—If the dark-red solution obtained by dissolving finely powdered vanadium pentoxide in strong boiling sulphuric acid be diluted with fifty times its bulk of water and then digested with metallic zinc, the liquid rapidly changes colour under the influence of the nascent hydrogen, passing through all shades of blue and green until, after lapse of some time, it assumes a permanent lavender or violet tint. The vanadium is then contained in solution in the lowest degree of oxidation (V_2O_2=134·6) as sulphate, and this compound absorbs oxygen with such avidity as to bleach indigo and other vegetable colouring-matters as quickly as chlorine itself, and acts far more powerfully than any other known reducing agent. In order to estimate the degree of oxidation of the dissolved vanadium, a standard solution of potassium permanganate was added to the reduced liquid until a permanent pink colour was produced, experiments with vanadium trioxide of known composition (given on page 15) having proved that this point is that of maximum oxidation (V_2O_5), and that it can be accurately reached provided an excess of sulphuric acid be present*.

* This method of estimation was proposed and used by Czudnowicz (Pogg. Ann. Bd. cxx. p. 37); the results described by him differ, however, altogether from my own. He did not obtain the lavender-coloured solution by the action of zinc, and it is clear that in his experiments the reducing action of the hydrogen was not complete.

The strength of the permanganate solution, prepared from the pure crystals, was accurately estimated by iron and oxalic acid; 1 cub. centim. of standard solution was found to be equal to 0·001413 grm. oxygen as a mean of three well-agreeing experiments; the strength of this solution, as tested from time to time, was found not to vary during the course of the determinations.

Reduction with Zinc.—

	Exp. 1.	Exp. 2.	Exp. 3.
Weight of V_2O_5 taken	0·1038	0·0963	0·1672
Cub. centims. permanganate required .	19·4	18·2	31·6
Loss of oxygen on 100 V_2O_5	26·4	26·6	26·6

The calculated loss of oxygen on 100 vanadium pentoxide, when reduced to the dioxide V_2O_2, is 26·3; the mean experimental number is 26·53.

In order to control the foregoing results, a fresh solution of permanganate was prepared; 1 cub. centim. represented 0·001301 grm. oxygen; 0·1397 grm. vanadium pentoxide dissolved in sulphuric acid and completely reduced by zinc required 27·7 cub. centims. of this solution for complete oxidation, corresponding to a loss of 25·8 on 100 parts. In ascertaining the reliability of this method, it was further found that a solution containing 0·1 grm. of vanadium pentoxide dissolved in sulphuric acid became of a permanent pink colour on the addition of 0·3 cub. centim. of permanganate solution; also that 5 grms. of zinc dissolved in sulphuric acid and diluted to the same strength as in the determinations, required only one drop of permanganate solution to produce the permanent pink tint.

Reduction with Cadmium and Sodium Amalgam.—If solutions of vanadium pentoxide in sulphuric acid be reduced by cadmium (platinum scraps being added to facilitate the evolution of hydrogen), or by an amalgam of sodium, the ultimate result is the formation of the lavender-coloured solution of the dioxide. Thus:

	Reduction by cadmium.	Reduction by sodium amalgam.
Weight of V_2O_5 taken	0·0897	0·0681
Cub. centims. permanganate required (1=·001413)	16·4	12·8
Loss of oxygen on 100 V_2O_5	25·8	26·6

The mean of these six determinations gives 26·3 as the loss on 100 parts of vanadium pentoxide, exactly corresponding to the calculated numbers.

The salt formed by the reducing action of hydrogen in presence of the above metals and sulphuric acid is doubtless a vanadium sulphate, that is, sulphuric acid in which the equivalent quantity of hydrogen is replaced by the metal. I have yet to determine the composition of these salts. When ammonia or caustic potash is added to the lavender solution, a brown-coloured precipitate, probably of vanadium hydroxide, is formed, and this instantaneously absorbs oxygen.

On allowing the neutralized lavender solution to stand exposed to air for a few seconds

the colour changes to a deep chocolate-brown; so rapid, indeed, is this change of colour, when nearly the whole of the free acid is neutralized by zinc, that such a lavender solution may serve as a reagent for the detection of free oxygen not inferior in delicacy to an alkaline pyrogallate. When a current of air is passed through the lavender solution of sulphate in presence of excess of acid, oxygen is absorbed, the colour of the liquid gradually changes to a bright and permanent blue, and the vanadium is contained in solution as tetroxide, two atoms of oxygen having been taken up (see tetroxide, page 16). If the free acid contained in the lavender solution of vanadium sulphate be completely neutralized by zinc and air passed through, the liquid assumes a permanent brown colour, which on addition of acids turns green, and the vanadium is contained in solution as trioxide (see trioxide, page 15). The point of the reduction at which the bleaching action commences is easily ascertained by testing the liquid from time to time with litmus paper. The changes in colour which the sulphuric-acid solution of vanadium pentoxide undergoes when treated with zinc may be divided into seven stages.

Stage.	Colour.	Reaction.	State of oxidation of the metal.
1	Green	Acid	Vanadium pentoxide to tetroxide.
2	Bluish green	Acid	Vanadium pentoxide to tetroxide.
3	Blue	Acid	Vanadium tetroxide.
4	Greenish blue	Acid	Vanadium tetroxide to trioxide.
5	Green	Bleaches slightly ..	Vanadium trioxide to dioxide.
6	Bluish violet	Bleaches strongly ..	Vanadium trioxide to dioxide.
7	Lavender or violet ..	Bleaches strongly ..	Vanadium dioxide.

This shows that the bleaching action commences in stage 5 with the formation of the dioxide. A quantitative experiment made in the same way showed that the bleaching action of the reduced solution ceased when to 100 parts of the dioxide 9·5 parts of oxygen had been added; in order to pass completely into trioxide, 11·9 parts of oxygen would have been needed.

The reduction of vanadium to dioxide by means of nascent hydrogen in presence of zinc serves as an easy and accurate method of estimating vanadium when mixed with certain other metals.

2. *Vanadium Trioxide* (the suboxide of Berzelius), V_2O_3=150·6.—The anhydrous trioxide is obtained, as is well known, by reducing vanadium pentoxide in a current of hydrogen at a red heat. It is perfectly stable up to temperatures approaching a white heat; and even when heated in a current of hydrogen to this temperature the trioxide does not lose weight. 0·411 grm. vanadium pentoxide, when reduced in hydrogen and heated for two hours to whiteness in a wind furnace, yielded 0·366 grm. of black trioxide, the theoretical quantity being 0·368 grm.

In addition to the complete description of the properties of the oxide given by Berzelius, I have only to add that it undergoes oxidation, not only rapidly glowing when exposed whilst warm to the air, but likewise slowly when exposed to the air at ordinary temperatures. Black amorphous trioxide, when allowed to stand exposed to the air for some months, undergoes a remarkable change; it takes up an

atom of oxygen, and is transformed to small dark indigo-coloured crystals of tetroxide (see page 16).

Vanadium Trioxide in solution.—The trioxide is insoluble in acids, but it may be obtained in solution as follows:—If solutions of vanadium pentoxide in sulphuric acid are diluted and treated with excess of metallic magnesium, the changes of colour which are observed when zinc, cadmium, or sodium amalgam are used do not continue beyond the green, and the liquid contains vanadium trioxide in solution.

In order to ascertain the reliability of the method of analysis described, viz. oxidation by means of a standard permanganate solution, a known weight of vanadium trioxide was prepared by reducing vanadium pentoxide in a current of dry hydrogen with the precautions which will be found described under the atomic weights determinations.

The resulting trioxide was placed in a flask containing dilute sulphuric acid and filled with carbonic acid; on the addition of permanganate the black finely divided oxide is rapidly oxidized, dissolving in the acid, and the point of complete oxidation can thus be accurately attained. 0·4043 grm. of vanadium pentoxide thus reduced required 50·4 cub. centims. of permanganate solution (1 cub. centim. = 0·001413 grm. oxygen) for complete oxidation; this corresponds to a gain of 21·35 on 100 parts of trioxide; the theoretical gain in passing from trioxide to pentoxide is 21·24.

The following numbers were obtained by the action of magnesium on the solution of vanadium pentoxide in sulphuric acid (1 cub. centim. standard solution represents 0·001413 grm. oxygen):—

Reduction with Magnesium.—

	Exp. 1.	Exp. 2.	Exp. 3.	Exp. 4.
Weight of V_2O_5 taken	0·0759	0·1593	0·1025	0·1187
Cub. centims. permanganate required .	9·4	19·6	13·0	15·0
Observed loss of oxygen on 100 V_2O_5 .	17·5	17·38	17·9	17·9
Ditto, calculated	17·5	17·5	17·5	17·5

Solutions of vanadium trioxide can also be obtained by partial oxidation of the lavender-coloured solution of dioxide. If a current of air be passed through a reduced solution in which the free acid has been neutralized by excess of zinc, and the remaining metallic zinc removed, the liquid attains a permanent brown colour, which, on the addition of a few drops of acid, turns to green. Both the brown and green liquids contain trioxide in solution, the dioxide having taken up one atom of oxygen, as the following analyses show:—

(1) 0·1672 V_2O_5, reduced by zinc and oxidized by air passing through for one hour, required 21·9 cub. centims. of the foregoing permanganate solution for complete oxidation.

(2) 0·1652 V_2O_5, treated in the same way, but exposed for four hours to a current of air, required 21·0 cub. centims. permanganate (1 cub. centim. = 0·001320 grm. oxygen).

Hence 100 parts of vanadium dioxide have absorbed (1) 13·05 and (2) 12·96 parts of oxygen; to pass to trioxide, 11·9 parts are required.

If a few drops of acid are added to the brown solution, which does not undergo change even when air is passed through for several days, it immediately turns green, but does not undergo any alteration as regards its oxygen; thus a brown solution was divided into two equal parts, the one which was oxidized whilst neutral and brown required 12·13 per cent. of oxygen, the other, to which a few drops of hydrochloric acid were added, became at once green and required 10·82 per cent. of oxygen to raise it to V_2O_5.

Action of Chlorine upon Vanadium Trioxide.—The action of chlorine upon this oxide led Berzelius to give to it the formula VO, (V=68·5); but, as has already been stated, the fact that one-third of the vanadium remains in combination with oxygen in the residual vanadic acid follows equally from the formula V_2O_3 when V=51·3.

Thus $$3(V_2O_3)+6Cl_2=V_2O_5+4VOCl_3.$$

Two experiments made in order to test the purity of the trioxide prepared by reduction of vanadic acid in hydrogen gave the following results:—

(1) 4·6004 grms. V_2O_3, treated with dry chlorine, gave a residue of 1·8307 grm. V_2O_5; theoretically it should yield 1·8597 grm.

(2) 0·2686 grm. V_2O_3 yielded 0·992 V_2O_5, instead of 1·0858 grm.

The composition and constitution of the salts forming the green solution still remain to be ascertained.

3. *Vanadium Tetroxide*, V_2O_4=166·6 (the vanadic oxide of Berzelius).—The anhydrous tetroxide was obtained by Berzelius (Pogg. Ann. Bd. xxii. p. 19) by precipitating the hydrated oxide from a sulphate by sodium carbonate, washing, and igniting *in vacuo* to free it from water. On reducing in hydrogen Berzelius found that 0·762 grm. yielded 0·691 grm., or 90·67 per cent. of his "suboxide" (V_2O_3, V=51·2); and hence he legitimately concludes that "vanadic oxide has lost as much oxygen as the suboxide contains,—that is, as much as he thought it contained, for V_2O_4 yields 90·36 per cent. of V_2O_3:

$$V_2O_2+O=V_2O_3,$$
$$V_2O_2+O_2=V_2O_4.$$

The anhydrous tetroxide can also be prepared by the slow oxidation of the black trioxide at the ordinary atmospheric temperature. It possesses an indigo-blue colour, and under the microscope is seen to consist of brilliant blue shining crystals. This remarkable change occurs when the black trioxide is exposed for several weeks to the air, and it has been repeatedly observed. 1·1686 grm. of the blue crystals thus obtained were dried completely under the air-pump, yielded 1·2688 grm. vanadium pentoxide, corresponding to a percentage increase of 8·57: 100 parts of the tetroxide require 9·59 parts of oxygen for conversion into the pentoxide. These blue crystals are anhydrous. A weighed portion of trioxide was exposed to the air and from time to time dried *in vacuo* and weighed. On November 13th, 1866, the weight of the black trioxide was 0·7507; on February 4th, 1867, the colour had changed to a bluish black, and the weight was

0·8112; on March 25th the colour had become blue and the weight of the oxide was 0·8267, showing a percentage increase of 10·13, or an absorption of nearly one atom of oxygen, as 100 of trioxide require 11·87 of oxygen for conversion into the pentoxide. On further exposure the blue oxide increases in weight, again changes colour to a dark olive-green, and a hydrated oxide is formed.

Vanadium Tetroxide in solution.—Solutions of the salts of vanadium tetroxide are bright blue as described by BERZELIUS. They may be obtained (1) by the action of moderate reducing agents, such as sulphur dioxide and sulphuretted hydrogen (probably also oxalic acid, sugar, alcohol, &c.) upon solutions of vanadium pentoxide in sulphuric acid, (2) by the action of a current of air upon the acid solution of vanadium sulphate.

(1) If sulphur dioxide or sulphuretted hydrogen be passed into the yellow solution of vanadium pentoxide in dilute sulphuric acid, the liquid becomes permanently blue, no green or violet colour being obtained by the continued action of these reducing agents. In order to determine the state of oxidation of the metal, these blue solutions were boiled in an atmosphere of carbonic acid until every trace of the reducing agent was expelled, and on cooling standard permanganate was added until the pink tint was noticed.

	Reduced by SO_2.		Reduced by SH_2.			
	(1)	(2)	(3)	(4)	(5)	(6)
Weight of V_2O_5 taken . .	0·1062	0·1106	0·1060	0·1248	0·1489	0·1239
Cub. centim. permanganate required (1 cub. centim. =0·0014130)	6·7	7·0	6·8	8·3	9·5	7·9
Loss of oxygen on 100 pentoxide	8·91	8·94	8·97	9·39	9·01	9·00

Giving a mean loss of 9·03, whereas theory requires a loss of 8·75.

(2) When a current of air is passed through acid solutions of the dioxide a permanent blue colour is attained, two atoms of oxygen have been absorbed, and the solution contains tetroxide.

(1) 0·1149 grm. vanadium pentoxide completely reduced by zinc, and the acid solution oxidized by a current of air passing through for fifteen hours, required 9·5 cub. centims. of permanganate solution (1 cub. centim.=0·001320 grm. oxygen) for complete oxidation.

(2) 0·0919 grm. vanadium pentoxide, treated in the same way, required 6·3 cub. centims. of the permanganate solution for complete oxidation.

Hence 100 of vanadium dioxide has absorbed (1) 20·9 and (2) 23·4 parts of oxygen; in order to pass from the dioxide to the tetroxide 23·78 per cent. is needed. The sulphate obtained by evaporating the blue solution containing this oxide was analyzed by BERZELIUS (*loc. cit.* p. 18), and it may be regarded as a vanadyl sulphate, or sulphuric acid in which hydrogen is replaced by vanadyl.

4. *Vanadium Pentoxide* (Vanadic acid, V_2O_5, molec. weight 182·6).—The properties

of vanadic acid have been fully described by BERZELIUS; the only important addition to our knowledge of this oxide made since his time is the determination of the crystalline form (rhombic prisms) made by NORDENSKJÖLD*. In the present communication I propose to discuss the properties and composition of vanadic acid and the vanadates only so far as is necessary for the purpose of elucidating the true atomic weight of the metal.

Constitution of the so-called Monovanadates.—The analyses made by BERZELIUS of the ammonium and barium vanadates† suffice to point out the true character of these salts. The formulæ which he there gives, assuming the atomic weight of the metal to be 68·5, show that these compounds must be considered as metavanadates when the atomic weight of the metal is taken to be 51·3.

BERZELIUS's formulæ. V=68·5, O=8.

Vanadiate of ammonia, NH_3VO_3+HO.

Vanadiate of baryta, $BaOVO_3$.

New formulæ. V=51·3, O=16.

NH_4VO_3, ammonium metavanadate, or $\left.\begin{matrix}NH_4\\VO\end{matrix}\right\}O_2$.

BaV_2O_6, barium metavanadate, or $\left.\begin{matrix}Ba\\2VO\end{matrix}\right\}O_4$.

In order to confirm BERZELIUS's analysis of the ammonium salts, the following analyses were made of the white anhydrous salt obtained by blowing ammonia gas into water containing suspended vanadic acid, and repeatedly crystallizing. The ammonium salt, on careful roasting in a platinum boat in a current of oxygen gas, yielded vanadium pentoxide, which was fused and weighed.

Nos.	Weight of ammonium salt taken.	Weight of pentoxide obtained.	Percentage of pentoxide found.	Calculated percentage.
1.	6·8531	5·3263	77·72	77·82
2.	2·0445	1·5902	77·78	
3.	2·6129	2·0314	77·75	

BERZELIUS found 77·59 per cent. of vanadic acid. The analysis of the ammonium metavanadate, if accurately made, serves likewise as a check on the atomic weight determinations. If a represent the weight of the salt taken, and b that of the vanadium pentoxide obtained, the atomic weight of vanadium is found from the equation

$$V=\frac{66b-40a}{a-b}.$$

From No. 1. V=50·65.

From No. 2. V=51·06.

From No. 3. V=50·82. Mean result 50·84.

Constitution of the so-called Bivanadates.—The bivanadates obtained by the cautious addition of acetic acid to the monovanadates, according to BERZELIUS's directions, have

* Pogg. Ann. Bd. cxii. p. 160. † Ibid. Bd. xxii. p. 49.

been analyzed by Von Hauer*. The ammonium and sodium bivanadates were found to correspond with the formulæ $NH_4O.2VO_3$ and $NaO2VO_3$, where V=68·5 and O=8. If we assume V=51·3 and O=16, these salts will be expressed by the formulæ $(NH_4)_2V_4O_{11}$ and $Na_2V_4O_{11}$; or they may be represented as *anhydro-salts*, either $2(NH_4VO_3)+V_2O_5$ and $2(NaVO_3)+V_2O_5$, or else as $Na_4V_2O_7+3(V_2O_5)$ as an anhydro-salt of sodium pyro-vanadate.

Constitution of the Normal Vanadates.—That vanadic acid in its normal state is tribasic is shown by the fact first pointed out by Czudnowicz†, that when vanadium pentoxide is fused with an excess of sodium carbonate three molecules of carbon dioxide ($3CO_2$) are displaced for every molecule of vanadium pentoxide (V_2O_5) entering into combination. In an experiment carefully conducted, in which sodium carbonate was heated to redness with 0·4323 grm. of vanadium pentoxide until no further loss of weight occurred, the pentoxide was found to expel 0·3078 grm. carbon dioxide (CO_2); this corresponds to a ratio of 2·957 molecules of carbon dioxide to 1 of V_2O_5, or nearly in the proportion of 3 to 1. Normal- or ortho-sodium vanadate is Na_3VO_4, or $\left.\begin{matrix}Na_3\\VO\end{matrix}\right\}O_3$. Hence there is no doubt that vanadic acid in its normal state acts as a tribasic acid.

It is my intention fully to investigate the constitution and properties of the vanadates at a future time.

V. VANADIUM OXYCHLORIDES.

1. *Vanadium Oxytrichloride*, or *Vanadyl Trichloride*, $VOCl_3$, molec. weight 173·67 (terchloride of vanadium of Berzelius).—The fact that the lemon-yellow-coloured liquid chloride of vanadium prepared by the action of chlorine upon the trioxide does contain oxygen, contrary to the distinct statements of previous experimenters, was ascertained in various ways:—

(1) A few grammes of the lemon-coloured liquid chloride was placed in a bulb attached to a long combustion-tube half filled with pure sugar-charcoal and half with metallic copper. A current of dry hydrogen, purified from oxygen according to the method previously described, was then passed over the bulb and cold tube until every trace of air was expelled; the carbon and copper were then heated to redness, and as soon as the escaping gases had ceased for fifteen minutes to render baryta-water turbid, Liebig's bulbs containing clear baryta-water were attached. The bulb containing the oxychloride, which up to this point had been kept cool, was now warmed, and the chloride distilled into the heated carbon in the tube; as soon as the vapour of the chloride was carried forward by the hydrogen, a dense precipitate of barium carbonate, which effervesced on the addition of hydrochloric acid, was thrown down in the Liebig's bulbs, proving that the liquid contains oxygen and undergoes partial decomposition when mixed with hydrogen and brought in contact with red-hot charcoal.

This experiment was repeated twice with identical results. It is not possible in this

* Journ. Prak. Chem. Bd. lxix. p. 388 (1856). † Pogg. Ann. Bd. cxx. p. 33.

manner to estimate the quantity of oxygen which the oxychloride contains, as only a small portion of the oxygen combines with the carbon, by far the greater portion remaining in combination with the metal to form dioxide or the lower solid oxychlorides yet to be described.

(2) The vapour of the lemon-yellow chloride was passed over metallic magnesium heated in a current of pure hydrogen; the action was violent, and the metal took fire. On cooling the tube the excess of magnesium was carefully removed, and the boat was found to contain a dark-coloured powder which, on extraction with water, contained both magnesium and chlorine; after repeated boiling with water, hydrochloric acid extracted from this powder large quantities of magnesia, the oxygen of which could only be derived from the vanadyl trichloride.

(3) The foregoing experiment, made with sodium instead of magnesium, showed that a thick crust of caustic soda was formed on the tube close to the boat in which the metal was placed.

(4) The vapour of the liquid oxychloride mixed with pure hydrogen was passed through a red-hot tube; the chloride was decomposed, black shining crystals of vanadium trioxide were deposited in the first portions of the tube, a mixture of lower solid oxychlorides filled up the further portions, and small quantities of a dark-red liquid were formed. The black crystals on analysis were found to be free from chlorine; 0·0860 grm. of these crystals gave on oxidation 0·1051 grm. vanadium pentoxide, or the substance contained 99·2 per cent. of vanadium trioxide.

(5) When vanadyl trichloride is rapidly distilled over heated carbon, or when it is prepared by treating a mixture of trioxide and carbon with chlorine, a dark reddish-brown liquid is formed consisting of a mixture of the oxychloride and another chloride. The analysis of this dark liquid yielded results which, when calculated for chlorine and vanadyl (V O=67·3), always added up to from 103 to 105 per cent. on the quantity taken, showing the presence in this liquid of a vanadium chloride containing no oxygen.

(i.) *Preparation of Vanadyl Trichloride.*—Finely divided vanadium pentoxide is intimately mixed with pure sugar-charcoal, and the mixture heated to redness in a current of hydrogen. After cooling in hydrogen the mixture of trioxide and carbon is removed to a hard glass retort, heated by a large Bunsen's lamp, and a current of dry chlorine gas passed in. The crude oxychloride comes off as a reddish-yellow* liquid, of which 180 grms. were collected in one operation. In order to purify this liquid it is distilled upwards in a current of carbonic acid for several hours, and afterwards rectified several times over clean sodium in a current of carbonic acid, when the sodium becomes coated with a dark substance due to the decomposition of some other chlorides. The liquid gradually assumes a light amber-colour, and on continued distillation it attains a permanent lemon-yellow tint; it began to boil at 126° C., and all came over before 130°.

* The dark colour of the chloride thus prepared is not due to the presence of vanadic acid as Schafarik supposes, as the acid is insoluble in the chloride, but rather to some dark-coloured vanadium chlorides containing no oxygen.

When perfectly pure the boiling-point was accurately determined with 100 grms. of substance; the temperature corrected for freezing-point of the thermometer and mean temperature of the column was found to be 126°·7 under 767·0 millims. of mercury.

(ii.) Vanadyl trichloride is obtained at once of a bright lemon-yellow colour, and free from the compounds which accompany its formation in the preceding reaction, by passing dry chlorine over the trioxide gently heated. The reaction is as follows, viz.—

$$3(V_2O_3) + 6Cl_2 = V_2O_5 + 4(VOCl_3).$$

The liquid thus obtained requires only to be distilled upwards for a short time in a current of carbonic acid, to remove excess of chlorine and hydrochloric acid, and then rectified once over sodium to obtain the substance chemically pure.

The general properties of vanadyl trichloride have been already described by BERZELIUS.

The specific gravity of this liquid was carefully determined at three temperatures, and found to be—

at 14°·5	1·841.
17°·5	1·836.
24°·0	1·828.

The trichloride remains liquid at temperatures above −15° C. As the coefficient of expansion of this liquid has not been yet determined above 24°, it is not possible to calculate the atomic volume of this liquid, or to compare it with that of phosphorus oxychloride ($POCl_3$).

The vapour-density of vanadyl trichloride was determined according to DUMAS's method with the following results:—

Weight of bulb and air at 11° C. and under 776·5 millims. . .	6·5172 grms.
Weight of bulb and vapour at 18°·6 C. and 780 millims. . .	7·0003 grms.

Residual air = 0·0 cub. centim. Capacity of bulb 135·13 cub. centims.

Hence the vapour-density is found to be 88·20 (H=1) or 6·108 (air =1); the calculated density is 86·8 (H=1) or 6·000 (air =1).

Analysis of Vanadyl Trichloride. Determination of the atomic weight of Vanadium. Method 2.—The difficulty attending determinations of the atomic weights of the metals from analysis of their volatile chlorides has already been pointed out by PIERRE* in the case of titanium. The errors here referred to, arising from absorption of moisture by these hygroscopic chlorides, may, however, be avoided by sealing up several portions of the chloride for analysis at once from a large mass of liquid.

The following seventeen determinations were most carefully made, and the numbers obtained, closely coinciding as they do with the results of the reduction experiments already given, serve as a most important control of the reliability of the original method.

Volumetric Determination of the Chlorine.—Before employing GAY-LUSSAC's method to this determination, it was ascertained that when the chloride is decomposed by water in

* Ann. de Ch. et de Phys. 3 sér. t. xx. p. 257.

presence of nitric acid and silver solution added, no trace of vanadium goes down with the silver.

Pure silver was prepared with all the precautions detailed by Stas* in his first memoir, and the analyses were conducted in the manner described by him†. The nitric acid was prepared by distilling a large quantity of pure acid until three-fifths had come over; the remainder was then distilled and collected, and on careful testing with silver showed not the slightest indication of chlorine. The distilled water used had been carefully redistilled, and likewise gave no indication of chlorine. The weight of silver to be taken was calculated from a careful gravimetric analysis, and several volumetric determinations were made at one time. The weighed quantity of silver was placed with ten times its weight of pure nitric acid, sp. gr. 1·2, in well-stoppered 300 cubic centimetre bottles, and heated to about 40° C. until all the silver had dissolved. After cooling, the bulb containing the weighed quantity of vanadyl trichloride was brought into the bottle, the bottle closed, and the bulb broken by shaking; distilled water was then added, so that the weight of the whole liquid amounted to from forty to fifty times that of the silver employed. The lower oxides of nitrogen present in solution in the acid reduced the vanadium to a bright blue solution, in which the slightest turbidity of silver chloride could be well seen.

In the volumetric determination of chlorine it has already been noticed, both by Gay-Lussac and Stas, that when one or two milligrammes of silver per litre still remain in solution, a turbidity is produced by the addition of both silver and chloride solution. This fact was also observed in the volumetric analyses of vanadyl trichloride; and in order to render the results comparable, the course proposed by Stas was invariably adopted, viz. that of adding the decimal chloride solution until all turbidity ceased. The error thus introduced is, however, extremely small (less than 0·0005), and is counterbalanced by the error arising from a trace of impurity contained in the silver.

In several cases the vanadium in the filtrate was estimated by precipitating the excess of silver, expelling the acid by evaporation, again filtering from the trace of silver chloride which separates out, and evaporating the filtrate to dryness in porcelain, transferring to platinum, oxidizing by nitric acid, and weighing the fused acid.

The following Table contains the results of nine chlorine determinations made according to the above method with trichlorides of various preparations. Nos. 1, 2, 3, and 4 were made with substances of one preparation, Nos. 5 and 6 of a second preparation, Nos. 7 and 8 of a third preparation.

* Recherches sur les Rapports réciproques des Poids atomiques. Bruxelles, 1860, p. 25.

† Idem, p. 62.

No. of experiment.	Weight of trichloride taken.	Weight of silver required for complete precipitation.	Percentage of chlorine*.	
1.	2·4322	4·5525	61·49	Analyst A.
2.	4·6840	8·7505	61·37	,,
3.	4·2188	7·8807	61·37	,,
4.	3·9490	7·3792	61·39	,,
5.	0·9243	1·7267	61·37	,,
6.	1·4330	2·6769	61·37	,,
7.	2·8530	5·2853	60·86	Analyst B.
8.	2·1252	3·9535	61·11	,,
9.	1·4248	2·6642	61·43	,,
		Mean result . . .	61·306	

Gravimetric Determination of the Chlorine.—As a check on the volumetric analysis, weight estimations were made in the usual way, it having been proved that no trace of vanadium is carried down with the silver in nitric acid-solution.

The trichloride used in these experiments was taken from four different preparations; analyses Nos. 4, 5, and 7 were made from the same sample by two different experimenters.

No. of experiment.	Weight of trichloride taken.	Weight of silver chloride obtained.	Percentage of chlorine.	
1.	1·8521	4·5932	61·33	Analyst B.
2.	0·7013	1·7303	61·01	,,
3.	0·7486	1·8467	61·00	,,
4.	1·4408	3·5719	61·30	,,
5.	0·9453	2·3399	61·21	,,
6.	1·6183	4·0282	61·55	,,
7.	2·1936	5·4309	61·22	Analyst A.
8.	2·5054	6·2118	61·31	,,
		Mean result . . .	61·241	

The atomic weight of vanadium calculated from the mean (61·276) of the above seventeen determinations is

$$\frac{(106{\cdot}371 \times 100) - (122{\cdot}371 \times 61{\cdot}276)}{61{\cdot}276} = 51{\cdot}29.$$

Taking the mean of the numbers found (1) by the reduction of vanadium pentoxide in hydrogen, viz. 51·37, and (2) by the determination of chlorine in vanadyl trichloride viz. 51·29, *we have the number* 51·33 *as the mean atomic weight of vanadium.*

Determinations of Vanadium.—The vanadium in vanadyl trichloride was directly estimated in some instances (Nos. 1 and 4) in separate portions of the trichloride, whilst in other cases (Nos. 2 and 3) it was estimated in the filtrate from the chlorine determinations.

* Stas's numbers are here used, viz. O = 16, Ag = 107·93, Cl = 35·457.

Nos.	Weight of trichloride taken.	Weight of vanadium pentoxide found.	Percentage of vanadium.
1.	1·4188	0·7368	29·20
2.	0·7013	0·3679	29·47
3.	0·7486	0·3959	29·71
4.	1·1731	0·6252	29·94
		Mean result	29·58

These results give as the composition of vanadyl trichloride,—

	Calculated.	Found.
V = 51·3	29·54	29·58
Cl_3=106·37	61·25	61·27
O = 16·00	9·21	——
173·67	100·00	

2. *Vanadium Oxydichloride*, or *Vanadyl Dichloride*, $VOCl_2$=138·21.—This substance is formed, together with other lower solid oxychlorides, when the vapour of vanadyl trichloride mixed with hydrogen is passed through a red-hot tube. It is easily obtained, however, in the pure state by the action of metallic zinc on the oxytrichloride in sealed tubes at 400°. For this purpose strips of zinc and slight excess of the liquid chloride are heated in a strong tube placed in an inclined position in an air-bath for three or four days in a temperature considerably above the boiling-point of mercury; the oxytrichloride is decomposed, a black oxide of vanadium is formed together with zinc chloride, and a sublimate of splendid grass-green tabular crystals of the oxydichloride collects in the upper part of the tube. On opening the tube, the portion containing the crystals was quickly pushed inside a wider tube heated to 130° in a paraffin bath through which a current of dry carbonic acid was passed; the excess of liquid oxychloride was thus easily expelled and the green crystals obtained of a constant weight. The black oxide formed together with the green crystals consists of the lowest oxide, V_2O_2, as on treatment with dilute acid it bleaches strongly.

Vanadium oxydichloride is a grass-green body crystallizing in thin plates and possessing an unctuous feel. It is slowly decomposed by water, and on exposure to moist air it deliquesces; it easily dissolves in dilute nitric acid. Its specific gravity at 13° C. is 2·88.

The composition of the oxydichloride is readily determined by precipitating the chlorine in nitric-acid solution, and estimating the vanadium in the filtrate.

Nos.	Weight of oxydichloride taken.	Ag Cl. Found.	V_2O_5. Found.	Percentages of chlorine.	Percentages of vanadium.
1.	0·3765	0·7683	0·2498	50·40	37·30
2.	0·3207	0·6631	0·2160	51·07	37·87

Hence we have as the composition of this substance:—

		Calculated.	Found.
V	= 51·3	37·12	37·58
Cl_2	= 70·91	51·39	50·73
O	= 16·0	11·59	——
	138·21	100·00	

3. *Vanadium Oxymonochloride*, or *Vanadyl Monochloride*, V O Cl=102·76.—Vanadyl monochloride is a brown, light, powdery solid, formed by the action of hydrogen upon vanadyl trichloride. It is obtained by passing the vapour of the trichloride together with hydrogen through a red-hot tube, and is deposited at the end nearest to the point where the trichloride enters the tube. It is insoluble in water, but easily soluble in nitric acid. It may readily be distinguished and separated from the preceding and following oxychlorides by its appearance and light flocculent nature. The analysis was made in the manner already described, the substance being dried at 130° in a stream of carbon dioxide till its weight was constant.

Nos.	Weight of substance taken.	Weight of silver chloride found.	Weight of vanadium pentoxide found.	Percentages of chlorine.	Percentages of vanadium.
1.	0·1393	0·1823	0·1240	32·35	50·03
2.	0·2631	0·3912	0·2285	36·71	48·82

Hence we have—

		Calculated.	Found.
V	= 51·3	49·92	50·21
Cl	= 35·46	34·51	34·53
O	= 16·0	15·57	——
	102·76	100·00	

4. *Divanadyl Monochloride*, V_2O_2Cl=170·06.—This substance is likewise formed in a similar manner to the foregoing oxychlorides. It has a bright, shining, metallic lustre and a brownish-yellow colour, closely resembling the tin disulphide known as "mosaic gold." This substance is always deposited in the portion of the heated tube furthest from the trichloride, and it adheres firmly to the glass. From its heavy coherent nature it can easily be separated from the preceding compound. Under the microscope it is seen to consist of brilliant-yellow metallic crystals, and it has been mistaken for metallic vanadium by SCHAFARIK.

It is insoluble in water, but dissolves like the other solid oxychlorides easily in nitric acid. The brown shining crystals were heated in carbon dioxide at 140° until their weight was constant, and then they were analyzed as above described.

Nos.	Weight of substance taken.	Weight of silver chloride found.	Weight of vanadium pentoxide found.	Percentages of chlorine.	Percentages of vanadium.
1.	0·2130	0·1777	0·2443	19·72	64·48
2.	0·6098	0·4767	0·6390	18·15	58·91

Hence we have—

	Calculated.	Found.
V_2=102·60	60·33	61·69
Cl= 35·46	20·84	18·93
O_2= 32·00	18·83	——
170·06	100·00	

VI. VANADIUM NITRIDES.

1. *Vanadium Mononitride*, V N=65·3.—The process described by BERZELIUS for preparing metallic vanadium by heating the ammonio-oxychloride in an atmosphere of ammonia does not yield the metal but a nitride.

The mononitride is obtained by passing dry ammonia gas over vanadyl trichloride contained in a bulb-tube, and igniting the ammonio-oxychloride formed until the ammonium chloride has volatilized. The black powder thus produced (vanadium dinitride?) is then placed in a platinum boat, which is exposed for some hours to the white heat of a wind-furnace in a porcelain tube through which dry ammonia (or rather a mixture of its constituent gases) passed. The substance thus obtained is a greyish-brown powder, which does not undergo change on exposure to air at ordinary temperatures, and remains unaltered when it is again strongly heated in ammonia. When roasted in the air it slowly oxidizes, glows, and forms a blue oxide; on further heating it melts, and ultimately forms pure vanadium pentoxide. Heated with soda-lime in a glass tube, the mono-nitride readily evolves ammonia.

The nitride employed in the following analyses had been repeatedly heated to whiteness in a current of ammonia until no further diminution of weight occurred.

Boat and nitride after second ignition in ammonia 6·285 grms.
" " third " for three hours 6·284 "

(1) *Determination of Vanadium.*—
Weight of nitride employed 0·3126 grm.
Constant weight of vanadium pentoxide, obtained by roasting, 0·4359.
Hence the nitride contained 77·8 per cent. of vanadium.
The formula V N requires 78·6 per cent. of metal.

(2) *Determination of Nitrogen.*—The nitrogen was determined according to SIMPSON'S method, by heating the nitride with a mixture of the oxides of mercury and copper, and passing the gases over red-hot metallic copper.

Analysis (1). Weight of nitride taken=0·2683 grm.

Observed vol. of nitrogen (measured dry).	Pressure.	Temp.	Vol. at 0° and 760 millims.
142·1 vols.	718 millims.	9°·2	43·2 cub. centims.

(1 vol.=0·331 cub. centim.).

This corresponds to a percentage of 20·3 nitrogen.

Analysis (2). Weight of nitride taken=0·2000 grm.

Observed vol. of nitrogen (measured moist).	Pressure.	Temp.	Vol. at 0° and 760 millims.
112 vols. (1 vol.=0·331 cub. centim.).	682·1 millims.	13°	31·8 cub. centims.

This corresponds to a percentage of 20·0 nitrogen.

	Calculated.		Found.		
Vanadium . .	51·3	78·6	77·8	——	——
Nitrogen . . .	14·0	21·4	——	20·3	20·0
	65·3				

2. *Vanadium Dinitride*, VN_2=79·3.—This substance is obtained as a black powder by passing dry ammonia over vanadyl trichloride heated in a glass tube to expel sal-ammoniac, washing with ammonia-water, and drying *in vacuo* over sulphuric acid.

This nitride was obtained in 1858 by Uhrlaub*, but this chemist made no direct determination of the nitrogen, by which alone the true nature of these compounds can be discovered; and he, adopting, as a matter of course, the atomic weight 68·5, showed that the body obtained as above described cannot be represented by any simple formula. If, however, Uhrlaub's experimental results be calculated with the true atomic weight 51·3, it is seen that, according to the mean of three determinations, the substance contains 64·1 per cent. of vanadium, the formula VN_2 requiring 64·6 per cent.

The existence of vanadium mononitride, VN, not only demonstrates with absolute certainty the true atomic weight of the metal, but it also serves as the starting-point from which to commence the study of the metal, as well as of an entirely new class of bodies, viz. the compounds of the metal vanadium with chlorine and the other halogens, which I have in vain endeavoured to obtain from vanadyl compounds. In the next communication I hope to describe these interesting substances.

* Pogg. Ann. Bd. ciii. p. 134.

II. *Computation of the Lengths of the Waves of Light corresponding to the Lines in the Dispersion Spectrum measured by* KIRCHHOFF. *By* GEORGE BIDDELL AIRY, *Astronomer Royal.*

Received March 2,—Read March 21, 1867.

THE greatest and most valuable system of measures which we possess of the lines in the Dispersion Spectrum, produced by the emanation of light from the Sun, by transmission through the Atmosphere, and by the combustion of Metals, is that published by Professor KIRCHHOFF in the Memoirs of the Berlin Academy for 1861 and 1862. This noble series includes about 1600 lines, taken in uninterrupted succession through the spectrum, beginning a little before FRAUNHOFER'S line A and ending a little after G. The same apparatus was used throughout, and in the same manner; and the system followed in the whole process was so nearly constant that we may speak of it generally as one consistent series of measures. This work has been made known to English readers by an accurate translation published by Professor ROSCOE.

The measures thus exhibited depend upon the form of the apparatus, the refractive and dispersive powers of the glass, the nature of the scale employed, and the value of its unit of measure. They are not therefore available for physical research, until they are cleared of the effects of these circumstances of experiment. There can be no hope of our arriving at, or even commencing, any mechanical theory on the formation of the spectral lines, until we have obtained the one natural measure of each, namely the length of the Wave of Light corresponding to each.

I have therefore undertaken the labour of computing from each spectral measure the corresponding length of the Wave of Light; and I trust that the result may not be unacceptable to the Royal Society.

I will now describe the method which I have used.

When I commenced this work, I could not learn (although I made inquiries) that any careful measures had been made of the Diffraction Spectrum since the original measures of FRAUNHOFER. I therefore determined to make FRAUNHOFER'S measures of the principal lines the basis of my computations. From GILBERT'S 'Annalen der Physik und der Physikalischen Chemie,' XIV. Band, Leipzig 1823, page 559, I took the following values of the Length of a Wave, omitting the value for B, which apparently was not determined in the same series of measures with the others:—

					in.
Fr_C =	Length of wave for	C,	in Paris inch,	=	0·00002422,
Fr_D =	,,	,,	D	,,	= 0·00002175,
Fr_E =	,,	,,	E	,,	= 0·00001945,

Fr_F = Length of wave for F, in Paris inch, = 0·00001794 in.,

Fr_G = " " G " = 0·00001587.

The lines are not designated, in KIRCHHOFF's tables of measures, by the received letters; the letters, however, as well as KIRCHHOFF's scale-numbers, are attached to his engravings of the spectrum, which appear to be carefully drawn. From these I took the following numbers:—

K_C = KIRCHHOFF's scale-measure for C = 694·1,

K_D = " " D = 1004·8,

K_E = " " E = 1527·7,

K_F = " " F = 2080·0,

K_G = " " G = 2854·3.

The next step was to construct a formula, which, when applied to KIRCHHOFF's values for C, D, E, F, G, should produce FRAUNHOFER's values for the same lines. I know no physical reason for adopting one formula in preference to another; and I therefore adopted the simplest algebraical formula. Putting $k_i = \frac{1}{1000} K_i$, and $f_i = 100000000 \times \text{Fr}_i$, I assumed

$$f_i = a + b \cdot k_i + c(k_i)^2 + d(k_i)^3 + e(k_i)^4,$$

and formed five equations numerically by putting successively on the left side f_C, f_D, &c., and on the right side k_C, k_D, &c., giving to those symbols the numerical values derived from the small tables above. Solving the equations, I found

$$\begin{aligned} f_i = {} & 3596{\cdot}688 \\ & - 2582{\cdot}018 \times k_i \\ & + 1587{\cdot}046 \times (k_i)^2 \\ & - 476{\cdot}676 \times (k_i)^3 \\ & + 52{\cdot}959 \times (k_i)^4. \end{aligned}$$

From this formula, values of f_i and Fr_i were tabulated for every 0·01 of k_i, or for every 10·0 of KIRCHHOFF's scale. It will be sufficient to give here every tenth number.

Kirchhoff.	Fraunhofer. in. 0·0000	Kirchhoff.	Fraunhofer. in. 0·0000	Kirchhoff.	Fraunhofer. in. 0·0000
0	35967	1000	21780	2000	18148
100	33539	1100	21199	2100	17888
200	31400	1200	20697	2200	17625
300	29525	1300	20262	2300	17358
400	27887	1400	19879	2400	17087
500	26462	1500	19538	2500	16814
600	25227	1600	19229	2600	16539
700	24162	1700	18942	2700	16269
800	23244	1800	18671	2800	16007
900	22456	1900	18407	2900	15760
				3000	15535

Between the numbers computed for every 10·0 of KIRCHHOFF's scale, the numbers were interpolated for every one of KIRCHHOFF's line-measures by simple proportion of

first differences. Thus I had a complete table of the values of the length of the wave of light expressed in Paris inches for every one of KIRCHHOFF's lines. Each number was then multiplied by 27·0700, and thus I had a complete table of the values of the length of the wave of light expressed in millimetres for every one of KIRCHHOFF's lines. In this state I very carefully revised all the final numbers. This is the table of results now offered to the Royal Society. It did not appear necessary to publish a long series of results expressed by a measure which is now so nearly obsolete as the Paris inch; although, as will appear below, the possession of those numbers in my own hands has proved to be of considerable utility.

From the description which I have given of the process by which I have computed the wave-lengths from KIRCHHOFF's measures, it will be perceived that the results are not rigorously accurate, unless the same system (whatever it may be) has been rigorously followed by KIRCHHOFF through the whole of his measures. Now KIRCHHOFF has distinctly stated that he did not in every case adapt his prisms carefully to the angle of minimum deviation. So far as I could learn by communication (through Professor ROSCOE) with M. KIRCHHOFF, no memorandum was preserved on the possible amount of the error thus introduced. I was confident, however, as well from the character of Professor KIRCHHOFF as from the absence of any apparent *saltus* in the measures, that the error must be small, perhaps imperceptible. It was a matter of great satisfaction to me therefore, after the completion of my work, to be referred to two late series of direct measures of wave-length for numerous lines. One is that by ÅNGSTRÖM, published in POGGENDORFF'S 'Annalen der Physik und Chemie,' CXXIII. Band, p. 498, in which I have found 51 lines apparently identifiable with lines measured by KIRCHHOFF. The measures in these series are expressed by the Paris inch; and the possession of my own calculations in terms of the Paris inch, to which I have alluded above, greatly facilitated the comparison of ÅNGSTRÖM'S measures with my calculations. The other is that by DITSCHEINER, published in the 'Sitzungsberichte der Mathematisch-Naturwissenschaftlichen Classe der Kaiserlichen (Wiener) Academie der Wissenschaften,' L. Band, II. Abtheilung, 1864, page 340: it contains direct measures of the wave-length, expressed in millimetres, for 107 of the lines measured by KIRCHHOFF. The comparison of these direct measures with my computed values of the wave-length exhibited small discrepancies, of which the greater part arises from discordances between the values for the fundamental lines found by FRAUNHOFER and those determined by ÅNGSTRÖM and DITSCHEINER (which those philosophers ascribe to an erroneous evaluation by FRAUNHOFER of his interference-grating); a part apparently is produced by faults inherent in every process of interpolation among a limited number of values, especially when that interpolation is so extended as to become extrapolation; but, as I believe, no sensible part, or a very small part, can arise from the suspected class of errors in the measures of the prismatic dispersion-spectrum.

I had arrived at this stage of my work, when I became acquainted with DITSCHEINER'S Essay contained in the 'Sitzungsberichte der Mathematisch-Naturwissenschaftlichen

Classe der Kaiserlichen Akademie der Wissenschaften,' LII. Band, II. Abtheilung, page 289, &c. This paper gives the details of a new measure of the grating with which FRAUNHOFER'S original measures of wave-lengths were made (as I understand M. DITSCHEINER'S words); and by means of this correction of measure, and the corresponding correction of wave-length for the line D common to FRAUNHOFER'S and DITSCHEINER'S measures, DITSCHEINER computed new values of wave-lengths for the entire series of lines, primary and secondary, which he had himself observed. On careful consideration of all the circumstances of formation of these new values, it appeared to me that it was indispensable for me to base my reduction of KIRCHHOFF'S measures upon DITSCHEINER'S new values. I should thus have the opportunity of introducing a value for B, which would enable me to adopt a function of the 5th order instead of the 4th order (upon which all my preceding calculations were founded); and I could use the same opportunity for making correction for a petty error which I had committed in the line E (I had inadvertently used 1527·7 instead of 1523·2 for KIRCHHOFF'S measure).

My immediate object, in calculation, now was, to make such an addition to the function embodied in my former calculations as would produce definite alterations in the wave-lengths computed for B, C, D, E, F, G. This, it was evident, could be done by a function or a sum of functions of the 5th order; and I had only to adopt the most convenient form. Remarking that KIRCHHOFF'S measures for B, C, D, E, F (with the decimal point thrown back three places) differ very little from 0·6, 0·7, 1·0, 1·5, 2·1, and that the measure for G is not very different from 2·9, I adopted the following form for general correction; where by [0·6], [0·7], [1·0], [1·5], [2·1], [2·9], are meant certain constants whose values will shortly be seen.

General correction for all values of k.

$$\frac{\text{correction for } k=0\cdot6}{[0\cdot6]}\times(k-0\cdot7)\times(k-1\cdot0)\times(k-1\cdot5)\times(k-2\cdot1)\times(k-2\cdot9)$$

$$+\frac{\text{correction for } k=0\cdot7}{[0\cdot7]}\times(k-0\cdot6)\times(k-1\cdot0)\times(k-1\cdot5)\times(k-2\cdot1)\times(k-2\cdot9)$$

$$+\frac{\text{correction for } k=1\cdot0}{[1\cdot0]}\times(k-0\cdot6)\times(k-0\cdot7)\times(k-1\cdot5)\times(k-2\cdot1)\times(k-2\cdot9)$$

$$+\frac{\text{correction for } k=1\cdot5}{[1\cdot5]}\times(k-0\cdot6)\times(k-0\cdot7)\times(k-1\cdot0)\times(k-2\cdot1)\times(k-2\cdot9)$$

$$+\frac{\text{correction for } k=2\cdot1}{[2\cdot1]}\times(k-0\cdot6)\times(k-0\cdot7)\times(k-1\cdot0)\times(k-1\cdot5)\times(k-2\cdot9)$$

$$+\frac{\text{correction for } k=2\cdot9}{[2\cdot9]}\times(k-0\cdot6)\times(k-0\cdot7)\times(k-1\cdot0)\times(k-1\cdot5)\times(k-2\cdot1).$$

The numerical values of the products following the fraction in each line are given, for every 0·1 of k from 0·0 to 3·0, in the last six columns of the following Table.

Corrections to the computed Wave-lengths, on different suppositions as to the parts of the series which are to be left without alteration.

Approximate places of principal lines.	Values of k.	No correction for k=0·7, 1·0, 1·5, 2·1, 2·9; Correction for k=0·6 being +1242.	No correction for k=0·6, 1·0, 1·5, 2·1, 2·9; Correction for k=0·7 being +730.	No correction for k=0·6, 0·7, 1·5, 2·1, 2·9; Correction for k=1·0 being +1254.	No correction for k=0·6, 0·7, 1·0, 2·1, 2·9; Correction for k=1·5 being +3024.	No correction for k=0·6, 0·7, 1·0, 1·5, 2·9; Correction for k=2·1 being +1100.	No correction for k=0·6, 0·7, 1·0, 1·5, 2·1; Correction for k=2·9 being +1077.
	0·0	+6395	−5481	+3837	−2558	+1827	− 132
	0·1	+4234	−3528	+2352	−1512	+1058	− 76
	0·2	+2668	−2134	+1334	− 821	+ 562	− 40
	0·3	+1572	−1179	+ 674	− 393	+ 262	− 18
A	0·4	+ 842	− 561	+ 281	− 153	+ 99	− 7
	0·5	+ 384	− 192	+ 115	− 58	+ 36	− 2
B	0·6	+ 124	0	0	0	0	0
C	0·7	0	+ 74	0	0	0	0
	0·8	− 38	+ 76	+ 38	− 11	+ 6	− 0
	0·9	− 29	+ 43	+ 86	− 14	+ 7	− 0
D	1·0	0	0	+ 125	0	0	0
	1·1	+ 29	− 36	+ 144	+ 36	− 14	+ 1
	1·2	+ 46	− 55	+ 138	+ 92	− 31	+ 2
	1·3	+ 46	− 54	+ 108	+ 161	− 40	+ 2
	1·4	+ 29	− 34	+ 59	+ 235	− 34	+ 2
E	1·5	0	0	0	+ 302	0	0
	1·6	− 35	+ 39	− 59	+ 351	+ 70	− 3
	1·7	− 67	+ 74	− 106	+ 370	+ 184	− 6
	1·8	− 87	+ 95	− 131	+ 348	+ 348	− 10
	1·9	− 86	+ 93	− 125	+ 281	+ 562	− 11
	2·0	− 59	+ 63	− 82	+ 164	+ 819	− 9
F	2·1	0	0	0	0	+1109	0
	2·2	+ 88	− 94	+ 118	− 202	+1411	+ 20
	2·3	+ 200	− 212	+ 261	− 424	+1697	+ 57
	2·4	+ 321	− 340	+ 413	− 643	+1928	+ 116
	2·5	+ 432	− 456	+ 547	− 821	+2052	+ 205
	2·6	+ 502	− 528	+ 627	− 912	+2006	+ 334
	2·7	+ 490	− 514	+ 605	− 857	+1714	+ 514
G	2·8	+ 344	− 360	+ 420	− 582	+1081	+ 757
	2·9	0	0	0	0	0	+1077
	3·0	− 621	+ 648	− 745	+ 994	−1656	+1490

It is easily seen now that [0·6] is the number corresponding to k=0·6 in the third column of the Table; that [0·7] is the number corresponding to k=0·7 in the fourth column of the Table; and so for the others.

It appeared now that the corrections to be applied to numbers which I had actually computed, in order to produce DITSCHEINER's numbers, were the following:—

For line B . . . +320
C . . . +147
D . . . +142
E . . . + 97
F . . . +123
G . . . +213.

And, after making a preliminary calculation, the following corrections were found to be necessary:—

For k=0·6, $+306 = +140 + 124 \times (1+\frac{1}{3})$
0·7, $+142 = +140 + 74 \times \frac{1}{30}$
1·0, $+140 = +140$
1·5, $+109 = +140 - 302 \times \frac{1}{10}$
2·1, $+149 = +140 + 1109 \times \frac{1}{130}$
2·9, $-98 = +140 - 1077 \times (\frac{1}{5} + \frac{1}{50})$.

The factors in the last column are [0·6], [0·7], &c. respectively.

Then, using the number +140 as a general correction, and using the numbers of the last column for "Correction for k=0·6," "Correction for k=0·7," &c. in the general formula (or, in other words, multiplying the 3rd, 4th, &c. columns of the Table, by $(1+\frac{1}{3})$, $\frac{1}{30}$, &c.), the following special numerical corrections are obtained.

Corrections to the wave-lengths as originally computed. (The unit of the corrections is the same as the unit of the last figure of the computed wave-lengths in millimetres.)

k.	Correction.	k.	Correction.	k.	Correction.
0·0	+8857	1·0	+140	2·0	+ 55
0·1	+5891	1·1	+174	2·1	+ 149
0·2	+3750	1·2	+191	2·2	+ 283
0·3	+2257	1·3	+184	2·3	+ 447
0·4	+1269	1·4	+154	2·4	+ 616
0·5	+ 654	1·5	+110	2·5	+ 761
0·6	+ 305	1·6	+ 61	2·6	+ 832
0·7	+ 141	1·7	+ 18	2·7	+ 770
0·8	+ 92	1·8	− 4	2·8	+ 493
0·9	+ 103	1·9	+ 6	2·9	− 97
				3·0	−1116

With these numbers, a curve was laid down, and the corrections for every 0·01 of k were obtained graphically. Between these, the corrections for all the individual lines were interpolated numerically, and were applied separately to each computed wave-length. Every number which is presented in the following Tables is affected with the correction.

The agreement of the computed numbers with DITSCHEINER'S, or with ÅNGSTRÖM'S (after multiplication by the proper factor), as far as they admit of comparison, is not so close as I expected. For the lines B, C, D, E, F, G, the accordance with DITSCHEINER'S

numbers is necessarily (from the nature of the process) almost perfect. But between each of these principal lines and the next, the discordance rises gradually from 0 to a considerable value and then drops gradually to 0. Between F and G the value is large. The discordances, however, are systematic. There is no appearance of any *saltus* between one principal line and the next; but there is nothing to enable us to decide positively whether there is a *saltus* at each principal line.

It appears to me that we must look to one of the three following causes for explanation:—

1. The actual inability of a formula of the fifth order to represent the wave-length in terms of the spectrum-measure with sufficient approximation.

2. A change in Professor KIRCHHOFF'S method of observation at each successive principal line.

3. A change in DITSCHEINER'S and ÅNGSTRÖM'S methods at each successive principal line.

The third of these suggestions is extremely improbable: and after remarking the harmonious flow in the course of the discordances in every interval between principal lines (a harmony which gives no small testimony to the care employed in both classes of measures, and to the accuracy of the interpolation-details in the computation which connects them), I am hardly inclined to advocate the second suggestion. I think it more probable that the real cause is to be found in the first.

I remark, however, that this doubt affects only the physical question on the broad scale. It does not affect the possibility of computing with accuracy the wave-length for any one of KIRCHHOFF'S lines. By means of the comparison between the computed wave-lengths and DITSCHEINER'S measures (at the end of the Tables) there is no difficulty in computing for any other line the correction that ought to be applied to the wave-length in the principal Tables, in order to exhibit the true wave-lengths on DITSCHEINER'S scale, without appreciable error.

I have now only to explain the Tables which follow.

The principal Table, headed "Conversion of KIRCHHOFF'S Spectral Measures into Wave-lengths in terms of the Millimetre," corresponds, in extent, to KIRCHHOFF'S Tables in the Berlin Memoirs, 1861 and 1862. Instead, however, of adopting the broken arangement of these Memoirs, I have placed the numbers in a continuous series, and I have omitted the repetitions in KIRCHHOFF'S successive Tables.

The Table consists throughout of four columns (one of which is occasionally blank). The first column contains simply KIRCHHOFF'S measures; and the second contains the corresponding wave-lengths, computed by the process described above. The third column contains KIRCHHOFF'S symbols expressing the darkness and the breadth of the lines; 1 expresses a slight darkness, 2 a little darker, and so on up to 6 which is very dark; *a* denotes a very narrow line, *b* a little broader, and so on up to *g*, which is a line of considerable breadth (the measure of the breadth is not given). The fourth column (where it is used) contains the contracted name of the metal which, by combustion, pro-

duces a line corresponding in spectral position to the line here measured. This Table is, in fact, KIRCHHOFF'S Table, with the insertion throughout of one column containing the computed values of the wave-lengths. In copying the first column, I have endeavoured to copy the braces and brackets of KIRCHHOFF'S Table, without in every case understanding their application.

The Table which next follows is KIRCHHOFF'S special Table for the lines produced by the metals Cerium, Didymium, Lanthanum, Palladium, Platinum, Ruthenium, Iridium, with the corresponding wave-lengths. Two of its compartments contain the lines in a combination of Lanthanum and Didymium, and in a combination of Ruthenium and Iridium. It is accompanied by a similar Table of lines produced by the atmosphere.

The next Table (formed by myself) contains the wave-lengths for each of the metals whose lines have been measured by KIRCHHOFF, and are contained either in the General Table of lines of the solar spectrum, or in the smaller special Table which I have just described. The numbers are simply extracted from those Tables. I may mention that my original intention was to confine my computations to these metal-lines, in the hope that they might give some assistance to the formation of a mechanical theory.

The two last Tables contain the comparison of direct measures of the wave-length made by ÅNGSTRÖM and DITSCHEINER with the computed measures given in the General Table, as far as the identity of the lines appears to be established. ÅNGSTRÖM'S measures were given in terms of the Paris inch: I have preserved these; and have also given their equivalents in terms of the millimetre.

I have only to add that, in my original communication to the Royal Society, dated 1867, March 2, the modifications of the first computed numbers, now introduced after consideration of DITSCHEINER'S later measures, had not been applied. I have now incorporated them in the finished Tables, with the permission of the President and Council of the Society.

G. B. AIRY.

Royal Observatory, Greenwich,
1867, *October* 15.

Conversion of KIRCHHOFF's Spectral Measures into Wave-Lengths, in terms of the Millimetre.

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000.		
381·7	77694	1 *e*		476·4	73266	1 *b*	
384·1	77574	2 *e*				2	
385·9	77479	2 *d*		477·0	73240	5 *b*	
387·5	77396	3 *d*				2	
388·9	77322	4 *d*		477·8	73206	4 *b*	
390·4	77249	4 *e*		479·1	73153	2 *c*	
392·1	77160	5 *e*				1	
393·6	77085	6 *e*		480·1	73113	6 *c*	
395·0	77012	6 *e*		480·4	73100	4 *d*	
396·2	76950	5 *e*		481·2	73067	4 *c*	
397·4	76890	4 *e*		482·1	73031	2 *d*	
398·4	76838	4 *d*		483·3	72979	4 *d*	
399·2	76799	4 *d*		484·1	72949	2 *d*	
399·8	76767	4 *d*		485·1	72906	3 *d*	
400·4	76743	3 *d*		486·2	72861	6 *e*	
401·9	76670	4 *c*		486·8	72836	2 *c*	
		3		from 488·2	72778	1	
402·4	76648	4		488·8	72754	5 *a*	
402·8	76625	5		489·6	72719	6 *c*	
A 403·2	76605	6		491·2	72652	3 *e*	
405·0	76517	5		491·5	72641	5 *b*	
405·6	76492	4		491·9	72621	4 *c*	
406·2	76462	3		493·1	72575	2 *e*	
		5 *e*		494·1	72531	3 *b*	
406·8	76436	1 *d*		495·4	72478	1 *e*	
408·5	76336	2 *b*		495·7	72466	2 *b*	
423·7	75641	2 *b*		497·2	72400	1 *b*	
426·6	75477	2 *c*		497·5	72388	2 *a*	
433·8	75147	2 *b*		498·4	72351	4 *c*	
437·0	75003	2 *d*		499·0	72324	5 *b*	
442·8	74746	2 *c*		499·9	72287	5 *d*	
444·6	74663	2 *b*		500·8	72253	3 *d*	
445·8	74611	2 *b*		501·8	72214	2 *c*	
446·1	74599	2 *a*		502·0	72207	5 *b*	
447·0	74560	1 *b*		502·6	72183	5 *c*	
448·4	74496	2 *c*		503·8	72132	6 *d*	
452·6	74308	1 *b*		504·3	72113	5 *b*	
453·0	74292	1 *b*		505·1	72082	6 *c*	
454·4	74231	1 *b*		506·2	72036	2 *b*	
460·0	73981	1 *c*		506·4	72029	5 *b*	
461·0	73937	1 *b*		506·6	72020	2 *b*	
462·2	73884	2 *b*		507·4	71990	5 *c*	
463·3	73837	2 *a*		508·2	71956	3 *b*	
466·0	73720	1 *b*		509·1	71918	3 *b*	
466·5	73698	2 *c*		509·9	71885	2 *b*	
467·0	73676	1 *b*		510·9	71846	1 *a*	
468·1	73627	2 *e*		512·9	71767	2 *b*	
470·0	73542	2 *b*		513·6	71741	3 *b*	
470·5	73520	3 *c*		517·1	71606	2 *b*	
470·9	73504	2 *b*		519·3	71513	2 *b*	
472·4	73437	2 *e*		521·6	71426	1 *b*	
472·7	73425	3 *c*		529·4	71129	1 *b*	
473·8	73379	4 *d*		530·4	71089	1 *c*	
		1		532·8	70997	1 *b*	
474·7	73341	3 *b*		536·9	70840	2 *b*	
from 475·7	73297	2		537·3	70826	1 *b*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000.			Kirchhoff's measure.	Wave-length, m.m. 0·000		
540·6	70705	3 *b*		597·4	68681	1 *b*	
541·1	70685	2 *c*		601·2	68556	1 *a*	
542·0	70649	1 *a*		601·8	68535	1 *b*	
543·6	70591	4 *b*		602·8	68501	1 *a*	
544·6	70554	3 *d*		606·0	68396	1 *b*	
547·0	70464	4 *c*		608·3	68317	1 *a*	
547·9	70430	2 *b*		612·4	68186	1 *b*	
549·6	70366	3 *e*		613·4	68151	1 *a*	
551·2	70310	3 *c*		623·4	67831	1 *b*	
552·5	70263	3 *c*		626·1	67744	1 *b*	
553·8	70213	1 *c*		631·4	67576	1 *b*	
554·0	70207	3 *b*		638·4	67357	1 *b*	
554·6	70183	2 *b*		639·8	67310	1 *b*	
557·0	70100	1 *a*		641·0	67273	2 *b*	Ca
557·7	70073	2 *b*		645·3	67140	1 *b*	
558·1	70058	1 *b*		648·1	67053	1 *b*	
559·7	69999	1 *c*		654·3	66865	2 *b*	
561·5	69937	1 *b*		659·3	66713	2 *a*	
562·5	69902	3 *b*		665·7	66524	2 *a*	
563·0	69883	2 *c*		669·5	66410	2 *b*	
564·1	69845	4 *c*		678·6	66150	1 *b*	
565·0	69809	2 *c*		681·4	66068	1 *a*	
566·0	69773	2 *c*		682·8	66028	1 *b*	
566·9	69741	2 *b*		683·1	66020	2 *a*	
567·4	69722	3 *b*		685·3	65956	1 *b*	
568·6	69680	2 *b*		689·8	65829	2 *b*	
		1		690·9	65798	1 *a*	
569·2	69660	2 *b*		692·1	65765	2 *a*	
		1		693·4	65729	1	
570·0	69629	3 *c*	C from	694·1	65710	6 *e*	Air
570·6	69608	2 *b*	to	694·8	65690	1	
572·2	69552	3 *b*		698·1	65598	2 *a*	
572·9	69526	1 *b*		700·0	65545	2 *a*	
573·6	69502	3 *c*		701·1	65514	2 *b*	
574·4	69473	1 *b*		702·1	65486	2 *a*	
575·1	69449	2 *d*		702·6	65472	1 *b*	
576·6	69398	2 *d*		705·5	65395	2 *a*	
578·1	69346	3 *d*		705·9	65383	2 *a*	
579·6	69290	3 *d*		707·5	65339	1 *b*	
581·1	69238	3 *e*		708·6	65311	2 *b*	
582·5	69192	3 *e*		710·5	65258	2 *e*	
583·8	69137	4 *e*		711·4	65233	3 *c*	
585·0	69106	4 *f*		712·0	65218	2 *b*	
586·2	69065	4 *e*		713·2	65185	1 *b*	
587·0	69035	3 *e*		714·4	65151	1 *c*	
587·9	69005	2 *b*		717·8	65059	2 *b*	Ca
589·0	68970	3 *b*		718·7	65035	2	Ba
589·4	68955	3 *b*	from	719·6	65011	3 *a*	
589·9	68938	3 *b*		720·1	65000	2 *e*	Ca
590·3	68925	3 *b*		721·1	64972	2 *b*	Fe
590·7	68911	3 *b*		723·7	64903	2 *c*	
591·1	68896	3 *b*		724·2	64890	1 *b*	
591·5	68882	4 *b*		725·1	64867	1 *b*	Air
591·9	68869	4 *b*		726·7	64825	3 *c*	
592·3	68856	3 *b*		727·8	64796	1 *e*	
B 592·7	68853	6 *c*		728·0	64791	2 *a*	
593·1	68829	4 *g*		729·0	64764	2 *b*	Ca
595·0	68764	1 *a*		731·7	64695	5 *b*	Ca
596·6	68707	1 *a*		734·0	64634	1 *d*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000.		
736·9	64559	3 *b*	Ca	838·2	62158	1 *b*	
740·9	64436	5 *b*	Ca, Cd	838·6	62150	2 *b*	
743·7	64384	2 *b*		839·2	62136	2 *b*	
744·3	64368	4 *b*		845·7	62001	2 *b*	
748·1	64271	4 *b*		849·7	61915	3 *c*	Fe
748·7	64257	3 *b*		851·2	61881	1 *a*	
750·1	64220	1 *a*		851·8	61868	1 *a*	
751·0	64199	1 *b*		855·0	61802	2 *a*	
752·3	64165	4 *b*		856·8	61764	2 *a*	
753·8	64129	3 *b*	Sr	857·5	61749	2 *a*	
756·9	64049	5 *b*	Fe	858·3	61734	2 *a*	
759·3	63989	3 *b*		859·7	61704	3 *a*	
764·2	63869	1 *a*		860·2	61693	3 *d*	Ca
771·8	63683	1 *a*	Zn	861·6	61662	2 *a*	
773·4	63644	2 *b*		862·2	61649	1 *a*	
774·8	63611	2 *b*		863·2	61631	2 *c*	
778·3	63525	1 *b*	(Ru, Ir)	863·9	61617	5 *b*	Ca
779·5	63498	1 *b*		864·4	61606	1 *d*	
781·9	63438	3 *b*		866·2	61568	2 *b*	
783·1	63411	4 *b*		867·1	61549	2 *b*	
783·8	63395	3 *b*		867·6	61539	1 *a*	
786·8	63323	1 *a*		869·2	61507	2 *b*	
788·9	63273	3 *b*		870·9	61472	1 *b*	
791·0	63223	1 *d*		871·4	61461	2 *b*	
791·4	63214	3 *b*		872·5	61439	1 *b*	
792·9	63180	2 *d*		874·0	61411	1 *b*	
794·5	63144	1 *d*		874·3	61405	4 *b*	Ba
798·1	63059	3 *a*		876·5	61358	4 *a*	
798·5	63049	4 *a*	Fe	877·0	61348	4 *c*	Fe
799·8	63019	2 *b*		879·8	61292	1 *b*	
800·3	63005	2 *b*		880·9	61271	1 *a*	
801·2	62987	1 *a*		881·6	61255	2 *a*	
801·5	62980	1 *a*		882·6	61237	1 *a*	
802·7	62952	1 *b*		883·2	61223	1 *b*	
803·5	62935	2 *a*		884·9	61191	4 *b*	Ca, Co
805·8	62881	1 *b*		887·7	61135	2 *a*	Ni
807·4	62845	2 *b*		890·2	61086	1 *b*	Ba
808·2	62826	2 *c*		891·7	61056	2 *a*	Ni
808·7	62815	1 *c*		894·9	60990	2 *e*	Ca, Li
809·5	62796	3 *b*	Au	896·1	60968	1 *a*	
809·9	62785	2 *d*		896·7	60957	1 *b*	
812·7	62723	1 *a*		898·9	60912	1 *a*	
813·1	62715	2 *a*		899·1	60909	1 *a*	
815·0	62672	4 *b*		900·2	60888	1 *a*	
816·8	62631	2 *b*		901·4	60864	1 *a*	
818·0	62606	3 *c*		901·6	60860	1 *a*	
819·0	62582	4 *b*		902·4	60844	1 *a*	
820·1	62558	4 *b*		903·1	60829	1 *a*	
820·9	62542	4 *b*		903·6	60821	1 *a*	
823·5	62482	1 *a*		904·6	60803	1 *a*	
824·0	62471	4 *b*		906·1	60773	2 *c*	
824·9	62452	1 *d*		912·1	60659	3 *b*	Fe
826·4	62420	2 *a*		*916·3	60578	2 *b*	
827·6	62393	1 *a*		923·0	60452	2 *b*	
828·0	62382	2 *a*		929·5	60330	2 *b*	
830·2	62333	3 *b*		931·3	60297	4 *b*	Fe
831·0	62317	4 *c*	Fe	932·5	60277	4 *b*	
831·7	62301	1 *b*		933·3	60262	4 *c*	
836·5	62196	2 *b*		935·1	60229	4 *b*	

* A large proportion of the measures, from 916·3 to 1006·8, are included in a subsequent Table of Atmospheric Lines.—G. B. A.

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
936·7	60199	4 *b*		1005·0	59017	2 *b*	Ni
937·4	60187	1 *b*		D*a* 1006·8	58988	6 *b*	Na
940·1	60134	3 *b*		1011·2	58916	3 *a*	
940·4	60128	2 *b*		1023·0	58726	1 *a*	
943·4	60075	3 *b*		1025·5	58686	3 *a*	
946·6	60018	3 *b*		1027·7	58653	2 *a*	
947·0	60011	1 *a*		1029·3	58626	3 *c*	Ca, Ni
949·4	59967	1 *b*		1031·8	58583	2 *a*	Ba
949·8	59960	1 *b*		1032·8	58567	1 *a*	
951·7	59925	1 *c*		1035·3	58528	1 *a*	
952·9	59904	3 *b*		1058·0	58180	2 *b*	
954·3	59880	3 *b*		1063·0	58104	2 *b*	
954·8	59872	3 *b*		1065·0	58072	2 *b*	
958·8	59801	3 *b*		1066·0	58059	1 *a*	
959·6	59788	3 *b*		1067·0	58046	2 *b*	
961·9	59745	1 *a*		1070·5	57990	2 *b*	
963·7	59713	1 *c*		1073·5	57946	1 *a*	
964·4	59703	1 *c*		1074·2	57936	1 *a*	
968·7	59628	2 *a*		1075·5	57916	3 *a*	
969·0	59623	2 *a*		1077·5	57888	1 *a*	
969·6	59613	3 *a*		from 1078·9	57866	1	
970·5	59597	1 *b*		to 1079·7	57854		
971·5	59578	2 *c*		1080·3	57845	1 *a*	
972·1	59568	1 *b*		1080·9	57836	1 *a*	
973·1	59551	3 *a*		1081·8	57824	2 *b*	Cu
973·5	59544	3 *a*		1083·0	57805	2 *a*	Ba
974·3	59531	2 *a*		1087·5	57738	2 *a*	
975·0	59519	2 *a*		1089·6	57709	2 *a*	
976·8	59487	3 *a*		1096·1	57616	3 *c*	Fe
977·4	59477	2 *a*		1096·8	57607	1 *a*	
977·7	59471	2 *a*		1097·8	57592	1 *a*	
979·1	59448	1 *b*		1100·4	57554	1 *a*	
980·8	59421	1 *a*		1102·1	57527	3 *b*	
981·2	59415	3 *b*		1102·9	57517	3 *a*	
982·0	59401	1 *a*		1103·3	57511	2 *b*	
982·3	59396	2 *a*		1104·1	57498	2 *b*	
983·0	59384	3 *c*		1107·1	57455	2 *c*	
984·5	59359	1 *c*		1111·4	57397	1 *a*	
986·3	59329	1 *a*		1119·0	57289	2 *a*	
986·7	59323	2 *c*		1122·6	57244	2 *a*	
987·4	59310	1 *b*		1128·3	57162	2 *b*	
988·9	59284	2 *a*		1130·9	57126	2 *b*	
989·2	59279	2 *a*		1133·1	57098	3 *c*	
989·6	59272	2 *a*		1133·9	57087	3 *c*	
990·8	59252	2 *a*		1135·1	57068	4 *d*	
991·2	59246	1 *a*		1135·9	57057	2 *c*	
991·9	59234	3 *b*	Fe	1137·0	57043	2 *b*	
992·4	59225	1 *a*		1137·8	57034	3 *b*	
993·9	59199	1 *b*		1141·3	56985	2 *c*	
994·3	59192	1 *b*		1143·6	56956	2 *c*	
995·0	59180	1 *a*		1146·2	56921	1 *b*	
997·2	59145	2 *b*		1147·2	56908	1 *b*	
998·1	59130	1 *a*		1148·6	56890	1 *b*	
998·9	59117	1 *a*		1149·4	56879	1 *b*	
999·2	59112	1 *a*		1151·1	56852	4 *b*	
1000·0	59099	1 *a*		1152·5	56834	2 *b*	
1000·4	59093	1 *a*		1154·2	56811	3 *b*	
1001·4	59078	1 *a*		1155·7	56790	3 *b*	
D*b* 1002·8	59053	6 *b*	Na	1155·9	56788	2 *c*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
1158·3	56756	2 *a*		1264·4	55439	1 *a*	
1160·9	56724	2 *a*		1264·9	55434	2 *a*	
1165·2	56666	1 *a*		1267·3	55406	3 *a*	
1165·7	56660	1 *a*		1268·0	55398	3 *a*	
1167·0	56644	1 *d*		1271·9	55352	1 *a*	
1168·3	56627	1 *a*		1272·4	55347	1 *a*	
1169·4	56612	1 *a*		1274·2	55326	3 *b*	Ba
1170·6	56596	2 *c*		1274·7	55320	3 *a*	Sr
1174·2	56550	5 *d*		1276·2	55303	2 *a*	
1175·0	56540	2 *a*		1276·7	55297	1 *a*	
1176·6	56518	3 *c*		1280·0	55259	6 *d*	
1177·0	56513	2 *a*		1281·3	55246	3 *c*	
1177·3	56509	1 *a*		1282·6	55231	2 *c*	
1177·6	56505	1 *a*		1285·3	55199	2 *c*	
1178·6	56494	1 *a*		1287·5	55175	1 *c*	Ba
1179·0	56489	1 *a*		1289·7	55149	2 *c*	
1179·4	56483	1 *a*		1291·9	55125	3 *c*	
1179·8	56478	1 *a*		1293·8	55105	3 *c*	
1180·2	56472	1 *a*		1294·5	55097	3 *c*	
1183·4	56432	2 *a*		1295·6	55083	1 *a*	
1184·8	56413	3 *a*		1296·3	55075	2 *c*	
1186·8	56386	2 *a*		1297·5	55061	1 *a*	
1187·1	56383	2 *a*		1298·9	55044	5 *c*	
1189·3	56356	3 *b*		1299·7	55036	2 *c*	
1190·1	56345	2 *b*		1302·0	55012	2 *c*	
1193·1	56307	3 *a*		1303·5	54994	5 *c*	
1199·6	56223	2 *d*		1306·7	54958	5 *c*	
1200·6	56210	4 *b*	Fe	1315·0	54865	4 *c*	
1201·0	56204	2 *a*		1315·7	54857	2 *b*	
1203·5	56175	2 *c*		1319·0	54821	3 *c*	Co
1204·2	56166	2 *c*		1320·6	54805	4 *c*	Sr
1204·9	56158	2 *d*		1321·1	54797	3 *b*	
1206·1	56143	1 *c*		1323·3	54773	2 *b*	
1207·3	56129	5 *g*	Fe	1324·0	54765	2 *b*	
1217·8	55997	5 *d*	Fe, Ca	1324·8	54757	4 *d*	Ni
1219·2	55981	3 *c*	Ca	1325·3	54751	2 *d*	
1220·1	55970	2 *c*		1327·7	54726	4 *b*	
1221·6	55951	5 *d*	Ca	1328·7	54715	2 *b*	
1224·7	55913	5 *d*	Ca	1330·4	54696	3 *b*	
1225·3	55904	1 *b*		1333·3	54665	1 *a*	
1226·6	55888	2 *d*		1334·0	54657	4 *b*	
1228·3	55868	2 *d*	Ca	1336·3	54634	1 *b*	
1229·6	55852	4 *c*	Ca	1337·0	54626	4 *d*	Fe
1230·5	55842	2 *c*		1337·8	54618	1 *b*	
1231·3	55833	5 *d*	Fe	1338·5	54609	1 *b*	
1232·8	55814	2 *b*		1343·5	54556	6 *c*	Fe
1235·0	55787	3 *d*	Ca	1351·1	54477	5 *d*	Fe
1237·8	55752	2 *c*		1352·7	54456	5 *b*	Fe
1239·9	55728	4 *a*	Fe	1356·5	54417	1 *a*	
1242·6	55696	6 *c*	Fe	1360·9	54373	1 *a*	
1245·6	55660	4 *d*	Fe	1361·6	54365	1 *a*	
1247·4	55639	3 *b*		1362·9	54351	5 *b*	Fe
1248·6	55625	3 *d*		1364·3	54337	1 *a*	
1250·4	55605	3 *c*		1364·7	54333	1 *a*	
1251·1	55597	2 *b*		1367·0	54309	6 *d*	Fe
1253·3	55571	2 *b*		1371·4	54262	1 *b*	Ba
1255·2	55548	2 *b*		1372·1	54255	1 *b*	
1257·5	55521	3 *c*		1372·6	54250	5 *b*	Fe
1258·5	55510	2 *b*		1374·8	54226	1 *c*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
1375·8	54214	2 *a*		1483·0	53158	4 *b*	
1377·4	54198	1 *a*		1487·7	53113	5 *b*	Fe
1379·0	54181	1 *a*		1489·2	53098	2 *c*	
1380·5	54167	4 *c*	Fe	1489·9	53093	1 *a*	
1384·7	54121	4 *c*	Fe	(1491·2	53081	1 *c*	
1385·7	54112	5 *b*	Cr	1491·6	53078	3 *c*	
1386·3	54107	2 *b*		1492·4	53070	4 *b*	
1387·4	54096	2 *b*		1493·1	53064	4 *b*	
1389·4	54076	6 *c*	Fe	1494·5	53050	1 *a*	
1390·9	54063	5 *d*	Fe	1495·9	53036	1 *a*	
1394·2	54028	4 *c*		1497·3	53024	1 *a*	Cu
1395·3	54017	1 *c*		1501·3	52988	2 *b*	
1396·4	54006	2 *c*		1504·8	52956	1 *a*	
1397·5	53995	5 *c*	Fe	1505·3	52950	1 *a*	
1400·2	53964	3 *b*		1505·7	52947	1 *a*	
1401·6	53949	4 *c*	Fe	1506·3	52941	5 *c*	Fe
1403·1	53936	3 *c*		1508·6	52919	5 *b*	Fe
1404·1	53927	1 *b*		1510·3	52904	2 *c*	Co
1405·2	53916	3 *b*		1515·5	52855	4 *d*	
1410·5	53862	4 *c*	Fe	1516·5	52847	4 *c*	
1412·5	53842	2 *b*		1519·0	52823	4 *d*	
1414·0	53828	2 *b*		E (1522·7	52792	6 *c*	Fe, Ca
1415·8	53809	2 *b*		1523·7	52782	6 *c*	Fe
1419·4	53771	2 *b*		1525·0	52770	1 *b*	Co
1421·5	53749	6 *c*	Fe	1527·7	52747	5 *c*	Fe, Co
1423·0	53736	5 *b*	Fe	1528·7	52739	5 *c*	Ca
1423·5	53730	2 *b*		1530·2	52725	4 *c*	Ca
1425·4	53713	5 *b*	Fe	1531·2	52715	4 *c*	
1427·5	53690	3 *b*		1532·5	52705	4 *b*	Ca
1428·2	53685	5 *b*	Fe	1533·1	52701	4 *b*	Ca
1430·1	53666	5 *b*		(1541·4	52622	1 *g*	
1431·2	53655	1 *b*		1541·9	52618	3 *b*	
1438·9	53579	4 *c*	Co	1543·7	52601	2 *a*	
1440·2	53568	1 *b*	Co	1545·5	52584	2 *a*	
1443·1	53541	2 *b*		1547·2	52569	3 *a*	
1443·5	53536	2 *b*	Ca	1547·7	52565	2 *a*	
1444·4	53528	4 *c*		1551·0	52539	2 *a*	
1446·7	53508	4 *c*		1551·6	52532	2 *a*	
1448·7	53488	2 *a*	Co	1555·6	52495	2 *a*	
1449·4	53479	1 *a*	Co	1557·3	52481	3 *a*	
1450·8	53463	5 *c*	Fe	1561·0	52450	1 *a*	
1451·8	53454	5 *b*	Fe	1564·2	52423	1 *a*	
1453·7	53436	1 *a*		1566·5	52401	2 *b*	Co
1454·7	53425	3 *b*		1567·5	52392	2 *b*	
1456·6	53408	1 *a*		1569·6	52375	5 *c*	Fe
1458·6	53388	3 *c*		1573·5	52341	5 *a*	
1461·5	53359	2 *c*		1575·4	52325	1 *b*	
1462·2	53351	2 *c*		(1577·2	52309	5 *c*	Fe
1462·8	53346	5 *c*	Fe	1577·6	52306	3 *c*	
1463·3	53343	5 *c*	Fe	1579·4	52292	2 *a*	
1464·8	53326	1 *a*		1580·1	52286	2 *a*	
1465·3	53323	1 *a*		1588·3	52215	1 *g*	Cu
1466·8	53309	5 *c*	Fe	1589·1	52208	3 *b*	
1468·8	53289	2 *b*		1590·7	52195	3 *b*	
1469·6	53280	1 *b*		1592·3	52180	3 *b*	
1473·9	53243	5 *b*	Fe	1598·9	52123	2 *b*	
1475·3	53229	1 *a*		(1601·4	52102	6 *b*	Cr
1476·8	53215	1 *a*		1601·7	52099	3 *d*	
1477·5	53208	1 *a*		1604·4	52077	5 *b*	Cr

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
1606·4	52060	5 *b*	Cr	1710·7	51213	5 *a*	
1609·2	52033	5 *b*		1712·2	51201	3 *b*	
1611·3	52017	1 *c*		1713·4	51193	5 *b*	
1613·9	51996	3 *b*		1715·2	51178	4 *b*	
1615·6	51982	2 *b*		1717·9	51157	4 *b*	
1616·6	51976	1 *b*		1719·4	51145	1 *c*	
1617·4	51968	2 *b*		1726·9	51087	1 *a*	
1618·2	51961	3 *b*		1727·3	51084	3 *b*	Ni
1618·9	51956	4 *b*		1733·6	51035	5 *b*	
1621·5	51932	1 *b*		1734·6	51027	3 *b*	
1622·3	51925	5 *c*	Fe	1737·7	51002	5 *d*	
1623·4	51915	5 *b*	Fe	1741·0	50976	4 *b*	Cu
1627·2	51883	5 *b*	Ca	1742·7	50964	1 *a*	
1628·2	51875	1 *b*		1743·1	50961	1 *a*	
1631·5	51848	1 *b*		1744·6	50950	2 *a*	
b 1633·5	51832	4 *g*		1748·9	50917	3 *c*	Ni
1634·1	51826	6 *g*	Mg	1749·6	50912	2 *d*	Ni
1634·7	51821	4 *g*		1750·4	50906	5 *c*	
1638·7	51790	1 *b*		1752·0	50896	2 *b*	
1642·1	51761	1 *b*		1752·8	50889	4 *c*	
1643·0	51754	1 *b*	Ni	1762·0	50821	3 *c*	
1647·3	51718	5 *a*		1771·5	50748	3 *c*	
1648·4	51709	4 *e*		1772·5	50741	3 *c*	
b_1 1648·8	51705	6 *f*	Mg	1774·0	50729	2 *b*	
1649·2	51702	4 *e*		1775·8	50716	3 *b*	Ni
1650·3	51693	6 *b*	Fe	1776·5	50712	3 *c*	Ni
b_2 1653·7	51664	6 *b*	Fe, Ni	1777·5	50704	3 *c*	
1654·0	51661	4 *c*		1778·5	50696	3 *e*	
1655·6	51647	6 *e*	Fe, Mg	1782·7	50665	3 *b*	
1655·9	51644	4 *d*		1784·4	50651	1 *b*	
1657·1	51635	5 *b*		1785·0	50647	4 *b*	
1658·3	51627	2 *b*		1787·7	50626	2 *c*	
to 1659·4	51618	1		1788·7	50618	3 *b*	
1662·8	51592	5 *b*	Fe	1793·8	50583	4 *b*	
1667·4	51554	3 *a*		1795·4	50571	1 *a*	
1670·3	51531	1 *a*		1796·0	50567	3 *a*	
1671·5	51522	3 *b*		1797·8	50555	1 *a*	
1672·2	51517	4 *a*	Ni	1799·0	50546	4 *c*	
1673·7	51506	4 *a*		1799·6	50542	3 *b*	
1674·7	51497	3 *c*	Cu	1806·4	50492	2 *b*	
1676·2	51484	2 *d*		1818·7	50403	5 *b*	
1676·5	51481	4 *b*		1821·4	50384	5 *b*	
1677·9	51469	4 *c*		1822·6	50376	3 *a*	
1681·6	51438	4 *c*		1823·2	50371	2 *a*	
1684·0	51421	4 *a*	Ni	1823·6	50368	2 *a*	
1684·4	51418	1 *b*		1828·6	50330	1 *b*	
1685·9	51404	2 *a*		1830·1	50320	3 *b*	
1686·3	51401	2 *a*		1832·8	50302	2 *a*	Ca
1689·5	51377	5 *c*		1833·4	50298	6 *c*	
1690·0	51373	5 *b*	Ni	1834·3	50292	6 *c*	
1691·0	51365	5 *b*		1835·9	50282	3 *b*	
1693·8	51343	6 *e*	Fe	1836·7	50276	3 *c*	
1696·5	51321	3 *c*		1837·5	50271	3 *c*	
1697·0	51317	3 *c*	Ni	1841·0	50244	4 *b*	
1701·8	51281	5 *c*	Fe	1841·6	50240	4 *b*	
1704·6	51259	2 *c*		1842·2	50236	4 *b*	Ni
1704·9	51257	3 *b*		1848·9	50189	2 *c*	
1707·6	51236	2 *c*		1851·0	50173	1 *c*	
1707·9	51234	3 *b*		1853·2	50157	3 *b*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
1854·0	50152	2 *b*		1966·2	49398	2 *b*	
1854·9	50146	4 *c*		1966·7	49394	2 *b*	
1856·9	50133	1 *c*		1970·1	49373	3 *b*	
1857·9	50127	2 *b*		1974·7	49344	4 *b*	
1860·4	50108	2 *b*		1975·7	49337	2 *d*	
1861·3	50102	3 *c*		1979·2	49316	3 *c*	
1862·3	50095	2 *b*		1982·8	49293	5 *a*	
1864·9	50079	3 *b*		1983·3	49289	5 *a*	
1867·1	50063	5 *d*	Fe	1983·8	49286	5 *a*	
1868·4	50054	5 *b*	Ni	1984·5	49282	4 *b*	
1869·5	50046	1 *c*		1985·8	49273	4 *b*	
1870·6	50038	3 *a*		1986·9	49266	2 *a*	
1872·4	50025	5 *b*		1987·5	49263	3 *a*	Ni
1873·4	50018	6 *b*		1989·5	49250	6 *c*	Ba
1874·2	50013	2 *a*		1990·4	49243	5 *b*	
1874·8	50009	2 *a*		1991·8	49234	1 *b*	
1875·8	50002	2 *c*		1994·1	49219	5 *b*	
1876·5	49997	6 *b*		1996·9	49201	2 *a*	
1884·3	49945	6 *b*		1997·5	49198	2 *a*	
1885·8	49933	6 *b*		1999·6	49184	2 *c*	
1886·4	49929	6 *b*		2000·6	49176	5 *a*	
1889·5	49908	1 *g*		2001·6	49169	5 *c*	Fe
1891·0	49898	3 *b*		2003·2	49159	3 *b*	
1892·5	49890	5 *b*		2003·7	49157	1 *a*	
1893·8	49879	1 *b*		2004·9	49149	2 *d*	
1894·8	49871	3 *b*		2005·2	49148	6 *d*	Fe
1896·2	49861	4 *b*		2007·2	49135	6 *c*	Fe
1897·9	49850	1 *c*		2008·1	49129	1 *b*	Ni
1900·0	49835	1 *c*		2008·6	49125	1 *b*	
1904·5	49806	4 *b*		2009·8	49118	2 *b*	
1905·1	49802	2 *c*		2013·9	49094	2 *a*	
1908·5	49778	5 *d*		2014·3	49092	2 *a*	
1911·9	49755	3 *c*		2015·7	49083	1	
1916·2	49726	1 *d*		to 2016·9	49076		
1917·5	49719	4 *b*		2017·7	49070	2 *b*	
1917·9	49716	4 *b*				1	
1919·8	49704	4 *b*		2018·5	49066	2 *b*	
1920·2	49701	4 *b*	Ni	2019·5	49061	2 *a*	
1921·1	49694	4 *b*		2021·2	49048	1 *g*	
1922·0	49689	4 *b*		2024·9	49025	1 *a*	
1922·4	49686	4 *b*		2025·7	49020	4 *a*	Ni
1923·5	49678	4 *b*		2026·8	49012	4 *b*	
1925·8	49662	4 *b*	Ni	2031·1	48986	2 *c*	Ba
1928·0	49647	4 *b*		2035·4	48959	1 *b*	
1931·2	49625	1 *c*		2039·6	48934	1 *b*	
1932·5	49618	1 *c*		2041·3	48923	6 *c*	Fe
1936·2	49593	3 *c*		2042·2	48917	6 *b*	Fe
1939·5	49574	2 *c*		2044·5	48904	5 *b*	
1940·6	49566	2 *c*		2045·0	48901	5 *b*	
1941·5	49561	3 *b*		2047·0	48888	3 *d*	
1943·5	49548	2 *c*		2047·8	48884	3 *b*	
1944·5	49543	3 *b*		2049·3	48874	3 *a*	
1947·6	49522	4 *c*		2049·7	48873	3 *a*	
1949·4	49510	1 *c*		2051·3	48862	3 *c*	
1953·6	49482	2 *b*		2053·0	48851	4 *b*	
c 1960·8	49433	6 *b*		2053·7	48846	4 *c*	
		4	Fe	2058·0	48820	6 *c*	FeCa
1961·2	49431	6 *b*		2060·0	48808	2 *b*	
1964·3	49412	2 *c*		2060·6	48804	2 *a*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000		
2061·0	48802	1 *a*	
2064·7	48778	2 *c*	Ni
2066·2	48769	5 *c*	Fe
2067·1	48764	5 *c*	Fe
2067·8	48761	3 *b*	
2068·8	48755	3 *b*	
2070·6	48743	1 *b*	
2071·3	48739	1 *b*	Co
2073·5	48728	3 *b*	Ni
2074 6	48721	2 *b*	
2076·5	48709	1 *b*	
2077·3	48704	2 *b*	
F 2079·5	48690	4 *e*	
2080·0	48689	6 *g*	
2080·5	48686	4 *e*	
2082·0	48676	6 *a*	Fe
2084·6	48661	2 *b*	
2086·0	48651	1	
to 2086·9	48646	1	
2086·9	48646	3 *b*	Ni
2087·6	48642	1 *a*	
2089·7	48632	1 *a*	
2090·9	48625	1 *a*	
2094·0	48606	2 *b*	
2096·8	48589	1 *b*	
2098·8	48577	1 *a*	
2099·8	48573	2 *a*	
2100·4	48569	1 *a*	
2102·6	48556	4 *a*	
2103·3	48552	4 *b*	
2104·0	48548	4 *a*	
2105·1	48543	4 *b*	
2107·0	48531	1 *a*	
2107·4	48528	2 *a*	
2109·1	48520	2 *b*	
2111·1	48507	3 *b*	
2112·7	48497	3 *b*	Ni
2115·0	48484	3 *a*	Ni
2115·4	48481	3 *a*	
2119·8	48456	1 *b*	
2121·2	48446	4 *b*	
2121·9	48442	5 *c*	
2124·3	48428	1 *b*	
2125·1	48424	2 *b*	
2127·7	48408	3 *b*	
2132·3	48379	2 *a*	Co
2132·7	48376	1 *a*	
2133·8	48369	2 *a*	
2134·3	48367	1 *a*	
2136·0	48357	5 *a*	Zn
2138·0	48346	2 *g*	
2138·4	48343	4 *a*	
2139·5	48336	4 *a*	
2140·4	48331	4 *a*	
2141·9	48322	2 *a*	
2142·4	48319	5 *a*	
2144·6	48307	4 *a*	
2146·9	48293	3 *a*	
2147·4	48291	4 *a*	

Kirchhoff's measure.	Wave-length, m.m. 0·000		
2148·5	48284	4 *a*	
2148·9	48282	3 *a*	
2150·1	48277	3 *a*	
2150·5	48276	3 *a*	
2157·0	48239	3 *a*	Co, Au
2157·4	48237	5 *a*	
2159·0	48229	1 *e*	
2160·6	48219	5 *a*	
2160·9	48217	4 *a*	
2161·7	48213	4 *a*	
2162·6	48207	3 *a*	
2163·7	48201	4 *a*	
2164·0	48198	4 *a*	Ni
2167·5	48178	6 *b*	
2171·5	48155	3 *b*	Co
2172·2	48151	2 *a*	
2175·7	48131	2 *b*	
2176·4	48127	1 *b*	
2179·9	48107	5 *b*	
2181·2	48100	3 *e*	
2184·9	48080	5 *b*	
2186·5	48072	3 *b*	
2187·1	48069	5 *a*	
2187·9	48064	5 *a*	
2188·5	48061	5 *a*	
2190·1	48052	5 *b*	
2191·9	48042	3 *e*	
2192·3	48039	5 *b*	
2193·3	48034	5 *a*	
2195·7	48017	2 *b*	
2197·1	48010	2 *b*	
2197·7	48007	2 *b*	
2198·8	48000	4 *a*	
2199·2	47998	3 *a*	Ni
2201·1	47988	2 *b*	
2201·9	47983	5 *c*	
2203·3	47975	2 *a*	
2203·8	47973	1 *a*	
2205·1	47965	1 *b*	
2206·4	47958	1 *a*	Co
2206·7	47956	1 *a*	
2209·1	47943	4 *c*	
2211·7	47928	4 *b*	
2213·4	47918	4 *b*	
2215·1	47909	1 *b*	
2216·7	47900	3 *b*	
2217·5	47895	3 *b*	
2218·3	47890	3 *a*	
2219·8	47882	3 *b*	
2221·3	47873	1 *a*	
2221·7	47871	1 *a*	
2222·3	47868	5 *c*	
2223·5	47862	3 *c*	
2225·4	47851	2 *b*	
2226·2	47845	4 *b*	
2227·6	47837	2 *a*	
2228·6	47832	2 *a*	
2229·1	47829	4 *a*	
2230·7	47820	4 *a*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
2231·2	47817	2 *a*		2316·0	47344	2 *b*	
2232·3	47811	4 *a*		2316·6	47341	1 *b*	
2233·7	47803	5 *c*		2322·0	47311	2 *b*	
2234·0	47801	2 *c*		2323·0	47306	2 *b*	
2237·4	47784	1 *b*		2325·3	47291	6 *d*	
2238·7	47778	1 *b*		2328·3	47275	5 *b*	
2240·0	47769	3 *b*	Zn	2329·5	47269	5 *b*	Cu
2241·4	47760	2 *b*		2332·8	47250	2 *b*	
2245·1	47739	3 *b*		2333·0	47248	5 *b*	
2246·2	47733	1 *b*		2334·1	47242	2 *d*	Ni
2248·2	47722	3 *c*		2335·0	47237	5 *b*	
2249·7	47715	6 *a*	Ni	2336·2	47231	2 *d*	
2250·0	47712	3 *d*		2336·8	47228	5 *b*	
2255·4	47682	4 *b*		2339·9	47213	4 *b*	
2256·2	47678	2 *b*		2342·5	47198	1 *d*	
2257·1	47673	4 *d*		from 2343·7	47190	1	
2257·6	47670	2 *b*		2345·1	47181	2 *d*	
2258·5	47666	2 *c*		2346·7	47172	4 *b*	
2259·4	47663	4 *c*		2347·3	47168	4 *b*	
2261·4	47652	1 *b*		2349·4	47157	1 *b*	
2262·1	47649	2 *a*		2349·9	47155	2 *b*	
2263·4	47642	2 *a*		2351·4	47145	1 *c*	
2264·3	47638	6 *d*		2352·2	47141	2 *b*	
2266·2	47627	2 *a*		2354·1	47130	6 *c*	
2266·6	47625	3 *a*		2357·4	47112	5 *a*	
2268·0	47616	3 *a*		2358·4	47104	5 *b*	
2269·1	47611	3 *a*		2361·0	47090	1 *d*	
2269·9	47607	3 *a*		2362·2	47084	1 *c*	
2270·2	47605	3 *a*		2362·6	47083	4 *b*	
2274·2	47587	1 *d*		2364·0	47074	4 *b*	
2278·4	47560	4 *c*		2365·9	47064	2 *b*	
2279·8	47554	2 *a*		2366·8	47059	1 *b*	
2280·7	47548	2 *a*		2367·7	47054	2 *b*	
2282·0	47541	1 *a*		2369·7	47044	2 *b*	
2282·3	47539	1 *b*		2371·4	47033	2 *b*	
2283·6	47531	2 *a*		2371·6	47032	4 *b*	
2284·9	47524	2 *b*		2372·4	47027	4 *b*	
2286·1	47518	2 *b*		2374·2	47016	3 *b*	
2288·1	47506	2 *a*		2375·0	47013	2 *b*	
from 2289·1	47501	1		2375·6	47010	4 *b*	
2289·9	47496	2 *b*		2376·1	47006	1 *b*	
2290·4	47493	1 *b*		2379·0	46989	6 *c*	
2291·8	47486	2 *g*	Zn	2381·6	46975	6 *c*	
2293·1	47478	2 *a*		2386·1	46951	3 *b*	
		1		2386·6	46948	2 *a*	
2293·6	47474	3 *b*		2388·7	46935	2 *c*	
2294·5	47469	2 *b*	Cd	2389·7	46929	2 *c*	
2301·7	47428	4 *c*		2390·7	46924	3 *a*	
2302·9	47420	3 *b*		2391·2	46920	1 *b*	
2305·3	47404	3 *d*		2393·1	46911	5 *b*	
2306·8	47397	4 *c*		2394·4	46904	4 *a*	
2307·8	47391	1 *b*		2395·8	46896	1 *f*	
2308·2	47389	5 *b*		2396·1	46895	3 *b*	
2309·0	47385	5 *c*		2396·7	46892	2 *a*	
to 2310·4	47378	1				1	
2310·9	47375	2 *c*		2397·4	46887	2 *a*	
2312·5	47365	3 *b*		2399·6	46875	3 *a*	
2313·7	47358	3 *b*		2399·9	46874	3 *a*	
2314·3	47354	3 *b*		2402·2	46858	3 *b*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 6·000		
2403·2	46851	3 *b*		2478·7	46404	2 *a*	
2404·9	46842	2 *b*		2479·7	46397	2 *a*	
2406·2	46835	2 *b*		2480·1	46395	2 *a*	
2406·6	46833	6 *c*		2481·1	46388	1 *a*	
2407·2	46828	1 *b*		2482·1	46384	1 *a*	
2408·2	46823	4 *b*		2482·4	46381	1 *c*	
2409·0	46818	1 *b*		2486·6	46355	5 *b*	
2410·2	46811	4 *b*		2487·0	46352	5 *b*	
2412·8	46799	3 *b*		2488·2	46346	4 *b*	
2414·7	46787	2 *b*		2489·4	46339	5 *d*	
(2416·0	46780	3 *d*		(2490·5	46333	5 *a*	
2416·3	46777	5 *b*		2490·8	46330	3 *d*	
2418·0	46768	3 *b*		2493·0	46318	3 *a*	
2419·3	46761	5 *b*		(2493·6	46314	5 *a*	Co
2420·6	46753	2 *b*		2493·9	46313	3 *f*	
2422·3	46744	6 *d*	Co	2495·8	46302	5 *b*	
2423·8	46735	3 *e*		2497·2	46292	6 *d*	
2424·4	46731	4 *b*		2499·0	46282	3 *b*	
2426·5	46720	4 *b*		2499·8	46277	3 *b*	
2428·4	46709	1 *a*		2500·3	46273	4 *c*	
2429·5	46703	3 *b*		(2502·2	46262	4 *e*	Ba
2431·9	46689	2 *b*		2502·4	46260	1 *b*	
2432·4	46686	1 *b*		2505·6	46240	4 *d*	
(2435·3	46668	2 *b*		2509·4	46218	2 *d*	
2435·5	46667	5 *c*		2512·1	46200	1 *c*	
(2435·7	46665	2 *b*		2512·5	46198	2 *a*	
2436·5	46660	5 *a*		2513·2	46194	2 *b*	
2438·5	46649	1 *a*		2513·5	46192	1 *b*	
2439·4	46644	2 *b*		2517·0	46170	3 *b*	
2440·0	46640	1 *a*		(2518·2	46161	2 *c*	
2441·8	46629	2 *a*		2518·4	46159	3 *a*	
2442·4	46626	1 *a*		2520·9	46145	3 *a*	
2443·9	46617	5 *a*		2522·3	46136	1 *a*	
2444·2	46615	5 *a*		2525·0	46120	2 *a*	
2445·3	46609	1 *c*		2525·4	46117	1 *b*	
2446·6	46601	5 *b*		2527·0	46106	4 *a*	
2452·1	46570	2 *c*		2532·0	46075	2 *b*	
2454·1	46555	4 *b*		2535·5	46052	2 *b*	
2457·5	46537	4 *b*		2535·9	46049	2 *b*	
2457·9	46534	4 *b*		2536·6	46045	1 *b*	
2458·6	46530	3 *a*		2537·1	46042	5 *c*	
2459·5	46524	2 *b*		2538·0	46035	1 *b*	
2460·4	46518	1 *c*		2538·3	46033	2 *a*	
2461·2	46513	6 *b*	Ba	2540·5	46021	2 *g*	Pt
2463·4	46499	4 *b*		2543·5	46002	4 *c*	
2466·0	46482	3 *a*		2544·5	45995	2 *d*	
(2467·3	46474	3 *c*		2545·4	45989	1 *c*	
2467·6	46472	5 *c*		2547·2	45976	6 *c*	
(2467·9	46470	3 *c*		2547·7	45972	2 *b*	
2468·7	46465	3 *a*		2548·4	45968	1 *c*	
2470·1	46457	4 *a*		2549·7	45959	1 *b*	
(2471·2	46450	2 *b*		2550·1	45956	1 *b*	
2471·4	46448	4 *a*		(2551·2	45948	1 *b*	
2472·9	46439	4 *a*		2551·4	45946	3 *a*	
2473·8	46433	2 *c*		(2552·4	45940	3 *a*	
2474·6	46428	4 *b*		2552·6	45938	1 *b*	
2475·5	46424	1 *c*		2553·6	45932	3 *a*	
2477·4	46411	2 *a*		2554·0	45929	3 *a*	
2477·8	46408	2 *a*					

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
(2554·9	45922	3 *a*		2624·1	45428	1 *b*	
2555·1	45921	2 *c*		2625·2	45419	5 *a*	
2556·3	45913	2 *c*		2625·9	45413	4 *a*	
2559·9	45890	3 *b*		2626·3	45410	2 *a*	
2562·1	45875	4 *b*		2627·0	45405	5 *b*	
2564·0	45862	3 *b*		2627·9	45398	2 *a*	
2565·0	45855	6 *c*		2628·9	45391	1 *c*	
2565·9	45848	2 *b*		2629·7	45386	1 *b*	
2566·3	45845	3 *d*		2630·5	45380	1 *a*	
2567·8	45836	3 *b*		2633·6	45356	2 *c*	
2568·4	45832	2 *b*		2634·4	45351	1 *d*	
2574·4	45790	5 *c*		2635·5	45343	3 *b*	
2579·3	45756	3 *d*		2636·4	45337	2 *c*	
2581·0	45744	1 *a*		2637·4	45329	4 *b*	
2581·5	45740	1 *a*		(2638·5	45320	4 *e* }	Ca
2582·0	45737	2 *a*		2638·8	45318	5 *a* }	
2582·4	45733	2 *a*		2639·6	45313	1 *c*	
2582·8	45730	1 *a*		2640·6	45305	2 *c*	
2584·0	45721	3 *e*		2641·6	45297	3 *c*	
2585·4	45712	5 *b*		2642·5	45291	2 *a*	
2587·9	45694	3 *a*		2643·2	45287	1 *a*	
2588·5	45690	5 *b*		2643·5	45285	1 *a*	
2589·7	45681	1 *b*		2645·6	45268	4 *b*	
2591·3	45669	4 *a*		2646·2	45263	2 *g*	(La, Di)
2591·7	45666	2 *c*		(2650·5	45233	5 *b*	
2593·0	45656	1 *e*		2650·7	45231	3 *c*	
(2594·9	45643	2 *b*		(2652·9	45213	1 *d* }	Ca
(		1		2653·2	45211	5 *b* }	
2595·4	45640	4 *a*		from (2656·7	45185	1	
(2595·9	45636	4 *a*		2657·9	45177	3 *b*	
(		1		2658·6	45171	1 *b*	
2596·4	45632	2 *c*		2664·9	45119	3 *a*	
2597·7	45622	3 *b*		2665·9	45109	3 *b*	
2598·5	45617	1 *b*		2666·7	45102	1 *b*	
(2599·4	45611	3 *c*		2667·6	45094	3 *a*	
(2599·7	45609	5 *b*		2668·0	45091	1 *b*	
2600·6	* 45602	2 *a*		2669·4	45079	3 *b*	
2601·0	45599	2 *c*		*f** 2670·0	45074	6 *e*	Fe
2602·1	45590	4 *b*		2673·8	45042	1 *a*	
2602·9	45583	1 *a*		2674·5	45036	2 *a*	
2603·6	45577	2 *b*		2675·6	45026	2 *c*	
2604·0	45574	1 *a*		2676·5	54018	2 *a*	
2604·8	45568	4 *b*		2677·2	45011	1 *a*	
(2605·8	45561	3 *b* }		2678·4	45001	1 *a*	
(		2 }	Ca	2679·0	44996	2 *a*	
2606·6	45555	5 *c* }		(2680·0	44988	5 *b*	
2607·1	45551	3 *c*		(2680·2	44986	3 *b*	
2608·2	45543	1 *e*		2681·2	44977	5 *a*	
2608·6	45540	1 *b*		2683·1	44961	4 *b*	
2608·9	45538	1 *a*		(2686·0	44936	3 *c*	
2610·2	45529	1 *a*		(2686·4	44933	6 *f*	Fe
2612·3	45514	3 *b*		(2686·8	44930	3 *e*	
2613·6	45504	2 *e*		2688·4	44915	2 *e*	
2614·1	45501	3 *c*		(2690·8	44894	5 *b*	
2616·5	45486	2 *b*		(2691·1	44890	3 *e*	
2619·1	45467	5 *b*		2692·3	44881	3 *c*	
2619·9	45461	3 *a*		2693·5	44871	4 *c*	
2620·3	45458	3 *a*		from 2695·2	44855	} 1	
2622·3	45443	1 *b*		to 2696·8	44841	}	

* The identification of *f* appears doubtful.—G. B. A.

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
2698·2	44830	1 *f*		2755·4	44291	1 *b*	
from 2699·8	44816	1		2755·8	44287	2 *b*	
2700·7	44810	2 *a*		2756·5	44279	1 *e*	
2702·1	44799	3 *b*		2757·2	44272	1 *e*	
2702·3	44797	4 *a*		2759·4	44249	1 *a*	
2702·5	44794	3 *b*		2760·1	44242	2 *a*	
2703·5	44786	3 *a*		2760·6	44236	2 *d*	
from 2703·8	44783	1		2762·0	44221	4 *e*	
to 2704·9	44773			2763·8	44202	3 *f*	
2707·4	44751	1 *f*		2767·2	44166	1 *d*	
2707·7	44748	3 *a*		2768·2	44155	2 *a*	
2708·9	44737	4 *b*		2768·5	44152	1 *a*	
2709·6	44731	2 *b*		2770·0	44136	2 *b*	
2710·6	44722	3 *a*		2770·8	44128	2 *b*	
2710·9	44719	1 *g*		2774·0	44095	5 *c*	
2711·9	44711	1 *a*		2775·4	44081	4 *c*	
2712·8	44702	2 *a*		2775·7	44077	6 *e*	
2713·3	44697	3 *a*		2776·0	44073	4 *c*	
2714·3	44688	2 *a*		2777·3	44059	3 *a*	
2715·2	44681	2 *b*		2777·8	44054	2	
2716·1	44673	1 *d*				1	
2718·5	44650	3 *g*		2778·5	44047		
2719·0	44646	4 *c*		2781·2	44019	2 *b*	
2720·2	44635	1		2782·2	44008	1 *b*	
2720·8	44629	2		2782·9	44001	3 *b*	
2721·6	44622	6	Fe	2783·9	43990	1 *b*	
2722·8	44611	3		2784·8	43981	1 *e*	
2725·5	44586	2 *d*		2785·1	43978	2 *e*	
2725·8	44582	3 *a*		2788·8	43939	1 *b*	
2726·8	44572	2 *a*		2789·1	43936	3 *c*	
2728·0	44560	4 *b*		2790·5	43921	1 *e*	
2728·4	44557	1 *b*		2791·1	43915	3 *b*	
2729·8	44543	2 *c*		2793·0	43895	1	
2730·7	44534	1 *b*		2794·0	43886	2	
2731·6	44526	3 *c*		2795·7	43868	6	
2732·4	44518	1 *c*		2796·7	43857	2	
2733·7	44506	5 *b*		2797·6	43848	3 *b*	
2734·1	44502	3 *b*				2	
2735·7	44487	1		2798·0	43844	3 *b*	
2736·5	44479	3 *b*				1	
2736·9	44475	3 *b*		2798·9	43835	2 *c*	
2737·4	44470	1 *a*				1	
2737·8	44466	2 *a*		2799·5	43829	2 *c*	
2739·2	44453	2 *c*				1	
2739·9	44446	1 *b*		2800·1	43823	3 *b*	
2741·3	44433	3 *d*				1	
2741·7	44429	3 *b*		2800·7	43816	3 *b*	
2743·8	44408	1 *f*				1	
2744·1	44406	4 *c*		2801·4	43807	4 *d*	
2744·3	44404	1 *d*		2804·5	43771	1 *b*	
2746·8	44379	1		2805·4	43761	1 *b*	
2747·2	44374			2806·9	43743	1 *c*	
2747·6	44370	3 *a*		2807·2	43739	2 *a*	
2748·0	44367	4 *c*		2808·6	43723	1 *b*	
2749·8	44350	3 *c*		2808·8	43721	2 *a*	
2750·6	44341	3 *a*		2809·0	43719	1 *b*	
2754·5	44300	2 *c*		2810·8	43697	2 *b*	

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000			Kirchhoff's measure.	Wave-length, m.m. 0·000		
2811·7	43687	2 *a*		2851·6	43202	3 *b*	
2812·0	43683	2 *a*				2	
2812·5	43678	2 *a*		2852·0	43197	4 *a*	
2812·8	43675	1 *e*				2	
2814·1	43659	1 *b*		2852·3	43193	4 *a*	
2817·7	43615	3 *e*				1	
2819·2	43598	3 *b*		2853·1	43181	3	
2819·6	43594	2 *b*		2853·6	43175	4	
2820·6	43582	2		2854·1	43168	6	} Fe
2821·0	43578	3		G 2854·7	43160	4	} Ca
2821·6	43571	6	Fe	2855·2	43153	3	
2822·3	43564	3		2855·7	43146		
				2856·9	43129	4 *d*	
2823·4	43552	4 *e*		from 2857·9	43118	3	
2824·2	43543	3 *a*		2858·5	43110	4 *a*	
		2					
2825·0	43534	4 *e*		2858·9	43104	2	Sr
		3					
2825·9	43523	4 *b*		2859·4	43098	3	
		3					
2826·5	43516	4 *e*		2860·2	43088	1	
2828·9	43488	3 *b*		2860·9	43078	2	
2830·7	43467	3 *g*				1	
2834·2	43426	5 *e*	Ca	2861·7	43069	4 *b*	
2837·7	43384	1 *g*		2861·9	43066	3 *b*	
2841·4	43339	5 *b*				1	
2841·7	43335	4 *e*		2863·1	43050	3 *b*	
2843·0	43318	3 *d*		2863·6	43044		
2843·3	43314	4 *a*				4	
2844·0	43304	3 *b*		2864·2	43036	5 *b*	
2845·3	43286	4 *f*				2	
		2		2864·7	43030	4 *b*	Ca
2846·1	43275	3 *e*				2	
		2		2865·3	43022	4 *e*	
2846·9	43264	4 *e*				1	
		1		2866·3	43009	5 *b*	
2847·7	43253	4 *a*				3	
		2		2867·1	42999		
2848·0	43249	4 *a*				2	
		2		2868·1	42985	4 *e*	
2848·4	43243	3 *b*				3	
		2		2869·7	42964	5 *e*	Ca
2848·9	43237	3 *b*				4	
		2		2871·2	42944		
2849·3	43232	3 *b*		2872·2	42930	4 *d*	
		2				1	
2849·8	43225	3 *b*		2873·4	42913	2 *b*	
		2				1	
2850·2	43221	3 *b*		2873·9	42907	2 *b*	
		2				1	
2850·7	43214	3 *b*		2874·3	42901	3 *b*	
		2				1	
2851·1	43209	3 *b*		2874·7	42897	2 *b*	
		2				1	
				2875·2	42890	4 *e*	

Conversion of KIRCHHOFF's Spectral Measures into Wave-Lengths in terms of the Millimetre, for the lines produced by Metals and Air.

(The lines marked with an asterisk appear to coincide with dark lines in the Solar Spectrum.)—Note by KIRCHHOFF.

Kirchhoff's measure.	Wave-length, m.m. 0·000	
Ce		
1190·1	56345	1
1249·9	55611	1
1256·7	55530	1
1329·1	54710	2
1332·4	54676	2
1336·2	54635	1
1385·0	54118	2
1401·7	53948	2
*1438·9	53580	3
1460·9	53363	1
1517·9	52834	3
from 1571·0	52364	1
to 1572·4	52351	
1573·0	52345	2
1623·1	51918	1
from 1629·2	51866	2
to 1630·4	51858	
1683·1	51427	1
1725·5	51098	1
*1777·5	50705	2
from 1782·4	50669	1
to 1784·5	50650	
1938·8	49579	2
2052·3	48856	1
2221·5	47871	1
Di		
1225·0	55908	2
1230·0	55847	1
from 1364·5	54335	1
to 1365·2	54327	
1431·9	53648	1
1471·1	53269	1
from 1518·6	52828	1
*to 1519·4	52820	
1536·0	52675	1
1541·4	52622	1
1548·9	52555	2
*1567·5	52392	1
1709·2	51224	2
La		
from 1411·6	53851	2
*to 1412·8	53839	
1416·8	53798	2
1451·0	53462	1
1606·8	52056	2
1627·9	51878	2
1634·8	51821	2
2136·8	48352	1

Kirchhoff's measure.	Wave-length, m.m. 0·000	
(La, Di)		
1025·0	58693	1
1064·5	58080	1
1066·1	58058	1
1071·1	57979	1
1075·6	57915	1
1077·0	57895	1
1092·1	57674	2
*1302·0	55012	1
*1303·4	54995	2
1317·6	54836	1
1345·4	54537	1
from 1486·8	53121	2
*to 1489·2	53098	
*1622·3	51925	1
*1623·3	51916	1
1716·6	51167	2
1728·8	51072	2
from 1894·5	49873	2
*to 1895·2	49869	
1903·0	49816	1
1940·2	49569	1
from 1988·6	49255	1
to 1989·5	49250	
2003·8	49156	1
2004·7	49151	2
2031·0	48987	2
2081·0	48682	2
2121·4	48444	1
2208·2	47948	2
2214·5	47912	2
2217·8	47894	2
Pd		
1114·7	57349	1
*1146·2	56921	2
1164·9	56670	2
1185·6	56403	1
1264·6	55438	2
1269·0	55386	2
1279·1	55270	1
from 1400·0	53965	2
*to 1400·7	53959	
1430·1	53666	1
1447·0	53504	1
1477·0	53212	1
1495·2	53042	3
1540·0	52636	1
from 1566·5	52401	2
to 1567·1	52396	
1601·4	52103	1

TABLE (continued).

Kirchhoff's measure.	Wave-length, m.m. 0·000		Kirchhoff's measure.	Wave-length, m.m. 0·000	
from 1660·0	·51613	} 3	from 1488·2	53108	} 3
to 1660·7	51608	}	to 1489·0	53100	}
1732·9	51040	2	1576·8	52313	1
1801·9	50525	1	from 1806·1	50494	} 2
2062·0	48795	2	*to 1806·9	50488	}
2123·6	48432	2	2057·0	48826	1
2162·0	48212	2			
			(Ru, Ir)		
Pt			1348·3	54504	2
1325·7	54747	1	*1489·9	53093	1

Atmospheric Lines.

Kirchhoff's measure.	Wave-length, m.m. 0·000		Kirchhoff's measure.	Wave-length, m.m. 0·000	
711·4	65232		977·7	59471	
948·0	59991		982·0	59401	
949·4	59967		982·3	59396	
949·8	59960		988·9	59284	
951·7	59925		989·2	59279	
954·2	59881		989·6	59272	
958·8	59801		993·1	59215	
959·6	59788		993·4	59206	
961·9	59746		998·1	59130	
963·7	59713		999·2	59112	
964·4	59703		1000·0	59099	
965·7	59681		1001·4	59078	
968·7	59624		1005·8	59004	
969·0	59623		1008·3	58963	
969·6	59613		1009·2	58949	
970·5	59597		1010·5	58927	
972·1	59568		1013·9	58873	
974·3	59531		1015·1	58854	
975·0	59519		1016·4	58833	
975·7	59505		1017·7	58812	
976·1	59499		1018·2	58803	
977·4	59477				

Measures of Wave-Lengths, in Millimetres, for the Spectral Lines produced by Air and different Metals: collected from the preceding Tables.

Air. m.m. 0·000	Ba m.m. 0·000	Ca m.m. 0·000	Cr m.m. 0·000	Fe m.m. 0·000	La m.m. 0·000	Na m.m. 0·000	Pd m.m. 0·000
65729	65035	43153	54112	54167	53851	59053	52396
65710	61405	43030	52102	54121	53839	58988	52103
65690	61086	42964	52077	54076	53798		51613
65232	58583		52060	54063	53462	Ni	51608
64867	57805	Cd		53995	52056	61135	51040
60011	55326	64456	Cu	53949	51878	61056	50525
59991	55175	47469	57824	53862	51821	59017	48795
59967	54262		53024	53749	48352	58626	48432
59960	49250	Ce	52215	53736		54757	48212
59925	48986	56345	51497	53713		51754	
59880	46513	55611	50976	53685	(La, Di)	51664	Pt
59801	46262	55530	47269	53463	58693	51517	54747
59788	46260	54710		53454	58080	51421	53108
59746		54676	Di	53346	58058	51373	53100
59713		54635	55908	53343	57979	51317	52313
59703	Ca	54118	55847	53309	57915	51084	50494
59681	67273	53948	54335	53243	57895	50917	50488
59624	65059	53580	54327	53113	57674	50912	48826
59623	65000	53363	53648	52941	55012	50716	46021
59613	64764	52834	53269	52919	54995	50712	
59597	64695	52364	52828	52792	54836	50236	(Ru, Ir)
59568	64559	52351	52820	52782	54537	50054	63525
59531	64456	52345	52675	52747	53121	49701	54504
59519	61693	51918	52622	52375	53098	49662	53093
59505	61617	51866	52555	52309	51925	49263	
59499	61191	51858	52392	51925	51916	49129	Sr
59477	60990	51427	51224	51915	51167	49020	64129
59471	58626	51098		51693	51072	48778	55320
59401	55997	50704	Fe	51664	49873	48728	54805
59396	55981	50669	64972	51647	49869	48646	43118
59284	55951	50651	64049	51592	49816	48497	43110
59279	55913	49579	63049	51343	49569	48484	43104
59272	55868	48856	62317	51281	49255	48198	
59215	55852	47871	61915	50063	49250	47998	Zn
59206	55787		61348	49433	49156	47715	63683
59130	53536	Co	60659	49431	49151	47242	48357
59112	52792	61191	60297	49169	48987		47769
59099	52739	54821	59234	49148	48682	Pd	47486
59078	52725	53579	57616	49135	48444	57349	
59004	52705	53568	56210	48923	47948	56921	
58963	52701	53488	56129	48917	47912	56670	
58949	51883	53479	55997	48820	47894	56403	
58927	50302	52904	55833	48769	45263	55438	
58873	48820	52770	55728	48764		55386	
58854	45561	52747	55696	48676		55270	
58833	45555	52401	55660	45074	Li	53965	
58812	45551	48739	54626	44933	60990	53959	
58803	45320	48379	54556	44629		53666	
	45318	48239	54477	44622		53504	
	45213	48155	54456	43564	Mg	53212	
Au	45211	47958	54351	43168	51826	53042	
62796	43426	46744	54309	43160	51705	52636	
48239	43160	46314	54250		51647	52401	

Comparison of the Wave-Lengths found by ÅNGSTRÖM with the Wave-Lengths found in the preceding Tables.

(For ÅNGSTRÖM's measures, see POGGENDORFF's 'Annalen der Physik und Chemie,' CXXIII. Band, page 498. ÅNGSTRÖM's measures were originally given in terms of the Paris inch, but are here converted into measures in terms of the millimetre.)

Designation of line and Kirchhoff's measure.	Wave-length, by Ångström in terms of the Paris inch. in. 0·0000	Wave-length, by Ångström in terms of the millimetre. m.m. 0·000	Wave-length, computed in preceding Tables. m.m. 0·000	Designation of line and Kirchhoff's measure.	Wave-length, by Ångström in terms of the Paris inch. in. 0·0000	Wave-length, by Ångström in terms of the millimetre. m.m. 0·000	Wave-length, computed in preceding Tables. m.m. 0·000
A 401·9—406·8	28120	76121	76561	1463·3	19696	53317	53343
B 592·7—593·1	25397	68750	68841	1466·8	19681	53276	53309
C 694·1	24863	65680	65710	1473·9	19653	53201	53243
849·7	22873	61917	61915	E 1522·7	19484	52743	52792
860·2	22796	61709	61693	E 1523·7	19480	52732	52782
863·9	22768	61633	61617	1527·7	19468	52700	52747
874·3	22694	61433	61405	1564·2	19364	52418	52423
877·0	22677	61387	61348	1569·6	19346	52370	52375
884·9	22621	61235	61191	1622·3	19196	51964	51925
894·9	22551	61046	60990	*b* 1633·5	19165	51880	51832
D 1002·8	21797	59004	59053	b_1 1648·8	19124	51769	51705
D 1006·8	21775	58945	58988	b_2 1653·7	19111	51733	51664
1200·6	20761	56200	56210	b_2 1655·6	19105	51717	51647
1207·3	20713	56070	56129	*c* 1960·8	18327	49611	49433
1217·8	20654	55910	55997	2001·6	18191	49243	49169
1231·3	20601	55767	55833	2005·2	18184	49224	49148
1337·0	20169	54597	54626	2041·3	18083	48951	48923
1343·5	20136	54508	54556	2066·2	18011	48756	48769
1362·9	20073	54338	54351	F 2079·5	17973	48653	48690
1367·0	20053	54283	54309	F 2080·5			48686
1389·4	19984	54097	54076	*f* 2670·0	16322	44184	45074
1390·9	19979	54083	54063	2686·4	16285	44083	44933
1421·5	19858	53756	53749	2720·8	16204	43864	44629
1423·0	19853	53742	53736	G 2854·7	15923	43104	43160
1425·4	19842	53712	53713				
1428·2	19835	53693	53685				
1450·8	19742	53442	53463				

Comparison of the Wave-Lengths found by DITSCHEINER with the Wave-Lengths found in the preceding Tables. (See 'Sitzungsberichte der Mathematisch-Naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften,' LII. Band. II. Abtheilung, 1865, p. 289, &c.)

Designation of line and Kirchhoff's measure.	Wave-length, by Ditscheiner. m.m. 0·000	Wave-length, computed in preceding Tables. m.m. 0·000	Designation of line and Kirchhoff's measure.	Wave-length, by Ditscheiner. m.m. 0·000	Wave-length, computed in preceding Tables. m.m. 0·000
B 593	68833	68835	1737·6	51180	51002
C 694	65711	65710	1750·4	51068	50906
711·5	65258	65233	1777·4	50879	50729
719·5	65054	65011	1799	50729	50546
783·5	63472	63403	1834	50493	50295
831	62407	62317	1854·5	50363	50149
850	62009	61915	1867	50264	50063
860	61796	61693	1873·5	50205	50018
864	61719	61617	1885·8	50145	49933
874·5	61520	61405	1908·5	50010	49778
877	61470	61348	1920	49914	49701
885	61322	61191	1961	49653	49432
895	61128	60990	1975·6	49541	49337
959	59859	59801	1983	49471	49289
Dδ 1002·8	59053	59053	1989·5	49412	49250
Dα 1006·8	58989	58988	2002	49269	49169
1029·4	58670	58626	2018	49178	49068
1096·1	57716	57616	2041·4	48990	48923
1103	57621	57517	2058	48854	48820
1135	57193	57068	2067	48791	48764
1155·7	56913	56790	F 2080·1	48687	48689
1174·4	56676	56550	2103·3	48499	48552
1200·4	56339	56210	2119·8	48317	48456
1207·5	56240	56129	2148·9	48092	48282
1218	56119	55997	2157·4	47991	48237
1231·6	55955	55833	2160·6	47959	48219
1242·5	55819	55696	2187·1	47717	48069
1280	55368	55259	2201·9	47626	47983
1303·7	55154	54994	2221·7	47470	47871
1307	55111	54958	2233·7	47371	47803
1324·8	54854	54757	2250	47161	47712
1337	54719	54626	2264·3	47106	47638
1343·5	54646	54556	2309	46742	47385
1351·3	54549	54477	2416	46097	46780
1367	54382	54309	2436·5	45901	46660
1389·6	54132	54076	2457·5	45714	46537
1410·5	53919	53862	2467·4	45606	46472
1421·6	53792	53749	2489·4	45409	46339
1451	53491	53463	2537·1	45089	46042
1463	53369	53345	2547·2	45046	45976
1492·5	53062	53070	2566·3	44880	45845
1506·5	52919	52941	2606	44633	45555
1515·5	52841	52855	2627	44498	45408
E 1523·5	52783	52782	2638·6	44418	45319
1542	52638	52618	2670	44222	45074
1569·8	52413	52375	2686·6	44121	44933
1577·5	52349	52308	2721·6	43908	44622
1589·1	52236	52208	2734·9	43813	44495
1601·6	52171	52101	2775·6	43600	44077
1622·4	52006	51925	2797	43466	43857
1634	51912	51826	2822·8	43314	43564
b 1648·8	51809	51705	G 2854·7	43170	43160
1655·6	51754	51647	2869·7	43070	42964
1693·8	51503	51343			

III. *An Account of the Observations on the Great Nebula in Orion, made at Birr Castle, with the 3-feet and 6-feet Telescopes, between* 1848 *and* 1867. *With a drawing of the Nebula.* *By Lord* Oxmantown. *Communicated by the Earl of* Rosse, *K.P., F.R.S.*

Received June 17,—Read June 20, 1867.

Many drawings of the Great Nebula in Orion have already been published by different astronomers; still as the present drawing was made with the advantage of instrumental power far exceeding that at the disposal of previous observers, and as great care has been taken to make it accurate, in fact every available hour during the winter months of seven seasons having been employed upon it, perhaps it may be of some interest to the Royal Society.

Several drawings of this wonderful object were published previous to the year 1825, but they were made with instruments of little power; however, in 1825, Sir J. Herschel published a drawing made with his celebrated 18-inch reflector (Memoirs of the Astronomical Society, 1826). Sir J. Herschel's second drawing with the same instrument, but under more favourable circumstances, together with a description and a catalogue of stars, was published in 1847* (Cape of Good Hope Observations). That was succeeded in 1848 by Mr. Bond's drawing, also with a description and catalogue of stars (Memoirs of the American Academy of Arts and Sciences, 1848).

In 1854 Mr. Lassell published his observations on the Nebula in Orion made at Malta in 1853 (Memoirs of the Royal Astronomical Society, 1854); and in 1862 MM. O. Struve and Liapounov's memoir, with a catalogue of stars, was published at St. Petersburg.

The observations upon this nebula, recorded in the Journal of the Observatory at Parsonstown, date from 1849. From that time till February 1858 there are entries of 54 observations.

In the year 1852 Mr. Bindon Stoney made a drawing of the Huygenian region, which is a very interesting record. Mr. Bindon Stoney was a highly educated civil engineer, well accustomed to use his pencil; he accepted the office of assistant till he was enabled to obtain an engineering appointment. His drawing was made with great care, and he was engaged upon it the whole season. It was compared by several persons with the nebula, and was considered exact. When we compare this drawing with the nebula as it is at (Plate I.) present, there are strong indications of change†.

* This drawing appears to have been executed principally during the years 1835, 1836 & 1837.

† An outline of the principal features of this drawing is given at the upper right-hand corner of the Skeleton Map (Plate II.).

Between February 1860 and February 1864 there are 74 entries of observations. In February 1860, Mr. HUNTER, who was then the assistant, being an accomplished artist, commenced a new drawing, and was engaged upon it till February 1864, when he was obliged to retire from ill health.

As a groundwork for his drawing, Mr. HUNTER laid down all the stars given in 'Observations de la grande Nebuleuse d'Orion faites à Cazan et à Pulkova, par O. STRUVE, St. Pétersbourg, 1862,' in the positions given at page 118 of that treatise, the nebulosity was gradually filled in by eye as correctly as possible with reference to the stars given in that memoir, and twenty-eight* additional stars from the 9th to the 15th magnitude were inserted by eye-estimation.

During the season 1864–65 the nebula was often examined with the view of verifying the drawing made by Mr. HUNTER, and in 1865–66 some additions were made to it in the neighbourhood of the stars 1_{IX}, 2, 3, 4, 1_{VII}, 2_{II}, 9_{II}, 27_{I}, 45_{I} (Plate II.); also the positions of a few stars not given in STRUVE's list were determined by rough micrometrical measurement, and others were laid down by eye-estimation. During the season 1866–67 these measures were completed, the additions of the previous season verified, and the drawing extended to the neighbourhood of the stars 1_{X}, 0_{VII}, 0_{VIII}, 46_{II}, 99_{II}, 143, 149, and 147.

The facts which seem to be of the most interest have been collected under the following heads:—

1st. List of stars which do not occur in OTTO STRUVE's memoir.

2nd. Distance to which the nebulosity extends in various directions.

3rd. Peculiarities of form in some parts.

4th. Evidence of variability in some of the stars.

5th. Evidence of change in the form and brightness of various part of the nebula.

6th. Resolvability of parts of the nebula.

7th. Spectrum of the nebula.

List of New Stars.

The following is a list of the stars which have been inserted in the drawing, but which are not contained in the list at page 118 of M. O. STRUVE's memoir.

In order to avoid altering Sir J. HERSCHEL's numbers and at the same time to keep the whole list of stars in regular order in right ascension, each of the stars in this list has affixed to it the same number as that of the star in HERSCHEL's list which next precedes it in R.A., one, two, three, &c. dots or Roman figures being placed under the number to distinguish each from HERSCHEL's star and from all other new stars. Some of these stars have been laid down by rough measures of their distance and position with respect to the nearest stars in STRUVE's list, but the greater number have been laid

* Three of these are not inserted either in the engraving or in the list of stars, as I was unable to find them in Mr. HUNTER's drawing. The positions given in Mr. HUNTER's list are $-1'\,5''$, $+0'\cdot52$; $+2'\,30''$, $+1'\,35''$; $7'\,13''$, $-3'\,20''$.

down by eye-measurement. The measures were taken with a wire micrometer without illumination, the parallel wires being bars $\frac{1}{10}$ inch in breadth, formed each of four wires laid side by side; the position wire was a silk thread about $\frac{1}{40}$ inch thick. The instrument generally used for these measures was an equatoreal of 18 inches aperture and 10 feet focal length, driven by a water-clock movement, as described in the Monthly Notices of the Astronomical Society for 1866. Two measures only were taken, when they were found to agree pretty well; the object being, not to place the stars with the utmost accuracy, but to lay them down so near their true position that the drawing of the nebula might not be distorted to any appreciable extent.

Those stars which are marked (M) have been determined by micrometrical measurement, the remaining ones by eye-estimation. The stars marked (H) have been inserted by Mr. HUNTER.

This list does not contain all the stars which can be seen within the limits of the drawing. The positions given in the following list are taken from the accompanying Skeleton Map by the method which HERSCHEL describes (Cape Observations, Sect. 58).

	Δ R from 69.	− Δ Declination.		Approximate magnitude.	
	′ ″	′ ″			
0_{I}	−36 40	−15 55	M	10	Distance only from 1_{VII} measured.
0_{II}	−36 25	+ 6 40	...	10	
0_{III}	−36 0	+13 30	M	10	
0_{IV}	−35 20	−17 20	...	13	
0_{V}	−34 55	−17 40	...	13	
0_{VI}	−34 55	+ 7 35	...	11	
0_{VII}	−33 45	− 4 15	...	12	
0_{VIII}	−33 20	−11 30	...	9	
1_{I}	−30 30	+ 3 35	...	13	
1_{II}	−29 5	− 4 50	...	11	
1_{III}	−28 40	+ 5 25	...	11	
1_{IIII}	−27 55	+ 1 45	...	10	
1_{V}	−26 55	+14 0	...	13	
1_{VI}	−26 45	− 1 20	...	12	
1_{VII}	−26 55	−22 25	M	10	
1_{VIII}	−26 30	−39 20	M	8	
1_{IX}	−26 10	−10 0	...	12	
1_{X}	−26 10	+17 10	...	12	
1_{XI}	−25 5	− 3 30	M	10	
1_{XII}	−23 0	+ 8 0	...	13	
1_{XIII}	−23 0	− 7 30	...	13	
1_{XIV}	−23 0	+13 20	M	8	
2_{I}	−22 5	−22 55	M	9	
2_{II}	−21 10	−13 55	...	14	
2_{III}	−21 0	−19 45	...	14	
2_{IV}	−20 45	−18 50	...	14	
4_{I}	−20 0	+ 5 45	H	9	
4_{II}	−18 50	−12 20	...	15	
4_{III}	−18 20	−13 5	...	14	
4_{IIII}	−18 10	−13 40	...	13	
4_{V}	−17 30	+12 30	H	9	
4_{VI}	−17 5	+ 8 55	H	12	
9_{I}	−14 30	−29 10	...	12	
13_{I}	−12 20	+ 8 15	H	12	
16_{I}	−12 5	− 1 55	H	13	
16_{II}	−12 0	+10 35	H	13	

List of New Stars (continued).

	Δ R from 69.	−Δ Declination.		Approximate magnitude.	
	s	′ ″			
18_{I}	−11 30	−18 30	...	14	
25_{I}	− 9 20	−36 5	...	9	
27_{I}	− 9 5	−27 40	M	12	
28_{I}	− 8 50	−18 50	...	14	
28_{II}	− 8 45	−18 0	...	14	
29_{I}	− 8 5	−20 0	...	14	
29_{II}	− 8 10	−36 30	M	7	
29_{III}	− 8 0	−37 0	...	7	
30_{I}	− 8 45	−23 10	M	11	
32_{I}	− 8 15	+12 15	...	12	
33_{I}	− 7 40	+11 45	...	12	
34_{I}	− 7 10	−29 20	M	12	
34_{II}	− 6 40	−21 50	M	11	
36_{I}	− 6 15	−20 10	...	14	
36_{II}	− 6 5	−19 15	...	13	
37_{I}	− 5 50	−35 40	...	9	
37_{II}	− 5 40	−20 40	M	11	
37_{III}	− 5 50	−39 10	...	9	
45_{I}	− 3 45	−20 5	...	14	
46_{I}	− 3 45	−37 20	M	5	
46_{II}	− 3 20	−36 45	M	5	
47_{I}	− 3 10	−35 50	...	9	
51_{I}	− 2 10	−28 20	M	8	
51_{II}	− 1 55	−32 0	M	7	
52_{I}	− 1 25	−38 25	...	9	
56_{I}	− 1 0	− 2 40	H	13½	
58_{I}	− 0 45	+ 1 30	H	14	
58_{II}	− 0 30	+ 3 50	H	14½	
59_{I}	− 0 40	−26 10	M	7	
61_{I}	− 0 20	−28 10	...	11	
69_{I}	0 0	−38 25	M	7	
75_{I}	+ 0 25	+ 1 20	H	15	
88_{I}	+ 1 0	−27 30	...	11	
88_{II}	+ 1 15	− 3 35	H	14	
91_{I}	+ 1 38	− 0 42	H	13½	
91_{II}	+ 1 42	− 0 20	H	13½	
96_{I}	+ 2 0	−25 15	M	7	
96_{II}	+ 2 10	−27 40	M	8	
98_{I}	+ 2 15	−26 0	...	10	
99_{I}	+ 2 20	−30 55	...	3½	Taken from B. A. Catalogue = ι Orionis.
100_{I}	+ 2 25	− 7 0	H	13½	
112_{I}	+ 4 0	− 5 50	H	13½	
112_{II}	+ 4 5	− 7 0	H	13½	
113_{I}	+ 4 40	− 5 45	H	13½	
113_{II}	+ 4 45	+ 7 5	H	14½	
113_{III}	+ 4 45	+ 7 30	H	15	
114_{I}	+ 4 25	−37 0	...	11	
115_{I}	+ 5 20	−40 25	...	8	
125_{I}	= V	.....	...	.....	Struve.
127_{I}	+ 7 25	+ 2 40	H		
129_{I}	+ 7 40	+ 0 30	H	15	
129_{II}	+ 7 50	−34 10	...	8	
135_{I}	+ 9 40	− 3 25	H		
135_{II}	+10 20	− 2 10	H	14	
138_{I}	+11 10	+ 0 20	H	14½	
138_{II}	+12 10	−35 20	...	8	
142_{I}	+14 20	− 0 45	H	14½	
150_{I}	+23 10	− 5 50	...	9	

Remarks on the foregoing List.

Of the existence of all the stars in the foregoing list which are not marked with an H I am fully satisfied, and of the remainder I have seen 4_v, 4_{vi}, 13, 16_i, 16_{ii}, 56_i.

The nebulosity I examined very carefully, and except in a few places I was able to verify most of the details. The three rays of nebulosity between the stars 13_i, 16_{ii}, 28, 39 (Plates II. & III.) I was never able to see well, and on only one occasion did I succeed in seeing two of them*. The patches of nebulosity in the region included between the stars 25, 56, 79, and 99 I never succeeded in seeing, nor did the rays in the region between 133, 142_i, and 149 come out as clearly as represented in the drawing; the greater part of the nebulosity on the south and preceding sides of the drawing is very faint and ill defined, but the curved outline above 20 is tolerably sharp, and the neighbourhood of 20 and 30_i is darker than anywhere in the surrounding parts, even as far as 99_i, 37_{iii}, and 1_{viii}.

Extreme limits of Nebulosity.

The distance to which the nebulosity can be traced is very great, but it fades away so very gradually that it is quite impossible to assign any definite limit to it.

M. Liapounov, with an achromatic telescope of $9\frac{1}{2}$ inches aperture, was able to trace it up to a line a little beyond 15 through 34, 74, 135, 126, a little outside 135_i near 82, 46, 6, 4_{vi}, and 5, and the spiral round 108†.

Mr. Bond does not mention the extreme limits to which he was able to trace the nebulosity except in two directions. He says, "The connexion of the main body of the nebula with that portion which surrounds C Orionis is traced by the north preceding route. It is quite decided," &c. He also says that the main body of the nebula is not connected with C on the following side; and again, "I was unable to satisfy myself how far it might be possible to trace it (the light) southward, but certainly beyond *ι*. Soon after this star it, however, becomes very faint"‡.

His drawing extends to the limits mentioned by Liapounov, and therefore he was probably able to trace it rather further on all sides.

Mr. Lassell, with his reflector of two-feet aperture, did not extend the nebula quite as far as Mr. Bond in his drawing, but probably he was able to trace it a good deal further.

Sir J. Herschel's drawing in the Cape Observations extends to the stars 6, 124, 145, 141, 96_i, a point halfway between 5 and 1_{xi}, and to 34 and 58, and he says (page 29), "The area of the figure (half a square degree in extent) comprises all the nebulous convolutions and appendages which I have been able to trace, with the exception of the terminal effusion of the greater proboscis beyond the star A (=135) southwards, which may be traced as far as the double star *ι* Orionis, which it involves and renders nebu-

* Mr. Hunter during the last season sometimes employed the 6-feet telescope as a Herschelian; I never did so; silvered glass was, however, occasionally substituted for the flat mirror of speculum-metal, which diminished the loss of light by the second reflexion, but it soon became dewed and the silver surface broke up.

† Struve's Memoir, page 2.

‡ Bond's description of the nebula about θ Orionis, page 93.

lous. It is, however, of little intensity, and offers nothing remarkable enough in respect of form to make it worth while to enlarge the dimensions of the engraving sufficiently to include the whole."

He also says (page 29), "Northwards between this nebula and C Orionis; . . . . no nebulous connexion has been traced."

With regard to the extent to which the nebulosity can be traced with the 6-feet telescope I do not find any record by Mr. BINDON STONEY. He appears to have confined his attention entirely to the central portions, except in the following instance, where he remarks—

"Nov. 25th, 1851. There is a long dark channel following the Huygenian region by about the diameter of the latter, in which no nebulosity exists, and twice on good nights with a freshly polished speculum it gave me the idea of immense depth, like a gulf without any bottom; ordinarily the light from the surrounding parts spreads a faint light over it. Comparatively few stars follow this part of the nebula for about 80′."

With reference to the extent of the nebulosity, Mr. HUNTER says, "It has been repeatedly traced up to the star ι Orionis on the south, and on several occasions to C Orionis on the north, while in the preceding direction the sky assumes a peculiar milkiness, at least one degree before the nebula comes into the field. In the following direction it does not seem to extend much (about 10′) beyond the limits in the sketch."

"Between the stars 135 and ι Orionis the nebulosity narrows to a band of about 5′ in breadth, and then again expands as it approaches ι; there has been no attempt to trace it further in this direction."

"Again, the nebulosity curves round from the star 6 in a north preceding direction until it joins a narrow band of faint nebulosity, passing in the preceding and following direction through the little group of stars of which C Orionis is the brightest."

On many occasions I have examined the neighbourhood of this nebula with the view of determining, as far as possible, the extent of the nebulosity. On the following side I was able to trace it 35′ following the trapezium; this nebulosity was, however, excessively faint, and of almost uniform intensity; in fact the only proof of its existence was the prolongation of the dark lane extending through it from the star 142_{II}, which made the surrounding region look slightly luminous by contrast. On the north side the nebulosity appears to be nearly cut off short by the same dark lane which extends by the stars 114, 56, 25, 1_{XIV}. At the last star this ceases, and a faint broad streak of nebulosity appears to curve round in the direction of C Orionis, nearly in the position described by Mr. HUNTER. On the north side of this lane the nebulosity seems to begin again, and gradually increases in intensity to the streak at C Orionis, the brightest part being perhaps rather fainter than the nebulosity round ι.

On the preceding side the nebulosity appears to extend to a great distance, and seems to be of the same character as that about the stars 0_{IV}, 0_{VIII}, 1_{II}; the streaks, however, gradually diminish in intensity, and though, when followed by the eye in the preceding direction, they may be traced for some distance (5′, or in some cases perhaps 10′) beyond

the drawing, yet when once lost sight of they cannot be recovered again without returning to the brighter parts and tracing them in the preceding direction again. The great difficulty in tracing them, however, appears to be due, not so much to their extreme faintness, as to the uniform intensity of the nebulosity in this region; for when the nebula is examined with the equatoreal of 18 inches, a very low power capable of taking in a field of 1° being used, the whole sky in this region has a general luminous appearance when contrasted with the following and north following region, which itself is not entirely free from nebulosity. In drawing the greater part of the outlying regions of the nebula an eyepiece of a magnifying power of about 230 only, and having a field of 26′, was generally used; higher powers do not seem to show as much as the lower power, although with the higher powers the whole of the light from the large mirror is received into the eye, which is not the case with the lower power. With higher powers the field does not appear to be sufficiently large to allow of each streak of nebulosity being compared with the surrounding parts where the nebulosity is not quite so bright. The higher powers, however, bring out minute stars with ease which are hardly visible with the lower power. On the south preceding side the nebulosity appears to be much of the same character as on the preceding side; long streaks have been traced from 1_{VII} and from a point about 3′ or 4′ south of it, which have been represented in the drawing just at their commencement. Another streak extends from the south side of 9_{I} through 46_{I}, 46_{II}, which seems to extend considerably further than the drawing,—also another from ι in a south following direction.

It is probable that the drawing might be extended considerably further in various directions, but the nebulosity is of such extreme faintness that the work would advance very slowly, the eye requiring so long, after each exposure to even very feeble lamp-light, to recover its full power.

Form.

Very little need be said on this subject as the drawing will speak for itself; it may, however, be well to call attention to the apparent connexion between some of the stars and the nebulosity near them.

In some places the stars appear to have either repelled or absorbed the nebulosity, for instance at the trapezium, at 32 and 35, and 80; and in other places the nebulosity is denser, as if the stars had attracted it, for instance at 2_{II}, 4, 34, and 108. Around the star 108 the nebulosity seems to have a spiral character, and the same appearance, though much less decided, may be seen round 4. Round the stars 46_{II}, 46_{III}, and 99_{I} the nebulosity seems to have been concentrated, but close to them there appears to be an absence of nebulosity; and in the case of 99_{I} the dark hole is situated excentrically with respect to the principal star, its nearer companion being close to the opposite side of the hole*; but in the case of the double star 46_{I} 46_{II} the hole is nearly symmetrically situated, but the nebulosity is brightest at the north preceding side. We can

* A drawing of the nebulosity around ι was published in the Philosophical Transactions for 1850, in which the hole is well shown.

hardly, therefore, account for these numerous coincidences, except by supposing some at least of the stars to be situated nearly at the same distance from us as the nebula—in fact immersed in the nebulous matter.

Evidence of Change.

1. *Variability of the Stars.*—The only remark concerning the variability of a star which I find in Mr. Bond's paper (page 94), refers to star 78, which he thinks is a variable star of short period.

In Mr. Lassell's paper I do not find anything on this subject.

A star marked x in Sir J. Herschel's diagram, published in 1825, was not found by him in 1837; perhaps this may be the same as 129_1, but as there is no list of positions of the stars in the paper of 1825, the identity is uncertain. Possibly this star may be variable.

Although M. Liapounov's observations were made about fourteen years after Sir J. Herschel's, Struve found that the magnitudes of the stars as given by Herschel agreed very fairly with his (Liapounov's) determinations, but one star (IV.), whose position was estimated by M. Liapounov, and at about the same time was measured by Mr. Lassell, does not occur in Sir John Herschel's list, although it is so situated that it would not be likely to be overlooked. M. Struve examined with considerable care Bond's Catalogue of Stars in this nebula with the view of identifying them, as far as possible, with those in Sir J. Herschel's list. B 26 and B 27 he considers identical, and they are near the position of 75, but Mr. Bond appears satisfied of the existence of two stars here of the 17th and 18th magnitudes respectively; their positions are only about $2\frac{1}{4}''$ apart, and therefore their combined light might easily be taken for that of one of $12\frac{1}{2}$ magnitude, which is that assigned to 75 by Struve; Herschel, however, assigns to 75 the 18th magnitude, so that the star is possibly variable.

Mr. Bond gives two stars about $11''$ apart near the position of 57, but no one else seems to have seen more than one. I have examined this part of the nebula repeatedly, but have never seen more than one star in this position. He also gives two stars south following 88, which may be identical with the two stars 91 and 91_1. Mr. Hunter gives the two stars in his drawing, but he is not quite sure whether 91_1 is not the same as 91, as he was not able to find the latter on the night on which he saw 91_1.

Bond's stars 42, 85, and 88 were not found by either Struve or Hunter; there are also several others which Struve could not identify and which are not given by Hunter, but Struve is of opinion that some errors exist in Bond's positions.

Struve next examines Lamont's catalogue of 34 stars, all of which he identifies more or less satisfactorily with stars of Herschel's list, with the exception of four, viz. 4′, 5′, 7′, and 28*; 4′ and 7′ I do not find in Hunter's list; 5′ is in very nearly the position of 75_{II} in Mr. Stoney's drawing† (I do not find any mention of this star in Mr. Hunter's observations, but I am almost certain that I have seen one in this position and on the edge of the nebulosity); 28 is not more than $15''$ distant from Struve's star II.

* See Struve's Memoir, p. 95; these numbers are Lamont's.

† See diagram (Plate II.).

STRUVE during his observations discovered a star (II.) which, strange to say, did not occur in HERSCHEL'S list, although it was brighter than several small stars in this neighbourhood which were given in that list; nine days afterwards he saw it with difficulty, while the other stars had preserved their brightness.

He repeatedly compared this star with 88, 51, 57, 75, 78, with the view of determining, if possible, the period of their variability, and gives the maximum and minimum brightness as follows, 88, whose brilliancy he found to be most constant, being taken as of the 12th magnitude:—

51	varies from	11·9	to 12·5 magnitude.
57	„ „	12·5	to 13·5
II.	„ „	11·8	to invisibility.
III.	„ „	12·5	to „
75	„ „	12·0	to „
78	„ „	12·5	to „

98 he also considers variable to a certain extent; and of the above list, II. he thinks is a variable star, whose maximum is of short duration and minimum of long duration, and 75 one whose maximum is of a long and whose minimum is of short duration.

Another instance of variability is IV., which STRUVE considers varies from the 11·5 to 13·5 magnitude.

With reference to the variability of the stars, our observations furnish very little information; our attention was directed principally to delineating, as carefully as possible, the various details of the nebulæ with reference to stars whose positions had been previously determined. I may, however, remark that 29, to which STRUVE has assigned the same ($7\frac{1}{2}$) magnitude as that of 20, 23, and 24, is now decidedly fainter than they are; it is about the 11th or 12th magnitude. HERSCHEL, on the other hand, gives the following magnitudes, 20=23=24=8th magnitude, 29=12th, which agrees with their present appearance; 29 also is not quite in the same relative position as given in HERSCHEL'S drawing; he represents it nearly in the same line with 20, 23, 24, whereas the line through 23, 24 is inclined through an angle of about 28 degrees from the line 24, 29. This discrepancy is probably due to an error in the position of 29, as HERSCHEL places this star in his fourth or least accurately placed class. The discrepancy seems far too great to be accounted for in any other way.

2. *Variability of Form and Intensity of the Nebulosity.*—On this subject it is impossible to speak decidedly.

On comparing the following six drawings,—

Sir J. HERSCHEL'S	of about the year		1825,
Sir J. HERSCHEL'S	„	„	1837,
Mr. BOND'S	„	„	1848,
M. LIAPOUNOV'S	„	„	1850,
Mr. LASSELL'S	„	„	1854,
Mr. HUNTER'S	„	„	1863,

great discrepancies exist in almost every part, but these are probably to be attributed in a great measure to the difference of power in the instruments used and the amount of labour expended on the drawings, as no continuous change seems to be shown by them. In the case of the spiral nebula round 108, Bond's, Lassell's, and Hunter's drawings appear to agree tolerably well, allowance being made for the difference of size of the instruments, but when we go back to Herschel's drawing of 1837 we find a considerable discrepancy. Herschel's drawing of 1825, however, as far as it goes, is in this place more like the latter drawings. With regard to the following extremity of the Huygenian region, all the former drawings, with the exception of Liapounov's, represent the "Frons" as curving round to meet the proboscis major, which latter also curves round to meet the former; whereas Mr. Hunter represents both these parts as curving slightly in the opposite direction. This I am satisfied is their present appearance. If, however, the night is not good, they acquire very much the appearance of the other drawings, the light of the brighter portions being scattered, to a certain extent, over the intervening space. With regard to the shape of the preceding edge of the proboscis, the drawings of Herschel (1825), Bond, and Liapounov represent it as of a uniform curve throughout the greater part of its length, one elbow only at star 126 being shown in those drawings, which extend far enough south; whereas in the case of the other drawings, more powerful means bring out more irregularity of outline. In the case of the Huygenian region, Herschel's drawing (1837) agrees much more nearly with Mr. Hunter's than any of the others, although the interval (30 years) is so much longer than in the case of Mr. Bond's and Mr. Lassell's drawings (15 and 9 years respectively).

With reference to the relative brightness of the various parts, I find recorded by Mr. Hunter, Feb. 22nd, 1861, "In bright moonlight the degrees of brightness are—

"1. The Huygenian region.
"2. The nebulosity immediately south preceding it.
"3. The Mairian region.
"4. The subnebulous region.
"5. The south Messierian branch, and the nebulosity immediately north of the Huygenian region."

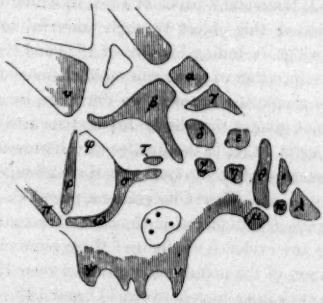
And again, "The Observation of February 22nd, 1861, gives very different degrees of brightness for the various regions from what they had this season (1863–64).

"1. The Huygenian region.
"2. The nebulosity immediately south preceding it.
"3. The nebulosity immediately north of it.
"4. Subnebulous region.
"5. The south Messierian branch and the Mairian region nearly equal."

Mr. Hunter on two occasions estimated, as nearly as he could, the relative brightness of the various masses of nebulosity of the Huygenian region. The following are his estimations (see diagram)*.

* To these estimates we may attach much importance, as Mr. Hunter had the advantage of a considerable amount of training as an artist.

February 13th, 1864.	March 1st, 1864.
σ, τ, ν, γ nearly equal; brightest of these is perhaps σ.	σ, brightest.
π.	ν, τ.
ω, ε, δ, β; β is the faintest of these four.	γ, ε, δ.
α, ι, ψ, λ.	θ, μ.
ζ, ϕ, faintest.	π, very faint.



There are several places where we have reason to suspect that a change of form may have taken place in the nebulosity since our observations commenced. 1st. In Mr. Bindon Stoney's drawing, of which an outline is given at the upper right-hand corner of the Skeleton Map, a dark lane exists running from 88 in a direction parallel to the "Frons," whereas at present the only break in the nebulosity at all in the same direction runs from 88 in a south following direction.

2nd. The projection of the nebulosity below 88 into the Sinus Magnus does not exist in Mr. Stoney's drawing.

3rd. The following outline of the nebulosity immediately below 75 is concave towards the following side in Mr. Stoney's drawing, but convex in Mr. Hunter's; in all these points I believe that Mr. Hunter gives as nearly as possible the present appearance.

4th. Mr. Hunter represents the outline of the nebulosity surrounding the dark region or lake round the stars 32, 35 as very marked; I often examined this part during the seasons 1864–65 and 1865–66, but never saw it quite as distinctly as it is represented on the following side, nor did I see the elbow just following 35; the nebulosity appeared to be more of the shape represented by the coarsely dotted line in the Skeleton Map.

5th. I was never able to see more than two of the three rays below this lake, and except on two or three occasions I could only make out one. Mr. Hunter has since told me that in the last season during which he was working, these rays were much fainter than they had been previously, and that they are represented too bright for their appearance during the season 1863–64.

In connexion with this subject it may not be uninteresting to compare the observations of former observers with each other and with our own.

Sir J. Herschel in his paper of 1825 discusses the differences between his own drawings and those of Huygens, Picard, Messier, and Le Gentil, and thinks that the first three, when compared with his own, tend to show a gradual diminution or condensation of the nebulosity; but Le Gentil's, which was older than Messier's, represents it just as he himself saw it.

We next come to Sir J. Herschel's paper of 1837, in which he says that although to any one who has not viewed this object through powerful telescopes the differences between the various drawings, including his own of 1824 and 1837, may seem great, and tend to convey a strong impression of great and rapid changes undergone by the nebula itself, yet, after carefully comparing his own two drawings, he comes to the conclusion that the differences are not greater than he is disposed to attribute to his own inexperience in such delineations in 1824, to the greater care bestowed on the later drawing, and especially to the advantage of better local situation and superior defining power, &c. of the telescope at the latter date (Cape Observations, page 31). There are three points, however, to which he directs attention, but in the case of two only of them is he inclined to conclude that there is any evidence of change; these points are—

1. The form and position of the nebula oblongata between 127_1 and 129_1.
2. The position of the nebulous spur between 111 and 122.
3. The form of the nebula round 108.

In 1824 Sir J. Herschel saw the nebula oblongata as a "tolerably regular oval," nearly in a line between the stars 120 and 136, whereas in his drawing of 1837 it is irregular in outline, and decidedly above the line through 120 and 136.

With respect to the form of the nebula oblongata, the brighter part forms a "tolerably regular oval," but when the fainter parts are included it seems to be more of the form given in Herschel's drawing of 1837. It is therefore quite possible, even probable, that Herschel would have seen it oval in 1825, but long and slightly curved upwards with the superior means at his disposal in 1837, without any change of form having taken place in the interval; but as regards its position it appears to be now entirely above the line 120–136.

With regard to the nebulous spur between 111 and 122, diagrams which he made in 1832 and 1834 represent it as "running directly from 135 to 111 and forming a complete hook no way disjoined from the proboscis." In 1837 he saw it "neither joined to the proboscis nor directed towards 135, but rather towards a point one-third the distance from 135 to 126" near the position of 131. Herschel's second drawing appears to agree very fairly with the accompanying one in this respect; perhaps the superior definition of Herschel's instrument in 1837, a better atmosphere, and the greater* meridian altitude of the object enabled Herschel to perceive the interval between this spur and the proboscis which had escaped his notice in 1832 and 1834.

* This last applies to the diagram of 1832 only.

With regard to the nebula round 108, the amount of detail in HERSCHEL's drawing of 1837 is so much greater than in that of 1824, and the detail in the accompanying drawing is so much greater than in HERSCHEL's of 1837, that it seems hardly possible to arrive at any conclusion by comparing them.

Resolvability.

On this subject HERSCHEL remarks in his paper of 1825 that the illumination of the Huygenian region is "extremely unequal and irregular," and compares it to "a curdling liquid, or a surface strewed over with flocks of wool, or to the breaking up of a mackerel sky when the clouds of which it consists begin to assume a cirrous appearance; not very unlike the mottling of the sun's disk, only the grain is much coarser and the intervals darker, and the flocculi instead of being generally round are drawn out into little wisps. They present, however, no appearance of being composed of small stars, and their aspect is altogether different from that of resolvable nebula." This describes very well the appearance on any very moderately good night, but at this time HERSCHEL does not appear to have seen clearly the coarser mottling or breaking up into nebulous masses which have since been seen and drawn. This curdled appearance he does not find described in any previous account.

The next mention of this subject appears to be by Mr. BOND, who, after quoting part of HERSCHEL's remarks given above, says, "To me it appears composed of several clusters of stars, the components being separately seen for a moment under favourable circumstances, more particularly north of 75*, and likewise in the vicinity of 8 and 91; but where the nebula assumes a cirrous character, as in the Messierian branch, I can see nothing of the kind."

The next mention of the subject is by Mr. LASSELL, who says that with a power of 1018 the whole nebula "seemed like large masses of cotton wool packed one behind the other, the edges pulled out so as to be very filmy;" also "the brightness of the minute points about the trapezium is strikingly greater than at Starfield, yet I could not mark the places of more than three or four new stars." "With a power of 1018 there is no appearance of resolvability." "With a power of 1018 the wool-like masses appear as I have previously described them, and there is no disposition whatever in them to turn into stars," &c.

We now come to our own observations made with the 3-feet and 6-feet telescopes, and find the following remarks:—

February 17th, 1849.—"Saw a multitude of stars in the bright part about the trapezium, but when they came to be drawn, only got in 9 certain and 6 uncertain, the state of the air having become worse."

January 22nd, 1851.—"At times the 5th and 6th stars of the trapezium seen, also red stars south following the trapezium," &c.

Mr. HUNTER on many occasions saw small stars in the brighter parts; the following are nearly all his remarks on this subject:—

* N.B. In all cases where not otherwise specified the numbers are those of HERSCHEL's list.

December 4th, 1861.—"I thought I could resolve the Huygenian region at ν, ζ, and ι."

February 22nd, 1862.—"3-feet speculum newly polished shows it much better than the former, indeed better than the 6-feet now in the tube. The part round the trapezium looks just like fine flour scattered over a grey surface, so that I have no hesitation in saying that it is composed of stars, many small ones seen through it," &c.

January 12th, 1864.—"The knots γ, δ, ϵ, ν are, I think, resolved"*.

February 3rd, 1864.—"κ is decidedly resolvable, ι is also resolvable; I believe about θ is also resolvable"*.

February 4th, 1864.—"At moments I could see stars through the Huygenian region glancing, and I have no doubt that all the bright knots of it are resolvable; at κ, δ, ϵ, γ I saw clearly at least one star in each, and at δ I believe for a short time I saw its stars separated." The little knots opposite τ and σ have each a resolvable look.

March 1st, 1864.—"The Huygenian region is clearly resolved; I could see the individual stars, though I could not count them; the stars are well separated in the triangular knot α. I strongly suspect the region at 113_{m} is resolved, especially at the edges of the bay."

March 10th, 1864.—"Saw stars clearly at σ and at ν*, at intervals also around 113_{m}."

March 24th, 1864.—"Huygenian region clearly resolved."

From these observations we may conclude that there are multitudes of small stars in the whole of the bright parts round the trapezium, and also at 113_{m}. Mr. HUNTER makes no mention of resolvability in the proboscis major, but he has marked the preceding edge from 126 to about 80″ below this star as resolvable. All the parts which have this appearance are marked over in the drawing with dots of Indian ink, and the rest of the nebula was done with a stump and blacklead pencil. It was, however, found almost impossible to reproduce this difference of appearance in the engraving, since the whole of the surface consists of minute black dots.

This resolvable appearance can be seen on good nights only, and with a very good speculum.

We now come to the last part of our subject, the knowledge of the nature of this nebula acquired by the use of

The Spectroscope.

Very little has been done with this instrument as yet. The clock-movement for the six-feet telescope is not yet finished, and consequently it is impossible to keep the slit steadily on any small object; we have, however, obtained the spectra of about twelve objects, and our results, as far as they go, fully confirm those of Mr. HUGGINS, namely, that some nebulæ give gaseous spectra, consisting of one, two, or three bright lines, while others give faint continuous spectra in their brighter parts. The telescope which was generally used was the three-feet, in the Herschelian form; of course without clock-movement. This instrument was found on the whole more convenient than the six-feet, as it could be kept with greater ease on the object.

The following was the method of observation employed. One observer kept the ob-

* See diagram, p. 67.

ject as well as he could on the slit by viewing with a lens its image in a diagonal reflector placed in front of the slit, and having its edge almost touching the slit, but not actually covering it, and swinging the tube till the brightest part of the nebula just passed the edge of the reflector, while the other observer looked into the spectroscope. The telescope was used in the Herschelian form, as it was desirable to get as bright an image as possible, but not of importance to get the best possible definition.

The following is a list of the objects which we have examined, some of which have also been observed by Mr. HUGGINS. All these objects were examined both by Mr. BALL the present Assistant, and myself alternately.

Gaseous Spectrum.	Continuous Spectrum.
Great nebula in Orion.	Great nebula in Andromeda.
2102	2373
2343	2377
4964	2786
	2203
	2207
	2211
	2347
	2786

No decided spectrum seen; spectrum suspected to be continuous.

Although the last six gave no decided spectrum, there can be very little doubt but that their spectra are continuous; they were examined before our eyes had been accustomed to a faint or continuous spectrum by examining those of brighter objects, such as the nebula in Andromeda and the bright cluster in Canes Venatici. If they had given a gaseous spectrum we could hardly have failed to have seen it.

Although in consequence of the smallness of the number of objects hitherto observed it would be premature to lay much stress on any inferences derived from these observations, it may not perhaps be out of place to mention that in addition to the results arrived at by comparing our observations with those of Mr. HUGGINS (viz. that no cluster or resolved nebula yet observed gives a gaseous spectrum, and of the remainder those which give a continuous spectrum are generally of a more resolvable character than those which give a gaseous spectrum*) we find—

1st. That no planetary or annular nebula yet observed has been found to give any but a gaseous spectrum, with in some cases a suspicion of a very faint continuous spectrum.

2nd. That no nebula of the class generally denominated "rays" has yet been found to give a gaseous spectrum.

3rd. That of the remaining objects observed, those which give a gaseous spectrum but are not denominated planetary are in one respect of the same character with them, viz. that they have in many places a well-defined termination to an almost uniformly bright

* See Mr. HUGGINS's paper, Philosophical Transactions, Part I. 1866.

nebulosity, whereas those nebulæ which give a continuous spectrum appear to fade away tolerably uniformly on all sides from their nuclei, and although in some cases they have dark lanes running through them, the edges do not generally appear very well defined.

Spectrum of Nebula in Orion.

With regard to the spectrum of the nebula in Orion, three bright lines were seen several times, both with 3-feet and 6-feet instruments, but no attempt was made to identify them except on one occasion, when the spectrum of the nebula was compared with that of the electric spark in a capillary vacuum-tube containing a trace of hydrogen, similar to those used by M. PLÜCKER in his researches on the spectra of gases, and one glimpse of the coincidence of the green line in the latter with the most refrangible of the three lines in the former was obtained. The least refrangible line was the brightest, the most refrangible was next in brightness, and the middle line the faintest. Both Mr. BALL* and I were almost certain that there was, in addition to the three bright lines, a faint continuous spectrum; to me there appeared to be a dark space on the less refrangible side of the least refrangible line, of breadth about equal to the distance from the same line to the second, and beyond this a faint light dying gradually away towards the red extremity. A continuous spectrum would probably explain this appearance, as the intensity of the bluish-green and green on the more refrangible side of (*b*) in a continuous spectrum is much less than that of the yellowish-green and yellow. We also once suspected a very feeble light at the other side of the three bright lines.

It might at first sight appear that these observations and those of Mr. HUGGINS on the spectrum of this nebula lead us to results which are completely at variance with those derived from our numerous observations, and those of Mr. BOND on the resolvable appearance of the Huygenian region and other parts of the nebula; but when we consider the subject carefully we shall see that this is far from being the case.

Reason why no continuous Spectrum was seen.

It is evident that when a spectroscope whose collimator and telescope have object-glasses of equal focal length is placed with its slit in the plane of the image of a nebula giving out perfectly homogeneous light, a line of light of length and breadth equal to the length and breadth of the slit will be found at the focus of the spectroscope-telescope, and of brightness equal to the brightness of the part of the original image on the slit at the time, multiplied by a constant quantity (E) less than unity, depending on the number of reflecting surfaces in the apparatus, &c.; and if the nebula give out light of three different refrangibilities, the mean brightness of the lines will be one-third of this quantity; whereas if the light emitted be of all refrangibilities the mean brightness will be less in the ratio of the length of the spectrum to once or three times the breadth of the slit, according as we compare it with a spectrum of one or three lines. This is fully confirmed in practice; for in the case of those nebulæ whose spectra are continuous, the

* The present Assistant.

spectrum is extremely faint, except just at the nucleus, and even there it is pretty faint. When therefore we consider that a great part of the light of the Huygenian region of the nebula in Orion, the brightest parts of which are probably not much, if at all brighter than the nuclei of such nebulæ as 2373, 2377, goes to form a gaseous spectrum, we can hardly expect that the remaining light could produce any but the feeblest continuous spectrum. As a further confirmation of this view, it may be mentioned that on one occasion the spectrum of this nebula was examined in bright moonlight; the light was so strong that the Huygenian region was scarcely visible; in fact its boundaries were not more apparent than the boundary of the proboscis major at 131 is on a dark night; yet although the three gaseous lines of the nebula were very fairly seen, no continuous spectrum from the moonlight, which was probably equal in intensity to the light of the nebula, could be detected.

This observation, however, was not so satisfactory as I could have wished, as a haze which soon after eclipsed the nebula was beginning to come on at the time.

Besides the central parts of the nebula, the nebula Mairiani was examined with the spectroscope, and a gaseous spectrum seen near the star 108. The proboscis major was also examined near 126, and a gaseous spectrum seen. ι Orionis was also examined, but no spectrum but that of the star itself detected.

A clock-movement for the 6-feet telescope is now in progress, and when this is completed we hope to examine the spectra of this and other nebulæ with more care.

Memorandum.

Through the kindness of Sir John Herschel I have been permitted to see his remarks on this paper, and gladly take advantage of his suggestions. The engraving is upon the whole very accurate; a little more softening off in the faint outlying parts would have been desirable, but Mr. Basire did not think that it would be practicable consistent with the reasonable durability of the plate; the forms, however, are correct. The sharpness of outline and the hard and marked character of the principal features are the result of the great light of the instrument; with a diminishing aperture these characteristics gradually fade away. The engraving faithfully represents the object as it may be seen on any clear night, and the details are so well marked that no material change can take place hereafter which will not at once be recognized with an instrument of similar power. The interior of the trapezium has not been examined recently with the view to the question whether it is absolutely dark. With the 6-feet instrument the eye is so dazzled by the light of the four stars that it is difficult to form an accurate opinion; and any nebulosity which may exist is probably too faint to affect the spectroscope. I am not certain that any part of the nebula is absolutely free from nebulosity, but the contrast is so great between the dark spaces alluded to by Sir John Herschel, and the contiguous portions of the nebula, that even in the drawing it was scarcely possible to indicate nebulosity so slightly as not to interfere with the proper gradation of light; in fact it was scarcely possible to represent the bright parts sufficiently bright.

IV. *On the Curves which satisfy given Conditions.* By Professor CAYLEY, *F.R.S.*

Received April 18,—Read May 2, 1867.

THE present Memoir relates to portions only of the subject of the curves which satisfy given conditions; but any other title would be too narrow: the question chiefly considered is that of finding the number of the curves which satisfy given conditions; the curves are either curves of a determinate order r (and in this case the conditions chiefly considered are conditions of contact with a given curve), or else the curves are conics; and here (although the conditions chiefly considered are conditions of contact with a given curve or curves) it is necessary to consider more than in the former case the theory of conditions of any kind whatever. As regards the theory of conics, the Memoir is based upon the researches of CHASLES and ZEUTHEN, as regards that of the curves of the order r, upon the researches of DE JONQUIÈRES: the notion of the quasi-geometrical representation of conditions by means of loci in hyper-space is employed by SALMON in his researches relating to the quadric surfaces which satisfy given conditions. The papers containing the researches referred to are included in the subjoined list. I reserve for a separate Second Memoir the application to the present question, of the Principle of Correspondence.

List of Memoirs and Works relating to the Curves which satisfy given conditions, with remarks.

DE JONQUIÈRES: Theorèmes généraux concernant les courbes géométriques planes d'un ordre quelconque, Liouv. t. vi. (1861) pp. 113–134. In this valuable memoir is established the notion of a series of curves of the *index* N; viz. considering the curves of the order n which satisfy $\frac{1}{2}n(n+3)-1$ conditions, then if N denotes how many there are of these curves which pass through a given arbitrary point, the series is said to be of the index N.

In Lemma IV it is stated that all the curves C_n of a series of the index N can be analytically represented by an equation $F(y, x)=0$, which is rational and integral of the degree N in regard to a variable parameter λ: this is not the case; see Annex No. 1.

CHASLES: Various papers in the Comptes Rendus, t. lviii. *et seq.* 1864–67. The first of them (Feb. 1864), entitled "Determination du nombre des sections coniques qui doivent toucher cinq courbes données d'ordre quelconque, ou satisfaire à diverses autres conditions," establishes the notion of the two characteristics (μ, ν) of a system of conics which satisfy four conditions; viz. μ is the number of the séconics which pass through a given arbitrary point, and ν the number of them which touch a given arbitrary line.

The Principle of Correspondence for points on a line is established in the paper of June–July 1864. Many of the leading points of the theory are reproduced in the present Memoir. The series of papers includes one on the conics in space which satisfy seven conditions (Sept. 1865), and another on the surfaces of the second order which satisfy eight conditions (Feb. 1866).

SALMON: On some Points in the Theory of Elimination, Quart. Math. Journ. t. vii. pp. 327–337 (Feb. 1866); On the Number of Surfaces of the Second Degree which can be described to satisfy nine Conditions, Ibid. t. viii. pp. 1–7 (June 1866),—which two papers are here referred to on account of the notion which they establish of the quasi-geometrical representation of conditions by means of loci in hyper-space.

ZEUTHEN: Nyt Bidrag . . . Contribution to the Theory of Systems of Conics which satisfy four conditions, 8°. pp. 1–97 (Copenhagen, Cohen, 1865), translated, with an addition, in the *Nouvelles Annales*.

The method employed depends on the determination of the line-pairs and point-pairs, and of the numerical coefficients by which these have to be multiplied, in the several systems of conics which satisfy four conditions of contact with a given curve or curves. It is reproduced in detail, with the enumeration called "ZEUTHEN'S Capitals," in the present Memoir.

CAYLEY: Sur les coniques determinées par cinq conditions d'intersection avec une courbe donnée.—Comptes Rendus, t. lxiii. pp. 9–12, July 1866. Results reproduced in the present Memoir.

DE JONQUIÈRES: Two papers, Comptes Rendus, t. lxiii. Sept. 1866, reproduced and further developed in the "Mémoire sur les contacts multiples d'ordre quelconque des courbes du degré r qui satisfont à des conditions données de contact avec une courbe fixe du degré m; suivi de quelques reflexions sur la solution d'un grand nombre de questions concernant les proprietés projectives des courbes et des surfaces algébriques," Crelle, t. lxvi. (1866), pp. 289–322,—contain a general formula for the number of curves C^r having contacts of given orders $a, b, c, \ldots$ with a given curve U^m &c., which formula is referred to and considered in the present Memoir.

DE JONQUIÈRES: Recherches sur les séries ou systèmes de courbes et de surfaces algébriques d'ordre quelconque; suivi d'une reponse &c. 4°. Paris, Gauthier Villars, 1866*.

On the quasi-geometrical representation of Conditions.—Article Nos. 1 to 23.

1. A condition imposed upon a subject gives rise to a relation between the parameters of the subject; for instance, the subject may be, as in the present Memoir, a plane curve of a given order, and the parameters be any arbitrary parameters contained in the equation of the curve. The condition may be onefold, twofold, ... or, generally, k-fold, and the corresponding relation is onefold, twofold, . . . or k-fold accordingly. Two or more conditions, each of a given manifoldness, may be regarded as forming together a

* The foregoing list is not complete, and the remarks are not intended to give even a sketch of the contents of the works comprised therein, but only to show their bearing on the present Memoir.

single condition of a higher manifoldness, and the corresponding relations as forming a single relation; and thus, though it is often convenient to consider two or more conditions or relations, this case is in fact included in that of a k-fold condition or relation. In dealing with such a condition or relation it is assumed that the number of parameters is at least $=k$; for otherwise there would not in general be any subject satisfying the condition: when the number of parameters is $=k$, the number of subjects satisfying the condition is in general determinate.

2. A subject which satisfies a given condition may for shortness be termed a solution of the condition; and in like manner any set of values of the parameters satisfying the corresponding relation may be termed a solution of the relation. Thus for a k-fold condition or relation, and the same number k of parameters, the number of solutions is in general determinate.

3. A condition may in some cases be satisfied in more than a single way, and if a certain way be regarded as the ordinary and proper one, then the others are *special* or *improper*: the two epithets may be used conjointly, or either of them separately, almost indifferently. For instance, the condition that a curve shall touch a given curve (have with it a two-pointic intersection) is satisfied if the curve have with the given curve a proper contact; or if it have on the given curve a node or a cusp (or, more specially, if it be or comprise as part of itself two coincident curves); or if it pass through a node or a cusp of the given curve: the first is regarded as the ordinary and proper way of satisfying the condition; the other two as special or improper ways; and the corresponding solutions are ordinary and proper solutions, or special or improper ones accordingly. This will be further explained in speaking of the locus which serves for the representation of a condition.

4. A set of any number, say ω, of parameters may be considered as the coordinates of a point in ω-dimensional space; and if the parameters are connected by a onefold, twofold, ... or k-fold relation, then the point is situate on a onefold, twofold, ... or k-fold locus accordingly; to the relation made up of two or more relations corresponds the locus which is the intersection or common locus of the loci corresponding to the several component relations respectively. A locus is at most ω-fold, viz. it is in this case a point-system. The relation made up of a k-fold relation, an l-fold relation, &c., is in general $(k+l+\&c.)$fold, and the corresponding locus is $(k+l+\&c.)$fold accordingly.

5. The order of a point-system is equal to the number of the points thereof, where, of course, coincident points have to be attended to, so that the distinct points of the system may have to be reckoned each its proper number of times. The locus corresponding to any linear j-fold relation between the coordinates is said to be a j-fold *omal* locus; and if to any given k-fold relation we join an arbitrary $(\omega-k)$fold linear relation, that is, intersect the k-fold locus by an arbitrary $(\omega-k)$fold omal locus, so as to obtain a point-system, the order of the k-fold relation or locus is taken to be equal to the number of points of the point-system, that is, to the order of the point-system. And this being so, if a k-fold relation, an l-fold relation, &c. are *completely* independent, that

is, if they are not satisfied by values which satisfy a less than $(k+l+\&c.)$fold relation, or, what is the same thing, if the k-fold locus, the l-fold locus, &c. have no common less than $(k+l+\&c.)$fold locus, then the relations make up together a $(k+l+\&c.)$fold relation, and the loci intersect in a $(k+l+\&c.)$fold locus, the orders whereof are respectively equal to the product of the orders of the given relations or loci. In particular if we have $k+l+\&c.=\omega$, then we have an ω-fold relation, and corresponding thereto a point-system, the orders whereof are respectively equal to the product of the orders of the given relations or loci.

6. A k-fold relation, an l-fold relation, &c., if they were together equivalent to a less than $(k+l+\&c.)$fold relation, would not be independent; but the relations, assumed to be independent, may yet *contain* a less than $(k+l+\&c.)$fold relation, that is, they may be satisfied by the values which satisfy a certain less than $(k+l+\&c.)$fold relation (say the common relation), and exclusively of these, only by the values which satisfy a proper $(k+l+\&c.)$fold relation, which is, so to speak, a residual equivalent of the given relations. This is more clearly seen in regard to the loci; the k-fold locus, the l-fold locus, &c. may have in common a less than $(k+l+\&c.)$fold locus, and besides intersect in a residual $(k+l+\&c.)$fold locus. (It is hardly necessary to remark that such a connexion between the relations is precisely what is excluded by the foregoing definition of *complete* independence.) In particular if $k+l+\&c.=\omega$, the several loci may intersect, say in an $(\omega-j)$fold locus, and besides in a residual ω-fold locus, or point-system. The order (in any such case) of the residual relation or locus is equal to the product of the orders of the given relations or loci, *less* a reduction depending on the nature of the common relation or locus, the determination of the value of which reduction is often a complex and difficult problem.

7. Imagine a curve of given order, the equation of which contains ω arbitrary parameters: to fix the ideas, it may be assumed that these enter into the equation rationally, so that the values of the parameters being given, the curve is uniquely determined. Suppose, as above, that the parameters are taken to be the coordinates of a point in ω-dimensional space; so long as the curve is not subjected to any condition, the point in question, say the parametric point, is an arbitrary point in the ω-dimensional space; but if the curve be subjected to a onefold, twofold, . . . or k-fold condition, then we have a onefold, twofold, . . . or k-fold relation between the parameters, and the parametric point is situate on a onefold, twofold, . . . or k-fold locus accordingly: to each position of the parametric point on the locus there corresponds a curve satisfying the condition, that is, a solution of the condition. In the case where the condition is ω-fold, the locus is a point-system, and corresponding to each point of the point-system we have a solution of the condition; the number of solutions is equal to the number of points of the point-system.

8. Considering the general case where the condition, and therefore also the locus, is k-fold, it is to be observed that every solution whatever, and therefore each special solution (if any), corresponds to some point on the k-fold locus; we may therefore have on

the k-fold locus what may be termed "special loci," viz. a special locus is a locus such that to each point thereof corresponds a special solution. A special locus may of course be a point-system, viz. there are in this case a determinate number of special solutions corresponding to the several points of this point-system. We may consider the other extreme case of a special k-fold locus, viz. the k-fold locus of the parametric point may break up into two distinct loci, the special k-fold locus, and another k-fold locus the several points whereof give the ordinary solutions: we can in this case get rid of the special solutions by attending exclusively to the last-mentioned k-fold locus and regarding it as the proper locus of the parametric point. But if the special locus be a more than k-fold locus, that is, if it be not a part of the k-fold locus itself, but (as supposed in the first instance) a locus on this locus, then the special solutions cannot be thus got rid of: we have the k-fold locus of the parametric point, a locus such that to every point thereof there corresponds a proper solution, save and except that to the points lying on the special locus there correspond special or improper solutions. It is to be noticed that the special locus may be, but that it is not in every case, a singular locus on the k-fold locus.

9. Suppose that the conditions to be satisfied by the curve are a k-fold condition, an l-fold condition, &c. of a total manifoldness $=\omega$. If the conditions are *completely* independent (that is, if the corresponding relations, *ante*, No. 5, are completely independent), we have a k-fold locus, an l-fold locus, &c., having no common locus other than the point-system of intersection, and the number of curves which satisfy the given conditions, or (as this has been before expressed) the number of solutions, is equal to the number of points of the point-system, or to the order of the point-system, viz. it is equal to the product of the orders of the loci which correspond to the several conditions respectively; among these we may however have special solutions, corresponding to points situate on the special loci upon any of the given loci; but when this is the case the number of these special solutions can be separately calculated, and the number of proper solutions is equal to the number obtained as above, less the number of the special solutions.

10. If, however, the given conditions are not completely independent (that is, if the corresponding relations are not completely independent), then the k-fold locus, the l-fold locus, &c. intersect in a common $(\omega-j)$fold locus, and besides in a residual point-system. The several points of the $(\omega-j)$fold locus give special solutions—in fact the very notion of the conditions being *properly* satisfied by a curve implies that the curve shall satisfy a true $(k+l+\&c.)$fold, that is, a true ω-fold condition; the proper solutions are therefore comprised among the solutions given by the residual point-system, and the number of them is as before equal to the order of the point-system, or number of the points thereof, *less* the number of points which give special solutions: the order of the point-system is, as has been seen, equal to the product of the orders of the k-fold locus, the l-fold locus, &c., *less* a reduction depending on the nature of the common $(\omega-j)$fold locus, and the difficulty is in general in the determination of the value of this reduction.

11. In all that precedes, the number of the parameters has been taken to be ω; but if

the parameters are taken to be contained in the equation of the curve homogeneously, then the parameters before made use of are in fact the ratios of these homogeneous parameters; and using the term henceforward as referring to the homogeneous parameters, the numbers of the parameters will be $=\omega+1$.

12. I assume also that the equation of the curve contains the parameters linearly: this being so, the condition that the curve shall pass through a given arbitrary point implies a linear relation between the parameters; and the condition that the curve shall pass through j given points, a j-fold linear relation between the parameters. It follows that the number of the curves which satisfy a given k-fold condition, and besides pass through $\omega-k$ given points, is equal to the order of the k-fold relation, or of the corresponding k-fold locus; and thus if we define the order of the k-fold condition to be the number of the curves in question, the condition, relation, and locus will be all of the same order, and in all that precedes we may (in place of the order of the relation or of the locus) speak of the order of the condition. Thus, subject to the modifications occasioned by common loci and special solutions as above explained, the order of the $(k+l+$ &c.$)$-fold condition made up of a k-fold condition, a l-fold condition, &c., is equal to the product of the orders of the component conditions; and in particular if $k+l+$ &c.$=\omega$, then the order of the ω-fold condition, or number of the solutions thereof, is equal to the product of the orders of the component conditions.

13. The conditions to be satisfied by the curve may be conditions of contact with a given curve or curves. In particular if the curve touch a given curve, the parametric point is then situate on a onefold locus. It is to be noticed in reference hereto that if the given curve have nodes or cusps, then we have special solutions, viz. if the sought for curve passes through a node or a cusp of the given curve; and each such node or cusp gives rise to a special onefold locus, presenting itself in the first instance as a factor of the onefold locus of the parametric point; this is, however, a case where the special locus is of the same manifoldness as the general locus (*ante*, No. 8), and is consequently separable; throwing off therefore all these special loci, we have a onefold locus which no longer comprises the points which correspond to curves passing through a node or a cusp of the given curve; the onefold locus, so divested of the special onefold factors, may be termed the "contact-locus" of the given curve. To each point of the contact-locus there corresponds a curve having with the given curve a two-pointic intersection, viz. this is either a proper contact, or it is a special contact, consisting in that the sought for curve has on the given curve a node or cusp, or (which is a higher speciality) in that the sought for curve is or contains as part of itself two or more coincident curves (*ante*, No. 3). To a point in general on the contact-locus there corresponds a curve having a proper contact with the given curve, save and except that to each point on any one of certain special loci on the contact-locus there corresponds a curve having some kind of special contact as above with the given curve. To fix the ideas, it may be mentioned that for the curves of the order r which touch a given curve of the order m and class n, the order of the contact-locus is $=n+(2r-2)m$.

14. If, then, the curve touch a given curve, the parametric point is situate on the contact-locus of that curve. If it touch a second given curve, the parametric point is in like manner situate on the contact-locus of the second given curve, that is, it is situate on the twofold locus which is the intersection of the two contact-loci; and the like in the case of any number of contacts each with a distinct given curve. But if the curve, instead of ordinary contacts with distinct given curves, has either a contact of the second, or third, any higher order, or has two or more ordinary or other contacts with the same given curve, then if the total manifoldness be $=k$, the parametric point is situate on a k-fold locus, which is given as a singular locus of the proper kind on the onefold contact-locus; so that the theory of the contact-locus corresponding to the case of a single contact with a given curve, contains in itself the theory of any system whatever of ordinary or other contacts with the same given curve, viz. the last-mentioned general case depends on the discussion of the singular loci which lie on the contact-locus. And similarly, if the curve has any number of ordinary or other contacts with each of two or more given curves, we have here to consider the intersections of singular loci lying on the contact-loci which correspond to the several given curves respectively, or, what is the same thing, to the singular loci on the intersection of these contact-loci; that is, the theory depends on that of the contact-loci which belong to the given curves respectively.

15. Suppose that the curve which has to satisfy given conditions is a line; the equation is $ax+by+cz=0$, and the parameters (a, b, c) are to be taken as the coordinates of a point in a plane. Any onefold condition imposed upon the line establishes a onefold relation between the coordinates (a, b, c), and the parametric point is situate on a curve; a second onefold condition imposed on the line establishes a second onefold relation between the coordinates (a, b, c), and the parametric point is thus situate on a second curve; it is therefore determined as a point of intersection of two ascertained curves. In particular if the condition imposed on the line is that it shall touch a given curve, the locus of the parametric point is a curve, the contact-locus; (this is in fact the ordinary theory of geometrical reciprocity, the locus in question being the reciprocal of the given curve;) and the case of the twofold condition of a contact of the second order, or of two contacts, with the given curve, depends on the singular points of the contact-locus, or reciprocal of the given curve; in fact according as the line has a contact of the second order, or has two contacts with the given curve (that is, as it is an inflexion-tangent, or a double tangent of the given curve), the parametric point is a cusp or a node on its locus, the reciprocal curve: this is of course a fundamental notion in the theory of reciprocity, and it is only noticed here in order to show the bearing of the remark (*ante*, No. 14) upon the case now in hand where the curve considered is a line.

16. If the curve which has to satisfy given conditions is a conic

$$(a, b, c, f, g, h)(x, y, z)^2=0,$$

we have here six parameters (a, b, c, f, g, h), which are taken as the coordinates of a point in 5-dimensional space. It may be remarked that in this 5-dimensional space we have the onefold cubic locus $abc-af^2-bg^2-ch^2+2fgh=0$, which is such that to

any position of the parametric point upon it there corresponds not a proper conic but a line-pair; this may be called the discriminant-locus. We have also the threefold locus the relation of which is expressed by the six equations

$$(bc-f^2=0,\ ca-g^2=0,\ ab-h^2=0,\ gh-af=0,\ hf-bg=0,\ fg-ch=0),$$

which is such that to any position of the parametric point thereon, there corresponds not a proper conic but a coincident line-pair. I call this the Bipoint-locus*, and I notice that its order is $=4$; in fact to find the order we must with the equations of the Bipoint combine two arbitrary linear relations,

$$(*\,)(a, b, c, f, g, h)=0,$$
$$(*'\,)(a, b, c, f, g, h)=0;$$

the equations of the locus are satisfied by

$$a:b:c:f:g:h=\alpha^2:\beta^2:\gamma^2:\beta\gamma:\gamma\alpha:\alpha\beta$$

(where $\alpha:\beta:\gamma$ are arbitrary); and substituting these values in the linear relations, we have two quadric equations in (α, β, γ), giving four values of the set of ratios $(\alpha:\beta:\gamma)$; that is, the order is $=4$, or the Bipoint is a threefold quadric locus.

17. The discriminant-locus does not in general present itself except in questions where it is a condition that the conic shall have a node (reduce itself to a line-pair); thus for the conics which have a node and touch a given curve (m, n), or, what is the same thing, for the line-pairs which touch a given curve (m, n), the parametric point is here situate on a twofold locus, the intersection of the discriminant-locus with the contact-locus. It may be noticed that this twofold locus is of the order $3(n+2m)$, but that it breaks up into a twofold locus of the order $3n$, which gives the proper solutions; viz. the nodal conics which touch the given curve properly, that is, one of the two lines of the conic touches the curve; and into a twice repeated twofold locus of the order $3m$ which gives the special solutions, viz. in these the nodal conic has with the given curve a special contact, consisting in that the node or intersection of the two lines lies on the given curve. By way of illustration see Annex No. 2. But the consideration of the Bipoint-locus is more frequently necessary.

18. Suppose that the conic satisfies the condition of touching a given curve; the parametric point is then situate on a onefold contact-locus $(a, b, c, f, g, h)^q=0$ (to fix the ideas, if the given curve is of the order m and class n, then the order q of the contact-locus is $=n+2m$). The contact-locus of any given curve whatever passes through the Bipoint-locus; in fact to each point of the Bipoint-locus there corresponds a coincident line-pair, that is, a conic which (of course in a special sense) touches the given curve whatever it be; and not only so, but inasmuch as we have a special contact at each of the points of intersection of the given curve with the coincident line-pair regarded as a single line, that is, in the case of a given curve of the m-th order, m special contacts, the Bipoint-locus is a multiple curve on the corresponding contact-locus.

* In framing the epithet Bipoint, the coincident line-pair is regarded as being really a point-pair: see *post*, No. 30.

19. If the conic has simply to touch a given curve of the order m_1 and class n_1, then the order of the condition (or number of the conics which satisfy the condition, and besides pass through four given points) is equal to the order of the contact-locus, that is, it is $=n_1+2m_1$. If the conic has also to touch a second given curve of the order m_2 and class n_2, then the order of the twofold condition (or number of the conics which satisfy the twofold condition, and besides pass through three given points) is equal to the order of the intersection or common locus of the two contact-loci; and these being of the orders n_1+2m_1 and n_2+2m_2 respectively, the order of the intersection and therefore that of the twofold condition is $=(n_1+2m_1)(n_2+2m_2)$. But in the next succeeding case it becomes necessary to take account of the singular locus.

20. If the conic has to touch three given curves of the order and class (m_1, n_1), (m_2, n_2), (m_3, n_3) respectively, we have here three contact-loci of the orders (n_1+2m_1), n_2+2m_2, n_3+2m_3 respectively; these intersect in a threefold locus, but since each of the contact-loci passes through the threefold Bipoint-locus, this is part of the intersection of the three contact-loci; and not only so, but inasmuch as they pass through the Bipoint-locus m_1, m_2, m_3 times respectively, the Bipoint-locus must be counted $m_1m_2m_3$ times, and its order being $=4$, the intersection of the contact-locus is made up of the Bipoint reckoning as a threefold locus of the order $4m_1m_2m_3$, and of a residual threefold locus of the order

$$(n_1+2m_1)(n_2+2m_2)(n_3+2m_3)-4m_1m_2m_3,$$
$$=n_1n_2n_3+2(n_1n_2m_3+\&c.)+4(n_1m_2m_3+\&c.)+4m_1m_2m_3;$$

and the order of the threefold condition (or number of the conics which touch the three given curves, and besides pass through two given points) is equal to the order of the residual threefold locus, and has therefore the value just mentioned.

21. In going on to the cases of the conics touching four or five given curves, the same principles are applicable; the contact-loci have the Bipoint (a certain number of times repeated) as a common threefold locus, and they besides intersect in a residual fourfold or (as the case is) fivefold locus, and the order of the condition is equal to the order of this residual locus; but the determination of the order of the residual locus presents the difficulties alluded to, *ante*, No. 10. I do not at present further examine these cases, nor the cases of the conics which have with a given curve or curves contacts of the second or any higher order, or more than a single contact with the same given curve.

22. The equation of the conic has been in all that precedes considered as containing the six parameters (a, b, c, f, g, h); but if the question as originally stated relates only to a class of conics the equation whereof contains linearly 2, 3, 4, or 5 parameters, or if, reducing the equation by means of any of the given conditions, it can be brought to the form in question, then in the latter case we may employ the equation in such reduced form, attending only to the remaining conditions; and in either case we have the equation of a conic containing linearly 2, 3, 4, or 5 parameters, which parameters are taken as the coordinates of a point in 1-, 2-, 3-, or 4-dimensional space, and the discussion relates to loci in such dimensional space. This is in fact what is done in Annex No. 2

above referred to, where the conics considered being the conics which pass through three given points, the equation is taken to be $fyz+gzx+hxy=0$, and we have only the three parameters (f, g, h); and also in Annex No. 3, where the conics pass through two given points, and are represented by an equation containing the four parameters (a, b, c, h): I give this Annex as a somewhat more elaborate example than any which is previously considered, of the application of the foregoing principles, and as an investigation which is interesting for its own sake. See also Annexes 4 and 5, which contain other examples of the theory. The remark as to the number of parameters is of course applicable to the case where the curve which satisfies the given conditions is a curve of any given order r; the number of the parameters is here at most $=\frac{1}{2}(r+1)(r+2)$, and the space therefore at most $\frac{1}{2}r(r+3)$dimensional; but we may in particular cases have $\omega+1$ parameters, the coordinates of a point in ω-dimensional space, where ω is any number less than $\frac{1}{2}r(r+3)$.

23. I do not at present consider the case of a curve of the order r, or further pursue these investigations; my object has been, not the development of the foregoing quasi-geometrical theory, so as to obtain thereby a series of results, but only to sketch out the general theory, and in particular to establish the notion of the order of condition, and to show that, as a rule (though as a rule subject to very frequent exceptions), the order of a compound condition is equal to the product of the orders of the component conditions. The last-mentioned theorem seems to me the true basis of the results contained in a subsequent part of this paper in connexion with the formulæ of DE JONQUIÈRES, *post*, No. 74 *et seq.* But I now proceed to a different part of the general subject.

Reproduction and Development of the Researches of CHASLES *and* ZEUTHEN.
Article Nos. 24 to 72.

24. The leading points of CHASLES's theory are as follows: he considers the conics which satisfy four conditions (4X), and establishes the notion of the *characteristics* (μ, ν) of such a system, viz. $\mu, =(4X\cdot)$, denotes the number of conics in the system which pass through a given (arbitrary) point, and $\nu, =(4X/)$, the number of conics in the system which touch a given (arbitrary) line. We may say that μ is the parametric order, and ν the parametric class of the system.

25. The conics

$$(::),\quad (\cdot:\cdot/),\quad (:///),\quad (\cdot///),\quad (////)$$

which pass through four given points, or which pass through three given points and touch a given line, &c., ... or touch four given lines, have respectively the characteristics

$$(1, 2),\quad (2, 4),\quad (4, 4),\quad (4, 2),\quad (2, 1).$$

26. A single condition (X) imposed upon a conic has two representative numbers, or simply representatives, (α, β); viz. if (4Z) be an *arbitrary* system of four conditions, and (μ, ν) the characteristics of (4Z), then the number of the conics which satisfy the five conditions (X, 4Z) is$=\alpha\mu+\beta\nu$.

27. As an instance of the use of the characteristics, if X, X′, X″, X‴, X⁗ be any five independent conditions, and $(\alpha, \beta), \ldots (\alpha''', \beta''')$ the representatives of these conditions respectively, then the number of the conics which satisfy the five conditions (X, X′, X″, X‴, X⁗) is

$$=(1, 2, 4, 4, 2, 1)(\alpha, \beta)(\alpha', \beta')(\alpha'', \beta'')(\alpha''', \beta''')(\alpha'''', \beta'''')$$

viz. this notation stands for $1\alpha\alpha'\alpha''\alpha'''\alpha'''' + 2\Sigma\alpha\alpha'\alpha''\alpha'''\beta'''' \ldots + 1\beta\beta'\beta''\beta'''\beta''''$.

28. In particular if X be the condition that a conic shall touch a given curve of the order m and class n, then the representatives of this condition are (n, m), whence the number of the conics which touch each of five given curves $(m, n), \ldots (m'''', n'''')$ is

$$=(1, 2, 4, 4, 2, 1)(n, m)(n', m')(n'', m'')(n''', m''')(n'''', m'''').$$

29. A system of conics (4X) having the characteristics (μ, ν), contains

$2\nu - \mu$ line-pairs, that is, conics each of them a pair of lines; and

$2\mu - \nu$ point-pairs, that is, conics each of them a pair of points (*coniques infiniment aplaties*).

30. I stop to further explain these notions of the line-pair and the point-pair; and also the notion of the line-pair-point.

A conic is a curve of the second order and second class; *quà* curve of the second order it may degenerate into a pair of lines, or line-pair (but the class is then $=0$): *quà* curve of the second class it may degenerate into a pair of points, or point-pair (but the order is then $=0$). The two lines of a line-pair may be coincident, and we have then a coincident line-pair; such a line-pair (it must I think be postulated) ordinarily arises, not from a line-pair the two lines of which become coincident, but from a proper conic, flattening by the gradual diminution of its conjugate axis, while its transverse axis remains constant or approaches a limit different from zero; the conic thus tends (not to an indefinitely extended but) to a terminated line*; in other words, the tangents of the conic become more and more nearly lines through two fixed points, the terminations of the terminated line; and these terminating points, which continue to exist up to the instant when the conjugate axis takes its limiting value $=0$, are regarded as still existing at this instant, and the coincident line-pair as being in fact the point-pair formed by the two terminating points. Similarly the two points of a point-pair may be coincident, and we have then a coincident point-pair; such a point-pair (it must in like manner be postulated) ordinarily arises, not from a point-pair the two points of which become coincident, but from a proper conic sharpening itself to coincide with its asymptotes, and so becoming ultimately a pair of lines through the coincident point-pair; and the coincident point-pair is regarded as being in fact the line-pair formed by some two lines through the coincident point-pair.

31. In accordance with the foregoing notions we may with propriety, and it will in

* A line is regarded as extending from any point A thereof to B, and then in the same direction, from B through infinity to A; it thus consists of two portions separated by these points; and considering either portion as removed, the remaining portion is a terminated line.

the sequel be found convenient to speak of a point-pair as a line terminated by two points on this line, and similarly to speak of a line-pair as a point terminated (that is, the pencil of lines through the point is terminated) by two lines through the point.

32. If in a point-pair thus considered as a line terminated by two points the two points become coincident (the line continuing to exist as a definite line), or, what is the same thing, if in a line-pair thus considered as a point terminated by two lines, the two lines become coincident (the point continuing to exist as a definite point), we have a "line-pair-point;" viz. this is at once a coincident line-pair and a coincident point-pair; it may also be regarded as the limit of a conic the axes of which, and the ratio of the conjugate to the transverse axis, all ultimately vanish: it may be described as a line terminated each way at a point thereof, or as a point terminated each way at a line through it. The notion of a line-pair-point first presents itself in ZEUTHEN's researches, as will presently appear; but it may be noticed here that line-pair-points, and these the same line-pair-points, may present themselves among the $2\nu - \mu$ line-pairs, and among the $2\mu - \nu$ point-pairs of the system of conics 4X.

33. Returning to the foregoing theory of characteristics, I remark that the fundamental notion may be taken to be, not the characteristics (μ, ν) of the conics which satisfy four conditions, but in every case the number of the conics which satisfy five conditions. Thus for the conics not subjected to any condition, we may consider the symbols

$$(\,{:}{:}\,),\quad (:{:}/),\quad (\therefore//),\quad (:///),\quad (\cdot\,////),\quad (/////)$$

denoting the number of the conics which pass through five given points, or which pass through four given points and touch a given line, &c. ..., or which touch five given lines; these numbers are respectively

$$=1,\quad 2,\quad 4,\quad 4,\quad 2,\quad 1.$$

So for the conics which satisfy a given condition X, or two conditions 2X, ..., or five conditions 5X, we have respectively the numbers

X,	(::),	(∴/),	(:/),	(·//),	(////)
2X,	(∴),	(:/),	(·//),	(//)	
3X,	(:),	(·/),	(//)		
4X,	(·),	(/)			
5X,					

where the X, 2X, &c. belong to the symbols which follow: read (X::), (X∴/), &c., or, as we may for shortness represent them,

μ''',	ν''',	ξ''',	σ''',	τ'''
μ'',	ν'',	ξ'',	σ''	
μ',	ν',	ξ'		
μ,	ν			
μ_0				

viz. the single condition X has the five characteristics $(\mu''', \ldots \tau''')$, ...; the four conditions 4X, the characteristics (μ, ν) as in the original theory; and the five conditions 5X a single characteristic μ_0.

34. We thus see the origin of the notion of the representatives (α, β) of a single condition X; for considering the arbitrary four conditions 4Z, the characteristics whereof are (μ, ν), and assuming that the single characteristic, or number of the conics (X, 4Z), is $=\alpha\mu+\beta\nu$, and taking for (4Z) successively the conditions

$$(::), \quad (\therefore/), \quad (:///), \quad (\cdot///), \quad (////),$$

having respectively the characteristics

$$(1, 2), \quad (2, 4), \quad (4, 4) \quad (4, 2) \quad (2, 1),$$

we have

$$\begin{aligned} \mu''' &= 1\alpha+2\beta, \\ \nu''' &= 2\alpha+4\beta, \\ \xi''' &= 4\alpha+4\beta, \\ \sigma''' &= 4\alpha+2\beta, \\ \tau''' &= 2\alpha+1\beta, \end{aligned}$$

that is, the characteristics $(\mu''', \nu''', \xi''', \sigma''', \tau''')$ of a single condition X are not independent, but are representable as above by means of two independent quantities (α, β); or, what is the same thing, we have

$$\nu'''=2\mu''', \quad \sigma'''=2\tau''', \quad \xi'''=\tfrac{3}{5}(\nu'''+\sigma'''),$$

which being satisfied, the representatives (α, β) are given by

$$\alpha=\tfrac{1}{3}(2\tau'''-\mu'''), \quad \beta=\tfrac{1}{3}(2\mu'''-\tau''').$$

35. I find that a like property exists as to the characteristics $(\mu'', \nu'', \xi'', \sigma'')$ of the two conditions 2X, viz. these are not independent but are connected by a single linear relation,

$$\mu''-\tfrac{3}{5}\nu''+\tfrac{3}{5}\xi''-\sigma''=0.$$

This may be proved in the case where the conditions 2X are two separate conditions (X, X'); viz. let the representatives of these be (α, β), (α', β') respectively, then combining with them the three arbitrary conditions X'', X''', X'''' having respectively the representatives (α'', β''), (α''', β'''), (α'''', β''''), we have the general equation

$$(X, X', X'', X''', X'''')=(1, 2, 4, 4, 2, 1)(\alpha, \beta)(\alpha', \beta')(\alpha'', \beta'')(\alpha''', \beta''')(\alpha'''', \beta'''');$$

taking herein

$$(X', X'', X''')=(\therefore), (:/), (\cdot//), (///)$$

successively, and observing that the representatives of $(\cdot)$ are (1, 0) and those of $(/)$ are (0, 1), we thus obtain for $(\mu'', \nu'', \xi'', \sigma'')$, characteristics of (X, X'), the values

$$\begin{aligned} \mu'' &= (1, 2, 4)(\alpha, \beta)(\alpha', \beta'), \\ \nu'' &= (2, 4, 4)(\alpha, \beta)(\alpha', \beta'), \\ \xi'' &= (4, 4, 2)(\alpha, \beta)(\alpha', \beta'), \\ \sigma'' &= (4, 2, 1)(\alpha, \beta)(\alpha', \beta'), \end{aligned}$$

(viz. $\mu''=1\alpha\alpha'+2(\alpha\beta'+\alpha'\beta)+4\beta\beta'$, &c.), and these values give identically

$$2\mu''-3\nu''+3\varrho''-2\sigma''=0,$$

which is the foregoing equation. And I assume that the theorem extends to the case of two inseparable conditions 2X, but in this case I do not even know where the proof is to be sought for.

The characteristics (μ', ν', ϱ') of the three conditions 3X are in general independent.

36. It has been mentioned that if (α, β) are the representatives of the condition X, and (μ, ν) the characteristics of the conditions 4Z, then

$$(X, 4Z)=\alpha\mu+\beta\nu;$$

this is the most convenient form of the theorem, but as (α, β) are known functions of the characteristics $(\mu''', \nu''', \varrho''', \sigma''', \tau''')$ of the condition X, the equation is in effect an expression for (X, 4Z) in terms of the characteristics of X and 4Z respectively.

There is, similarly, an expression for (2X, 3Z) in terms of the characteristics $(\mu', \nu', \varrho', \sigma')$ of 3Z (satisfying the relation $\mu'-\frac{3}{2}\nu'+\frac{3}{2}\varrho'-\sigma'=0$) and the characteristics (μ, ν, ϱ) of 2X, viz. we have

$$\begin{aligned}(2X, 3Z)= \quad & \mu(\quad\quad - \tfrac{1}{4}\varrho'+\tfrac{1}{2}\sigma')\\ & +\nu\,(-\tfrac{3}{8}\mu'+\tfrac{5}{16}\nu'+\tfrac{5}{16}\varrho'-\tfrac{3}{8}\sigma')\\ & +\varrho\,(\ \tfrac{1}{4}\mu'-\tfrac{1}{4}\nu' \quad\quad).\end{aligned}$$

This may be easily proved in the case where the conditions 2X are two separable conditions X, X' having the representatives (α, β), (α', β') respectively, and the conditions 3Z three separable conditions Z'', Z''', Z'''' having the representatives (α'', β''), $(\alpha''')(\beta''')$, (α'''', β'''') respectively; we have, in fact,

$$\begin{aligned}\mu'&=(1, 2, 4\}(\alpha, \beta)(\alpha', \beta'), & \mu&=(1, 2, 4, 4\}(\alpha'', \beta'')(\alpha''', \beta''')(\alpha'''', \beta''''),\\ \nu'&=(2, 4, 4\}\ ,,\ \ ,,\ ), & \nu&=(2, 4, 4, 2\}\ ,,\ \ ,,\ \ ,,\ \ ,),\\ \varrho'&=(4, 4, 2\}\ ,,\ \ ,,\ ), & \varrho&=(4, 4, 2, 1\}\ ,,\ \ ,,\ \ ,,\ \ ,),\\ \sigma'&=(4, 2, 1\}\ ,,\ \ ,,\ ); \end{aligned}$$

and with these values the function

$$(X, X', Z'', Z''', Z'''')=(1, 2, 4, 4, 2, 1\}(\alpha, \beta)(\alpha', \beta')(\alpha'', \beta'')(\alpha''', \beta''')(\alpha'''', \beta'''')$$

is found to be expressible as above in terms of (μ, ν, ϱ), $(\mu', \nu', \varrho', \sigma')$; but I do not know how to conduct the proof for the inseparable conditions 2X and 3Z.

37. It may be remarked by way of verification that writing successively

$$(3Z)=(\therefore), \ (\because/), \ (\cdot//), \ (///),$$

that is,

$$(\mu, \nu, \varrho)=(1, 2, 4), \ (2, 4, 4), \ (4, 4, 2), \ (4, 4, 1),$$

we have in the first case

$$\begin{aligned}(2X\cdot\cdot)= &\quad -\tfrac{1}{4}\varrho'+\tfrac{1}{2}\sigma'\\ &-\tfrac{3}{4}\mu'+\tfrac{3}{8}\nu'+\tfrac{3}{8}\varrho'-\tfrac{3}{4}\sigma'\\ &+2\mu'-\ \nu'\\ =&\mu'+\tfrac{1}{4}(\mu'-\tfrac{3}{2}\nu'+\tfrac{3}{2}\varrho'-\sigma'),=\mu',\end{aligned}$$

and similarly in the other three cases,

$$(2X\cdot//)=\nu',\ (2X\cdot//)=\varrho',\ (2X///)=\sigma'.$$

38. Let $(\mu, \nu, \varrho, \sigma)$ be the characteristics of 2Z, $(\mu-\frac{3}{2}\nu+\frac{3}{2}\varrho-\sigma=0)$, and $(\mu', \nu', \varrho', \sigma')$ the characteristics of 2X, $(\mu'-\frac{3}{2}\nu'+\frac{3}{2}\varrho'-\sigma'=0)$. Then in the formula for (2X, 3Z), writing successively for 3X

$$(2X\cdot),\ \text{characteristics}\ (\mu, \nu, \varrho),$$

and

$$(2X/),\quad ,,\quad (\nu, \varrho, \sigma),$$

we obtain expressions for the characteristics (2X, 2Z·) and (2X, 2Z/) of (2X, 2Z), viz. eliminating from the formulæ, first the (σ, σ') and secondly the (μ, μ'), each of these may be expressed in two different forms as follows:—

(2X, 2Z·)	(2X, 2Z/)
$= \frac{1}{2}\mu\mu'$	$= \frac{1}{2}\mu\mu'$
$+\frac{7}{8}\nu\nu'$	$-\frac{3}{8}\nu\nu'$
$-\frac{3}{4}(\mu\nu'+\mu'\nu)$	$-\frac{1}{4}\varrho\varrho'$
$+\frac{1}{2}(\mu\varrho'+\mu'\varrho)$	$-\frac{1}{4}(\mu\nu'+\mu'\nu)$
$-\frac{1}{4}(\nu\varrho'+\nu'\varrho)$	$+\frac{1}{2}(\nu\varrho'+\nu'\varrho)$
$= \frac{1}{2}\sigma\sigma'$	$= \frac{1}{2}\sigma\sigma'$
$-\frac{3}{8}\varrho\varrho'$	$+\frac{7}{8}\varrho\varrho'$
$-\frac{1}{4}\nu\nu'$	$-\frac{3}{4}(\varrho\sigma'+\varrho'\sigma)$
$-\frac{1}{4}(\varrho\sigma'+\varrho'\sigma)$	$+\frac{1}{2}(\nu\sigma'+\nu'\sigma)$
$+\frac{1}{2}(\nu\varrho'+\nu'\varrho),$	$-\frac{1}{4}(\nu\varrho'+\nu'\varrho),$

the two expressions of the same quantity being of course equivalent in virtue of the relations between $(\mu, \nu, \varrho, \sigma)$ and $(\mu', \nu', \varrho', \sigma')$ respectively.

The characteristics of (X, Z), (X, 2Z), (X, 3Z) are at once deducible from the before-mentioned expression $\alpha\mu+\beta\nu$ of (X, 4Z).

39. ZEUTHEN's investigations are based upon the before-mentioned theorem, that in a system of conics (4X), characteristics (μ, ν), there are $2\mu-\nu$ point-pairs and $2\nu-\mu$ line-pairs. If in the given system the number of point-pairs is$=\lambda$ and the number of line-pairs is $=\varpi$, then, conversely, the characteristics of the system are

$$\mu=\tfrac{1}{3}(2\lambda+\varpi),\ \nu=\tfrac{1}{3}(\lambda+2\varpi).$$

And by means of this formula he investigates the characteristics of the several systems of conics which satisfy four conditions (4X) of contact with a given curve or curves, viz. these are the conics

$$\begin{array}{llllll} (1)(1)(1)(1), & (1,1)(1)(1), & (1,1)(1,1), & (1,1,1)(1), & (1,1,1,1), \\ (2)(1)(1) & , (2)(1,1) & , (2,1)(1) & , (2,1,1), & \\ (2)(2) & , (2,2) & , & , & \\ (3)(1) & , (3,1) & , & & \\ (4) & , & & & \end{array}$$

where (1) denotes contact of the first order, (2) of the second order, (3) of the third order, (4) of the fourth order, with a given curve; (1)(1) denotes contacts of the first order with each of two given curves, (1, 1) two such contacts with the same given curve, and so on. A given curve is in every case taken to be of the order m and class n, with δ nodes, $\varkappa$ cusps, τ double tangents, and ι inflexions (m_1, n_1, δ_1, $\varkappa_1$, τ_1, ι_1; m_2, n_2, &c., as the case may be). The symbols (1) &c. might be referred to the corresponding curves by a suffix; thus $(1)_m$ would denote that the contact is with a given curve of the order m (class n, &c.); but this is in general unnecessary.

40. In a system of conics satisfying four conditions of contact, as above, it is comparatively easy to see what are the point-pairs and line-pairs in these several systems respectively; but in order to find the values of λ and ϖ, each of these point-pairs and line-pairs has to be counted not once, but a proper number of times; and it is in the determination of these multiplicities that the difficulty of the problem consists. I do not enter into this question, but give merely the results.

41. For the statement of these I introduce what I call the notation of ZEUTHEN'S Capitals. We have to consider several classes of point-pairs and the reciprocal classes of line-pairs. A point-pair may be described (*ante*, No. 31) as a terminated line, and a line-pair as a terminated point; and we have first the following point-pairs, viz.:—

A, line terminated each way in the intersection of two curves or of a curve with itself (node).

B, tangent to a curve, terminated in a curve, and in the intersection of two curves or of a curve with itself.

C, common tangent of two curves, or double tangent of a curve, terminated each way in a curve.

D, inflexion tangent of a curve terminated each way in a curve:

and the corresponding line-pairs, viz.:—

A′, point terminated each way in the common tangent of two curves or the double tangent of a curve.

B′, point of a curve terminated by the tangent of a curve, and by the common tangent of two curves or double tangent of a curve.

C′, intersection of two curves, or of a curve with itself (node), terminated each way by the tangent to a curve.

D′, cusp of a curve terminated each way by the tangent to a curve:

all which is further explained by what follows; thus in the case $(1)(1)(1)(1)$, $=(1)_{m_1}(1)_{m_2}(1)_{m_3}(1)_{m_4}$, the value of A is given as $\Sigma m_1 m_2 . m_3 m_4 (= 3 m_1 m_2 m_3 m_4)$. Here A is the number of the point-pairs terminated one way in the intersection of any two m_1, m_2 of the four curves, and the other way in the intersection of the remaining two m_3, m_4 of the four curves. But in the case $(1, 1)(1)(1)$, $=(1, 1)_m(1)_{m_1}(1)_{m_2}$, the value of A is given as $=\delta m_1 m_2 + m m_1 . m m_2$. Here A denotes the number of the point-pairs, which are either $(\delta m_1 m_2)$ terminated one way at a node of m, and the other way at an intersection of m_1, m_2, or else $(m m_1 . m m_2)$ terminated one way at an intersection of m, m_1, and the other way at an intersection of m, m_2: and so in other cases.

42. This being so, we have

$(1)(1)(1)(1), =(1)_{m_1}(1)_{m_2}(1)_{m_3}(1)_{m_4}$

$A = \Sigma m_1 m_2 . m_3 m_4 (= 3\ m_1 m_2 m_3 m_4)$,	1	$A' = \Sigma n_1 n_2 . n_3 n_4$	$(= 3 n_1 n_2 n_3 n_4)$,
$B = \Sigma m_1 m_2 . m_3 . n_4 (= 3 \Sigma m_1 m_2 m_3 n_4)$,	2	$B' = \Sigma n_1 n_2 . n_3 . m_4$	$(= 3 \Sigma n_1 n_2 n_3 m_4)$,
$C = \Sigma m_1 . m_2 . n_3 n_4$	$(= \Sigma m_1 m_2 n_3 n_4)$. 4	$C' = \Sigma n_1 . n_2 . m_3 m_4$	$(= \Sigma n_1 n_2 m_3 m_4)$.

$(1, 1)(1)(1), =(1, 1)_m(1)_{m_1}(1)_{m_2}$.

$A = \delta m_1 m_2 + m m_1 . m m_2$,	1	$A' = \tau n_1 n_2 + n n_1 . n n_2$,	
$B = \delta n_1 m_2 + \delta n_2 m_1$	2	$B' = \tau m_1 n_2 + \tau m_2 n_1$	
$+ m m_1 (n-2) m_2 + m m_2 (n-2) m_1$		$+ n n_1 (m-2) n_2 + n n_2 (m-2) n_1$	
$+ m m_1 n_2 (m-1) + m m_2 n_1 (m-1)$		$+ n n_1 m_2 (n-1) + n n_2 m_1 (n-1)$	
$+ m_1 m_2 n (m-2)$,		$+ n_1 n_2 m (n-2)$,	
$C = \tau m_1 m_2$	4	$C' = \delta n_1 n_2$	
$+ n n_1 (m-2) m_2 + n n_2 (m-2) m_1$		$+ m m_1 (n-2) n_2 + m m_2 (n-2)$	
$+ n_1 n_2 . \frac{1}{2} m (m-1)$,		$+ m_1 m_2 . \frac{1}{2} n (n-1)$,	
$D = \iota m_1 m_2$.	3	$D' = \varkappa n_1 n_2$.	

$(1, 1)(1, 1), =(1, 1)_m(1, 1)_{m_1}$.

$A = \delta\delta_1 + \frac{1}{2} m m_1 (m m_1 - 1)$,	1	$A' = \tau\tau_1 + \frac{1}{2} n n_1 (n n_1 - 1)$,	
$B = \delta n_1 (m_1 - 2) + \delta_1 n (m-2)$	2	$B' = \tau m_1 (n-2) + \tau_1 m (n-2)$	
$+ m m_1 (n-2)(m_1 - 1) + n m_1 (n_1 - 2)(m-1)$,		$+ n n_1 (m-2)(n_1 - 1) + n n_1 (m_1 - 2)(n-1)$,	
$C = \tau . \frac{1}{2} m_1 (m_1 - 1) + \tau_1 . \frac{1}{2} m (m-1$	4	$C' = \delta . \frac{1}{2} n_1 (n_1 - 2) + \delta_1 . \frac{1}{2} n (n-1)$	
$+ n n_1 (m-2)(m_1 - 2)$,		$+ m m_1 (n-2)(n_1 - 2)$,	
$D = \iota . \frac{1}{2} m_1 (m_1 - 1) + \iota_1 . \frac{1}{2} m (m-1)$,	3	$D' = \varkappa . \frac{1}{2} n_1 (n_1 - 1) + \varkappa_1 . \frac{1}{2} n (n-1)$.	

$(1, 1, 1)(1) = (1, 1, 1)_m(1)_{m_1}$.

$A = \delta m m_1$,	1	$A' = \tau n n_1$,
$B = \delta(n-4)m, + \delta n_1(m-2)$, $+ m m_1(n-2)(m-3)$,	2	$B' = \tau(m-4)n_1 + \tau m_1(n-2)$, $+ n n_1(m-2)(n-3)$,
$C = \tau(m-4)m_1 + n n_1 . \frac{1}{2}(m-2)(m-3)$,	4	$C' = \delta(n-4)n_1 + m m_1 . \frac{1}{2}(n-2)(n-3)$,
$D = \iota(m-3)m_1$,	3	$D' = \varkappa(n-3)n_1$.

$(1, 1, 1, 1), = (1, 1, 1, 1)_m$.

$A = \frac{1}{2}\delta(\delta-1)$,	1	$A' = \frac{1}{2}\tau(\tau-1)$,
$B = \delta(n-4)(m-4)$,	2	$B' = \tau(m-4)(n-4)$,
$C = \tau . \frac{1}{2}(m-4)(m-5)$,	3	$C' = \delta . \frac{1}{2}(n-4)(n-5)$,
$D = \iota . \frac{1}{2}(m-3)(m-4)$.	4	$D' = \varkappa . \frac{1}{2}(n-3)(n-4)$.

43. Secondly, we have the point-pairs:—

E, tangent to curve from intersection of two curves or of a curve with itself (node), and terminated at the point of contact and the last-mentioned point.

F, tangent to a curve at intersection with another curve or with itself, and terminated there and at a curve.

G, common tangent of two curves or double tangent of a curve, terminated at one of the points of contact and at a curve.

D, *ut suprà*.

H, line joining cusp of a curve with intersection of two curves or of a curve with itself, and terminated at these points.

I, line from cusp of a curve touching a curve, and terminated at the cusp and at a curve.

J, Inflexion tangent of a curve, terminated there and at a curve:

and the corresponding line-pairs, viz.

E′, point on a curve in common tangent of two curves or double tangent of a curve, and terminated by this tangent and by tangent to a curve.

F′, point on a curve in common tangent of this and another curve or in double tangent of this curve, and terminated by this tangent and by tangent to a curve.

D′, *ut suprà*.

H′, intersection of inflexion tangent of a curve with common tangent of two curves or double tangent of a curve, and terminated by these lines.

I′, intersection of inflexion tangent of a curve with a curve, and terminated by this tangent and by tangent of a curve:

and this being so,

$(2)(1)(1), =(2)_m(1)_{m_1}(1)_{m_2}$.

$E = n.m_1m_2,$	3	$E' = m.n_1n_2,$
$F = mm_1.m_2 + mm_2.m_1,$	3	$F' = nn_1.m_2 + nn_2.m_1,$
$G = nn_2.m_1 + nn_1.m_2,$	6	$G' = mm_2.n_1 + mn_1.n_2,$
$D = \iota m_1m_2,$	2	$D' = \varkappa n_1n_2,$
$H = \varkappa m_1m_2,$	1	$H' = \iota n_1n_2,$
$I = \varkappa n_1m_2 + \varkappa n_2m_1.$	2	$I' = \iota m_2n_1 + \iota m_1n_1.$

$(2)(1, 1) = (2)_m(1, 1)_{m_1}$.

$E = \delta_1 n,$	3	$E' = \tau_1 m,$
$F = m.m_1(m_1 - 1),$	3	$F' = n.n_1\ (n_1 - 1),$
$G = nn_1\ (m_1 - 2),$	6	$G' = mm_1\ (n_1 - 2),$
$D = \iota.\frac{1}{2}m_1(m_1 - 1),$	2	$D' = \varkappa.\frac{1}{2}n_1(n_1 - 1),$
$H = \varkappa\delta_1,$	1	$H' = \iota\tau_1,$
$I = \varkappa n_1\ (m_1 - 2).$	2	$I' = \iota m_1\ (n_1 - 2).$

$(2, 1)(1), =(2, 1)_m(1)_{m_1}$.

$E = (n-2).mm_1,$	3	$E' = (m-2).nn_1,$
$F = mm_1(m-2) + 2\delta m_1,$	3	$F' = nn_1(n-2) + 2\tau n_1,$
$G = nn_1(m-2) + 2\tau m_1,$	6	$G' = mm_1(n-2) + 2\delta n_1,$
$D = \iota(m-3)m_1,$	2	$D' = \varkappa(n-3)n_1,$
$H = \varkappa mm_1,$	1	$H' = \iota nn_1,$
$I = \varkappa(n-3)m_1 + \varkappa n_1(m-2),$	2	$I' = \iota(m-3)n_1 + \iota m_1(n-2),$
$J = \iota m_1.$	5	$J' = \varkappa n_1.$

$(2, 1, 1), =(2, 1, 1)_m$.

$E = \delta(n-4),$	3	$E' = \tau(m-4),$
$F = 2\delta(m-3),$	3	$F' = 2\tau(n-3),$
$G = 2\tau(m-4),$	6	$G' = 2\delta(n-4),$
$D = \iota.\frac{1}{2}(m-3)(m-4),$	2	$D' = \varkappa.\frac{1}{2}(n-3)(n-4),$
$H = \delta\varkappa,$	1	$H' = \iota\tau,$
$I = \varkappa(n-3)(m-4),$	2	$I' = \iota(m-3)(n-4),$
$J = \iota(m-3).$	5	$J' = \varkappa(n-3).$

44. Thirdly, we have the point pairs:—

K, common tangent of two curves or double tangent of a curve, terminated at points of contact.

L, line from cusp of a curve touching a curve, and terminated at cusp and point of contact.

M, line joining cusp of a curve with cusp of a curve, and terminated by the two cusps.

N, inflexion tangent terminated each way at inflexion, viz. this is a *line-pair-point*.

O, cuspidal tangent terminated each way at cusp, viz. this is a *line-pair-point*:

and the corresponding line-pairs:—

K′, intersection of two curves or of curve with itself (node), and terminated by the two tangents.

L′, intersection of inflexion tangent of a curve with a curve, and terminated by the inflexion tangent and the tangent at the intersection.

M′, intersection of inflexion tangent of a curve with inflexion tangent of a curve, and terminated by the two inflexion tangents.

N′, =O, *line-pair-point* as above.

O′, =N, *line-pair-point* as above:

which being so, we have

$(2)(2), = (2)_m\ (2)_{m_1}$.

$K = nn_1$,	9	$K' = mm_1$,
$L = \varkappa n_1 + \varkappa_1 n$,	3	$L' = \iota m_1 + \iota_1 m$,
$M = \varkappa\varkappa_1$.	1	$M' = u_1$.

$(2, 2), = (2, 2)_m$.

$K = \tau$,	9	$K' = \delta$,
$L = \varkappa(n-3)$,	3	$L' = \iota(m-3)$,
$M = \frac{1}{2}\varkappa(\varkappa-1)$,	1	$M' = \frac{1}{2}\iota(\iota-1)$,
$N = \iota$,	2	$N' = \varkappa$,
$O = \varkappa$.	1	$O' = \iota$.

45. Fourthly, we have the point-pairs:—

P, tangent of a curve at its intersection with another curve or itself, terminated each way at the point of contact—*line-pair-point*.

Q, common tangent of two curves or double tangent of a curve, terminated each way at one of the points of contact—*line-pair-point*.

J, *ut suprà*.

R, cuspidal tangent terminated at cusp and at a curve:

and the corresponding line-pairs:—

P′, =Q, *line-pair-point*.
Q′, =P, *line-pair-point*.
J′, *ut suprà*.
R′, inflexion of curve terminated by the inflexion tangent and by tangent to a curve: which being so, we have

$(3)(1), =(3)_m(1)_{m_1}$.

$P=mm_1$,	2	$P'=nm_1$,
$Q=nn_1$,	2	$Q'=mm_1$,
$J=\iota m_1$,	5	$J'=\varkappa n_1$,
$R=\varkappa m_1$.	4	$D'=\iota m_1$.

$(3, 1), =(3, 1)_m$.

$P=2\delta$,	2	$Q'=2\tau$,
$Q=2\tau$,	2	$P'=2\delta$,
$J=\iota(m-3)$,	5	$J'=\varkappa(n-3)$,
$R=\varkappa(m-3)$.	4	$R'=\iota(n-3)$.

46. And lastly, we have the point-pairs N, O (*line-pair-points*) and the line-pairs N′, O′ (*line-pair-points*), *ut suprà*, and

$(4), =(4)_m$.

$N=\iota$,	4	$N'=\varkappa$,
$O=\varkappa$.	2	$O'=\iota$.

47. Where in all cases the central column of figures gives the numerical factors which multiply the corresponding capitals, thus we have

for (1)(1)(1)(1)

$$\lambda=2\nu-\mu=A+2B+4C,$$
$$\varpi=2\mu-\nu=A'+2B'+4C';$$

for (1, 1)(1)(1),

$$\lambda=2\nu-\mu=A+2B+4C+3D,$$
$$\varpi=2\mu-\nu=A'+2B'+4C'+3D',$$

and so on.

48. The elements $(m, n, \delta, \varkappa, \tau, \iota)$ of a curve satisfy PLÜCKER'S six equations, and ZEUTHEN uses these equations, in a somewhat unsystematic way, to simplify the form of his results.

It is convenient in his formulæ to write $3m+\iota, =3n+\varkappa, =\alpha$, and to express every thing in terms of (m, n, α), viz. we have for this purpose

$$2\delta=m^2-m+8n-3\alpha,$$
$$2\tau=n^2-8m-n-3\alpha.$$

But I make another alteration in the form of his results; he gives, for instance, the characteristics of $(1,1)(1)(1)$ as

$$\mu = \mu''' m_1 m_2 + \mu''(m_1 n_2 + m_2 n_1) + \mu' n_1 n_2,$$
$$\nu = \nu''' m_1 m_2 + \nu''(m_1 n_2 + m_2 n_1) + \nu' n_1 n_2,$$

where

$$\mu' \quad = 2m(m+n-3) + r, = (1,1\,\therefore),$$
$$\mu'' = \nu' = 2n(m+2n-5) + 2r, = (1,1\,:/),$$
$$\mu''' = \nu'' = 2n(2m+n-5) + 2\delta, = (1,1\cdot //),$$
$$\nu''' = 2n(m+n-3) + \delta, = (1,1///),$$

viz. the four components have really the significations $(1,1\,\therefore)$ set opposite to them respectively; and accordingly, instead of giving the formulæ for the two characteristics of $(1,1)(1)(1)$, I give those for the four characteristics $(1,1\,\therefore)$, &c. of $(1,1)$, thus in every case obtaining formulæ which relate to a single curve only. Subject to the last-mentioned variation of form, I give ZEUTHEN'S original expressions in Annex 6; but here in the text I express them as above in terms of (m, n, α), viz.

49. We have the formulæ

(1)
$$(::) = n+2m,$$
$$(\therefore/) = 2n+4m,$$
$$(://) = 4n+4m,$$
$$(\cdot ///) = 4n+2m,$$
$$(////) = 2n+2m;$$

$(1,1)$
$$(\therefore) = 2m^2+2mn+\tfrac{1}{2}n^2-2m-\tfrac{1}{2}n-\tfrac{3}{2}\alpha,$$
$$(:/) = 2m^2+4mn+n^2-2m-n-3\alpha,$$
$$(\cdot //) = m^2+4mn+2n^2-m-2n-3\alpha,$$
$$(///) = \tfrac{1}{2}m^2+2mn+2n^2-\tfrac{1}{2}m-2n-\tfrac{3}{2}\alpha.$$

$(1,1,1)$
$$(:) = \tfrac{3}{2}m^3+2m^2n+mn^2+\tfrac{1}{2}n^3-2m^2-3mn-\tfrac{1}{2}n^2-\tfrac{29}{8}m-\tfrac{29}{8}n+\alpha(-3m-\tfrac{3}{2}n+13),$$
$$(\cdot/) = \tfrac{1}{2}m^3+2m^2n+2mn^2+\tfrac{1}{2}n^3-m^2-4mn-n^2-\tfrac{45}{8}m-\tfrac{45}{8}n+\alpha(-3m-3n+20),$$
$$(//) = \tfrac{1}{6}m^3+m^2n+2mn^2+\tfrac{3}{8}n^3-\tfrac{1}{2}m^2-3mn-2n^2-\tfrac{29}{8}m-\tfrac{29}{8}n+\alpha(-\tfrac{3}{2}m-3n+13);$$

$(1,1,1,1)$
$$(\cdot) = \tfrac{1}{12}m^4+\tfrac{3}{8}m^3n+m^2n^2+\tfrac{1}{3}mn^3+\tfrac{1}{24}m^4$$
$$-\tfrac{1}{2}m^3-3m^2n-2mn^2-\tfrac{1}{4}n^3-\tfrac{181}{12}m^2-21mn-\tfrac{329}{24}n^2+\tfrac{191}{12}m+\tfrac{403}{24}n$$
$$+\alpha(-\tfrac{3}{2}m^2-3mn-\tfrac{3}{2}n^2+\tfrac{43}{3}m+\tfrac{55}{4}n-\tfrac{347}{4})+\alpha.\tfrac{9}{8},$$
$$(/) = \tfrac{1}{24}m^4+\tfrac{1}{3}m^3n+m^2n^2+\tfrac{3}{8}mn^3+\tfrac{1}{12}n^4$$
$$-\tfrac{1}{4}m^3-2m^2n-3mn^2-\tfrac{1}{8}n^3-\tfrac{329}{24}m^2-21mn-\tfrac{181}{12}n^2+\tfrac{403}{24}m+\tfrac{191}{12}n$$
$$+\alpha(-\tfrac{3}{4}m^2-3mn-\tfrac{3}{2}n^2+\tfrac{55}{4}m+\tfrac{43}{3}n-\tfrac{347}{4})+\alpha^2.\tfrac{9}{8};$$

(2)

$$(\therefore) = \alpha,$$
$$(:/) = 2\alpha,$$
$$(\cdot//) = 2\alpha,$$
$$(///) = \alpha;$$

(2, 1)

$$(:) = 12m + 12n + (2m + n - 14)\alpha,$$
$$(\cdot/) = 24m + 24n + (2m + 2n - 24)\alpha,$$
$$(//) = 12m + 12n + (m + 2n - 14)\alpha;$$

(2, 1, 1)

$$(\cdot) = 24m^2 + 36mn + 12n^2 - 168m - 168n + \alpha(m^2 + 2mn + \tfrac{1}{2}n^2 - 25m - \tfrac{29}{2}n + 138) - \tfrac{3}{2}\alpha^2,$$
$$(/) = 12m^2 + 36mn + 24n^2 - 168m - 168n + \alpha(\tfrac{1}{2}m^2 + 2mn + n^2 - \tfrac{29}{2}m - 25n + 138) - \tfrac{3}{2}\alpha^2;$$

(2, 2)

$$(\cdot) = 27m + 24n - 20\alpha + \tfrac{1}{2}\alpha^2,$$
$$(/) = 24m + 27n - 20\alpha + \tfrac{1}{2}\alpha^2;$$

(3)

$$(:) = -4m - 3n + 3\alpha,$$
$$(\cdot/) = -8m - 8n + 6\alpha,$$
$$(//) = -3m - 4n + 3\alpha;$$

(3, 1)

$$(\cdot) = -8m^2 - 12mn - 3n^2 + 56m + 53n + \alpha(6m + 3n - 39),$$
$$(/) = -3m^2 - 12mn - 8n^2 + 53m + 56n + \alpha(3m + 6n - 39);$$

(4)

$$(\cdot) = -10m - 8n + 6\alpha,$$
$$(/) = -8m - 10n + 6\alpha.$$

50. By means of the foregoing formulæ I obtain, as will presently be shown, the following formulæ for the number of the conics which satisfy five conditions, viz. :—

$$(5) = -15m - 15n + 9\alpha;$$
$$(4, 1) = -8m^2 - 20mn - 8n^2 + 104m + 104n + \alpha(6m + 6n - 66);$$
$$(3, 2) = 120m + 120n + \alpha(-4m - 4n - 78) + 3\alpha^2;$$
$$(3, 1, 1) = -\tfrac{3}{2}m^3 - 10m^2n - 10mn^2 - \tfrac{3}{2}n^3 + \tfrac{129}{2}m^2 + 116mn + \tfrac{129}{2}n^2 - 434m - 434n;$$
$$+ \alpha(\tfrac{3}{2}m^2 + 6mn + \tfrac{3}{2}n^2 - \tfrac{69}{2}m - \tfrac{69}{2}n + 291) - \tfrac{3}{2}\alpha^2;$$
$$(2, 2, 1) = 24m^3 + 54mn + 24n^3 - 468m - 468n$$
$$+ \alpha(-8m - 8n + 327) + \alpha^2(\tfrac{1}{2}m + \tfrac{1}{2}n - 12);$$

$$(2,1,1,1) = 6m^3+30m^2n+30mn^2+6n^3-174m^2-348mn-174n^2+1320m+1320n$$
$$+\alpha(\tfrac{1}{2}m^2+m^2n+mn^2+\tfrac{1}{6}n^3-\tfrac{15}{2}m^2-26mn-\tfrac{15}{2}n^2+\tfrac{338}{3}m+\tfrac{338}{3}n-960)$$
$$+\alpha^2(-\tfrac{3}{2}m-\tfrac{3}{2}n+28);$$

$$(1,1,1,1,1)^* = \tfrac{1}{120}m^5+\tfrac{1}{12}m^4n+\tfrac{1}{8}m^3n^2+\tfrac{1}{8}m^2n^3+\tfrac{1}{12}mn^4+\tfrac{1}{120}n^5$$
$$-\tfrac{1}{12}m^4-\tfrac{5}{6}m^3n-2m^2n^2-\tfrac{5}{6}mn^3-\tfrac{1}{12}n^4$$
$$-\tfrac{11}{12}m^3-\tfrac{209}{12}m^2n-\tfrac{209}{12}mn^2-\tfrac{11}{12}n^3$$
$$+\tfrac{1321}{12}m^2+\tfrac{593}{3}mn+\tfrac{1321}{12}n^2-\tfrac{3159}{5}m-\tfrac{3159}{5}n$$
$$+\alpha(-\tfrac{1}{4}m^3-\tfrac{3}{2}m^2n-\tfrac{3}{2}mn^2-\tfrac{1}{4}n^3+\tfrac{29}{4}m^2+23mn+\tfrac{29}{4}n^2-\tfrac{337}{4}m-\tfrac{337}{4}n+486)$$
$$+\alpha^2(\tfrac{3}{2}m+\tfrac{3}{2}n-15);$$

51. I observe that by means of the abovementioned expressions of (X, 4Z) and (2X, 3Z), the foregoing results, other than those for (5), (4, 1), &c., may be presented in a somewhat different form, viz. we have

$$(4Z)(1) = n(\,\cdot\,)+m(/),$$

where $(\,\cdot\,)$ denotes $(4Z\,\cdot\,)$, $(\,/\,)$ denotes $(4Z\,/)$, and so in other cases, the understood term being 3Z or 2Z, as the case may be.

$$(3Z)(2) = (\cdot/)\tfrac{1}{3}\alpha;$$

$$(3Z)(1,1) = (\,:\,)(\tfrac{1}{2}n^2-\tfrac{1}{2}n)$$
$$+(\cdot/)(mn-\tfrac{3}{4}\alpha)$$
$$+(\,//\,)(\tfrac{1}{2}m^2-\tfrac{1}{2}m);$$

$$(2Z)(3) = (\,\therefore\,)(\tfrac{3}{2}m+n-\tfrac{3}{4}\alpha)$$
$$+(\,:/\,)(-\tfrac{7}{4}m-\tfrac{3}{2}n+\tfrac{9}{8}\alpha)$$
$$+(\,\cdot\,//)(-\tfrac{3}{2}m-\tfrac{7}{4}n+\tfrac{9}{8}\alpha)$$
$$+(///)(\quad m+\tfrac{3}{2}n-\tfrac{3}{4}\alpha);$$

$$(2Z)(2,1) = (\,\therefore\,)\{-3m-3n+\alpha(-\tfrac{1}{4}m+\tfrac{1}{4}n+2)\}$$
$$+(\,:/)\{\tfrac{9}{2}m+\tfrac{9}{2}n+\alpha(\tfrac{3}{8}m+\tfrac{1}{8}n-4)\}$$
$$+(\,\cdot\,//)\{\tfrac{9}{2}m+\tfrac{9}{2}n+\alpha(\tfrac{1}{8}m+\tfrac{3}{8}n-4)\}$$
$$+(///)\{-3m-3n+\alpha(\tfrac{1}{4}m-\tfrac{1}{4}n+2)\};$$

$$(2Z)(1,1,1) =$$
$$(\,\therefore\,)\{-\tfrac{1}{32}m^3-\tfrac{1}{32}m^2n+\tfrac{1}{32}mn^2+\tfrac{5}{32}n^3+\tfrac{1}{8}m^2+0mn-\tfrac{5}{8}n^2+\tfrac{11}{12}m+\tfrac{29}{48}n+\alpha(\tfrac{3}{8}m-\tfrac{3}{8}n-1)\}$$
$$+(\,:/)\{\tfrac{1}{16}m^3+\tfrac{3}{16}m^2n+\tfrac{1}{16}mn^2-\tfrac{1}{16}n^3-\tfrac{5}{16}m^2-\tfrac{1}{8}mn+\tfrac{5}{16}n^2-\tfrac{19}{96}m-\tfrac{25}{96}n+\alpha(-\tfrac{9}{16}m-\tfrac{3}{16}n+3)\}$$
$$+(\,\cdot\,//)\{\tfrac{1}{16}m^3+\tfrac{1}{16}m^2n+\tfrac{3}{16}mn^2+\tfrac{1}{16}n^3+\tfrac{5}{16}m^2-\tfrac{1}{8}mn-\tfrac{5}{16}n^2-\tfrac{25}{96}m-\tfrac{19}{96}n+\alpha(-\tfrac{3}{16}m-\tfrac{9}{16}n+3)\}$$
$$+(///)\{\tfrac{5}{32}m^3+\tfrac{1}{32}m^2n-\tfrac{1}{32}mn^2-\tfrac{5}{32}n^3-\tfrac{5}{8}m^2+0mn+\tfrac{5}{8}n^2+\tfrac{29}{48}m+\tfrac{11}{12}n+\alpha(-\tfrac{3}{8}m+\tfrac{3}{8}n-1);$$

in all which formulæ it is to be recollected that we have

$$(\therefore)-\tfrac{3}{2}(:/)+\tfrac{3}{2}(\,\cdot\,//)-(///)=0.$$

* In my paper in the Comptes Rendus, I gave erroneously the coefficients $-\frac{3329}{5}m-\frac{3329}{5}n..+\alpha(\ldots+\frac{1331}{5})$.

to which may be joined

$$(Z)(4X)=a(4X\cdot)+b(4X/),$$

where a, b are the representatives of the condition (Z), and where (4X) is to be considered as standing successively for (4), (3, 1), (2, 2), (2, 1, 1), and (1, 1, 1, 1), the values of $(4X\cdot)$ and $(4X/)$ being in each case given by the foregoing Table.

52. The formulæ are very convenient for the calculation of the numbers of the conics which satisfy five conditions of contact with two given curves; thus if, for example, $(3Z),=(3)_m$, denotes the condition of a contact of the third order with a given curve (m_1), then writing for symmetry $(2)_m$ in place of (2), we have

$$\begin{aligned}(3)_m(2)_m&=\tfrac{1}{2}\alpha(3\cdot/)_m,\\ &=\alpha(-4m_1-4n_1+3\alpha_1).\end{aligned}$$

53. To obtain the foregoing expressions of (5), (4, 1), (3, 2), (3, 1, 1), (2, 2, 1), (2, 1, 1, 1), and (1, 1, 1, 1, 1), I assume that the given curve breaks up into two curves (m, n, α) and (m', n', α'), or, as we may for shortness express it, into two curves m and m'.

We have then

$$(5)_{m+m'}=(5)_m+(5)_{m'},$$

viz. the conics which have contact of the 5th order with the aggregate curve $m+m'$ are made up of the conics which have this contact with the curve m and the conics which have this contact with the curve m'. Writing this under the form

$$(5)_{m+m'}-(5)_m-(5)_{m'}=0,$$

and observing that $(5)_m$ is a function $\phi(m, n, \alpha)$, and that consequently this is a functional equation $\phi(m+m', n+n', \alpha+\alpha')-\phi(m, n, \alpha)-\phi(m', n', \alpha')=0$, the solution is

$$\phi(m, n, \alpha)=am+bn+c\alpha,$$

where a, b, c are arbitrary constants; but as the solution should be symmetrical in regard to m, n, we have $a=b$, or the solution is $\phi(m, n, \alpha)=a(m+n)+c\alpha$.

54. Similarly we have

$$(4, 1)_{m+m'}-(4, 1)_m-(4, 1)_{m'}=(4)_m(1)_{m'}+(4_{m'})(1)_m,$$

viz. the conics which have with the aggregate curve $m+m'$ the contacts (4, 1) are made up of the contacts which have the two contacts 4 and 1 with the one curve or with the other curve, or the contact 4 with the one curve and the contact 1 with the other curve. The expression on the right-hand side is a known function of (m, n, α), (m', n', α'); hence the form of the functional equation is

$$\phi(m+m', n+n', \alpha+\alpha')-\phi(m, n, \alpha)-\phi(m', n', \alpha')=\mathrm{F}(m, n, \alpha, m', n', \alpha');$$

and any particular solution of this equation being obtained, the general solution is found by adding to it the term $am+bn+c\alpha$. Assuming that the particular solution is symmetrical in regard to (m, n), then the term to be added is as before $=a(m+n)+c\alpha$. And similarly for (3, 2), (3, 1, 1), &c.; that is, in every case we have a solution containing two arbitrary constants a, c, which remain to be determined.

55. Now in every case except $(5)_m$ the number of intersections of the conic with the curve is >6 (viz. for $(4,1)_m$ and $(3,2)_m$ the number is 7, for $(3,1,1)$ and $(2,2,1)$ it is 8, and for the remaining two cases it is 9 and 10 respectively); hence if the given curve m be a cubic, the number of conics satisfying the prescribed conditions is $=0$; and since a cubic may be the general cubic or a nodal or a cuspidal cubic, we have the three cases $(m, n, \alpha)=(3, 6, 18)$, $(3, 4, 12)$, and $(3, 3, 10)$. We have thus in each case three conditions for the determination of the constants a, c; so that there is in each case a verification of the resulting formula.

56. In the omitted case $(5)_m$, when the curve m is a cubic, the theory of the conics $(5)_m$ is a known one, viz. the points of contact of these conics, or the "sextactic" points of the cubic, are the points of contact of the tangents from the points of inflexion; the number of the conics $(5)_m$ is thus $=(n-3)$, viz. in the three cases respectively it is $=27$, 3, and 0. Hence for determining the constants we have the three equations

$$9a+18c=27,$$
$$7a+12c=\ 3,$$
$$6a+10c=\ 0,$$

which are satisfied by $a=-15$, $c=9$, and the resulting formula is

$$(5)=-15m-15n+9\alpha.$$

In the particular case of a curve without nodes or cusps, this is $(5)=12n-15m$, $=m(12m-27)$, which agrees with the result obtained in my memoir "On the Sextactic Points of a Plane Curve," Phil. Trans. vol. clv. (1865) pp. 545–578.

57. The subsidiary results required for the remaining cases $(4,1)$, &c. are at once obtained from the foregoing formulæ for $(4Z)(1)$, $(3Z)(2)$, &c.; for example, we have

$$\begin{aligned}(4)_m(1)_{m'}=\ & n'(-10m-\ 8n+6\alpha)\\ &+m'(-\ 8m-10n+6\alpha),\end{aligned}$$

with like expressions for $(3,1)_m(1)_{m'}$, &c.,

$$\begin{aligned}(3)_m(2)_{m'}\ &=\qquad \tfrac{1}{2}\alpha'\ (-8m-8n+6\alpha),\\ (3)_m(1,1)_{m'}&=(\tfrac{1}{2}n'^2-\tfrac{1}{2}n')(-4m-3n+3\alpha)\\ &\quad+(m'n'-\tfrac{3}{4}\alpha')(-8m-8n+6\alpha)\\ &\quad+(\tfrac{1}{2}m'^2-\tfrac{1}{2}m')(-3m-4n+3\alpha);\end{aligned}$$

with like expressions for $(2,1)_m(2)_{m'}$, $(2,1)_m(1,1)_{m'}$, &c. &c.

58. Calculation of $(4,1)$. We have

$$\begin{aligned}(4,1)_{m+m'}-(4,1)_m-(4,1)_{m'}&=(4)_m(1)_{m'}+(4)_{m'}(1)_m\\ &=-16mm'-20(mn'+m'n)-16nn'\\ &\quad+6(\alpha n'+\alpha' n)+6(\alpha m'+\alpha' m),\end{aligned}$$

the integral of which is

$$(4,1)_m=-8m^2-20mn-8n^2+a(m+n)+\alpha(6m+6n+c).$$

The particular cases $(m, n, \alpha)=(3, 6, 18), (3, 4, 12), (3, 3, 10)$ give respectively

$$0=252+9a+18c,$$
$$0=\ 64+7a+12c,$$
$$0=\ 36+6a+10c,$$

satisfied by $a=104$, $c=-66$.

59. Calculation of $(3, 2)$. We have

$$(3, 2)_{m+m'}-(3, 2)_m-(3, 2)_{m'}=(3)_m(2)_{m'}+(3)_{m'}(2)_m$$
$$=-4(ma'+m'\alpha)-4(na'+n'\alpha)+6aa':$$

the integral is

$$(3, 2)_m=a(m+n)+\alpha(-4m-4n+c)+3\alpha^2,$$

and, as before,

$$0=324+9a+18c,$$
$$0=\ 96+7a+12c,$$
$$0=\ 60+6a+10c,$$

satisfied by $a=120$, $c=-78$.

60. For the calculation of $(3, 1, 1)$ we have similarly

$$(3, 1, 1)_{m+m'}-(3, 1, 1)_m-(3, 1, 1)_{m'}=(3)_m(1, 1)_{m'}+(3)_{m'}(1, 1)_m$$
$$+(3, 1)_m(1)_{m'}+(3, 1)_{m'}(1)_m.$$

The function on the right-hand side was of course calculated from the values of $(3)_m(1, 1)_m$, &c.; but there is no use in this (and the more complicated cases which follow) in actually writing down the values of the function in question; it can in each case be calculated *backwards* from the foregoing expressions of $(3, 1, 1)$ &c., and the values so obtained be verified by actual substitution. But assuming it to be known, the solution of the functional equation gives of course the foregoing expression for $(3, 1, 1)$, except that the terms in $m+n$ and α are therein $a(m+n)+c\alpha$; and I shall in this and the subsequent cases give only the three equations which determine the constants. In the present case these are

$$-332+9a+18c=0,$$
$$-454+7a+12c=0,$$
$$-306+6a+10c=0,$$

satisfied by $a=-434$, $c=291$.

61. Remaining cases $(2, 2, 1)$, $(2, 1, 1, 1)$ and $(1, 1, 1, 1, 1)$. We have

$$(2, 2, 1)_{m+m'}-(2, 2, 1)_{m'}=\ (2, 2)_m(1)_{m'}+(2, 2)_{m'}(1)_m$$
$$+(2, 1)_m(2)_{m'}+(2, 1)_{m'}(2)_m,$$

and

$$-1674+9a+18c=0,$$
$$-\ 648+7a+12c=0,$$
$$-\ 462+6a+10c=0,$$

satisfied by $a=-468$, $c=327$.

Again,

$$(2, 1, 1, 1)_{m+m'}-(2, 1, 1, 1)_m-(2, 1, 1, 1)_{m'}= \begin{array}{l}(2, 1, 1)_m(1)_{m'}+(2, 1, 1)_{m'}(1)_m \\ +(2, 1)_m(1, 1)_{m'}+(2, 1)_{m'}(1)_m \\ +(2)_m(1, 1, 1)_{m'}+(2)_{m'}(1, 1, 1)_m,\end{array}$$

and

$$\begin{aligned} 5400+9a+18c&=0, \\ 2280+7a+12c&=0, \\ 1680+6a+10c&=0, \end{aligned}$$

satisfied by $a=1320$, $c=-960$; and finally,

$$(1, 1, 1, 1, 1)_{m+m'}-(1, 1, 1, 1, 1)_m-(1, 1, 1, 1, 1)_{m'}= \begin{array}{l}(1, 1, 1, 1)_m(1)_{m'}+(1, 1, 1, 1)_{m'}(1)_m \\ +(1, 1, 1)_m(1, 1)_{m'}+(1, 1, 1)_{m'}(1, 1)_m,\end{array}$$

and

$$\begin{aligned} -30618+90a+180c&=0, \\ -14094+70a+120c&=0, \\ -10692+60a+120c&=0, \end{aligned}$$

satisfied by $10a=6318$, $10c=4860$, that is, $a=-\frac{21}{5}\frac{59}{2}$, $c=486$.

62. The contacts of a conic with a given curve which have been thus far considered are contacts at unascertained points of the curve; but a conic may have with the given curve *at a given point thereof* a contact of the first order, the condition will be denoted by $(\overline{2})$; or a contact of the second order, the condition will be denoted by $(\overline{3})$, and so on. It is to be observed that the conditions $(\overline{2})$, $(\overline{3})$, &c. are sibireciprocal, the contact at a given point of the curve is the same thing as contact with a given tangent of the curve; but if we write $(\overline{1})$ to denote the condition of passing through a given point of the curve, this is *not* the same thing as the condition of touching a given tangent of the curve; and this last condition, if it were necessary to deal with it, might be denoted by $(\underline{1})$. But I attend only to the condition $(\overline{1})$. The expressions for the number of conics which satisfy such conditions as $(\overline{1})$, $(\overline{2})$, &c. are obtainable in several ways.

63. (1°) When the total number of conditions is 4, the question may be solved by ZEUTHEN'S method, viz. by determining the line-pairs and point-pairs of the system 4Z, with the proper numerical coefficients, and thence deducing the values of the characteristics $(4Z\cdot)$ and $(4Z/)$. A few cases are in fact thus solved in ZEUTHEN'S work.

64. (2°) By the foregoing functional method. It is to be observed that there is a difference in the form of the functional equation, and that the general solution is always given in the form, *Particular Solution* + *Constant*, so that there is only a single constant to be determined by special considerations. To take the simplest example, let it be required to find the number of the conics $(3Z)(\overline{1}, 1)$: writing for shortness in place hereof $(\overline{1}, 1)$, or (in order to mark the curve (m) to which the symbol has reference) $(\overline{1}, 1)_m$, let the curve (m) be the aggregate of the curves (m) and (m'). Regarding the point $\overline{1}$ as a given point on the curve (m), that is, an arbitrary point in regard to the

curve (m'), we have thus the equation

$$(\overline{1}, 1)_{m+m'} - (\overline{1}, 1)_m = (\cdot\, 1)_{m'},$$

where the right-hand side is known; and so in general the form of the functional equation is always $\varphi(m+m') - \varphi(m) =$ *given value*, that is,

$$\varphi(m+m', n+n', \alpha+\alpha') - \varphi(m, n, \alpha) = \text{given function of } (m, n, \alpha, m', n', \alpha');$$

whence, as stated, the general solution is Particular Solution + Constant. In the case in hand, taking successively $(3Z) = (\therefore)$, $(:/)$, $(\cdot\,//)$, and $(///)$, we have in the first of these cases

$$(\overline{1}, 1)_{m+m'} - (\overline{1}, 1)_m = n' + 2m',$$

whence $(\overline{1}, 1)_m = n + 2m +$ const. $= (\overline{1}, 1)(\therefore)$; and the value of the constant being in any way ascertained to be $= -2$, we have $(\overline{1}, 1)(\therefore) = n + 2m - 2$; and the like for the other three cases.

65. (3^e) The expressions for the number of conics which satisfy such conditions as $(\overline{1})$, $(\overline{2})$, &c. are deducible with more or less facility from the corresponding expressions wherein $(\overline{1})$, $(\overline{2})$, &c. are replaced by $(\cdot)$, $(:)$, &c.; thus from $(::1)^*$ we deduce

$$(\therefore \overline{1}, 1) = (::/) - 2(\therefore \overline{2}) = n + 2m - 2,$$

viz. if one of the four arbitrary points of $(::/)$ becomes a point on the curve, then the condition $(::/)$ is satisfied specially by the conic $(\therefore \overline{2})$ which passes through the remaining three points and touches the curve at the point in question; 2 of the conics $(::/)$ coincide with the conic in question. We have thus a reduction $2(\therefore \overline{2}), = 2$, and the number of the conics $(\therefore \overline{1}, 1)$ is $= n + 2m - 2$. Similarly, we have the system

$$(\therefore \overline{1}, 1\quad) = n + 2m - 2,$$
$$(:\ \overline{1}, \overline{1}, 1\quad) = n + 2m - 4,$$
$$(\cdot\ \overline{1}, \overline{1}, \overline{1}, 1) = n + 2m - 6,$$
$$(\overline{1}, \overline{1}, \overline{1}, \overline{1}, 1) = n + 2m - 8.$$

Again, two or even three of the given points on the curve may come together without any reduction being thereby caused, that is, we have

$$(:\ \overline{2}, 1\quad) \qquad\qquad = n + 2m - 4,$$
$$(\cdot\ \overline{2}, \overline{1}, 1,\ ) = (\cdot\ \overline{3}, 1\ ) = n + 2m - 6,$$
$$(\ \overline{2}, \overline{1}, \overline{1}, 1) = (\ \overline{3}, \overline{1}, 1) = n + 2m - 8;$$

but if the four points on the curve coincide in pairs, or, what is the same thing, if in $(\overline{2}, \overline{1}, \overline{1}, 1)$ the points $\overline{1}$ and $\overline{1}$ come to coincide, then there is a special reduction, and we have

$$(\overline{2}, \overline{2}, 1) = n + 2m - 8\ [-(m-2)] = m + n - 6,$$

* I write indifferently $(1)(::)$, $(1::)$ or $(::1)$; and so in other cases.

viz. here $(m-2)$ of the conics come to coincide with the two points considered as a point-pair or infinitely thin conic. If the points $\overline{2}$ and $\overline{2}$ come to coincide, that is, if the four given points on the curve all coincide, there is no further reduction, but we have

$$(\overline{4}, 1)=m+n-6.$$

66. The expressions involving a single $(\overline{1})$ may in every case be reduced by the foregoing method to depend upon other expressions; thus we have

$$\begin{aligned}
&(3Z\ )(\overline{1}, 1) &&=(\cdot 1) &&-2(\overline{2}) &&,\\
&(2Z\ )(\overline{1}, 2) &&=(\cdot 2) &&-3(\overline{3}) &&,\\
&\ \ \,,\quad (\overline{1}, 1, 1) &&=(\cdot 1, 1) &&-2(\overline{2}, 1) &&,\\
&(Z\ )(\overline{1}, 1, 2) &&=(\cdot 1, 2) &&-2(\overline{2}, 2) &&-3(1, \overline{3}),\\
&\ \ \,,\quad (\overline{1}, 1, 1, 1) &&=(\cdot 1, 1, 1) &&-2(\overline{2}, 1, 1), &&\\
&\ \ \,,\quad (\overline{1}, 3) &&=(\cdot 3) &&-4(\overline{4}) &&,\\
&(\overline{1}, 4) &&=(\cdot 4) &&-5(\overline{5}) &&,\\
&\&c., &&&&&&
\end{aligned}$$

where, comparing for example the equations for $(Z)(\overline{1}, 1, 2)$ and $(2Z)(\overline{1}, 1, 1)$, it will be observed that in the first case the contacts 1, 2 of the symbol $(\overline{1}, 1, 2)$ successively coalesce with the point $\overline{1}$, giving respectively $2(\overline{2}, 2)$ and $3(1, \overline{3})$, the exterior factor being in each case the barred number, whereas the second case, where the contacts 1, 1 of the symbol $(\overline{1}, 1, 1)$ are of the *same* order, we do not consider each of these symbols separately (thus obtaining $2(\overline{2}, 1)+2(1, \overline{2})$, $=4(\overline{2}, 1)$), but the identical symbol is taken only once, giving $2(\overline{2}, 1)$. Thus we have also

$$(\overline{1}, 1, 1, 1, 1)=(\cdot 1, 1, 1, 1)-2(\overline{2}, 1, 1, 1).$$

67. The value of a symbol involving $(\overline{2})$, say the symbol $(3Z)(\overline{2})$, is connected with that of $\frac{1}{3}(3Z\cdot/)$; but as an instance of the correction which is sometimes required I notice the equation

$$(\overline{2}, 1, 1, 1)=\tfrac{1}{3}(1, 1, 1\cdot/)-\{\tfrac{1}{3}(m-2)(m-3)+\tfrac{1}{2}(n-2)(n-3)+3(\overline{3}, 1, 1)+2(\overline{4}, 1)\},$$

which I have verified by other considerations.

68. We obtain the series of results:

$$\begin{aligned}
&(\overline{1})(\ ::)=1,\\
&(\ \therefore/)=2,\\
&(\ :///)=4,\\
&(\cdot///)=4,\\
&(\ ////)=2;
\end{aligned}$$

$$(\bar{1},1)(\therefore) = n+2m-2,$$
$$(:/) = 2n+4m-4,$$
$$(\cdot//) = 4n+4m-4,$$
$$(///) = 4n+2m-2;$$

$$(\bar{1},2)(:) = \alpha-3,$$
$$(\cdot/) = 2\alpha-6,$$
$$(//) = 2\alpha-3;$$

$$(\bar{1},1,1)(:) = 2m^2+2mn+\tfrac{1}{3}n^2-6m-\tfrac{5}{3}n+8-\tfrac{3}{3}\alpha,$$
$$(\cdot/) = 2m^2+4mn+n^2-6m-5n+12-3\alpha,$$
$$(//) = m^2+4mn+2n^2-3m-6n+8-3\alpha;$$

$$(\bar{1},3)(\cdot) = -4m-3n-4+3\alpha,$$
$$(/) = -8m-8n-4+6\alpha;$$

$$(\bar{1},1,2)(\cdot) = 6m+9n+30+\alpha(2m+n-16),$$
$$(/) = 21m+18n+30+\alpha(2m+2n-26);$$

$$(\bar{1},1,1,1)(\cdot) = \tfrac{3}{3}m^3+2m^2n+mn^2+\tfrac{1}{3}n^3-4m^2-7mn-\tfrac{3}{3}n^2+\tfrac{2}{3}\tfrac{3}{3}m-\tfrac{3}{3}n-36+\alpha(-3m-\tfrac{3}{3}n+16),$$
$$(/) = \tfrac{1}{3}m^3+2m^2n+2mn^2+\tfrac{1}{3}n^3-2m^2-8mn-3n^2-\tfrac{1}{3}\tfrac{9}{3}m-\tfrac{4}{3}n-36+\alpha(-3m-3n+23);$$

$$(\bar{2})(\therefore) = 1,$$
$$(:/) = 2,$$
$$(\cdot//) = 2,$$
$$(///) = 1;$$

$$(\bar{2},1)(:) = 2m+n-4,$$
$$(\cdot/) = 2m+2n-6,$$
$$(//) = m+2n-4;$$

$$(\bar{2},2)(\cdot) = \alpha-6,$$
$$(/) = \alpha-6;$$

$$(\bar{2},1,1)(\cdot) = m^2+2mn+\tfrac{1}{3}n^2-7m-\tfrac{8}{3}n+18-\tfrac{3}{3}\alpha,$$
$$(/) = \tfrac{1}{3}m^2+2mn+n^2-\tfrac{8}{3}m-7n+18-\tfrac{3}{3}\alpha;$$

$$\begin{aligned}
(\overline{3})(:) &= 1,\\
(\cdot) &= 2,\\
(//) &= 1;\\
\\
(\overline{3}, 1)(\cdot) &= n+2m-6,\\
(/) &= 2n+m-6;\\
\\
(\overline{4})(\cdot) &= 1,\\
(/) &= 1;
\end{aligned}$$

which are the several cases for the conics which satisfy not more than four conditions, and

69. For the conics satisfying 5 conditions, we have

$$\begin{aligned}
(\overline{5}) &= 1,\\
(\overline{4}, 1) &= m+n-6,\\
(\overline{3}, 2) &= -9+\alpha,\\
(\overline{3}, 1, 1) &= \tfrac{1}{2}m^2+2mn+\tfrac{1}{2}n^2-\tfrac{13}{2}m-\tfrac{13}{2}n+27-\tfrac{3}{2}\alpha,\\
(\overline{2}, 3) &= -4m-4n-6+3\alpha,\\
(\overline{2}, 2, 1) &= 6m+6n+54+\alpha(m+n-15),\\
(\overline{2}, 1, 1, 1) &= \tfrac{1}{6}m^3+m^2n+mn^2+\tfrac{1}{6}n^3-\tfrac{5}{2}m^2-8mn-\tfrac{5}{2}n^2+\tfrac{37}{3}m+\tfrac{37}{3}n-75\\
&\qquad +\alpha(-\tfrac{3}{2}m-\tfrac{3}{2}n+\tfrac{29}{2}),\\
(\overline{1}, 4) &= -10m-8n-5+6\alpha,\\
(\overline{1}, 1, 3) &= -8m^2-12mn-3n^2+60m+57n+36+\alpha(6m+3n-45),\\
(\overline{1}, 2, 2) &= 27m+24n+27-23\alpha+\tfrac{1}{2}\alpha^2,\\
(\overline{1}, 1, 1, 2) &= \tfrac{45}{2}m^2+30mn+\tfrac{21}{2}n^2-\tfrac{321}{2}m-\tfrac{321}{2}n-189\\
&\qquad +\alpha(m^2+2mn+\tfrac{1}{2}n^2-27m-\tfrac{33}{2}n+\tfrac{345}{2})-\tfrac{3}{2}\alpha^2,\\
(\overline{1}, 1, 1, 1, 1) &= \tfrac{1}{12}m^4+\tfrac{1}{3}m^3n+m^2n^2+\tfrac{1}{3}mn^3+\tfrac{1}{24}n^4-\tfrac{5}{6}m^3-5m^2n-4mn^2-\tfrac{7}{2}n^3\\
&\qquad -\tfrac{131}{12}m^2-5mn-\tfrac{209}{4}n^2+\tfrac{425}{6}m+\tfrac{913}{4}n+150\\
&\qquad +\alpha(-\tfrac{3}{2}m^2-3mn-\tfrac{3}{2}n^2+\tfrac{49}{2}m+\tfrac{67}{4}n-\tfrac{473}{4})+\tfrac{9}{8}\alpha^2.
\end{aligned}$$

70. The given point on the curve to which the symbols $\overline{1}$, $\overline{2}$, &c. refer may be a singular point, and in particular it is proper to consider the case where the point is a cusp. I use in this case an appropriate notation; a conic which simply passes through a cusp, in fact meets the curve at the cusp in two points; and I denote the condition of passing through the cusp by $\overline{1\times 1}$; similarly, a conic which touches the curve at the cusp, in fact there meets it in three points, and I denote the condition by $\overline{2\times 1}$; $\overline{1\times 1}$, $\overline{2\times 1}$ are thus special

forms of $\overline{1}$, $\overline{2}$, and the annexed $\overline{1}$ indicates the additional point of intersection arising *ipso facto* from the point $\overline{1}$ or $\overline{2}$ being a cusp. Similarly, we should have the symbols $\overline{3\times 1}$, $\overline{4\times 1}$, $\overline{5\times 1}$; but it is to be observed that at a cusp of the curve there is no *proper* conic having a higher contact than $\overline{2\times 1}$; thus if the symbol contains $\overline{3\times 1}$, or *à fortiori*, if it contain $\overline{4\times 1}$ or $\overline{5\alpha 1}$, the number of the conics is in every case $=0$; it is thus only the cases $\overline{1\times 1}$ and $\overline{2\times 1}$ which need to be considered.

71. The several modes of investigation which apply to the case of contact at a given ordinary point of the curve are applicable to the case of contact at a cusp: we may if we please employ the functional method; we have here a functional equation of the foregoing form, $\varphi(m+m')-\varphi m=$ *given value* (that is, $\varphi(m+m', n+n', \alpha+\alpha')-\varphi(m, n, \alpha)=$ given function of $(m, n, \alpha, m', n', \alpha')$, and the general solution is as before $=$ Particular Solution $+$ Constant; so that there is in each case a single arbitrary constant to be determined by special considerations. The determination of the constant is in some instances conveniently effected by means of the case of the cuspidal cubic: see Annexes Nos. 4 and 5.

The formation of the functional equation itself is similar to that in the corresponding case where the given point on the curve is an ordinary point. For example, we have

$$\begin{aligned}(2Z)(1, \overline{1}, 1)_{m+m'}-(1, \overline{1}, 1)_m= \;& (1, \overline{1})_m(1)_{m'} \;\; = \;\; n'(\overline{1}, 1\cdot)_m+m'(\overline{1}, 1/)_m \\ & +(\overline{1})_m(1, 1)_{m'} \quad +\tfrac{1}{2}(n'^2-n')(\overline{1}:)_m \\ & \qquad\qquad\quad + \;(m'n'-\tfrac{3}{4}\alpha')(\overline{1}\cdot/)_m \\ & \qquad\qquad\quad +\tfrac{1}{2}(m'^2 \;-m')(\overline{1}\;//)_m,\end{aligned}$$

and we may herein simply change $\overline{1}$ into $\overline{1\times 1}$. Writing successively $2Z=(:), (\cdot/)$ and $(//)$, we find

$$\begin{aligned}(\overline{1\times 1}, 1, 1:)_{m+m'}-(:\;)_m &= n'(\;n+2m-3)+m'(2n+2m-6)+(\tfrac{1}{2}n'^2-\tfrac{1}{2}n')1+(m'n'-\tfrac{3}{4}\alpha')2+(\tfrac{1}{2}m'^2-\tfrac{1}{2}m')4, \\ (\qquad\cdot/)_{m+m'}-(\cdot/)_m &= n'(2n+4m-6)+m'(4n+4m-6)+(\tfrac{1}{2}n'^2-\tfrac{1}{2}n')2+(m'n'-\tfrac{3}{4}\alpha')4+(\tfrac{1}{2}m'^2-\tfrac{1}{2}m')4, \\ (\qquad //)_{m+m'}-(\;//)_m &= n'(4n+4m-6)+m'(4n+2m-3)+(\tfrac{1}{2}n'^2-\tfrac{1}{2}n')4+(m'n'-\tfrac{3}{4}\alpha')4+(\tfrac{1}{2}m'^2-\tfrac{1}{2}m')4,\end{aligned}$$

which only differ from the corresponding expressions with $\overline{1}$ in that they contain

$$n+2m-3,\; 2n+4m-6,\; 4n+4m-6,\; 4m+2n-3$$

in place of

$$n+2m-2,\; 2n+4m-4,\; 4n+4m-4,\; 4m+2n-2$$

repectively, and they lead to the expressions for $(\overline{1\times 1}, 1, 1:)$, &c., the arbitrary constant being in each case properly determined.

72. We have

$$\begin{aligned}(\overline{1\times 1})(\;::\;) &= 1, \\ (\therefore/\;) &= 2, \\ (:/\!/\;) &= 4, \\ (\cdot/\!/\!/) &= 4, \\ (\;/\!/\!/\!/) &= 2;\end{aligned}$$

$$(\overline{1x}1, 1)\begin{cases}(\therefore) & = n+2m-3,\\ (:/) & =2n+4m-6,\\ (\cdot//) & =4n+4m-6,\\ (///) & =4n+2m-3;\end{cases}$$

$$(\overline{1x}1, 2)\begin{cases}(:) & = \alpha-4,\\ (\cdot/) & =2\alpha-8,\\ (//) & =2\alpha-4;\end{cases}$$

$$(\overline{1x}1, 1, 1)\begin{cases}(:) & =2m^2+2mn+\tfrac{1}{2}n^2-8m-\tfrac{7}{2}n+13-\tfrac{3}{2}\alpha,\\ (\cdot/) & =2m^2+4mn+\ n^2-8m-7n+18-3\alpha,\\ (//) & =\ m^2+4mn+2n^2-4m-8n+12-3\alpha;\end{cases}$$

$$(\overline{1x}1, 3)\begin{cases}(\cdot) & =-4m-3n-5+3\alpha,\\ (/) & =-8m-8n-6+6\alpha;\end{cases}$$

$$(\overline{1x}1, 1, 2)\begin{cases}(\cdot) & =\ 4m+\ 8n+44+\alpha(2m+\ n-17),\\ (/) & =20m+16n+42+\alpha(2m+2n-27);\end{cases}$$

$$(\overline{1x}1, 1, 1, 1)\begin{cases}(\cdot) & =\tfrac{2}{3}m^3+2m^2n+\ mn^2+\tfrac{1}{3}n^3-5m^2-\ 9mn-2n^2+\tfrac{43}{6}m+\tfrac{29}{6}n-57+\alpha(-3m-\tfrac{5}{2}n+\tfrac{35}{2}),\\ (/) & =\tfrac{1}{3}m^3+2m^2n+2mn^2+\tfrac{1}{3}n^3-\tfrac{5}{2}m^2-10mn-4n^2-\tfrac{11}{6}m+\tfrac{17}{3}n-54+\alpha(-3m-3n+\tfrac{49}{2});\end{cases}$$

$$(\overline{2x}1)\begin{cases}(\therefore) & =1,\\ (:/) & =2,\\ (\cdot//) & =2,\\ (///) & =1;\end{cases}$$

$$(\overline{2x}1, 1)\begin{cases}(:) & =2m+\ n-5,\\ (\cdot/) & =2m+2n-6,\\ (//) & =\ m+2n-4;\end{cases}$$

$$(\overline{2x}1, 2)\begin{cases}(\cdot) & =\alpha-7,\\ (/) & =\alpha-6;\end{cases}$$

$$(\overline{2x}1, 1, 1)\begin{cases}(\cdot) & =\ m^2+2mn+\tfrac{1}{2}n^2-7m-\tfrac{11}{2}n+21-\tfrac{5}{2}\alpha,\\ (/) & =\tfrac{1}{2}m^2+2mn+\ n^2-\tfrac{9}{2}m-\ 7n+18-\tfrac{3}{2}\alpha.\end{cases}$$

73. The remainder of this table, being the part where the symbols (·) and (/) do not occur, I present under a somewhat different form as follows:—

$$
\begin{aligned}
&(5\overline{2\varkappa 1}) && =0,\\
&(4\overline{2\varkappa 1}, 1) && =0,\\
&(3\overline{2\varkappa 1}, 2) && =0,\\
&(3\overline{2\varkappa 1}, 1, 1) && =0,\\
&(\overline{2}, 3) && -(\overline{2\varkappa 1}, 3) && =0,\\
&(\overline{2}, 2, 1) && -(\overline{2\varkappa 1}, 2, 1) && =n-3,\\
&(\overline{2}, 1, 1, 1) && -(\overline{2\varkappa 1}, 1, 1, 1) && =\tfrac{1}{2}(n-3)(n-4),\\
&(\overline{1}, 4) && -(\overline{1\varkappa 1}, 4) && =1,\\
&(\overline{1}, 1, 3) && -(\overline{1\varkappa 1}, 1, 3) && =(\overline{2\varkappa 1}, 3)+(n-3),\\
&(\overline{1}, 2, 2) && -(\overline{1\varkappa 1}, 2, 2) && =3(n-3)+\varkappa-1,\\
&(\overline{1}, 1, 1, 2) && -(\overline{1\varkappa 1}, 1, 1, 2) && =(\overline{2\varkappa 1}, 1, 2)+\tfrac{1}{2}(n-3)(n-4)+\delta+\overline{2n-3}\ \overline{m-4},\\
&(\overline{1}, 1, 1, 1, 1)-(\overline{1\varkappa 1}, 1, 1, 1, 1) && =(\overline{2\varkappa 1}, 1, 1, 1).
\end{aligned}
$$

These results relating to a cusp, are useful for the investigations contained in the Second Memoir.

It will be noticed that the symbols which contain $\overline{2\varkappa 1}$ are not, like those which contain $\overline{2}$, symmetrical in regard to (m, n): the interchange of (m, n) would of course imply the change of a cusp into an inflexion, and would therefore give rise to a new symbol such as $\underline{2\varkappa 1}$; but I have not thought it necessary to consider the formulæ which contain this new symbol.

Investigations in extension of those of DE JONQUIÈRES *in relation to the contacts of a Curve of the order* r *with a given curve.*—Nos. 74 to 93.

74. DE JONQUIÈRES has given a formula for the number of curves C^r of the order r which have with a given curve U^m of the mth order t contacts of the orders a, b, c, &c. respectively, which besides pass through p points distributed at pleasure on the curve U^m (this includes the case of contacts of any orders at given points of the curve U^m), and which moreover satisfy any other $\frac{r(r+3)}{2}-(a+b+c+\&c.)-p$ conditions; viz. the number of the curves C^r is $=\mu(a+1)(b+1)(c+1)\ldots$ into

$$
\left\{
\begin{aligned}
&[rm-(a+b+c\ldots)-p\quad]'\\
&+[rm-(a+b+c\ldots)-p-1]'^{-1}(a+b+c\ \ldots)[\mathrm{D}]'\\
&+[rm-(a+b+c\ldots)-p-2]'^{-2}(ab+ac+bc\ldots)[\mathrm{D}]'^{2}\\
&\quad\vdots\\
&+[rm-(a+b+c\ldots)-p-t]^{\circ}\ (abc\ldots\quad\ )[\mathrm{D}]',
\end{aligned}
\right.
$$

where the curve U^m is a curve *without cusps*, and having therefore a deficiency $D=\frac{1}{2}(m-1)(m-2)-\delta$; the numbers $a, b, c, \ldots$ are assumed to be all of them unequal, but if we have α of them each $=a$, β of them each $=b$, &c., then the foregoing expression is to be divided by $[\alpha]^\alpha[\beta]^\beta\ldots$; and μ denotes the number of the curves C^r which satisfy the system of conditions obtained from the given system by replacing the conditions of the t contacts of the orders a, b, c, &c. respectively by the condition of passing through $a+b+c\ldots$ arbitrary points. In order that the formula may give the number of the *proper curves* C^r which satisfy the prescribed conditions, it is sufficient that the $\frac{1}{2}r(r+3)-(a+b+c\ldots)-p$ conditions shall include the conditions of passing through at least a certain number T of arbitrary points: this restriction applies to all the formulæ of the present section.

75. I will for convenience consider this formula under a somewhat less general form, viz. I will put $p=0$, and moreover assume that the $\frac{1}{2}r(r+3)-(a+b+c\ldots)$ conditions are the conditions of passing through this number of arbitrary points; whence $\mu=1$.

We have thus a curve C^r having with the given curve U^m t contacts of the orders $a, b, c\ldots$ respectively, and besides passing through $\frac{1}{2}r(r+3)-(a+b+c\ldots)$ arbitrary points; and the number of such curves is by the formula $=(a+1)(b+1)(c+1),\ldots$ into

$$\left\{\begin{array}{l} [rm-(a+b+c\ldots)\quad]^t \\ +[rm-(a+b+c\ldots)-1]^{t-1}(a+b+c\ldots)[D]^1 \\ +[rm-(a+b+c\ldots)-2]^{t-2}(ab+ac+bc\ldots)[D]^2 \\ \quad\vdots \\ +[rm-(a+b+c\ldots)-t]^0\ (abc\ldots\quad\ )[D]^t, \end{array}\right.$$

where, as before, in the case of any equalities between the numbers $a, b, c, \ldots$, the expression is to be divided by $[\alpha]^\alpha[\beta]^\beta\ldots$

76. I have succeeded in extending the formula to the case of a curve with cusps: instead of writing down the general formula, I will take successively the cases of a single contact a, two contacts a, b, three contacts a, b, c, &c.; and then denoting the numbers of the curves C^r by (a), (a, b), (a, b, c), &c. in these cases respectively, I say that we have

$$(a)= (a+1)\left\{\begin{array}{l} [rm-a \\ +aD \end{array}\right\} -a\ .\ .\ \varkappa;$$

$$(a,b)= (a+1)(b+1)\left\{\begin{array}{l} [rm-a-b]^2 \\ +[rm-a-b-1]^1(a+b)[D]^1 \\ +\qquad ab\quad [D]^2 \end{array}\right\} - \left[\begin{array}{l} a(b+1)\left\{\begin{array}{l} [rm-a-b-1]^1 \\ +\qquad bD \end{array}\right. \\ +b(a+1)\left\{\begin{array}{l} [rm-a-b-1]^1 \\ +\qquad aD \end{array}\right. \end{array}\right][\varkappa]^1$$

$$+\quad ab\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ [\varkappa]^1$$

$$(a, b, c) = (a+1)(b+1)(c+1)\left\{\begin{array}{l} [rm-a-b-c\quad]^3 \\ +[rm-a-b-c-1]^2(a+b+c)[D]^1 \\ +[rm-a-b-c-2]^1(ab+acbc)[D]^2 \\ +\qquad\qquad abc\quad [D]^3 \end{array}\right\}$$

$$-[\Sigma c(a+1)(b+1)\left\{\begin{array}{l} [rm-a-b-c-1]^3 \\ +[rm-a-b-c-2]^1(a+b)[D]^1 \\ +\qquad\quad ab\quad [D]^2 \end{array}\right\}][x]^1$$

$$+[\Sigma bc(a+1)\left\{\begin{array}{l} [rm-a-b-c-2]^1 \\ +\qquad\quad aD \end{array}\right\}][x]^2$$

$$-abc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [x]^3$$

77. The foregoing examples are sufficient to exhibit the law; but as I shall have to consider the cases of four and five contacts, I will also write down the formula for (a, b, c, d), putting therein for shortness

$$a+b+c+d=\alpha,\ ab+\ldots+cd=\beta,\ abc\ldots+bcd=\gamma,\ abcd=\delta,$$

$$a+b+c=\alpha',\ ab+ac+bc=\beta',\ abc=\gamma',\ a+b=\alpha'',\ \ ab+\beta'',\ a=\alpha''';$$

and also the formula for (a, b, c, d, e), putting therein in like manner

$$(\alpha, \beta, \gamma, \delta, \varepsilon),\ (\alpha', \beta', \gamma', \delta'),\ (\alpha'', \beta'', \gamma''),\ (\alpha''', \beta'''),\ (\alpha'''')$$

for the combinations of (a, b, c, d, e), (a, b, c, d), (a, b, c), (a, b) and (a) respectively. We have

$$(a, b, c, d) = (a+1)(b+1)(c+1(d+1)\left\{\begin{array}{l} [rm-\alpha\quad]^4 \\ +[rm-\alpha-1]^3\alpha[D]^1 \\ +[rm-\alpha-2]^2\beta[D]^2 \\ +[rm-\alpha-3]^1\gamma[D]^3 \\ +\qquad\quad \delta[D]^4 \end{array}\right\}$$

$$[\Sigma d(a+1)(b+1)(c+1)\left\{\begin{array}{l} [rm-\alpha-1]^3 \\ +[rm-\alpha-2]^2\alpha'\ [D]^1 \\ +[rm-\alpha-3]^1\beta'\ [D]^2 \\ +\qquad\quad \gamma'\ [D]^3 \end{array}\right\}][x]^1$$

$$+[\Sigma cd.(a+1)(b+1)\left\{\begin{array}{l} [rm-\alpha-2]^2 \\ +[rm-\alpha-3]^1\alpha''\ [D]^1 \\ +\qquad\quad \beta''\ [D]^2 \end{array}\right\}][x]^2$$

$$-[\Sigma bcd(a+1)\left\{\begin{array}{l} [rm-\alpha-3]^1 \\ +\qquad\quad \alpha'''\ [D]^1 \end{array}\right\}][x]^3$$

$$+\ abcd \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [x]^4$$

$$(a, b, c, d, e)=(a+1)(b+1)(c+1)(d+1)(e+1)\left\{\begin{array}{ll} & [rm-\alpha\ \ ]^{s} \\ +[rm-\alpha-1]^{s}\alpha & [\mathrm{D}]^{1} \\ +[rm-\alpha-2]^{s}\beta & [\mathrm{D}]^{2} \\ +[rm-\alpha-3]^{s}\gamma & [\mathrm{D}]^{3} \\ +[rm-\alpha-4]^{1}\delta & [\mathrm{D}]^{4} \\ + & \epsilon\ [\mathrm{D}]^{5} \end{array}\right.$$

$$-[\Sigma e(a+1)(b+1)(c+1)(d+1)\left\{\begin{array}{ll} & [rm-\alpha-1]^{4} \\ +[rm-\alpha-2]^{3}\alpha' & [\mathrm{D}]^{1} \\ +[rm-\alpha-3]^{2}\beta' & [\mathrm{D}]^{2} \\ +[rm-\alpha-4]^{1}\gamma' & [\mathrm{D}]^{3} \\ + & \delta'\ [\mathrm{D}]^{4} \end{array}\right.][\varkappa]^{1}$$

$$+[\Sigma de(a+1)(b+1)(c+1)\left\{\begin{array}{ll} & [rm-\alpha-2]^{3} \\ +[rm-\alpha-3]^{2}\alpha'' & [\mathrm{D}]^{1} \\ +[rm-\alpha-4]^{1}\beta'' & [\mathrm{D}]^{2} \\ + & \gamma''\ [\mathrm{D}] \end{array}\right.][\varkappa]^{2}$$

$$-[\Sigma cde(a+1)(b+1)\left\{\begin{array}{ll} & [rm-\alpha-3]^{3} \\ +[rm-\alpha-4]^{1}\alpha''' & [\mathrm{D}]^{1} \\ + & \beta'''[\mathrm{D}]^{2} \end{array}\right.][\varkappa]^{3}$$

$$+[\Sigma bcde(a+1)\left\{\begin{array}{ll} & [rm-\alpha-4]^{1} \\ + & \alpha'''[\mathrm{D}]^{1} \end{array}\right.][\varkappa]^{4}$$

$$-\ abcde\ \ .\ \ .\ \ .\ \ .\ \ .\ \ [\varkappa]^{5}.$$

78. In all these formulæ there is, as before, a numerical divisor in the case of any equalities among the numbers a, b, c, &c. And D denotes, as before, the deficiency, viz. its value now is $\mathrm{D}=\frac{1}{2}(m-1)(m-2)-\delta-\varkappa$; or observing that the class n is $=m^{2}-m-2\delta-3\varkappa$, we have $\mathrm{D}=\frac{1}{2}n-m+1+\frac{1}{2}\varkappa$, or say $\mathrm{D}=1-m+\frac{1}{2}n+\frac{1}{2}\varkappa$, $=1+\Delta$ if $\Delta=-m+\frac{1}{2}n+\frac{1}{2}\varkappa$.

79. It is to be observed with reference to the applicability of these formulæ within certain limits only, that the formulæ are the *only* formulæ which are *generally* true; thus taking the simplest case, that of a single contact a, the only algebraical expression for the number of the curves C^{r} which have with a given curve U a contact of the order a, and besides pass through the requisite number $\frac{1}{2}r(r+3)-a$ of arbitrary points, is that given by the formula, viz.

$$(a)=(a+1)(rm-a+a\mathrm{D})-a\varkappa.$$

Considering the curve U^{m} and the order r of the curve C^{r} as given, if a has successively

the values 1, 2, ... up to a limiting value of α, the formula gives the number of the proper curves C^r which have with the given curve U^m a contact of the required order α: beyond this limiting value the formula no longer gives the number of the proper curves C^r which satisfy the required condition, and it thus ceases to be applicable; but there is no algebraic function of α which would give the number of the proper curves C^r as well beyond as up to the foregoing limiting value of α.

80. The formulæ are applicable provided only the conditions include the conditions of passing through a sufficient number of arbitrary points; viz. when the number of arbitrary points is sufficiently great, it is not possible to satisfy the conditions specially by means of improper curves C^r, being or comprising a pair of coincident curves. Thus to take a simple example, suppose it is required to find the number of the conics which touch a given curve t times and besides pass through $5-t$ given points: if the number of the given points be 4 or 3 there is no coincident line-pair through the given points, and therefore no coincident line-pair satisfying the given conditions; if the number of the given points is $=2$, then the line joining these points gives a coincident line-pair having at each of its m intersections with the given curve a special contact therewith, that is, having in $\frac{1}{6}m(m-1)(m-2)$ ways three special contacts with the given curve; if the number of the given points is 1 or 0, then in the first case any line whatever through the given point, and in the second case any line whatever, regarded as a coincident line-pair, has m special contacts with the given curve; and so in general there is a certain value for the number of given points, for which value the conditions of contact may be satisfied by a determinate number of improper curves C^r, and for values inferior to it the conditions may be satisfied by infinite series of improper curves C^r. It is by such considerations as these that De Jonquières has determined the minimum value T of the number of arbitrary points to which the conditions should relate in order that the formulæ may be applicable: I refer for his investigation and results to paragraphs XVII and XVIII of his memoir. I remark that in the case where the number of improper solutions is finite, the formula can be corrected so as to give the number of proper solutions by simply subtracting the number of the improper solutions: but this is not so when the improper solutions are infinite in number; the mode of obtaining the approximate formula is here to be sought in the considerations contained in the first part of the present Memoir; see in particular *ante*, Nos. 8, 9 & 10.

81. The expressions for (a), (a, b), &c. may be considered as functions of rm, $1+\Delta$, and $\varkappa$, and they vanish upon writing therein $rm=0$, $\Delta=0$, $\varkappa=0$; they are consequently of the form $(rm, \Delta, \varkappa)^1+(rm, \Delta, \varkappa)^2+$ &c., and I represent by $[a]$, $[a, b]$, &c. the several terms $(rm, \Delta, \varkappa)^1$, which are the portions of (a), (a, b), &c. respectively, linear in rm, Δ, and $\varkappa$. The terms in question are obtained with great facility; thus, to fix the ideas, considering the expressions for (a, b, c, d),—

1°. To obtain the term in rm, we may at once write $D=1$, $\varkappa=0$, the expression is thus reduced to

$$(a+1)(b+1)(c+1)(d+1)\{[rm-\alpha]^1+[rm-\alpha-1]^2\alpha\},$$

and the factor in { } being $=rm[rm-\alpha-1]^3$, the coefficient of rm is

$$(a+1)(b+1)(c+1)(d+1)[-\alpha-1]^3,$$

which is

$$=-(a+1)(b+1)(c+1)(d+1)\,.\,(\alpha+1)(\alpha+2)(\alpha+3).$$

2°. To obtain the term in Δ, writing $rm=0$, $\varkappa=0$, and observing that $[D]^1=\Delta+1$, $[D]^2=(\Delta+1)\Delta$, $[D]^3=(\Delta+1)\Delta(\Delta-1)$, $[D]^4=(\Delta+1)\Delta(\Delta-1)(\Delta-2)$, &c. give the terms Δ, Δ, $-\Delta$, $+2\Delta$, -6Δ, &c. respectively, the term in Δ is

$$(a+1)(b+1)c+1)(d+1)\left\{\begin{array}{rr} [-\alpha-1]^3\alpha\,. & 1 \\ +[-\alpha-2]^2\beta\,. & 1 \\ +[-\alpha-3]^1\gamma\,. & -1 \\ + \qquad \delta\,. & 2 \end{array}\right\}\Delta$$

$$=(a+1)(b+1)(c+1)(d+1)\left\{\begin{array}{rr} - & \alpha(\alpha+1)(\alpha+2)(\alpha+3) \\ + & \beta \qquad (\alpha+2)(\alpha+3) \\ + & \gamma \qquad\qquad (\alpha+3) \\ +2 & \delta \end{array}\right\}\Delta$$

3°. For the term in $\varkappa$, writing $rm=0$, $D=1$, and observing that $[\varkappa]^1$, $[\varkappa]^2$, $[\varkappa]^3$, $[\varkappa]^4$ give respectively the terms $\varkappa$, $-\varkappa$, $2\varkappa$, $-6\varkappa$, this is

$$=\left[\begin{array}{llll} -\Sigma d & (a+1)(b+1)(c+1) & \{[-\alpha-1]^3+[-\alpha-2]^2\alpha'\}\,. & 1 \\ +\Sigma cd & (a+1)(b-1) & \{[-\alpha-2]^2+[-\alpha-3]^1\alpha''\}\,. & -1 \\ -\Sigma bcd & (a-1) & \{[-\alpha-3]^1+\qquad \alpha'''\}\,. & 2 \\ + \quad abcd & & & -6 \end{array}\right]\varkappa,$$

where the terms in { } are

$$-(\alpha+1-\alpha')(\alpha+2)(\alpha+3),\ (\alpha+2-\alpha'')(\alpha+3) \text{ and } -(\alpha+3-\alpha'''),$$

that is,

$$-(d+1)(\alpha+2)(\alpha+3),\ (c+d+2)(\alpha+3) \text{ and } -(b+c+d+3)$$

respectively: whence the whole expression is

$$=\left[\begin{array}{lll} \Sigma d & (a+1)(b+1)(c+1)(d+1)\,.\,(\alpha+2)(\alpha+3) \\ -\ \Sigma cd & (c+d+2)(a+1)(b+1). \qquad (\alpha+3) \\ +2\Sigma bcd & (b+c+d+3)(a+1) \\ -\ 6abcd & \end{array}\right]\varkappa,$$

the expression multiplying $(\alpha+2)(\alpha+3)$ is

$$(a+1)(b+1)(c+1)(d+1)\Sigma d=(a+1)(b+1)(c+1)(d+1)\alpha;$$

and we have moreover

$$(a+1)(b+1)(c+1)(d+1)=(1+\alpha+\beta+\gamma+\delta);$$

the other lines are of course expressible in terms of $(\alpha, \beta, \gamma, \delta)$, but as the law of their formation would then be hidden, I abstain from completing the reduction.

82. The series of formulæ is

$$\begin{aligned}[a] &= (a+1)rm \\ &\quad +(a+1)a\Delta \\ &\quad - \quad a\varkappa,\end{aligned}$$

$$\begin{aligned}[a, b] = &-(a+1)(b+1)(\alpha+1) \quad ..rm \\ &-(a+1)(b+1)\begin{Bmatrix} \alpha(\alpha+1) \\ - \quad \beta \end{Bmatrix}\Delta \\ &+ \quad \begin{Bmatrix} \Sigma b(a+1)(b+1) \\ - \ ab \end{Bmatrix}\varkappa,\end{aligned}$$

where $\alpha = a+b$, $\beta = ab$; and coeff. of $\varkappa$ expressed in terms of α, β is $= \alpha(1+\alpha+\beta)-\beta$.

$$\begin{aligned}[a, b, c] = &\quad (a+1)(b+1)(c+1)(\alpha+1)(\alpha+2) \quad ..rm \\ &+(a+1(b+1)(c+1)\begin{Bmatrix} \alpha(\alpha+1)(\alpha+2) \\ - \quad \beta(\alpha+2) \\ - \quad \gamma \end{Bmatrix}\Delta \\ &+\begin{Bmatrix} -\Sigma\ c(a+1)(b+1)(c+1)(\alpha+2) \\ +\Sigma bc(b+c+2)(a+1) \\ -2\,abc \end{Bmatrix}\varkappa,\end{aligned}$$

where $\alpha = a+b+c$, $\beta = ab+ac+bc$, $\gamma = abc$; and the coefficient of $\varkappa$ expressed in terms of α, β, γ is $= -\alpha^3-\alpha^2\beta-\alpha^2\gamma-3\alpha^2-\alpha\beta-2\alpha+2\beta+\gamma$.

$$\begin{aligned}[a, b, c, d] = &-(a+1)(b+1)(c+1)(d+1)\ (a+1)(\alpha+2)(\alpha+3) \quad ..rm \\ &-(a+1)(b+1)(c+1)(d+1)\begin{Bmatrix} \alpha(\alpha+1)(\alpha+2)(\alpha+3) \\ - \ \beta \quad (\alpha+2)(\alpha+3) \\ - \ \gamma \quad (\alpha+3) \\ -2\delta \end{Bmatrix}\Delta \\ &+\begin{Bmatrix} + \ \Sigma \quad d(a+1)(b+1)(c+1)(d+1)(\alpha+2)(\alpha+3) \\ - \ \Sigma \ cd(c+d+2) \quad (a+1)(b+1) \quad (\alpha+3) \\ +2\Sigma \ bcd(b+c+d+3)(a+1) \\ -6 \ abcd \end{Bmatrix}\varkappa,\end{aligned}$$

where $\alpha = a+b+c+d$, .. $\delta = abcd$.

$$[a, b, c, d, e] = (a+1)(b+1)(c+1)(d+1)(e+1)(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4) \quad rm$$

$$+(a+1)(b+1)(c+1)(d+1)(e+1)\begin{vmatrix} \alpha(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4) \\ -\beta & (\alpha+2)(\alpha+3)(\alpha+4) \\ -\gamma & (\alpha+3)(\alpha+4) \\ -2\delta & (\alpha+4) \\ -6\varepsilon \end{vmatrix} \Delta$$

$$+\begin{vmatrix} -\Sigma e & (a+1)(b+1)(c+1)(d+1)(e+1) & (\alpha+2)(\alpha+3)(\alpha+4) \\ +\Sigma de & (d+e+2)(a+1)(b+1)(c+1) & (\alpha+3)(\alpha+4) \\ -2\Sigma cde & (c+d+e+3)(a+1)(b+1) & (\alpha+4) \\ +6\Sigma bcde & (b+c+d+e+4)(a+1) \\ -24abcde \end{vmatrix} x,$$

where $\alpha = a+b+c+d+e$, $\beta =$ &c., $\ldots$ $\varepsilon = abcde$.

83. The complete functions (a), (a, b), (a, b, c), &c. may be expressed by means of the linear terms $[a]$, $[a, b]$, $[a, b, c]$, &c. as follows, viz. we have

$$\begin{aligned}
(a) &= [a] \\
(a, b) &= [a][b] \\
&\quad + [a, b], \\
(a, b, c) &= [a][b][c] \\
&\quad + [a][b, c]+[b][a, c]+c[a, b] \\
&\quad + [a, b, c], \\
(a, b, c, d) &= [a][b][c][d] \\
&\quad +\Sigma[a][b][c, d] \\
&\quad +\Sigma[a, b][c, d] \\
&\quad +\Sigma[a][b, c, d] \\
&\quad + [a, b, c, d],
\end{aligned}$$

and so on: this is easily verified for (a, b), and without much difficulty for (a, b, c), but in the succeeding cases the actual verification would be very laborious.

84. The theoretical foundation is as follows. Writing for greater distinctness $(a)_m$ in place of (a), we have $(a)_m$ to denote the number of the curves C^r which have with a given curve U^m a contact of the order a, and which besides pass through $\frac{1}{2}r(r+3)-a$ points. Let the curve U^m be the aggregate of two curves of the orders m, m' respectively, or say let the curve U^m be the two curves m, m', then we have

$$(a)_{m+m'} = (a)_m + (a)_{m'},$$

a functional equation, the solution of which is

$$(a)_m = [a]_m,$$

where $[a]_m$ is a linear function of n, m, $\varkappa$, or, what is the same thing, of m, Δ, $\varkappa$. I assume for the moment that when the coefficients are determined $[a]_m$ would be found to have the value $=[a]$.

Similarly, if $(a, b)_m$ denote the number of the curves C^r which have with the given curve U^m contacts of the orders a and b respectively, and which besides pass through $\frac{1}{2}r(r+3)-a-b$ points, then if the given curve break up into the curves m, m', then we have

$$(a, b)_{m+m'}-(a, b)_m-(a, b)_{m'}=\{(a)_m(b)_{m'}\}+\{(a)_{m'}(b)_m\},$$

where $\{(a)_m(b)_{m'}\}$ is the number of the curves C^r which have with m a contact of the order a and with m' a contact of the order b, and which pass through the $\frac{1}{2}r(r+3)-a-b$ points; and the like for $\{(a)_{m'}(b)_m\}$. Then, not universally, but for values of a and b which are not too great, the order of the aggregate condition is equal to the product of the orders of the component conditions (*antè*, No. 12), that is, we have

$$\{(a)_m(b)_{m'}\}=(a)_m\,.\,(b)_{m'}=[a]_m[b]_{m'},$$
$$\{(a)_{m'}(b)_m\}=(a)_{m'}\,.\,(b)_m=[a]_{m'}[b]_m,$$

and thence the functional equation

$$(a, b)_{m+m'}-(a, b)_m-(a, b)_{m'}=[a]_m[b]_{m'}+[a]_{m'}[b]_m.$$

But $[a]_m$ &c. being linear functions of m, Δ, $\varkappa$, we have

$$[a]_{m+m'}=[a]_m+[a]_{m'},\ [b]_{m+m'}=[b]_m+[b]_{m'},$$

and thence a particular solution of the equation is at once seen to be $[a]_m[b]_m$; the general solution is therefore

$$(a, b)_m=[a]_m[b]_m+[a, b]_m,$$

where $[a, b]_m$ is an arbitrary linear function of m, Δ, $\varkappa$. Hence, assuming for the present that if determined its value would be found to be $=[a, b]$, we have the required formula $(a, b)=[a][b]+[a, b]$.

The investigation of the expression for $(a, b, c)_m$ depends in like manner on the assumption that we have

$$\{(a)_m(b, c)_{m'}\}=(a)_m\,.\,(b, c)_{m'}=[a]_m\{[b]_{m'}[c]_{m'}+[b, c]_{m'}\},$$

and so in the succeeding cases; and we thus, within the limits in which these assumptions are correct, obtain the series of formulæ for (a, b), (a, b, c)....

85. It is to be observed in the investigation of (a, b) that if $a=b$, the two terms $[a]_m[b]_{m'}$ and $[a]_{m'}[b]_m$ become equal, and the equal value must be taken not twice but only once, that is, the functional equation is

$$(a, a)_{m+m'}-(a, a)_m-(a, a)_{m'}=[a]_m[a]_{m'},$$

and the solution, writing $\frac{1}{2}[a, a]_m$ for the arbitrary linear function, is

$$(a, a)_m=\tfrac{1}{2}[a]_m[a]_m+\tfrac{1}{2}[a, a]_m,$$

in which solution it would appear, by the determination of the arbitrary function, that $[a, a]$ has the value obtained from $[a, b]$ by writing therein $b=a$. Writing the equation in the form

$$(a, a)=\tfrac{1}{2}[a][a]+\tfrac{1}{2}[a, a],$$

and comparing with the equation for (a, b), we see that $[a, b]$ is *not* to be considered as acquiring any divisor when b is put $=a$, but that the divisor is introduced as a divisor of the whole right-hand side of the equation in virtue of the remark as to the divisor of the functions (a, b), $(a, b, c)\ldots$ in the case of any equalities between the numbers $(a, b, c\ldots)$. This is generally the case, and the foregoing expressions for $[a, b]$, $[a, b, c]$, &c. are thus to be regarded as true without modification even in the case of any equalities among the numbers $a, b, c\ldots$.

86. To complete according to the foregoing method the determination of the expressions for (a), (a, b), . . , we have to determine the linear functions $[a]$, $[a, b]$, &c., which are each of them of the form $fm+g\Delta+h\varkappa$, where (f, g, h) are functions of r and of a, b, &c.; and I observe that the determination can be effected if we know the values of (a), (a, b), &c. in the cases of a unicursal curve without cusps and with a single cusp respectively. Thus assume that in these two cases respectively we have

$$(a)=(a+1)(rm-a),$$
$$(a)=(a+1)(rm-a)-a.$$

Writing first $\Delta=-1$, $\varkappa=0$, and secondly $\Delta=-1$, $\varkappa=1$, we have

$$(a+1)(rm-a) \quad =fm-g,$$
$$(a+1)(rm-a)-a=fm-g+h,$$

whence

$$f=(a+1)r,\quad g=(a+1)a,\quad h=-a,$$

giving the foregoing value

$$[a]=(a+1)rm+(a+1)a\Delta-a\varkappa.$$

Similarly, for two contacts assume that we have in the two cases respectively

$$(a, b)=(a+1)(b+1)[rm-a-b]^2,$$
$$(a, b)=(a+1)(b+1)[rm-a-b]^2-\{a(b+1)+b(a+1)\}[rm-a-b-1]^1.$$

Starting here from the formula $[a, b]=(a, b)-[a][b]=fm+g\Delta+h\varkappa$, and writing successively $\Delta=-1$, $\varkappa=0$, and $\Delta=-1$, $\varkappa=1$, we have

$$(a+1)(b+1)[rm-a-b]^2-\{(a+1)(rm-a)\}\{(b+1)(rm-b)\} \quad =fm-g,$$
$$(a+1)(b+1)[rm-a-b]^2-\{a(b+1)+b(a+1)\}[rm-a-b-1]^1$$
$$-\{(a+1)(rm-a)-a\}\{(b+1)(rm-b)-b\}=fm-g+h;$$

the first of which, putting therein $a+b=\alpha$, $ab=\beta$, is at once reduced to

$$(a+1)(b+1)\{rm(-\alpha-1)+\alpha(\alpha+1)-\beta\}=fm-g,$$

whence $f=-(a+1)(b+1)(\alpha+1)r$, $g=-(a+1)(b+1)(\alpha(\alpha+1)-\beta)$. And taking the difference of the two equations, we have

$$-\{a(b+1)+b(a+1)\}(rm-a-b-1)$$
$$+a(b+1)(rm-b)+b(a+1)(rm-a)-ab=h,$$

that is $h=(a+1)(b+1)(a+b)-ab$; whence $[a, b]$ has the value above assigned to it.

87. The actual calculation of $[a, b, c]$ would be laborious, and that of the subsequent

terms still more so; but it is clear that the principle applies, and that the foregoing values, assuming them to be correct, would be obtained if only we know, for a unicursal curve without cusps, that

$$(a, b, c, \ldots) = (a+1)(b+1)(c+1)\ldots[rm-(a+b+c, \ldots)]^t$$

(t the number of contacts $a, b, c, \ldots$), and for a unicursal curve with a single cusp, that

$$(a, b, c, \ldots) = (a+1)(b+1)(c+1)\ldots[rm-(a+b+c\ldots)\quad]^t$$
$$-\Sigma a(b+1)(c+1)\ldots[rm-(a+b+c\ldots)-1]^{t-1},$$

viz. that the diminution of $a, b, c, \ldots$) occasioned by the single cusp is

$$=[rm-(a+b+c, \ldots)-1]^{t-1}.\Sigma\{a(b+1)(c+1)\ldots\}.$$

88. Consider a unicursal curve U^m, and a curve C^r having therewith t contacts of the orders $a, b, c, \ldots$ respectively. The coordinates (x, y, z) of any point of the unicursal curve are given as functions of the order m of a variable parameter θ; and substituting these values in the equation of the curve C^r, we have an equation of the degree rm in θ, but containing the coefficients of C^r linearly; this equation gives of course the values of θ which correspond to the rm intersections of the two curves. Hence in order that the curve C^r may have the prescribed contacts with U^m, the equation of the degree rm in θ must have t systems of equal roots, viz. a system of a equal roots, another system of b equal roots, &c.: this implies between the coefficients of the equation an $(a+b+c, \ldots)$-fold relation, which may be shown to be of the order $(a+1)(b+1)(c+1)\ldots.$ $[rm-(a+b+c, \ldots)]^t$; and since the coefficients in question are linear in regard to the coefficients in the equation of the curve C^r, the order of the relation between the last-mentioned coefficients has the same value; that is, the number of the curves C^r which have the prescribed contacts with the unicursal curve U^m and besides pass through the requisite number of given points, is $=(a+1)(b+1)(c+1)\ldots.[rm-(a+b+c, \ldots)]^t$.

89. The reduction in the case of a cusp appears to be caused as follows:—Consider on the curve U^m a points indefinitely near to the cusp, and let the condition of the curve C^r having the contact of the a-th order be replaced by the condition of passing through the a points; that is, consider the curves C^r which have with the curve U^m $(t-1)$ contacts of the orders $b, c, \ldots$ respectively, which pass through the a points on the curve U^m in the neighbourhood of the cusp, and which also pass through the requisite number of arbitrary points. The number of these curves is $=(b+1)(c+1)\ldots[rm-a-(b+c+\ldots)]^{t-1}$ (the term $rm-a$ instead of rm, on account of the given a points on the curve: compare herewith De Jonquières' formula containing $rm-p$). Each of these curves, in that it passes through a points in the neighbourhood of the cusp, will *ipso facto* pass through $a+1$ points (viz. a curve which simply passes through the cusp of a cuspidal curve meets the cuspidal curve there in two points, a curve which touches the cuspidal tangent meets the curve in three points, &c.), and be consequently, in an improper sense, a curve having a contact of the a-th order with the given curve U^m. I assume that it counts as such curve a times, and this being so, we have, on account of the curves in question, a reduction

$=a(b+1)(c+1)\ldots[rm-(a+b+c\ldots)]^{t-1}$. We have in like manner for the curves passing through b points in the neighbourhood of the cusp a reduction $=b(a+1)(c+1)\ldots[rm-(a+b+c\ldots)]^{t-1}$, &c., and hence when the given unicursal curve U^m has a cusp, the total reduction on account of the cusp in the number of the curves C^r which have with the given curve the t contacts of the orders $a, b, c, \ldots$ and besides pass through the requisite number of given points, is

$$=\{a(b+1)(c+1)\ldots+b(a+1)(c+1)+\&c.\}[rm-(a+b+c\ldots)]^{t-1},$$

which is the auxiliary theorem in question; some of the steps require however to be more completely made out.

90. I have calculated the following numerical results, wherein, as before, $\Delta=D-1=-m+\frac{1}{2}n+\frac{1}{2}\varkappa$. I find also their values in the case where the curve C^r is a conic (that is $r=2$) first in terms of $m, n, \varkappa$, and finally in terms of m, n, α ($\alpha=3n+\varkappa$, as above). The results are—

				for $r=2$, that is, curve C^r a conic.							
$[1]=$	$2rm+$	$2\Delta-$	$\varkappa$	$=$	$2m+$	n		$=$	$2m+$	n	
$[2]=$	$3rm+$	$6\Delta-$	$2\varkappa$	$=$		$3n+$	$\varkappa$	$=$			α
$[3]=$	$4rm+$	$12\Delta-$	$3\varkappa$	$=-$	$4m+$	$6n+$	$3\varkappa$	$=-$	$4m-$	$3n+$	3α
$[4]=$	$5rm+$	$20\Delta-$	$4\varkappa$	$=-$	$10m+$	$10n+$	$6\varkappa$	$=-$	$10m-$	$8n+$	6α
$[5]=$	$6rm+$	$30\Delta-$	$5\varkappa$	$=-$	$18m+$	$15n+$	$10\varkappa$	$=-$	$18m-$	$15n+$	10α
$[1,1]=-$	$12rm-$	$20\Delta+$	$7\varkappa$	$=-$	$4m-$	$10n-$	$3\varkappa$	$=-$	$4m-$	$n-$	3α
$[1,2]=-$	$24rm-$	$60\Delta+$	$16\varkappa$	$=$	$12m-$	$30n-$	$14\varkappa$	$=$	$12m+$	$12n-$	14α
$[1,3]=-$	$40rm-$	$136\Delta+$	$29\varkappa$	$=$	$56m-$	$68n-$	$39\varkappa$	$=$	$56m+$	$49n-$	39α
$[1,4]=-$	$60rm-$	$260\Delta+$	$46\varkappa$	$=$	$140m-$	$130n-$	$84\varkappa$	$=$	$140m+$	$122n-$	84α
$[2,2]=-$	$45rm-$	$144\Delta+$	$32\varkappa$	$=$	$54m-$	$72n-$	$40\varkappa$	$=$	$54m+$	$48n-$	40α
$[2,3]=-$	$72rm-$	$288\Delta+$	$54\varkappa$	$=$	$144m-$	$144n-$	$90\varkappa$	$=$	$144m+$	$126n-$	90α
$[1,1,1]=$	$160rm+$	$352\Delta-$	$98\varkappa$	$=-$	$32m+$	$176n+$	$78\varkappa$	$=-$	$32m-$	$58n+$	78α
$[1,1,2]=$	$360rm+$	$1056\Delta-$	$240\varkappa$	$=-$	$336m+$	$528n+$	$288\varkappa$	$=-$	$336m-$	$336n+$	288α
$[1,1,3]=$	$672rm+$	$2528\Delta-$	$478\varkappa$	$=-$	$1184m+$	$1264n+$	$786\varkappa$	$=-$	$1184m-$	$1094n+$	786α
$[1,2,2]=$	$756rm+$	$2700\Delta+$	$530\varkappa$	$=-$	$1188m+$	$1350n+$	$820\varkappa$	$=-$	$1188m-$	$1110n+$	820α
$[1,1,1,1]=-$	$3360rm-$	$8928\Delta+$	$2106\varkappa$	$=$	$2208m-$	$4464n-$	$2358\varkappa$	$=$	$2208m+$	$2610n-$	2358α
$[1,1,1,2]=-$	$8064rm-$	$26784\Delta+$	$5376\varkappa$	$=$	$10656m-$	$13392n-$	$8016\varkappa$	$=$	$10656m+$	$10656n-$	8016α
$[1,1,1,1,1]=$	$96768rm+$	$296448\Delta-$	$61464\varkappa$	$=-$	$102912m+$	$148224n+$	$86760\varkappa$	$=-$	$10912m-$	$112056n+$	86760α

(It may be noticed as a curious circumstance that in the last column in the expressions of [2], [1, 2], [1, 1, 2] and [1, 1, 1, 2] respectively, the coefficients of m and n are in each case equal.)

91. In the case of the conic, (1), (2), &c. are the expressions denoted in the former part of this Memoir by (1 ::), (2 ∴), &c., the number of points being in each case such as to make in all five conditions; calculating these functions by means of the formulæ $(a)=[a]$, &c., the comparison of the resulting values with the values previously obtained will show *à posteriori* the limits within which the formulæ are applicable; where they cease to be applicable I find the difference, and annex it as a correction to the formula value: I have in some cases given what seems to be the proper theoretical form of this difference. We have

$$(1::) = 2m+n;$$

$$(2\therefore) = \alpha;$$

$$(3:) = -4m-3n+3\alpha;$$

$$(4\cdot) = -10m-8n+6\alpha;$$

$$(5) = -18m-15n+10\alpha-[-3m+\alpha] \quad (=-[\iota]);$$

$$2(1,1\therefore) = (2m+n)^2 - 4m-n-3\alpha;$$

$$(1,2:) = (2m+n)\alpha + 12m+12n-14\alpha;$$

$$(1,3\cdot) = (2m+n)(-4m-3n+3\alpha) + 56m+49n-39\alpha;$$

$$(1,4) = (2m+n)(-10m-8n+6\alpha) + 140m+122n-84\alpha - [(m-3)(-12m-6n+6\alpha)] \quad (=-[(m-3)(4\iota+2\varkappa)]);$$

$$2(2,2\cdot) = \alpha^2 + 54m+48n-40\alpha;$$

$$(2,3) = \alpha(-4m-3n+3\alpha) + 144m+126n-90\sigma - [24m+6n+(n-12)\alpha] \quad (=-[6\tau+(n-3)\varkappa]);$$

$$6(1,1,1:) = (2m+n)^3 + 3(2m+n)(-4m-n-3\alpha) - 32m-58n+78\alpha - [4m(m-1)(m-2)];$$

$$\begin{aligned}
2(2,1,1\cdot) &= (2m+n)^2\alpha \\
&\quad +2(2m+n)(12m+12n-14\alpha)+\alpha(-4m-n-3\alpha) \\
&\quad -336m-336n+288\sigma \\
&\quad -[2\alpha(m-2)(m-3)] \quad (=-[6n(m-2)(m-3)+2\varkappa(m-2)(m-3)];
\end{aligned}$$

$$\begin{aligned}
2(3,1,1) &= (2m+n)^2(-4m-3n+3\alpha) \\
&\quad +2(2m+n)(56m+49n-39\alpha) \\
&\quad +(-4m-n-3\alpha)(-4m-3n+3\alpha) \\
&\quad -1184m-1094n+786\alpha \\
&\quad -\begin{bmatrix} -13m^2-8m^2n+4mn^2+131m^2+92mn-8n^2-316m-226n \\ +\alpha(9m^2-87m-3n+204) \end{bmatrix};
\end{aligned}$$

$$\begin{aligned}
2(2,2,1) &= (2m+n)\alpha^2 \qquad \left(=-\begin{bmatrix} \delta(2n+10m-38) \\ +\varkappa(3m^2-19m+30) \\ +\iota(6m^2-41m+69) \\ +\tau(8m-32) \end{bmatrix}\right) \\
&\quad +2\alpha(12m+12n-14\alpha) \\
&\quad +(2m+n)(54m+48n-40\alpha) \\
&\quad -1188m-1110n-820\alpha \\
&\quad -\begin{bmatrix} 60m^2+42mn-252m-174n \\ +\alpha(-40m+166) \\ +\alpha^2(m-4) \end{bmatrix} = (-\begin{bmatrix} (m-4)(\varkappa^2-\varkappa) \\ +6(m-4)(n-3)\varkappa \\ +18(m-4)\tau \\ +(m-3)(4\iota+2\varkappa) \end{bmatrix});
\end{aligned}$$

$$\begin{aligned}
24(1,1,1,1\cdot) &= (2m+n)^4 \\
&\quad +6(2m+n)^2(-4m-n-3\alpha) \\
&\quad +3(-4m-n-3\alpha)^2 \\
&\quad +4(2m+n)(-32m-58n+78\alpha) \\
&\quad +2208m+2610n-2358\alpha \\
&\quad -\begin{bmatrix} 14m(m-1)(m-2)(m-3) \\ +16n(m+2)(m-2)(m-3) \\ -36\alpha(m-2)(m-3) \end{bmatrix} \\
&\quad (=-[(m-2)(m-3)(14m^2+16mn-14m-76n-36\varkappa)]);
\end{aligned}$$

$$\begin{aligned}
6(2,1,1,1) &= (2m+n)^3\alpha \\
&\quad +3(2m+n)^2(12m+12n-14\alpha) \\
&\quad +3(2m+n)\alpha(-4m-n-3\alpha) \\
&\quad +3(2m+n)(-336m-336n+288\alpha) \\
&\quad +\alpha(-32m-58n+78\alpha) \\
&\quad +3(-4m-n-3\alpha)(12m+12n-14\alpha) \\
&\quad +10656m+10656n-8016\alpha
\end{aligned}$$

$$-\left[\begin{array}{l} 108m^3+108m^2n-1116m^2-1116mn+2736m+2736n \\ +\alpha(7m^3+6m^2n-147m^2-30mn+1040m+24n-2256) \\ +\alpha^2(-9m+36), \end{array}\right]$$

where the correction is

$$=-(m-4)\left\{\begin{array}{l} 108m^2+108mn-684m-684n \\ +\alpha(7m^2-119m+564+6n(m-1)) \\ -9\alpha^2 \end{array}\right\}$$

$$=-(m-4)\left\{\begin{array}{l} 2\delta(21n-36) \\ +2\sigma(18m-126) \\ +\ \varkappa(7m^2+6mn-65m-13n+165)-9(\varkappa^2-\varkappa) \\ +\ \iota(+16n-96) \end{array}\right\}$$

$$\begin{aligned} 120\ (1,1,1,1,1) = \quad & (2m+n)^5 \\ & +10(2m+n)^3(-4m-n-3\alpha) \\ & +10(2m+n)^3(-32m-58n+78\alpha) \\ & +10(-4m-n-3\alpha)(-32m-58n+78\alpha) \\ & +\ 5(2m+n)(2208m+2610n-2358\alpha) \\ & +15(2m+n)(-4m-n-3\alpha)^2 \\ & -102912m-112056n+86760\alpha \end{aligned}$$

$$-\left[\begin{array}{l} \quad 31m^5+\quad 70m^4n+\ 40m^3n^2 \\ -\ 310m^4-\ 460m^3n-120m^2n^2 \\ -\ 235m^3-1030m^2n-400\,mn^2 \\ +10690m^3+16060m\,n+960\quad n^2 \\ +\alpha\left[\begin{array}{l} -\ 210m^3-180m^2n \\ +\ 2970m^2+900mn \\ -15630m\ -720n \\ +28440 \end{array}\right] \\ +\alpha^2(135m-540), \end{array}\right]$$

where the correction is

$$=-(m-4)\left[\begin{array}{l} 31m^4-186m^3-979m^2+6774m \\ +n(70m^2-180m^2-1750m+9060) \\ +40n^2(m+3)(m-2) \\ +\alpha\left(\begin{array}{l} -210m^2+2130m-7110 \\ -180n(m-1) \end{array}\right) \\ +\alpha^2.135, \end{array}\right]$$

which is

$$= -(m-4)\left[\begin{array}{l} +31(2\delta+3\varkappa)^2+110(2\delta+3\varkappa)(2\tau+3\iota) \\ +(\tfrac{70}{3}m^2+1142m+3174)(\varkappa-\iota) \\ +(-14m-638n-1524)(2\delta+3\varkappa) \\ +(-390m+110n+4272)(2\tau+3\iota) \\ +(-210m^2-180mn+2130m+990n-7110)\varkappa+135\varkappa^2 \end{array}\right]$$

but I have not sought to further reduce this expression, not knowing the proper form in which to present it.

92. The question which ought now to be considered is to determine the corrections or supplements which should be applied to the foregoing expressions (a), (a, b), &c., or to their equivalents $[a]$, $[a][b]+[a, b]$, &c. in order to obtain formulæ for the cases beyond the limits within which the present formulæ are applicable; but this I am not in a position to enter upon. If the extended formulæ were obtained, it would of course be an interesting verification or application of them to deduce from them the complete series of expressions $(1::)$, $(2\,.\,.)$ ... $(1, 1, 1, 1, 1)$ for the number of the conics which satisfy given conditions of contact with a given curve, and besides pass through the requisite number of given points. It will be recollected that throughout these last investigations, I have put DE JONQUIÈRES' $p=0$; that is, I have not considered the case of the curves C^r which (among the conditions satisfied by them) have with the curve U^m contacts of given orders at given points of the curve; it is probable that the general formulæ containing the number p admit of extensions and transformations analogous to the formulæ in which p is put$=0$, but this is a question which I have not considered.

93. The set of equations $(a)=[a]$, $(a, b)=[a][b]+[a, b]$, &c., considered irrespectively of the meaning of the symbols contained therein, gives rise to an analytical question which is considered in Annex No. 7.

The question of the conics satisfying given conditions of contact is considered from a different point of view in my Second Memoir above referred to.

Annex No. 1 (referred to in the notice of DE JONQUIÈRES' memoir of 1861).—*On the form of the equation of the curves of a series of given index.*

To obtain the general form of the equation of the curves C^n of a series of the index N, it is to be observed that the equation of any such curve is always included in an equation of the order n in the coordinates, containing linearly and homogeneously certain parameters $a, b, c\,..$; this is universally the case, as we may, if we please, take the parameters $(a, b, c\,..)$ to be the coefficients of the general equation of the order n; but it is convenient to make use of any linear relations between these coefficients so as to reduce as far as possible the number of the parameters. Assume that the number of the parameters is $=\omega+1$, then in order that the curves should form a

series (that is, satisfy $\frac{1}{2}n(n+3)-1$ conditions), we must have a $(\omega-1)$fold relation between the parameters, or, what is the same thing, taking the parameters to be the coordinates of a point in ω-dimensional space, say the parametric point, the point in question must be situate on a $(\omega-1)$fold locus. Moreover, the condition that the curve shall pass through a given point establishes between the parameters a linear relation (viz. that expressed by the original equation of the curve regarding the coordinates therein as belonging to the given point, and therefore as constants); that is, when the curve passes through a given point, the corresponding positions of the parametric point are given as the intersections of the $(\omega-1)$fold locus by an omal onefold locus; the number of the curves is therefore equal to the number of these intersections, that is, to the order of the $(\omega-1)$fold locus; or the index of the series being assumed to be $=N$, the order of the $(\omega-1)$fold locus must be also $=N$. That is, the general form of the equation of the curves C^n which form a series of the index N, is that of an equation of the order n containing linearly and homogeneously the $\overline{\omega+1}$ coordinates of a certain $(\omega-1)$fold locus of the order N. It is only in a particular case, viz. that in which the $(\omega-1)$fold locus is unicursal, that the coordinates of a point of this locus can be expressed as rational and integral functions of the order N of a variable parameter θ; and consequently only in this same case that the equation of the curves C^n of the series of the index N can be expressed by an equation $(*)(x, y, z)^n=0$, or $(*)(x, y, 1)^n=0$, rational and integral of the degree N in regard to a variable parameter θ.

If in the general case we regard the coordinates of the parametric point as irrational functions of a variable parameter θ, then rationalising in regard to θ, we obtain an equation rational of the order N in θ, but the order in the coordinates instead of being $=n$, is equal to a multiple of n, say qn. Such an equation represents not a single curve but q distinct curves C^n, and it is to be observed that if we determine the parameter by substituting therein for the coordinates their values at a given point, then to each of the N values of the parameter there corresponds a system of q curves, only one of which passes through the given point, the other $\overline{q-1}$ curves are curves not passing through the given point, and having no proper connexion with the curves which satisfy this condition.

Returning to the proper representation of the series by means of an equation containing the coordinates of the parametric point, say an equation $(*)(x, y, 1)^n=0$, involving the two coordinates (x, y), it is to be noticed that forming the derived equation and eliminating the coordinates of the parametric point, we obtain an equation rational in the coordinates (x, y), and also rational of the degree N in the differential coefficient $\frac{dy}{dx}$; in fact since the number of curves through any given point (x_0, y_0) is $=N$, the differential equation must give this number of directions of passage from the point (x_0, y_0) to a consecutive point, that is, it must give this number of values of $\frac{dy}{dx}$, and must consequently be of the order N in this quantity.

Conversely, if a given differential equation rational in $x, y, \frac{dy}{dx}$, and of the degree N in the last-mentioned quantity $\frac{dy}{dx}$, admit of an algebraical general integral, the curves represented by this integral equation may be taken to be irreducible curves, and this being so they will be curves of a certain order n forming a series of the index N; whence the general integral (assumed to be algebraical) is given by an equation of the above-mentioned form, viz. an equation rational of a certain order n in the coordinates, and containing linearly and homogeneously the $\omega+1$ coordinates of a variable parametric point situate on an $(\omega-1)$fold locus. The integral equation expressed in the more usual form of an equation rational of the order N in regard to the parameter or constant of integration, will be in regard to the coordinates of an order equal to a multiple of n, say $=qn$, and for any given value of the parameter will represent not a single curve C^n, but a system of q such curves: the first-mentioned form is, it is clear, the one to be preferred.

Annex No. 2 (referred to, No. 17).—*On the line-pairs which pass through three given points and touch a given conic.*

Taking the given points to be the angles of the triangle formed by the lines $(x=0, y=0, z=0)$, we have to find (f, g, h) such that the conic $(0, 0, 0, f, g, h)(x, y, z)^2=0$, or, what is the same thing, $fyz+gzx+hxy=0$, shall reduce itself to a line-pair, and shall touch a given conic $(1, 1, 1, \lambda, \mu, \nu)(x, y, z)^2=0$. The condition for a line-pair is that one of the quantities f, g, h shall vanish, viz. it is $fgh=0$; the condition for the contact of the two conics is found in the usual manner by equating to zero the discriminant of the function $1-(\lambda+\theta f)^2-(\mu+\theta g)^2-(\nu+\theta h)^2+2(\lambda+\theta f)(\mu+\theta g)(\nu+\theta h)=(a, b, c, d)(\theta, 1)^3$ suppose; the values of a, b, c, d being

$$\begin{aligned} a&= 2fgh,\\ b&=-\tfrac{1}{3}(f^2+g^2+h^2-2\lambda gh-2\mu hf-2\nu fg),\\ c&=\tfrac{2}{3}((\mu\nu-\lambda)f+(\nu\lambda-\mu)g+(\lambda\mu-\nu)h),\\ d&=1-\lambda^2-\mu^2-\nu^2+2\lambda\mu\nu. \end{aligned}$$

Hence considering (f, g, h) as the coordinates of the parametric point, we have the discriminant locus $a=0$, and the contact-locus

$$a^2d^2+4ac^3+4b^3d-3b^2c^2-6abcd=0,$$

and at the intersection of the two loci, $a=0$, $b^2(4bd-3c^2)=0$, equations breaking up into the system $(a=0, b=0)$ twice, and the system $a=0$, $4bd-3c^2=0$; the former of these is

$$fgh=0,\ f^2+g^2+h^2-2\lambda gh-2\mu hf-2\nu fg=0,$$

which expresses that the intersection of the two lines of the line-pair intersect on the given conic; in fact the system is satisfied by $f=0, g^2+h^2-2\lambda gh=0$, giving a line-pair

$x(hy+gz)=0$, the two lines whereof intersect on the conic $(1, 1, 1, \lambda, \mu, \nu)(x, y, z)^2=0$; and similarly, if $g=0$, then $h^2+f^2-2\mu hf=0$, or if $h=0$, then $f^2+g^2-2\lambda fg=0$. As noticed above this system occurs twice.

The second system is

$$fgh=0, \qquad (f^2+g^2+h^2-2\lambda gh-2\mu hf-2\nu fg)(1-\lambda^2-\mu^2-\nu^2+2\lambda\mu\nu)$$
$$+((\mu\nu-\lambda)f+(\nu\lambda-\mu)g+(\mu\lambda-\nu)h)^2=0,$$

or, as the second equation may also be written,

$$f^2(1-\mu^2)(1-\nu^2)+g^2(1-\nu^2)(1-\lambda^2)+h^2(1-\lambda^2)(1-\mu^2)$$
$$+2gh(1-\lambda^2)(\mu\nu-\lambda)+2hf(1-\mu^2)(\nu\lambda-\mu)+2fg(1-\nu^2)(\lambda\mu-\nu)=0,$$

which expresses that a line of the line-pair touches the conic; in fact the system is satisfied by $f=0$, $g^2(1-\nu^2)+h^2(1-\mu^2)+2gh(\mu\nu-\lambda)=0$, viz. we have here the line-pair $x(hy+gz)=0$, in which the line $hy+gz=0$ touches the conic $(1, 1, 1, \lambda, \mu, \nu)(x, y, z)^2=0$; and the like if $g=0$, or if $h=0$. This system it has been seen occurs only once.

Annex No. 3 (referred to, No. 22).—*On the conics which pass through two given points and touch a given conic.*

Consider the conics which pass through two given points and touch a given conic. We may take $Z=0$ as the equation of the line through the two given points, and then taking the pole of this line in regard to the given conic and joining it with the two given points respectively, the equations of the joining lines may be taken to be $X=0$ and $Y=0$ respectively. This being so, we have for the given points $(X=0, Z=0)$ and $(Y=0, Z=0)$ respectively, and for the given conic

$$aX^2+bY^2+2hXY+cZ^2=0;$$

and since the required conic is to pass through the two given points its equation will be of the form

$$wX^2+2xYZ+2yZX+2zXY=0,$$

where (x, y, z, w) are variable parameters which must satisfy a single condition in order that the last-mentioned conic may touch the given conic. The condition is at once seen to be that obtained by making the equation

$$(a+\lambda w)bc$$
$$-(a+\lambda w)(h+\lambda z)^2$$
$$-b\lambda^2y^2$$
$$-c\lambda^2z^2$$
$$+2\lambda^2xy(h+\lambda z)=0,$$

considered as a cubic equation in λ, have a pair of equal roots; or if we write

$$\begin{aligned} A &= 3c(ab-h^2), \\ B &= (ab-h^2)w - 2chz, \\ C &= -ax^2 - by^2 - cz^2 + 2h(xy-zw), \\ D &= 3z(2xy-wz), \end{aligned}$$

then the required condition is

$$A^2D^2 + 4AC^3 + 4B^3D - 6ABCD - 3B^2C^2 = 0.$$

Hence the conic

$$wX^2 + 2xYZ + 2yZX + 2zXY = 0$$

satisfies the prescribed conditions, if only the parameters (x, y, z, w) satisfy the last-mentioned equation, that is, if (x, y, z, w) are the coordinates of a point on the sextic surface represented by this equation.

The surface has upon it a cuspidal curve the equations whereof are

$$\left\| \begin{matrix} A, & B, & C \\ B, & C, & D \end{matrix} \right\| = 0;$$

this may be considered as the intersection of the quadric surface $AC - B^2 = 0$ and the cubic surface $AD - BC = 0$; and the cuspidal curve is consequently a sextic.

The surface has also a nodal curve made up of two conics; to prove this I write for shortness $k = h - \sqrt{ab}$, $k_1 = h + \sqrt{ab}$; the values of A, B, C, D then are

$$\begin{aligned} A &= -3ckk_1, \\ B &= -kk_1w - 2chz, \\ C &= -ax^2 - by^2 - cz^2 + 2h(xy-zw), \\ D &= 3z(2xy-zw). \end{aligned}$$

And it is in the first place to be shown that the surface contains the conic

$$x : y : z : w = \theta\sqrt{b} : \theta\sqrt{a} : 1 : -k\theta^2 + \frac{c}{k},$$

where θ is a variable parameter. Substituting these values, we have

$$\begin{aligned} A &= -3ckk_1, \\ B &= k^2k_1\theta^2 - c(3h + \sqrt{ab}), \\ C &= 2kk_1\theta^2 - \frac{c}{k}(3h - \sqrt{ab}), \\ D &= 3\left(k_1\theta^2 - \frac{c}{k}\right); \end{aligned}$$

and hence

$$AD-BC=-2k\left(kk_1\theta^2+\frac{2c\sqrt{ab}}{k}\right)^2,$$

$$AC-B^2=-k^2\left(kk_1\theta^2+\frac{2c\sqrt{ab}}{k}\right)^2,$$

$$BD-C^2=-\left(kk_1\theta^2+\frac{2c\sqrt{ab}}{k}\right)^2,$$

values which satisfy identically the equation of the surface written under the form

$$(AD-BC)^2-4(AC-B^2)(BD-C^2)=0.$$

Moreover, proceeding to form the derived equation, and to substitute therein the foregoing values of (x, y, z, w), we have

$$\partial A:\partial B:\partial C:\partial D=0:k^2:2k:3,$$

and then the derived equation is

$$\begin{aligned}&(AD-BC)(\ 3A-2kB-k^2C)\\ -&2(AC-B^2)(\ 3B-4kC+k^2D)\\ -&2(BD-C^2)(2kA-2k^2B\quad)=0,\end{aligned}$$

that is,

$$\begin{aligned}&-k(\ 3A-2kB-k^2C)\\ &+k^2(\quad 3B-4kC+k^2D)\\ &+\ (2kA-2k^2B),\end{aligned}$$

or finally

$$-k(A-3Bk+3k^2C-k^3D)=0,$$

which is satisfied by the foregoing values of A, B, C, D; hence the conic is a nodal curve on the sextic; and by merely changing the sign of one of the radicals $\sqrt{a}$, $\sqrt{b}$ (and therefore interchanging k, k_1) we obtain another conic which is also a nodal curve on the surface, that is, we have as nodal curves the two conics

$$\begin{cases}x:y:z:w=\theta\sqrt{b}:\quad\theta\sqrt{a}:1:-k\theta^2+\frac{c}{k}, \text{ and}\\ x:y:z:w=\theta\sqrt{b}:-\theta\sqrt{a}:1:-k_1\theta^2+\frac{c}{k_1}.\end{cases}$$

It is to be remarked that each of the nodal conics meets the cuspidal curve in two points, viz. writing for shortness $\Theta=\frac{1}{k}\sqrt{\frac{-2c\sqrt{ab}}{k_1}}$, $\Theta_1=\frac{1}{k_1}\sqrt{\frac{2c\sqrt{ab}}{k}}$, for the intersections of the first conic we have

$$x:y:z:w=\Theta\sqrt{a}:\quad\Theta\sqrt{b}:1:\frac{c}{k_1} \text{ and } =-\Theta\sqrt{a}:-\Theta\sqrt{b}:1:\frac{c}{k_1},$$

and for the intersections with the second conic

$$x:y:z:w=\Theta_1\sqrt{a}:-\Theta_1\sqrt{b}:1:\frac{c}{k} \text{ and } =-\Theta_1\sqrt{a}:\quad\Theta_1\sqrt{b}:1:\frac{c}{k}.$$

The condition of passing through any arbitrary point establishes a linear relation be-

tween the parameters (x, y, z, w). Hence, if the conic in addition to the prescribed conditions passes through two other given points, the point (x, y, z, w) is given as the intersection of a line with the sextic surface; the number of intersections is $=6$. If (x, y, z, w) is situate on the cuspidal curve, then the conic instead of simply touching the given conic will have with it a contact of the second order, and if we besides suppose that the conic passes through a given point, then the point (x, y, z, w) is given as the intersection of the cuspidal curve with a plane; the number is $=6$. Similarly, if the conic has two contacts with the given conic, and besides passes through a given point, then the point (x, y, z, w) is given as the intersection of the nodal curve by a plane; the number is $=4$. Finally (observing that in the case in question of the contacts of a conic with a conic we cannot have three simple contacts, or a simple contact and one of the second order), a point of intersection of the nodal and cuspidal curves answers to a contact of the third order; and the number is $=4$. That is, the theory of the sextic surface leads to the following values (agreeing with those obtained from the formulæ by writing therein $m=n=2$, $\alpha=6$), viz.

$$\begin{aligned}
(1::) \quad &=6, \quad = \quad 2m+n,\\
(1, 1\therefore) &=4, \quad = \quad 2m^2+2mn+\tfrac{1}{2}n^2-2m-\tfrac{1}{2}n-\tfrac{3}{2}\alpha,\\
(2\therefore) \quad &=6, \quad = \quad \alpha,\\
(3:) \quad &=4, \quad =-4m-3n+3\alpha.
\end{aligned}$$

I remark that the section by an arbitrary plane is a sextic curve having 6 cusps and 4 nodes; it is therefore a *unicursal* sextic; this suggests the theorem that the sextic surface is also *unicursal*, viz. that the coordinates are expressible rationally in terms of two parameters; I have found that this is in fact the case. In doing this there is no loss of generality in supposing that $a=b=c=1$; and assuming that this is so, and putting also $-1+h=k$, $1+h=k_1$, and therefore $2h=k+k_1$, we have

$$\begin{aligned}
A&=-3kk_1,\\
B&=-\ kk_1w-(k+k_1)z,\\
C&=-\ x^2-y^2-z^2+(k+k_1)(xy-zw),\\
D&=\ 3z(2xy-zw).
\end{aligned}$$

The equation of the sextic surface being, as before,

$$A^2D^2+4AC^3+4B^3D-3B^2C^2-6ABCD=0,$$

I say that this equation is satisfied on writing therein

$$\begin{aligned}
x+y&=\sqrt{-\frac{2}{k_1}}(1-k_1\alpha)\sin\phi,\\
x-y&=\ \sqrt{\frac{2}{k}}(1+k\alpha)\cos\phi,\\
z\ &=1,\\
w\ &=\left(2\alpha-\frac{1}{k}\right)\cos^2\phi+\left(2\alpha-\frac{1}{k_1}\right)\sin^2\phi,
\end{aligned}$$

where (α, φ) are arbitrary. In fact these values give

$$\begin{aligned}
\tfrac{1}{3}A &= -kk_1 \quad \cos^3\varphi - kk_1 \quad \sin^3\varphi,\\
B &= -k(2\alpha k_1+1)\cos^3\varphi - k_1\ (2\alpha k+1)\sin^3\varphi,\\
C &= -k(\ \alpha k_1+2)\cos^3\varphi - k_1\alpha(\ \alpha k+2)\sin^3\varphi,\\
\tfrac{1}{3}D &= -k\alpha^3 \quad \cos^3\varphi - k_1\alpha^3 \quad \sin^3\varphi,
\end{aligned}$$

whence ω being arbitrary, we have

$$\begin{aligned}
&\tfrac{1}{3}(A, B, C, D)(\omega, 1)^3\\
&\quad = -[k\cos^3\varphi(k_1\omega+1)+k_1\sin^3\varphi(k\omega+1)](\omega+\alpha)^3,
\end{aligned}$$

viz. the equation $(A, B, C, D)(\omega, 1)^3=0$, considered as a cubic equation in ω, has the twofold root $\omega=-\alpha$, that is, we have the above relation between (A, B, C, D). Whence also writing $\sin\varphi=\frac{2\lambda}{1+\lambda^2}$, $\cos\varphi=\frac{1-\lambda^2}{1+\lambda^2}$, the equation of the surface is satisfied by the values

$$\begin{aligned}
x+y : x-y : z : w = &\sqrt{-\tfrac{2}{k_1}}(1-k_1\alpha)2\lambda(1+\lambda^2)\\
&: \sqrt{\tfrac{2}{k}}(1+k\alpha \quad (1-\lambda^4)\\
&: \quad (1+\lambda^2)^2\\
&: \left(2\alpha-\frac{1}{k}\right)(1-\lambda^2)^2+\left(2\alpha-\frac{1}{k_1}\right)4\lambda^2,
\end{aligned}$$

or the coordinates are expressed rationally in terms of α, λ.

Annex No. 4 (referred to, Nos. 22 and 71).—*On the Conics which touch a cuspidal cubic.*

In the cuspidal cubic, if $x=0$ be the equation of the tangent at the cusp, $y=0$ that of the line joining the cusp with the inflexion, and $z=0$ that of the tangent at the cusp, then the equation of the curve is $y^3=x^2z$; the coordinates of a point on the cubic are given by $x : y : z=1 : \theta : \theta^3$, where θ is a variable parameter; and we have, at the cusp $\theta=\infty$, at the inflexion $\theta=0$. In the cubic, $m=n=3$, $\alpha(=3n+\varkappa)=10$.

Considering now the conic

$$(a, b, c, f, g, h)(x, y, z)^2=0,$$

this meets the cubic in the 6 points the parameters of which are determined by the equation

$$(a, b, c, f, g, h)(1, \theta, \theta^3)^2=0,$$

or, what is the same thing,

$$(c, 0, 2f, 2g, b, 2h, a)(\theta, 1)^6=0.$$

The discriminant of this sextic function contains the factor c, hence equating the residual factor to zero, we obtain the equation of the contact-locus in the form

$$(c, f, g, b, h, a)^2=0.$$

It follows that the number of the conics $(1::)$ is $=9$, which agrees with the general value $(1::)=2m+n$. If the conic pass through the cusp we have $c=0$, and the equation in δ is reduced to a quartic; it is convenient to alter the letters in such wise that the quartic equation may be obtained in the standard form $(a, b, c, d, e)(\theta, 1)^4=0$; viz. this will be the case if the equation of the conic is taken to be

$$(e, 6c, 0, \tfrac{1}{2}a, 2b, 2d)(x, y, z)^2=0,$$

and we then obtain the equation of the contact-locus in the form

$$(ae-4bd+3c^2)^3-27(ace+2bcd-ad^2-b^2e-c^3)^2=0,$$

which is a onefold locus of the order 6. It follows that we have

$$(\overline{1\times 1}, 1\,\therefore)=6, \text{ agreeing with } (\overline{1\times 1}, 1\,\therefore)=n+2m-3.$$

The condition in order that the conic may touch a given line is given by an equation of the form

$$(*)(a^2, ab, b^2, 2ce-3d^2, ae-8bd, ad-12bc)^1=0,$$

which is a onefold locus of the order 2; it at once follows that we have

$$(\overline{1\times 1}, 1, :/)=12, \text{ agreeing with } (\overline{1\times 1}, 1, :/)=2n+4m-6.$$

It is a matter of some difficulty to show that we have

$$(\overline{1\times 1}, 1, \cdot\,//)=18, \text{ agreeing with } (\overline{1\times 1}, 1\cdot\,//)=4n+4m-6;$$

but I proceed to effect this, first remarking that I do not attempt to prove the remaining case

$$(\overline{1\times 1}, 1\,///)=15, \text{ agreeing with } (\overline{1\times 1}, 1\,///)=4n+2m-3.$$

Investigation of the value $(1\overline{\times 1}, 1\cdot\,//)=18$:

We have the sextic locus

$$(ae-4bd+3c^2)^3-27(ace+2bcd-ad^2-b^2e-c^3)^2=0,$$

and combined therewith two quadric loci,

$$(*)(a^2, ab, b^2, 2ce-3d^2, ae-8bd, ad-12bc)^1=0,$$
$$(*')(a^2, ab, b^2, 2ce-3d^2, ae-8bd, ad-12bc)^1=0,$$

which intersect in a threefold locus of the order 24; it is to be shown that this contains as part of itself the quadric threefold locus $(a=0, b=0, 2ce-3d^2=0)$ taken three times, leaving a residual threefold locus of the order $24-6=18$.

We may imagine the coordinates a, b, c, d, e expressed as linear functions of any four coordinates, and so reduce the problem from a problem in 4-dimensional space to one in ordinary 3-dimensional space. We have thus a sextic surface, and two quadric surfaces; the sextic is a developable surface or torse, having for one of its generating lines the line $a=0, b=0$, and for the tangent plane along this line the plane $a=0$; the two quadric surfaces meet in a quartic curve passing through the two points $(a=0, b=0, 3ce-2d^2=0)$, which are points on the torse; it is to be shown that each of these points

count three times among the intersections of the torse with the quartic curve, the number of the remaining intersections being therefore $24-6, =18$; and in order thereto it is to be shown that each of the points in question ($a=0, b=0, 3ce-2d^2=0$) is situate on the nodal line of the torse, and that the quartic curve touches there the sheet which is not touched by the tangent plane $a=0$; for this being so the quartic curve touching one sheet and simply meeting the other sheet meets the torse in three consecutive points, or the two points of intersection each count three times.

The torse has the cuspidal line

$$S=ae-4bd+3c^2=0,\ T=ace+2bcd-ad^2-b^2e-c^3=0,$$

and the nodal line

$$\left\| \begin{matrix} 6(ac-b^2), & 3(ad-bc), & ae+2bd-3c^2, & 3(be-cd), & 6(ce-d^2) \\ a, & b, & c, & d, & e \end{matrix} \right\|$$

and the equations of the nodal line are satisfied by the values ($a=0, b=0, 3ce-2d^2=0$) of the coordinates of the points in question. To find the tangent planes at these points, starting from the equation $S^3-27T^2=0$ of the torse, taking (A, B, C, D, E) as current coordinates, and writing

$$\partial = A\partial_a + B\partial_b + C\partial_c + D\partial_d + E\partial_e,$$

then the equation of the tangent plane is in the first instance given in the form $S^2\partial S-18T\partial T=0$, which writing therein ($a=0, b=0, 3ce-2d^2=0$) assumes, as it should do, the form $0=0$; the left-hand side is in fact found to be $9c^3(3ce-2d^2)A$. Proceeding to the second derived equation, this is $S^2\partial^2 S+2S(\partial S)^2-18T\partial^2 T-18(\partial T)^2=0$, or substituting the values of the several terms, the equation is

$$\begin{aligned} & 9c^4(AE-4BD+3C^2) \\ + & \ 3c^3(eA-4dB+6cC)^2 \\ + & 18c^3\{e(AC-B^2)+2d(BC-AD)+c(AE+2BD-3C^2)\} \\ - & \ 9\ \{(ce-d^2)A+2cdB-3c^2C\}^2=0; \end{aligned}$$

the terms in BC, BD, C^2 vanish identically, that in B^2 is $(48-36=)12c^3d^2-18c^3e, = -6c^3(3ce-2d^2)B^2$, which also vanishes; hence there remain only the terms divisible by A, giving first the tangent plane $A=0$, and secondly the other tangent plane,

$$\begin{aligned} & A(-\ 6c^3e^2+18cd^2e-9d^4) \\ + & B(-\ 60c^2de+36cd^3) \\ + & C(\ 108c^3e-54c^2d^2) \\ + & D(-\ 36c^3d) \\ + & E.\,27c^4 \qquad\qquad =0. \end{aligned}$$

Taking the equations of the quadric surfaces to be

$$(\lambda, \mu, \nu, \varrho, \sigma, \tau)(a^2, b^2, ab, 3ce-2d^2, ae-8bd, ad-12bc)=0,$$
$$(\lambda', \mu', \nu', \varrho', \sigma', \tau')\quad ,, \qquad ,, \qquad ,, \qquad ,, \qquad)=0,$$

the equations of the tangent planes are

$$\rho(3cE+3eC-4dD)+\sigma(eA-8dB)+\tau(dA-12cB)=0,$$
$$\rho'(\quad\ ,,\quad\ )+\sigma'(\quad\ ,,\quad\ )+\tau'(\quad\ ,,\quad\ )=0,$$

in all which equations we have $3ce-2d^2=0$; and if to satisfy this equation we write $c:d:e=2:3\beta:3\beta^2$, then the equations of the tangent planes become

$$\beta^2(A\beta-8B)+8(3C\beta^2-4D\beta+2E)=0,$$
$$\rho(3CB^2-4D\beta+2E)+(\sigma\beta+\tau)(A\beta-8B)=0,$$
$$\rho'(\quad\ ,,\quad\ )+(\sigma'\beta-\tau)(\quad\ ,,\quad\ )=0,$$

or the three tangent planes intersect in the line $A\beta-8B=0$, $3C\beta^2-4D\beta+2E=0$, which completes the proof.

Reverting to the sextic locus,

$$(ae+4bd-3c^2)^2-27(ace+2bcd-ad^2-b^2e-c^3)^2=0,$$

considered as a locus in 4-dimensional space depending on the five coordinates (a, b, c, d, e), this has upon it the twofold locus

$$ae-4bd+3c^2=0,\ \ ace+2bcd-ad^2-b^2e-c^3=0,$$

say the cuspidal locus, of the order 6, and the twofold locus

$$\left\| \begin{matrix} 6(ac-b^2), & 3(ad-bc), & ae+2bd-3c^2, & 3(be-cd), & 6(ce-d^2) \\ a\ , & b\ , & c\ , & d\ , & e \end{matrix} \right\| =0,$$

say the nodal locus, of the order 4: there is also a threefold locus,

$$\left\| \begin{matrix} a, & b, & c, & d \\ b, & c, & d, & e \end{matrix} \right\| =0,$$

say the supercuspidal locus, of the order 4. We thence at once infer

$(\overline{1\times}1, 2\ :)=6$, agreeing with $(\overline{1\times}1, 2\ :)=\alpha-4$,

$(\overline{1\times}1, 1, 1:)=4$, ,, ,, $(\overline{1\times}1, 1:)=2m^2+2mn+\frac{1}{2}n^2-8m-\frac{7}{2}n+13-\frac{3}{2}\alpha$

$(\overline{1\times}1, 3\ :)=4$, ,, ,, $(\overline{1\times}1, 3\ :)=-4m-3n-5+3\alpha$;

but I have not investigated the application to the symbols with $\cdot/$ or $//$.

If the conic, instead of simply passing through the cusp, touches the cuspidal tangent, then in the equation $(a, b, 0, f, g, h)(x, y, z)^2=0$ of the conic we have $f=0$, or, what is the same thing, in the equation $(e, 6c, 0, \frac{1}{3}a, 2b, 2d)(x, y, z)^3=0$ of the conic we have $a=0$. The equation in θ is thus reduced to $4b\theta^3+6c\theta^2+4d\theta+e=0$. For the independent discussion of this case it is convenient to alter the coefficients so that the equation in θ may be in the standard form $(a, b, c, d)(\theta, 1)^3=0$, viz. we assume the equation of the conic to be $(d, 3b, 0, 0, \frac{1}{3}a, \frac{3}{2}c)(x, y, z)^2=0$. The equation of the contact-locus then is

$$a^3d^2+4ac^3+4b^3d-6abcd-3b^2c^2=0,$$

viz. this is a developable surface, or torse, of the order 4, and we at once infer

$$(\overline{2\times}1, 1:)=4, \text{ agreeing with } (\overline{2\times}1, 1:)=2m+n-5.$$

I will show also that we have

$$(\overline{2x1}, 1\cdot/)=6, \text{ agreeing with } (\overline{2x1}, 1\cdot/)=2m+2n-6,$$

$$\text{and } (\overline{2x1}, 1\ //)=5, \quad \text{„} \quad \text{„} \quad (\overline{2x1}, 1\ //)= m+2n-4.$$

The condition that the conic may touch an arbitrary line $\alpha x+\beta y+\gamma z=0$, is in fact

$$(0, -\tfrac{1}{4}a^2, \tfrac{3}{4}(4bd-3c^2), \tfrac{1}{4}ac, -\tfrac{3}{4}ab, 0)(\alpha, \beta, \gamma)^2=0,$$

which, considering therein (a, b, c, d) as coordinates, is the equation of a quadric surface passing through the conic $a=0$, $4bd-3c^2=0$; the quartic torse also passes through this conic; hence the quadric surface and the torse intersect in this conic, which is of the order 2, and in a residual curve of the order 6; and the number of the conics $(\overline{2x1}, 1\cdot/)$ is equal to the order of this residual curve, that is, it is $=6$.

If the conic touch a second arbitrary line $\alpha'x+\beta'y+\gamma'z=0$, then we have in like manner the quadric surface

$$(0, -\tfrac{1}{4}a^2, \tfrac{3}{4}(4bd-3c^2), \tfrac{1}{4}ac, -\tfrac{3}{4}ab, 0)(\alpha', \beta', \gamma')^2=0;$$

that is, we have the quartic torse and two quadric surfaces, each passing through the conic $a=0$, $4bd-3c^2=0$, and it is to be shown that the number of intersections not on this conic is $=5$. The two quadric surfaces intersect in the conic and in a second conic; this second conic meets the torse in 8 points, but 2 of these coincide with the point $a=0$, $b=0$, $c=0$, which is one of the intersections of the two conics (the point $a=0$, $b=0$, $c=0$ is in fact a point on the cuspidal edge of the torse, and, the conic passing through it, reckons for 2 intersections), and 1 of the 8 points coincides with the other of the intersections of the two conics; there remain therefore $8-2-1$, $=5$ intersections, or we have $(\overline{2x1}, 1\ //)=5$.

Annex No. 5 (referred to, Nos. 22 and 71).—*On the Conics which have contact of the third order with a given cuspidal cubic, and two contacts (double contact) with a given conic.*

Let the equation of the cuspidal cubic be $x^2z-y^3=0$ ($x=0$ tangent at cusp, $z=0$ tangent at inflexion, $y=0$ line joining cusp and inflexion; equation satisfied by

$$x : yz=1 : \theta : \theta^2);$$

and let the equation of the given conic be

$$U=(a, b, c, f, g, h)(x, y, z)^2=0;$$

then writing

$$\Theta=(a, b, c, f, g, h)(1, \theta, \theta^2)^2$$
$$=c\theta^4+2f\theta^3+2g\theta^2+b\theta^2+2h\theta+c,$$

the equation of a conic having with the given cubic at a given point $(1, \theta, \theta^2)$ contact of the second order, and having double contact with the given conic, is

$$\begin{vmatrix} \sqrt{U}, & x, & y, & z \\ \sqrt{\Theta} & 1, & \theta, & \theta^2 \\ (\sqrt{\Theta})' & . & 1, & 3\theta^2 \\ (\sqrt{\Theta})'' & . & . & 6\theta \end{vmatrix} =0,$$

viz. in the rational form this is

$$36\theta^3 U = \begin{vmatrix} & x, & y, & z \\ \sqrt{\Theta}, & 1, & \theta, & \theta^2 \\ (\sqrt{\Theta})', & . & 1, & 3\theta^2 \\ (\sqrt{\Theta})'', & . & . & 6\theta \end{vmatrix}^3 = 0,$$

and this will have at the point $(1, \theta, \theta^2)$ a contact of the third order if θ be determined by

$$\begin{vmatrix} \sqrt{\Theta}, & 1, & \theta, & \theta^2 \\ (\sqrt{\Theta})', & . & 1, & 3\theta^2 \\ (\sqrt{\Theta})'', & . & . & 6\theta \\ (\sqrt{\Theta})''', & . & . & 6 \end{vmatrix} = 0,$$

viz. this is

$$\Theta(\sqrt{\Theta})''' - (\sqrt{\Theta})'' = 0;$$

or developing and multiplying by $\Theta^{\frac{1}{2}}$, this is

$$\theta\{\Theta^3\Theta''' - \tfrac{3}{2}\Theta\Theta'\Theta'' + \tfrac{3}{4}\Theta'^3\} - (\Theta^3\Theta'' - \tfrac{1}{2}\Theta\Theta'^2) = 0,$$

or, what is the same thing,

$$\begin{aligned} &\Theta^3(\theta\Theta''' - \Theta'') \\ &+\Theta\Theta'(-\tfrac{3}{2}\theta\Theta'' + \tfrac{1}{2}\Theta') \\ &+\Theta'^2.\tfrac{3}{4}\theta\Theta' = 0; \end{aligned}$$

and substituting for Θ its value, this is

$$\begin{aligned} &(c\theta^4 + 2f\theta^3 + 2g\theta^2 + b\theta + 2h + a)^3(45c\theta^4 + 12f\theta^2 - b) \\ &+(c\theta^4 + 2f\theta^3 + 2g\theta^2 + b\theta + 2h + a)(3c\theta^4 + 4f\theta^3 + 3g\theta^2 + b\theta + h)(-42c\theta^3 - 32f\theta^2 \\ &\qquad\qquad -15g\theta^2 - 2b\theta + h) \\ &+3b(3c\theta^4 + 4f\theta^3 + 3g\theta^2 + b\theta + h)^2 = 0. \end{aligned}$$

The coefficients of the powers 16, 15, 14, 13 of θ all vanish, so that this is in fact an equation of the twelfth order $(*)(\theta, 1)^{12} = 0$; and putting, as usual,

$$(bc - f^2, ca - g^2, ab - h^2, gh - af, hf - bg, fg - ch) = (\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{F}, \mathrm{G}, \mathrm{H}),$$

the equation is found to be

$$\begin{aligned} &-4c\mathrm{A}\ \theta^{12} \\ &+30c\mathrm{H}\ \theta^{11} \\ &-36c\mathrm{B}\ \theta^{10} \\ &+16f\mathrm{A}\ \theta^{10} \\ &-10c\mathrm{G}\ \theta^{9} \\ &+40g\mathrm{A}\ \theta^{9} \\ &+20b\mathrm{A}\ \theta^{8} \\ &-60f\mathrm{B}\ \theta^{8} \\ &-90g\mathrm{H}\ \theta^{8} \\ &+72h\mathrm{A}\ \theta^{7} \\ &+9g\mathrm{B}\ \theta^{7} \\ &-22b\mathrm{H}\ \theta^{7} \\ &+40a\mathrm{A}\ \theta^{6} \\ &-130h\mathrm{H}\ \theta^{6} \\ &+10g\mathrm{G}\ \theta^{6} \\ &+40f\mathrm{F}\ \theta^{6} \\ &+33h\mathrm{B}\ \theta^{5} \\ &+2b\mathrm{G}\ \theta^{5} \\ &-108a\mathrm{H}\ \theta^{5} \\ &+45a\mathrm{B}\ \theta^{4} \\ &-20f\mathrm{C}\ \theta^{4} \\ &+10h\mathrm{G}\ \theta^{4} \\ &+5h\mathrm{F}\ \theta^{3} \\ &+20a\mathrm{G}\ \theta^{3} \\ &-4b\mathrm{C}\ \theta^{3} \\ &-12a\mathrm{F}\ \theta^{3} \\ &-5h\mathrm{C}\ \theta^{2} \\ &-a\mathrm{C} = 0, \end{aligned}$$

where the form of the coefficients may be modified by means of the identical equations

$$\begin{aligned}
(A, H, G\mathcal{X}a, h, g) &= K,\\
(H, B, F\mathcal{X}\ \ ,,\ \ ) &= 0,\\
(G, F, C\mathcal{X}\ \ ,,\ \ ) &= 0,\\
(A, H, G\mathcal{X}h, b, f) &= 0,\\
(H, B, F\mathcal{X}\ \ ,,\ \ ) &= K,\\
(G, F, C\mathcal{X}\ \ ,,\ \ ) &= 0,\\
(A, H, G\mathcal{X}g, f, c) &= 0,\\
(H, B, F\mathcal{X}\ \ ,,\ \ ) &= 0,\\
(G, F, C\mathcal{X}\ \ ,,\ \ ) &= K.
\end{aligned}$$

There is consequently a conic answering to each value of θ given by this equation, or we have in all 12 conics.

In the case where the given conic breaks up into a pair of lines, or say,

$$(a, b, c, f, g, h\mathcal{X}x, y, z)^2 = \Omega(\lambda x+\mu y+\nu z)(\lambda' x+\mu' y+\nu' z),$$

then, writing for shortness

$$\mu\nu'-\mu'\nu,\ \nu\lambda'-\nu'\lambda,\ \lambda\mu'-\lambda'\mu = X, Y, Z,$$

we have

$$(A, B, C, F, G, H) = (X^2, Y^2, Z^2, YZ, ZX, XY).$$

Substituting these values, but retaining (a, b, c, f, g, h) as standing for their values $a=2\lambda\lambda'$, &c., the equation in θ is found to contain the cubic factor $2X\theta^3-3Y\theta^2+Z$, where it is to be observed that this factor equated to zero determines the values of θ which correspond to the points of contact with the cuspidal cubic of the tangents from the point (X, Y, Z), which is the intersection of the lines $\lambda x+\mu y+\nu z=0$, and $\lambda' x+\mu' y+\nu' z=0$; and omitting the cubic factor, the residual equation is found to be

$$\left(\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
2cX & -12cY & -8fX & \begin{array}{l}-20gX\\-12fY\\+\ 4cZ\end{array} & \begin{array}{l}-10bX\\+\ 3gY\end{array} & \begin{array}{l}-40hX\\-\ 8bY\end{array} & \begin{array}{l}-20aX\\+17hY\\+\ 7gZ\end{array} & \begin{array}{l}+15aY\\+\ 4hZ\end{array} & +5bZ \quad +aZ\\
\hline
\end{array}\mathcal{X}(\theta, 1)^9\right.$$

$$= 0,$$

where the form of the coefficients may be modified by means of the identical equations

$$\begin{aligned}
aX+hY+gZ &= 0,\\
hX+bY+fZ &= 0,\\
gX+fY+cZ &= 0.
\end{aligned}$$

The equation is of the 9th order, and there are consequently 9 conics.

Annex No. 6 (referred to, No. 48).—*Containing, with the variation referred to in the text,* ZEUTHEN'S *forms for the characteristics of the conics which satisfy four conditions.*

$$
\begin{aligned}
(1)\quad &(\ ::\ ) = n+2m,\\
&(\ \therefore /) = 2n+4m,\\
&(\ ://) = 4n+4m,\\
&(\cdot ///) = 4n+2m,\\
&(\ ////) = 2n+\ m;
\end{aligned}
$$

$$
\begin{aligned}
(1,1)\quad &(\therefore) \ = 2m(\ m+\ n-3)+\ \tau,\\
&(:/) \ = 2m(\ m+2n-5)+2\tau,\\
&(\cdot //) = 2n\,(2m+\ n-5)+2\delta,\\
&(///) \ = 2n\,(\ m+\ n-3)+\ \delta;
\end{aligned}
$$

$$
\begin{aligned}
(1,1,1)\quad &(:) \ = \tfrac{1}{2}[2m^3+6m^2n-n^3-30m^2-18mn+13n^2+84m-42n+(6m+3n-26)\tau],\\
&(\cdot /) = \tfrac{1}{2}[(m+n)\big(-(m+n)^2-7(m+n)+48\big)+4mn(3m+3n-13)+2(3m+3n-20)(\delta+\tau)],\\
&(//) \ = \tfrac{1}{2}[-m^3+6mn^2+2n^3+13m^2-18mn-30n^2-42m+84n+(3m+6n-26)\delta];
\end{aligned}
$$

$$
\begin{aligned}
(1,1,1,1)\quad &(\cdot) = \tfrac{1}{2}\{\ 2(m-3)(m-4)(n^2-m-n)+(n-3)(n-4)(m^2-m-n)\\
&\qquad +4(m^2-11m+28)\tau \qquad +2(n^2-11n+28)\delta\\
&\qquad +(4(n-4)(m-4)-1)(2\delta+\tau)+2\delta^2+\tau^2\},\\
&(/) = \tfrac{1}{2}\{\ (m-3)(m-4)(n^2-m-n)\ +2(n-3)(n-4)(m^2-m-n)\\
&\qquad +2(m^2-11m+28)\tau \qquad +4(n^2-11n+28)\delta\\
&\qquad +(4(n-4)(m-4)-1)(\delta+2\tau)+\delta^2+2\tau^2\};
\end{aligned}
$$

$$
\begin{aligned}
(2)\quad &(\therefore) = \ 3m+\iota,\\
&(:/) = 2(3m+\iota),\\
&(\cdot //) = 2(3m+\iota),\\
&(///) = \ 3m+\iota;
\end{aligned}
$$

$$
\begin{aligned}
(2,1)\quad &(:) = 3(2mn+n^2+4m-10n)+(2m+n-14)\varkappa,\\
&(\cdot /) = 2(3m+\iota)(m+n-12)\ +24(m+n),\\
&(//) = 3(m^2+2mn-10m+4n)+(m+2n-14)\iota
\end{aligned}
$$

$$
\begin{aligned}
(2,1,1)\quad &(\cdot) = \ (2m+n-7)(6\tau+(n-3)\varkappa)\\
&\qquad +((m-n)(m+n-5)+\tau)(3m+\iota-36)\\
&\qquad +12(m-n)(m+n-3),\\
&(/) = \ (m+2n-7)(6\delta+(m-3)\mu)\\
&\qquad +((n-m)(m+n-5)+\delta)(3m+\iota-36)\\
&\qquad +12(n-m)(m+n-3);
\end{aligned}
$$

$$(2,2)(\cdot)=\tfrac{1}{2}(3m+\iota)^2-3(3m+\iota)-9\tau-8\delta,$$
$$(/)=\tfrac{1}{2}(3m+\iota)^2-3(3m+\iota)-8\tau-9\delta;$$

$$(3)(:)=6n-4m+3\varkappa=5m-3n+3\iota,$$
$$(\cdot/)=10n-8m+6\varkappa=10m-8n+6\iota,$$
$$(//)=5n-3m+3\varkappa=6m-4n+3\iota;$$

$$(1,3)(\cdot)=2(-4m^2+3mn+3n^2+28m-32n)+3(2m+n-13)\varkappa,$$
$$(/)=2(3m^2+3mn-4m^2-32m+28n)+3(m+2n-13)\iota;$$

$$(4)(\cdot)=10n-10m+6\varkappa=8m-8n+6\iota,$$
$$(/)=8n-8m+6\varkappa=10m-10n+6\iota.$$

Annex No. 7 (referred to, No. 93).

In connexion with DE JONQUIÈRES' formula, I have been led to consider the following question.

Given a set of equations:

$$a=a \quad (\text{viz. } b=b,\ c=c,\ \&c.),$$
$$ab=ab+(11)a.b \quad \left(\text{viz. } ac=ac+(11)a.c,\ \&c., \text{ and the like in all the subsequent equations}\right),$$
$$abc=abc+(12)(a.bc+b.ac+c.ab)+(111)\,a.b.c,$$
$$abcd=abcd+(13)(a.bcd+\&c.)+(22)(ab.cd+\&c.)+(112)(a.b.cd+\&c.)+1111\ a.b.c.d,$$

and so on indefinitely (where the $(\cdot)$ is used to denote multiplication, and *ab*, *abc* &c., and also ab, abc &c. are so many separate and distinct symbols not expressible in terms of *a*, *b*, *c* &c., a, b, c &c.), then we have conversely a set of equations

$$a=a \quad (\text{viz. } b=b,\ c=c\ \&c.,$$
$$ab=ab+[11]a.b \quad \left(\text{viz. } ac=ac+[11]a.c,\ \&c., \text{ and the like in all the subsequent equations}\right),$$
$$abc=abc=+[12\ ](a.bc+b.ac+c.ab)+[111]a.b.c,$$

$$\begin{aligned} abcd = {} & abcd \\ & + [13](a.bcd + \&c.) \\ & + [22](ab.cd + \&c.) \\ & + [112](a.b.cd + \&c.) \\ & + [1111]\, a.b.c.d, \end{aligned}$$

and so on; and it is required to find the relation between the coefficients ( ) and [ ]; we find, for example,

$$\begin{aligned} [11] &= - (11), \\ [12] &= - (12), \\ [111] &= 3 (11)(12) \\ &\quad - (111), \\ [13] &= - (13), \\ [22] &= - (22), \\ [112] &= 2 (13)(12) \\ &\quad + (22)(11) \\ &\quad - (112), \\ [1111] &= -12 (13)(12)(11) \\ &\quad + 4 (13)(111) \\ &\quad - 3 (22)(11)(11) \\ &\quad + 6 (112)(11) \\ &\quad - (1111). \end{aligned}$$

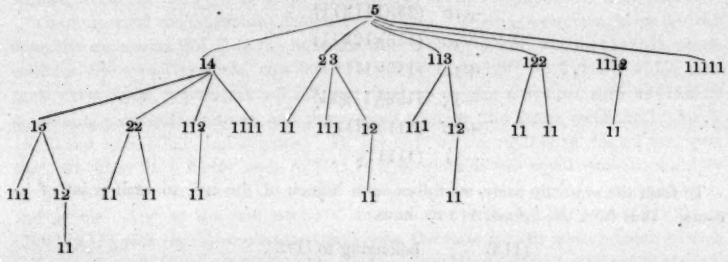
And it is to be noticed that, conversely, the coefficients ( ) are given in terms of the coefficients [ ] by the like equations with the very same numerical coefficients; in fact from the last set of equations, this is at once seen to be the case as far as (112); and for the next term (1111) we have

$$\begin{aligned} (1111) = {} & +12[13][12][11] \\ & - 4[13]\{3[12][11] - [111]\} \\ & + 3[22][11][11] \\ & - 6[11]\left\{\begin{matrix} 2[13][12] \\ +[22][11] \end{matrix}\right\} \\ & -[1111]\left\{\begin{matrix} -[112] \end{matrix}\right\} \end{aligned} \qquad \begin{aligned} = {} & (12) - 12 + 12 =) - 12 \; [13][12][11] \\ & + 4 \; [13][111] \\ & + (3 - 6 = \quad) - 3 \; [22][11][11] \\ & + 6 \; [112][11] \\ & - \quad [1111] \end{aligned}$$

having the same coefficients -12, $+4$, -3, $+6$, -1 as in the formula for [1111] in terms of the coefficients ( ); it is easy to infer that the property hold goods generally.

To explain the law for the expression of the coefficients of either set in terms of the other set, I consider, for example, the case where the sum of the numbers in the ( ), or

[ ] is =5; and I form a kind of tree as follows:



the formation of which is obvious; and I derive from it in the manner about to be explained the expressions for the coefficients [14], [23] &c. in terms of the corresponding coefficients in ( ); viz. we have

$$
\begin{aligned}
[14] &= - \quad (14),\\
[23] &= - \quad (23),\\
[113] &= \quad 2 \quad (14)(13)\\
&\quad + \quad (23)(11)\\
&\quad - \quad (113),\\
[122] &= \quad (14)(22)\\
&\quad + \; 2 \quad (23)(12)\\
&\quad - \quad (112),\\
[1112] &= - \; 6 \quad (14)(13)(12)\\
&\quad - \; 3 \quad (14)(22)(11)\\
&\quad + \; 3 \quad (14)(112)\\
&\quad - \; 6 \quad (23)(12)(11)\\
&\quad + \; 3 \quad (113)(12)\\
&\quad + \; 1 \quad (23)(111)\\
&\quad + \; 3 \quad (122)(11)\\
&\quad - \; 1 \quad (1112),\\
[11111] &= +60 \quad (14)(13)(12)(11)\\
&\quad -20 \quad (14)(13)(111)\\
&\quad +15 \quad (14)(22)(11)(11)\\
&\quad -30 \quad (14)(112)(11)\\
&\quad + \; 5 \quad (14)(1111)
\end{aligned}
$$

$+30$ (23)(12)(11)(11)
-10 (23)(111)(11)
-30 (113)(12)(11)
$+10$ (113)(111)
-15 (122)(11)(11)
$+10$ (1112)(11)
-1 (11111).

To form the symbolic parts, we follow each branch of the tree to each point of its course: thus from the branch 113 we have

(113) belonging to [113],
(113)(111) „ [11111],
(113)(12) „ [1112],
(113)(12)(11) „ [11111];

viz. (113) belongs to [113]; (113)(111), read 11(3 replaced by)111, belongs to [11111]; (113)(12), read 11(3 replaced by) 12, belongs to 1112; (113)(12)(11), read 11(3 replaced by) 1(2 replaced by) 11, belongs to [11111].

And observe that where (as, for example, with the symbol 122) there are branches derived from two or more figures, we pursue each such branch separately, and also all or any of them simultaneously to every point in the course of such branch or branches; thus for the branch 122 we have

(122) belonging to [122],
(122)(11) } (same twice) „ [1112],
(122)(11) }
(122)(11)(11) „ [11111].

Similarly for the branch 23 we have

(23) belonging to [23],
(23)(111) „ [1112],
(23)(12) „ [122],
(23)(12)(11) (same as *infrà*) „ [1112],
(23)(11)(111) „ [11111],
(23)(11)(12) (same as *suprà*) „ [1112],
(23)(11)(12)(11) „ [11111].

We thus obtain the symbolic parts of the several expressions for [14], [23]....[11111] respectively: the sign of each term is + or − according as the number of factors in ( )

is even or odd; thus in the expression for [11111], the term (14)(13)(12)(11) has four factors, and is therefore +, the term (113)(12)(11) has three factors, and is therefore −.

The numerical coefficients are obtained as follows. There is a common factor derived from the expression in [ ] on the left-hand side of the equation; viz. for [11111], which contains five equal symbols, this factor is 1.2.3.4.5, =120; for [1112], which contains three equal symbols, it is 1.2.3, =6; and so on (for a symbol such as [11222] containing two equal symbols, and three equal symbols, the factor would be 1.2.1.2.3, =12, and so in other similar cases). In any term on the right-hand side of the equation, we must for a factor such as (11), which contains two equal symbols, multiply by $\frac{1}{2}$; for a factor such as (111), which contains three equal symbols, multiply by $\frac{1}{6}$, and so on. And in the case where a term (as, for example, the term (122)(11) or (23)(12)(11), *vide suprà*) occurs more than once, the term is to be taken account of each time that it occurs; or, what is the same thing, since the coefficient obtained as above is the same for each occurrence, the coefficient obtained as above is to be multiplied by the number of the occurrences of the term. For example, taking in order the several terms of the expression for [1112], the common factor is =6, and the several coefficients are

$$6,\quad 6.\tfrac{1}{2},\quad 6.\tfrac{1}{3},\quad 6.\tfrac{1}{2}\times 2,\quad 6.\tfrac{1}{2},\quad 6.\tfrac{1}{6},\quad 6.\tfrac{1}{2}.\tfrac{1}{2}\times 2,\quad 6.\tfrac{1}{6};$$

and similarly in the expression for [11111] the common factor is 120, and the coefficients taken in order are

$$120.\tfrac{1}{2},\quad 120.\tfrac{1}{6},\quad 120.\tfrac{1}{2}.\tfrac{1}{2}.\tfrac{1}{2},\ \&c.,$$

without there being in this case any coefficient with a factor arising from the plural occurrence of the term.

The foregoing result was established by induction, and I have not attempted a general proof.

I observe by way of a convenient numerical verification, that in each equation the sum of the coefficients (taken with their proper signs) is $(-)^{n-1}1.2..(n-1)$; if n be the number of parts in the [ ] ($n=5$ for [11111], $=4$ for [1112] &c.), and moreover, that the sum of these sums each multiplied by the proper polynomial coefficient and the whole increased by unity is =0; viz. for

$$[14]\ [23]\ [113]\ [122]\ [1112]\ [11111],$$

the sums of the coefficients are

$$-1,\ -1,\ +2,\ +2,\ -6,\ +24 \text{ respectively,}$$

and we have

$$1+5(-1)+10(-1)+10(2)+15(2)+10(-6)+1(24),\ =75-75,\ =0.$$

If we have any five distinct things (a, b, c, d, e), then the polynomial coefficients 5, 10, 10, 15, 10, 1 denote respectively the number of ways in which these can be partitioned in the forms 14, 23, 113, 122, 1112, 11111 respectively, and the last-mentioned theorem is thus a theorem in the Partition of Numbers.

V. *Second Memoir on the Curves which satisfy given conditions; the Principle of Correspondence.* By Professor CAYLEY, *F.R.S.*

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In the present Memoir I reproduce with additional developments the theory established in my paper "On the Correspondence of two points on a Curve" (London Math. Society, No. VII., April 1866); and I endeavour to apply it to the determination of the number of the conics which satisfy given conditions; viz. these are conditions of contact with a given curve, or they may include arbitrary conditions Z, 2Z, &c. If, for a moment, we consider the more general question where the Principle is to be applied to finding the number of the curves C^r of the order r, which satisfy given conditions of contact with a given curve, there are here two kinds of special solutions; viz., we may have proper curves C^r touching (specially) the given curve at a cusp or cusps thereof, and we may have improper curves, that is, curves which break up into two or more curves of inferior orders. In the case where the curves C^r are lines, there is only the first kind of special solution, where the sought for lines touch at a cusp or cusps. But in the case to which the Memoir chiefly relates, where the curves C^r are conics, we have the two kinds of special solutions, viz., proper conics touching at a cusp or cusps, and conics which are line-pairs or point-pairs. In the application of the Principle to determining the number of the conics which satisfy any given conditions, I introduce into the equation a term called the "Supplement" (denoted by the abbreviation "Supp."), to include the special solutions of both kinds. The expression of the Supplement should in every case be furnished by the theory; and this being known, we should then have an equation leading to the number of the conics which properly satisfy the prescribed conditions; but in thus finding the expression of the Supplements, there are difficulties which I am unable to overcome; and I have contented myself with the reverse course, viz., knowing in each case the number of the proper solutions, I use these results to determine *à posteriori* in each case the expression of the Supplement; the expression so obtained can in some cases be accounted for readily enough, and the knowledge of the whole series of them will be a convenient basis for ulterior investigations.

The Principle of Correspondence for points in a line was established by CHASLES in the paper in the Comptes Rendus, June–July 1864, referred to in my First Memoir; it is extended to unicursal curves in a paper of the same series, March 1866, "Sur les courbes planes ou à double courbure dont les points peuvent se déterminer individuellement—Application du Principe de Correspondance dans la théorie de ces courbes," but not to the case of a curve of given deficiency D considered in my paper of April 1866

above referred to. The fundamental theorem in regard to unicursal curves, viz. that in a curve of the order m with $\frac{1}{2}(m-1)(m-2)$ double points (nodes or cusps) the coordinates (x, y, z) are proportional to rational and integral functions of a variable parameter θ,—as a case of a much more general theorem of RIEMANNS—dates from the year 1857, but was first explicitly stated by CLEBSCH in the paper "Ueber diejenigen ebenen Curven deren Coordinaten rationale Functionen eines Parameters sind," Crelle, t. 64 (1864), pp. 43–63. See also my paper "On the Transformation of Plane Curves," London Mathematical Society, No. III., Oct. 1865.

The paragraphs of the present Memoir are numbered consecutively with those of the First Memoir.

On the Correspondence of two points on a Curve.—Article Nos. 94 to 104.

94. In a unicursal curve the coordinates (x, y, z) of any point thereof are proportional to rational and integral functions of a variable parameter θ. Hence if two points of the curve correspond in such wise that to a given position of the first point there correspond α' positions of the second point, and to a given position of the second point α positions of the first point, the number of points which correspond each to itself is $=\alpha+\alpha'$. For let the two points be determined by their parameters θ, θ' respectively, then to a given value of θ there correspond α' values of θ', and to a given value of θ there correspond α values of θ; hence the relation between (θ, θ') is of the form $(\theta, 1)^{\alpha}(\theta', 1)^{\alpha'}=0$; and writing therein $\theta'=\theta$, then for the points which correspond each to itself, we have an equation $(\theta, 1)^{\alpha+\alpha'}=0$, of the order $\alpha+\alpha'$; that is, the number of these points is $=\alpha+\alpha'$.

Hence for a unicursal curve we have a theorem similar to that of M. CHASLES' for a line, viz. the theorem may be thus stated:—

If two points of a unicursal curve have an (α, α') correspondence, the number of united points is $=\alpha+\alpha'$. But a unicursal curve is nothing else than a curve with a deficiency $D=0$, and we thence infer—

Theorem. If two points of a curve with deficiency D have an (α, α') correspondence, the number of united points is $=\alpha+\alpha'+2k\mathrm{D}$; in which theorem $2k$ is a coefficient to be determined.

95. Suppose that the corresponding points are P, P', and imagine that when P is given the corresponding points P' are the intersections of the given curve by a curve Θ (the equation of the curve Θ will of course contain the coordinates of P as parameters, for otherwise the position of P' would not depend upon that of P). I find that if the curve Θ has with the given curve k intersections at the point P, then in the system of points (P, P') the number of united points is

$$\mathrm{a}=\alpha+\alpha'+2k\mathrm{D},$$

whence in particular if the curve Θ does not pass through the point P, then the number of united points is $=\alpha+\alpha'$, as in the case of a unicursal curve. (I have in the paper of

April 1866 above referred to, proved this theorem in the particular case where the k intersections at the point P take place in consequence of the curve Θ having a k-tuple point at P, but have not gone into the more difficult investigation for the case where the k intersections arise wholly or in part from a contact of the curve Θ, or any branch or branches thereof, with the given curve at P.)

96. It is to be observed that the general notion of a united point is as follows: taking the point P at random on the given curve, the curve Θ has at this point k intersections with the given curve; the remaining intersections are the corresponding points P′; if for a given position of P one or more of the points P′ come to coincide with P, that is, if for the given position of P the curve Θ has at this point more than k intersections with the given curve, then the point in question is a united point.

It might at first sight appear that if for a given position of P a number 2, 3, . . or j of the points P′ should come to coincide with P, then that the point in question should reckon for 2, 3 . . . or j (as the case may be) united points: but this is not so. This is perhaps most easily seen in the case of a unicursal curve; taking the equation of correspondence to be $(\theta, 1)^{\alpha}(\theta', 1)^{\alpha'}=0$, then we have $\alpha+\alpha'$ united points corresponding to the values of θ which satisfy the equation $(\theta, 1)^{\alpha}(\theta, 1)^{\alpha'}=0$; if this equation has a j-tuple root $\theta=\lambda$, the point P which answers to this value λ of the parameter is reckoned as j united points. But starting from the equation $(\theta, 1)^{\alpha}(\theta', 1)^{\alpha'}=0$, if on writing in this equation $\theta=\lambda$, the resulting equations $(\lambda, 1)^{\alpha}(\theta', 1)^{\alpha'}=0$ has a root $\theta'=\lambda$, it follows that the equation $(\theta, 1)^{\alpha}(\theta, 1)^{\alpha'}=0$ has a root $\theta=\lambda$, and that the point which belongs to the value $\theta=\lambda$ is a united point; if on writing in the equation $\theta=\lambda$, the resulting equation $(\lambda, 1)^{\alpha}(\theta', 1)^{\alpha'}=0$ has a j-tuple root $\theta'=\lambda$, *it does not follow* that the equation $(\theta, 1)^{\alpha}(\theta, 1)^{\alpha'}=0$ has a j-tuple root $\theta=\lambda$, nor consequently that the point answering to $\theta=\lambda$ in anywise reckons as j united points.

97. This may be further illustrated by regarding the parameters θ, θ' as the coordinates of a point in a plane; the equation $(\theta, 1)^{\alpha}(\theta', 1)^{\alpha'}=0$ is that of a curve of the order $\alpha+\alpha'$, having an α-tuple point at infinity on the axis $\theta=0$, and an α'-tuple point at infinity on the axis $\theta'=0$; the united points are given as the intersections of the curve with the line $\theta=\theta'$; a j-fold intersection, whether arising from a multiple point of the curve or from a contact of the line $\theta=\theta'$ with the curve, gives a point which reckons as j united points. But if $\theta=\lambda$ gives the j-fold root $\theta'=\lambda$, this shows that the line $\theta=\lambda$ has with the curve j intersections at the point $\theta=\theta'=\lambda$; *not* that the line $\theta=\theta$ has with the curve j intersections at the point in question.

98. Reverting to the notion of a united point as a point P which is such that one or more of the corresponding points P′ come to coincide with P; in the case where P is at a node of the given curve, it is necessary to explain that the point P must be considered as belonging to one or the other of the two branches through the node, and that the point P is not to be considered as a united point unless we have *on the same branch of the curve* one or more of the corresponding points P′ coming to coincide with the point P. If, to fix the ideas, $k=1$, that is, if the curve Θ simply pass through the point P, then if

P be at a node the curve Θ passes through the node and has therefore at this point two intersections with the given curve; but the second intersection belongs to the other branch, and the node is not a united point; in order to make it so, it is necessary that the curve Θ should at the node touch the branch to which the point P is considered to belong. The thing appears very clearly in the case of a unicursal curve; we have here two values $\theta=\lambda$, $\theta=\lambda'$ answering to the node according as it is considered as belonging to one or the other branch of the curve; and in the equation of correspondence $(\theta, 1)^{\kappa}(\theta', 1)^{\kappa'}=0$, writing $\theta=\lambda$, we have an equation $(\lambda, 1)^{\kappa}(\theta', 1)^{\kappa'}=0$ satisfied by $\theta'=\lambda'$ but not by $\theta'=\lambda$, and the equation $(\theta, 1)^{\kappa}(\theta, 1)^{\kappa'}=0$ is thus not satisfied by the value $\theta=\lambda$. The conclusion is that a node *quà* node is not a united point.

99. But it is otherwise as regards a cusp. When the point P is at a cusp, the curve Θ (which has in general with the given curve k intersections at P) has here more than k intersections, and (as in this case there is no distinction of branch) the cusp reckons as a united point. In the case of a unicursal curve, there is at the cusp a single value $\theta=\lambda$ of the parameter, and the equation $(\theta, 1)^{\kappa}(\theta, 1)^{\kappa'}=0$ is satisfied by the value $\theta=\lambda$. But for the very reason that the cusp *quà* cusp reckons as a united point, the cusp is a united point only in an improper or special sense, and it is to be rejected from the number of true united points. We may include the cusps, along with any other special solutions which may present themselves, under a head "Supplement," and instead of writing as above $\mathrm{a}-\alpha-\alpha'=2k\mathrm{D}$, write $\mathrm{a}-\alpha-\alpha'+\mathrm{Supp.}=2k\mathrm{D}$.

Before going further I apply the theorem to some examples in which the curve Θ is a system of lines.

100. Investigation of the class of a curve of the order m with δ nodes and $\varkappa$ cusps. Take as corresponding points on the given curve two points such that the line joining them passes through a fixed point O; the united points will be the points of contact of the tangents through O; that is, the number of the united points will be equal to the class of the curve. The curve Θ is here the line OP which has with the given curve a single intersection at P; that is, we have $k=1$. The points P′ corresponding to a given position of P are the remaining $m-1$ intersections of OP with the curve, that is, we have $\alpha'=m-1$; and in like manner $\alpha=m-1$. Each of the cusps is (specially) a united point, and counts once, whence the Supplement is $=\varkappa$. Hence, writing n for the class, we have $n+2(m-1)+\varkappa=2\mathrm{D}$, or writing for 2D its value $=m^2-3m+2-2\delta-2\varkappa$, we have $n=m^2-m-2\delta-3\varkappa$, which is right.

101. Investigation of the number of inflexions. Taking the point P′ to be a tangential of P (that is, an intersection of the curve by the tangent at P), the united points are the inflexions; and the number of the united points is equal to the number of the inflexions. The curve Θ is the tangent at P having with the given curve two intersections at this point; that is, $k=2$; P′ is any one of the $m-2$ tangentials of P, that is, $\alpha'=m-2$; and P is the point of contact of any one of the $n-2$ tangents from P′ to the curve, that is, $\alpha=n-2$. Each cusp is (specially) a united point, and counts once, whence the Supplement is $=\varkappa$. Hence, writing ι for the number of inflexions, we have

$$\iota-(m-2)-(n-2)+\varkappa=4\mathrm{D};$$

or substituting for 2D its value expressed in the form $n-2m+2+\varkappa$, we have

$$\iota = 3n-3m+\varkappa,$$

which is right.

102. For the purpose of the next example it is necessary to present the fundamental equation under a more general form. The curve Θ may intersect the given curve in a system of points P′, each p times, a system of points Q′, each q times, &c. in such manner that the points (P, P′), the points (P, Q′), &c. are pairs of points corresponding to each other according to distinct laws; and we shall then have the numbers $(\mathrm{a}, \alpha, \alpha')$, $(\mathrm{b}, \beta, \beta')$, &c. corresponding to these pairs respectively, viz. (P, P′) are points having an (α, α') correspondence, and the number of united points is $=\mathrm{a}$; (P, Q′) are points having a (β, β') correspondence, and the number of united points is $=\mathrm{b}$, and so on. The theorem then is

$$p(\mathrm{a}-\alpha-\alpha')+q(\mathrm{b}-\beta-\beta')+\&\mathrm{c.}+\mathrm{Supp.}=2k\mathrm{D},$$

being in fact the most general form of the theorem for the correspondence of two points on a curve, and that which will be used in all the investigations which follow.

103. Investigation of the number of double tangents. Take P′ an intersection of the curve with a tangent from P to the curve (or, what is the same thing, P, P′ cotangentials of any point of the curve): the united points are here the points of contact of the several double tangents of the curve; or if τ be the number of double tangents, then the number of united points is $=2\tau$. The curve Θ is the system of the $n-2$ tangents from P to the curve; each tangent has with the curve a single intersection at P, that is, $k=n-2$; each tangent besides meets the curve in the point of contact Q′ twice, and in $(m-3)$ points P′; hence if $(\mathrm{a}, \alpha, \alpha')$ refer to the points (P, Q′), and $(2\tau, \beta, \beta')$ to the points (P, P′), we have

$$2\{\mathrm{a}-\alpha-\alpha'\}+\{2\tau-\beta-\beta'\}+\mathrm{Supp.}=2(n-2)\mathrm{D}.$$

From the foregoing example the value of $\mathrm{a}-\alpha-\alpha'$ is $=4\mathrm{D}-\varkappa$. In the case where the point P is at a cusp, then the $n-2$ tangents become the $n-3$ tangents from the cusp, and the tangent at the cusp; hence the curve Θ meets the given curve in $2(n-3)+3$, $=2n-3$ points, that is, $(n-2)+(n-1)$ points; this does not prove (*ante*, No. 96), but the fact is, that the cusp counts in the Supplement $(n-1)$ times, and the expression of the Supplement is $=(n-1)\varkappa$. It is clear that we have $\beta=\beta'=(n-2)(m-3)$, so that the equation is

$$8\mathrm{D}-2\varkappa+2\tau-2(n-2)(m-3)+(n-1)\varkappa=(n-2)2\mathrm{D},$$

that is,

$$2\tau=2(n-2)(m-3)+(n-6)2\mathrm{D}+(-n+3)\varkappa;$$

or substituting for 2D its value $=n-2m+2+\varkappa$ and reducing, this is

$$2\tau=n^2+8m-10n-3\varkappa,$$

which is right.

104. As another example, suppose that the point P on a given curve of the order m and the point Q on a given curve of the order m' have an (α, α') correspondence, and let

it be required to find the class of the curve enveloped by the line PQ. Take an arbitrary point O, join OQ, and let this meet the curve m in P'; then (P, P') are points on the curve m having a $(m'\alpha, m\alpha')$ correspondence; in fact to a given position of P there correspond α' positions of Q, and to each of these m positions of P'; that is, to each position of P there correspond $m\alpha'$ positions of P'; and similarly to each position of P' there correspond $m'\alpha$ positions of P. The curve Θ is the system of the lines drawn from each of the α' positions of Q to the point O, hence the curve Θ does not pass through P, and we have $k=0$. Therefore the number of the united points (P, P'), that is, the number of the lines PQ which pass through the point O, is $=m\alpha'+m'\alpha$, or this is the class of the curve enveloped by PQ.

It is to be noticed that if the two curves are curves in space (plane, or of double curvature), then the like reasoning shows that the number of the lines PQ which meet a given line O is $=m\alpha'+m'\alpha$, that is, the order of the scroll generated by the line PQ is $=m\alpha'+m'\alpha$.

Application to the Conics which satisfy given conditions, one at least arbitrary.—
Article Nos. 105 to 111.

105. Passing next to the equations which relate to a conic, we seek for (4Z)(1), the number of the conics which satisfy any four conditions 4Z and besides touch a given curve, (3Z)(2) and (3Z)(1, 1), the number of the conics which satisfy three conditions, and besides have with the given curve a contact of the second order, or (as the case may be) two contacts of the first order; and so on with the conditions 2Z, Z, and then finally (5), (4, 1), ... (1, 1, 1, 1, 1), the numbers of the conics which have with the given curve a contact of the fifth order, or a contact of the fourth and also of the first order ..., or five contacts of the first order.

106. As regards the case (4Z)(1), taking P an arbitrary point of the given curve m, and for the curve Θ the system of the conics $(4Z)(\overline{1})$ which pass through the given point P and besides satisfy the four conditions, then the curve Θ has with the given curve $(4Z)(\overline{1})$ intersections at P, and the points P' are the remaining $(2m-1)(4Z)(\overline{1})$ intersections: in the case of a united point (P, P'), some one of the system of conics becomes a conic (4Z)(1); and the number of the united points is consequently equal to that of the conics (4Z)(1); we have thus the equation

$$\{(4Z)(1)-2(2m-1)(4Z)(\overline{1})\}+\text{Supp.}\,(4Z)(\overline{1})=(4Z)(\overline{1})\,.\,2D.$$

107. It is in the present case easy to find *à priori* the expression for the Supplement. 1. The system of conics (4Z) contains $2(4Z\,.\,)-(4Z/)$ point-pairs*; each of these, regarded as a line, meets the given curve in m points, and each of these points is (specially) a united point (P, P'); this gives in the Supplement the term $m\{2(4Z\,.\,)-(4Z/)\}$. 2. The number of the conics (4Z) which can be drawn through a cusp of the given curve is

* The expression a point-pair is regarded as equivalent to and standing for that of a coincident line-pair: see First Memoir, No. 30.

$=(4Z\cdot)$; and the cusp is in respect of each of these conics a united point; we have thus the term $\varkappa(4Z\cdot)$, and the Supplement is thus $=m\{2(4Z\cdot)-(4Z/)\}+\varkappa(4Z\cdot)$. We have moreover $(4Z)(\overline{1})=(4Z\cdot)$, $2D=n-2m+2+\varkappa$; and substituting these values, we find

$$\begin{aligned}(4Z)(1)= &\ (4m-2)(4Z\cdot)\\ &-m\{2(4Z\cdot)-(4Z/)\}-\varkappa(4Z\cdot)\\ &+(n-2m+2+\varkappa)(4Z\cdot)\\ = &\ n(4Z\cdot)+m(4Z/),\end{aligned}$$

which is right.

108. It is clear that if, instead of finding as above the expression of the Supplement, the value of $(4Z)(1),=n(4Z\cdot)+m(4Z/)$, had been taken as known, then the equation would have led to

$$\text{Supp.}(4Z)(\overline{1})=m\{2(4Z\cdot)-(4Z/)\}+\varkappa(4Z\cdot);$$

and this, as in fact already remarked, is the course of treatment employed in the remaining cases. It is to be observed also that the equation may for shortness be written in the form

$$\begin{aligned}(4Z)\quad &\{(1)-2(2m-1)(\overline{1})\}\\ &+\text{Supp.}(\overline{1})=(\overline{1})2D;\end{aligned}$$

viz. the $(4Z)$ is to be understood as accompanying and forming part of each symbol; and the like in other cases.

109. We have the series of equations

$$\begin{aligned}(4Z)\quad &\{(1)-(\overline{1})(2m-1)-(\overline{1})(2m-1)\}\\ &+\text{Supp.}(\overline{1}) &&=(\overline{1})2D;\\ (3Z)\quad &\{(2)-(\overline{2})(2m-2)-(\overline{1},1)\}\\ &+\text{Supp.}(\overline{2}) &&=2(\overline{2})2D;\\ (3Z)\quad &2\{(2)-(\overline{1},1)-(\overline{2})(2m-2)\}\\ &+\{2(1,1)-(\overline{1},1)(2m-3)-(\overline{1},1)(2m-3)\}\\ &+\text{Supp.}(\overline{1},1) &&=(\overline{1},1)2D;\\ (2Z)\quad &\{(3)-(\overline{3})(2m-3)-(\overline{1},2)\}\\ &+\text{Supp.}(\overline{3}) &&=3(\overline{3})2D;\\ (2Z)\quad &2\{(3)-(\overline{2},1)-(\overline{2},1)\}\\ &+\{(2,1)-(\overline{2},1)(2m-4)-(\overline{1},1,1)2\}\\ &+\text{Supp.}(\overline{2},1) &&=2(\overline{2},1)2D;\\ (2Z)\quad &3\{(3)-(\overline{1},2)-(\overline{3})(2m-3)\}\\ &+\{(1,2)-(\overline{1},2)(2m-4)-(\overline{1},2)(2m-4)\}\\ &+\text{Supp.}(\overline{1},2) &&=(\overline{1},2)2D;\end{aligned}$$

(2Z) $2\{(2, 1)-(\overline{1}, 1, 1)2-(\overline{2}, 1)(2m-4)\}$
$+\ \{3(1, 1, 1)-(\overline{1}, 1, 1)(2m-5)-(\overline{1}, 1, 1)(2m-5)\}$
$+$ Supp. $(1, 1, 1)$ $=(\overline{1}, 1, 1)2D$;

(Z) $\{(4)-(\overline{4})(2m-4)-(\overline{1}, 3)\}$
$+$ Supp. $(\overline{4})$ $=4(\overline{4})2D$;

(Z) $2\{(4)-(\overline{3}, 1)-(\overline{2}, 2)\}$
$+\{(3, 1)-(\overline{3}, 1)(2m-5)-(\overline{1}, 1, 2)\}$
$+$ Supp. $(\overline{3}, 1)$ $=3(\overline{3}, 1)2D$;

(Z) $3\{(4)-(\overline{2}, 2)-(3, \overline{1})\}$
$+\{2(2, 2)-(\overline{2}, 2)\,(2m-5)-(\overline{1}, 1, 2)\}$
$+$ Supp. $(\overline{2}, 2)$ $=2(\overline{2}, 2)2D$;

(Z) $2\{(3, 1)-(\overline{2}, 1, 1)2-(\overline{2}, 1, 1)2\}$
$+\{(2, 1, 1)-(\overline{2}, 1, 1)(2m-6)-(\overline{1}, 1, 1, 1)3\}$
$+$ Supp. $(\overline{2}, 1, 1)$ $=2(\overline{2}, 1, 1)2D$;

(Z) $4\{(4)-(\overline{1}, 3)-(\overline{4})(2m-4)\}$
$+\{(1, 3)-(\overline{1}, 3)(2m-5)-(\overline{1}, 3)(2m-5)\}$
$+$ Supp. $(\overline{1}, 3)$ $=(\overline{1}, 3)2D$;

(Z) $3\{(3, 1)-(\overline{1}, 1, 2)-(\overline{3}, 1)(2m-5)\}$
$+2\{2(2, 2)-(\overline{1}, 1, 2)-(\overline{2}, 2)(2m-5)\}$
$+\{2(1, 1, 2)-(\overline{1}, 1, 2)(2m-6)-(\overline{1}, 1, 2)(2m-6)\}$
$+$ Supp. $(\overline{1}, 1, 2)$ $=(\overline{1}, 1, 2)2D$;

(Z) $2\{(2, 1, 1)-(\overline{1}, 1, 1, 1)3-(\overline{2}, 1, 1)(2m-6)\}$
$+\{4(1, 1, 1, 1)-(\overline{1}, 1, 1, 1)(2m\ \ 7)-(\overline{1}, 1, 1, 1)(2m-7)\}$
$+$ Supp. $(\overline{1}, 1, 1, 1)$ $=(\overline{1}, 1, 1, 1)2D$.

110. I content myself with giving the expressions of only the following supplements.

$$\text{Supp. } (4Z)(\overline{1}) \quad = m[2(\cdot)-(/)]+\varkappa(\cdot).$$

$$\text{Supp. } (3Z)(\overline{2}) \quad = \tfrac{1}{2}n[2(:)-(\cdot/)]+\tfrac{1}{2}\varkappa(\cdot/).$$

$$\begin{aligned}\text{Supp. } (3Z)(\overline{1}, 1) = \quad & (\qquad 2mn-3n^2- \quad n+n\alpha)(:) \\ & +(2m^2-4mn \qquad -2m+2n+(m-\tfrac{1}{2})\alpha)(\cdot/) \\ & +(-m^2 \qquad\quad + \quad m \qquad\quad)(//).\end{aligned}$$

$$\begin{aligned}\text{Supp. } (2Z)(\overline{3}) &= -\tfrac{1}{4}m[2(\cdot\cdot)-(:/)]\\ &\quad +\tfrac{1}{4}n[2(\cdot\cdot)-(:/)+2(2(:/)-(\,\cdot\,//))]\\ &\quad +\tfrac{1}{2}\varkappa(:/).\\ \text{Supp. } (Z)(\overline{4}) &= a\varkappa+b(2\varkappa+2\iota),\end{aligned}$$

where a, b are the representatives of the condition Z.

It may be added that we have in general

$$\text{Supp. } (Z)(4X)=a \text{ Supp. } (4X\,\cdot\,)+b \text{ Supp. } (4X/),$$

where (4X) stands for any one of the symbols $(\overline{4})$, $(\overline{3}, 1)$ . . . . $(\overline{1}, 1, 1, 1)$.

111. The expression of Supp. (4Z)$(\overline{1})$ has been explained *suprà*, No. 108. That of Supp. (3Z)(2) may also be explained. 1. The point-pairs of the system of conics (3Z), regarding each point-pair as a line, are a set of lines enveloping a curve; the class of this curve is equal to the number of the lines which pass through an arbitrary point, that is, as at first sight would appear, to the number of point-pairs in the system $(3Z\,\cdot\,)$, or to $2(3Z:)-(3Z\,\cdot\,/)$: it is, however, necessary to admit that the number of distinct lines, and therefore the class of the curve, is one-half of this, or $=\frac{1}{2}[2(3Z:)-(3Z\,\cdot\,/)]$; which being so, the number of the point-pairs (3Z) which, regarded as lines, touch the given curve (of the order m and class n) is $=\frac{1}{2}n[2(3Z:)-(3Z\,\cdot\,/)]$. The point of contact of any one of these lines with the given curve is (specially) a united point, and we have thus the term $\frac{1}{2}n[2(3Z:)-(3Z\,\cdot\,/)]$ of the Supplement. 2. The number of the conics (3Z) which touch the given curve at a given cusp thereof, or, say, the conics $(3Z)(\overline{2\varkappa}1)$, is $=\frac{1}{2}(3Z\,\cdot\,/)$, and the cusp is in respect of each of these conics a united point; we have thus the remaining term $\frac{1}{2}\varkappa(3Z\,\cdot\,/)$ of the Supplement.

Application to the Conics which satisfy five conditions of contact with a given Curve.—Articles Nos. 112 to 135.

112. We have twelve equations, which I first present in what I call their original forms; viz. these are—

First equation:

$$\begin{aligned}&\{(5)-(\overline{5})(2m-5)-(\overline{1}, 4)\}\\ &+ \text{ Supp. } (\overline{5}) \qquad\qquad =5(\overline{5})2D.\end{aligned}$$

Second equation:

$$\begin{aligned}&2\{(5)-(\overline{4}, 1)-(\overline{2}, 3)\}\\ &+ \{(4, 1)-(\overline{4}, 1)(2m-6)-(\overline{1}, 1, 3)\}\\ &+ \text{ Supp. } (\overline{4}, 1) \qquad\qquad =4(\overline{4}, 1)2D.\end{aligned}$$

Third equation:

$$\begin{aligned} & 3\{(5)-2(\overline{3}, 2)-2(\overline{3}, 2)\} \\ + & \{(3, 2)-(\overline{3}, 2)(2m-6)-2(\overline{1}, 2, 2)\} \\ + & \text{Supp.}(\overline{3}, 2) \end{aligned} \qquad = 3(\overline{3}, 2)2\text{D}.$$

Fourth equation:

$$\begin{aligned} & 2\{(4, 1)-2(\overline{3}, 1, 1)-(\overline{2}, 2, 1)\} \\ + & \{(3, 1, 1)-(\overline{3}, 1, 1)(2m-7)-(\overline{1}, 1, 1, 2)\} \\ + & \text{Supp.}(\overline{3}, 1, 1) \end{aligned} \qquad = 3(\overline{3}, 1, 1)2\text{D}.$$

Fifth equation:

$$\begin{aligned} & 4\{(5)-(\overline{2}, 3)-(\overline{4}, 1)\} \\ + & \{(3, 2)-(\overline{2}, 3)(2m-6)-(\overline{1}, 1, 3)\} \\ + & \text{Supp.}(\overline{2}, 3) \end{aligned} \qquad = 2(\overline{2}, 3)2\text{D}.$$

Sixth equation:

$$\begin{aligned} & 3\{(4, 1)-2(\overline{2}, 2, 1)-2, (\overline{3}, 1, 1)\} \\ + & 2\{(3, 2)-(\overline{2}, 2, 1)-(\overline{2}, 2, 1)\} \\ + & \{2(2, 2, 1)-(\overline{2}, 2, 1)(2m-7)-2(\overline{1}, 1, 1, 2)\} \\ + & \text{Supp.}(\overline{2}, 2, 1) \end{aligned} \qquad = 2(\overline{2}, 2, 1)2\text{D}.$$

Seventh equation:

$$\begin{aligned} & 2\{(3, 1, 1)-3(\overline{2}, 1, 1, 1)-3(\overline{2}, 1, 1, 1)\} \\ + & \{(2, 1, 1, 1)-(\overline{2}, 1, 1, 1)(2m-8)-4(\overline{1}, 1, 1, 1, 1)\} \\ + & \text{Supp}\ (\overline{2}, 1, 1, 1) \end{aligned} \qquad = 2(\overline{2}, 1, 1, 1)2\text{D}.$$

Eighth equation:

$$\begin{aligned} & 5\{(5)-(\overline{1}, 4)-(\overline{5})(2m-5)\} \\ + & \{(4, 1)-(\overline{1}, 4)(2m-5)-(\overline{1}, 4)(2m-5)\} \\ + & \text{Supp.}(\overline{1}, 4) \end{aligned} \qquad = (\overline{1}, 4)2\text{D}.$$

Ninth equation:

$$\begin{aligned} & 4\{(4, 1)-(\overline{1}, 1, 3)-(\overline{4}, 1)(2m-6)\} \\ + & 2\{(3, 2)-(\overline{1}, 1, 3)-(\overline{2}, 3)(2m-6)\} \\ + & \{2(3, 1, 1)-(\overline{1}, 1, 3)(2m-7)-(\overline{1}, 1, 3)(2m-7)\} \\ + & \text{Supp.}(\overline{1}, 1, 3) \end{aligned} \qquad = (\overline{1}, 1, 3)2\text{D}.$$

Tenth equation:

$$\begin{aligned} &3\{(3,2)-2(\bar{1},2,2)-(\bar{3},2)(2m-6)\} \\ &+\{(2,2,1)-(\bar{1},2,2)(2m-7)-(\bar{1},2,2)(2m-7)\} \\ &+\text{Supp.}\,(\bar{1},2,2) \qquad\qquad =(\bar{1},2,2)2\text{D}. \end{aligned}$$

Eleventh equation:

$$\begin{aligned} &3\{(3,1,1)-(\bar{1},1,1,2)-(\bar{3},1,1)(2m-7)\} \\ &+2\{2(2,2,1)-2(\bar{1},1,1,2)-(\bar{2},2,1)(2m-7)\} \\ &+\{3(2,1,1,1)-(\bar{1},1,1,2)(2m-8)-(\bar{1},1,1,2)(2m-8)\} \\ &+\text{Supp.}\,(\bar{1},1,1,2) \qquad\qquad =(\bar{1},1,1,2)2\text{D}. \end{aligned}$$

Twelfth equation:

$$\begin{aligned} &2\{(2,1,1,1)-4(\bar{1},1,1,1,1)-(\bar{2},1,1,1)(2m-8)\} \\ &+\{5(1,1,1,1,1)-(\bar{1},1,1,1,1)(2m-9)-(\bar{1},1,1,1,1)(2m-9)\} \\ &+\text{Supp.}\,(\bar{1},1,1,1,1) \qquad\qquad =(\bar{1},1,1,1,1)2\text{D}. \end{aligned}$$

113. I alter the forms of these equations by substituting for 2D its value $=n-2m+2+\varkappa$, and by writing for the expressions with $(\bar{1})$ their values,

$$(\bar{1},4)=(\cdot\,4)-5(\bar{5}),\ \&\text{c.},$$

and except in the terms $\{\text{Supp.}\,(\bar{5})-\varkappa(\bar{5})\}$, &c., by writing for $\varkappa$ its value $-3n+\alpha$. The resulting equations, if the Supplements were known, would serve to determine the values of (5), (4, 1), &c.; but I assume instead that the last-mentioned expressions are known (First Memoir, No. 50), and use the equations to determine the Supplements, or, what comes to the same thing, the values of the terms in $\{\}$ which contain these Supplements. We have thus the twelve reduced equations, with resulting values of the supplements.

114. First equation:

$$=0=$$

(5)	$-15m-15n+9\alpha$
$+\{\text{Supp.}\,(\bar{5})-\varkappa(\bar{5})\}$	$-\ 3m\ \ +\ \alpha$
$+(\bar{5})(8m+7n-4\alpha)$	$\ \ 8m+\ 7n-4\alpha$
$-(\cdot\,4)$	$+10m+\ 8n-6\alpha$

(that is, we have

$$\text{Supp.}\,(\bar{5})-\varkappa(\bar{5})=-3m+\alpha,$$

and so in the subsequent cases, the equation gives the value of the term in $\{\}$ which contains the Supplement).

115. Second equation:

$$=0=$$

2(5)	$-30m-30n+\alpha($ 18)
$+(4,1)$	$-8m^2-20mn-8n^2+104m+104n+\alpha(6m+6n-66)$
$+\{\text{Supp. }(\overline{4},1)-\varkappa(\overline{4},1)\}$	$-6m^2-3mn+18m+9n+\alpha(3m-9)$
$+(\overline{4},1)(6m+5n-3\alpha)$	$+6m^2+11mn+5n^2-36m-30n+\alpha(-3m-3n+18)$
$-(\cdot 1,3)$	$+8m^2+12mn+3n^2-56m-53n+\alpha(-6m-3n+39).$

I stop for a moment to notice a very convenient verification of the term in { }; putting therein $\alpha=3n$, the term is

$$-6m^2-3mn+18m+9n+(9mn-27n);$$

and if in this we write $m=n=1$, $m^2=mn=n^2=2$, and when any higher terms enter $m^3=m^2n=mn^2=n^3=4$, $m^4=m^3n=m^2n^2=mn^3=n^4=8$, &c., the value is $-12-6+18+9+18-27$, $=0$, viz. we should always obtain a sum $=0$. The reason is that the term in question should always admit of being expressed in the form $p\delta+q\varkappa+r\tau+s\iota$; the reduction to this form might be effected by the substitutions $m=\frac{1}{2}(m+n)+\frac{1}{6}(\varkappa-\iota)$, $n=\frac{1}{2}(m+n)-\frac{1}{6}(\varkappa-\iota)$, $m^2=2.\frac{1}{2}(m+n)+2\delta+3\varkappa$, $n^2=2.\frac{1}{2}(m+n)+2\tau+3\iota$, giving a result $=A(m+n)+$terms in $(\delta,\varkappa,\tau,\iota)$, where A is a numerical coefficient calculable as above by simply writing $m=n=1$, $m^2=mn=n^2=2$, &c., and which is $=0$ when the term is of the proper form $p\delta+q\varkappa+r\tau+s\iota$. The complete reduction to the form in question is material in the sequel, but I advert to the point here only for the sake of the numerical verification.

116. Third equation:

$$=0=$$

3(5)	$-45m-45n+\alpha(+27)$
$+(3,2)$	$+120m+120n+\alpha(-4m-4n-78)+3\alpha^2$
$+\{\text{Supp. }(\overline{3},2)-\varkappa(\overline{3},2)\}$	$+15m+\alpha(n-7)$
$+(\overline{3},2)(4m+3n-2\alpha)$	$-36m-27n+\alpha(4m+3n+18)-2\alpha^2$
$-2(\cdot 2,2)$	$-54m-48n+\alpha(+40)-\alpha^2.$

Verification is $15+3(1.2-7)=0$.

117. Fourth equation:

$$=0=$$

2(4, 1)	$-16m^2-40mn-16n^2$	(1)
$+(3,1,1)$	$-\frac{3}{2}m^3-10m^2n-10mn^2-\frac{3}{2}n^3+\frac{19}{2}m^2+116mn+\frac{19}{2}n^2$	(2)
$+\{\text{Supp. }(\overline{3},1,1)-\varkappa(\overline{3},1,1)\}$	$-\frac{1}{2}m^2+\frac{1}{2}m^2n+2mn^2+\frac{23}{2}m^2+\frac{11}{2}mn-7n^2$	(3)
$+(\overline{3},1,1)(4m+3n-2\alpha)$	$2m^3+\frac{19}{2}m^2n+8mn^2+\frac{3}{2}n^3-26m^2-\frac{21}{2}mn-\frac{39}{2}n^2$	(4)
$-(\cdot 2,1,1)$	$-24m^2-36mn-12n^2$	(5)

(1) $+208m+208n+\alpha(\quad +12m+12n-132)$

(2) $-434m-434n+\alpha(\ \frac{8}{3}m^2+6mn+\frac{8}{3}n^2-\frac{62}{3}m-\frac{62}{3}n+291)-\frac{8}{3}\alpha^2$

(3) $-\ 50m-\ 23n+\alpha(\ \frac{1}{3}m^2\quad -\frac{1}{3}^0m-\ \frac{1}{3}n+\ 33)$

(4) $+108m+\ 81n+\alpha(-m^2-4mn-\ n^2+\ 7m+\frac{1}{3}^7n-\ 54)+3\alpha^2$

(5) $+168m+168n+\alpha(-m^2-2mn-\frac{1}{3}n^2+25m+\frac{2}{3}^5n-138)+\frac{8}{3}\alpha^2.$

Verification is $(-\frac{1}{3}+\frac{1}{3}+2)4+(\frac{3}{3}^3+\frac{1}{3}^1-7)2-50-23+3(\frac{1}{3}.4-(\frac{1}{3}^0+\frac{1}{3})2+33)=0.$

118. Fifth equation:

$=0=$

4(5)	$-\ 60m-\ 60n+\alpha(\quad 36)$	
$+$ (3, 2)	$+120m+120n+\alpha(-\ 4m-4n-78)+3\alpha^2$	
$+\{$Supp. $(\overline{2}, 3)-\varkappa(\overline{2}, 3)\}$	$n^2+\ 8m-\ \ n+\alpha(\quad -\ 3)$	
$+$ $(\overline{2}, 3)(2m+n-\alpha)$	$-8m^2-12mn-4n^2-\ 12m-\ \ 6n+\alpha(\ 10m+7n+\ 6)-3\alpha^2$	
$-$ $(\cdot 1, 3)$	$+8m^2+12mn+3n^2-\ 56m-\ 53n+\alpha(-\ 6m-3n+39).$	

Verification is $2+8-1+3(-3)=0.$

119. Sixth equation:

$=0=$

3(4, 1)	$-24m^2-\ 60mn-24n^2+312m+312n$	(1)
$+2$(3, 2)	$+240m+240n$	(2)
$+2$(2, 2, 1)	$+48m^2+108mn+48n^2-936m-936n$	(3)
$+\{$Supp. $(\overline{2}, 2, 1)-\varkappa(\overline{2}, 2, 1)\}$	$+12m^2+\ \ 6mn-\ \ 6n^2-\ 60m-\ \ 6n$	(4)
$+$ $(\overline{2}, 2, 1)(2m+n-\alpha)$	$+12m^2+\ 18mn+\ \ 6n^2+108m+\ 54n$	(5)
$-2(\cdot 2, 1, 1)$	$-48m^2-\ 72mn-24n^2+336m+336n$	(6)

(1) $+\alpha(\quad +18m+18n-198)$

(2) $+\alpha(\quad -\ 8m-\ 8n-156)+\alpha^2(\quad +\ 6)$

(3) $+\alpha(\quad -16m-16n+654)+\alpha^2(\ m+n-24)$

(4) $+\alpha(\quad mn\quad -\ 8m-\ 2n+\ 30)$

(5) $+\alpha(\ 2m^2+3mn+n^2-36m-21n-\ 54)+\alpha^2(-m-n+15)$

(6) $+\alpha(-2m^2-4mn-n^2+50m+29n-276)+\alpha^2(\quad +\ 3).$

Verification is $(12+6-6)2-60-6+3(4-(8+2)2+30)=0.$

120. Seventh equation:

$=0=$

2(3, 1, 1)		(1)
$+$(2, 1, 1, 1)		(2)
$+\{$Supp.$(\overline{2}, 1, 1, 1)-\varkappa(\overline{2}, 1, 1, 1)\}$	$\frac{1}{3}m^3n+\ m^2n^2$	(3)
$+$ $(\overline{2}, 1, 1, 1)(2m+n-\alpha)$	$\frac{1}{3}m^4+\frac{1}{6}^3m^3n+3m^2n^2+\frac{8}{3}mn^3+\frac{1}{3}n^4$	(4)
$-(\cdot 1, 1, 1, 1)$	$-\frac{1}{3}m^4-\ \frac{8}{3}m^3n-4m^2n^2-\frac{8}{3}mn^3-\frac{1}{3}n^4$	(5)

$$
\begin{array}{lrrrrrrrrrrrr}
(1) & -3m^3 & -20m^2n & -20mn^2 & -3n^3 & +109m^2 & +232mn & +109n^2 & -868m & -868n & & (1)\\
(2) & 6m^3 & +30m^2n & +30mn^2 & +6n^3 & -174m^2 & -348mn & -174n^2 & +1320m & & & (2)\\
(3) & & -\tfrac{7}{4}m^2n & -5mn^2 & -\tfrac{3}{4}n^3 & -20m^2 & -5mn & +\tfrac{29}{3}n^2 & +80m & +26n & & (3)\\
(4) & -5m^3 & -\tfrac{37}{3}m^2n & -13mn^2 & -\tfrac{5}{3}n^3 & +\tfrac{74}{3}m^2 & +37mn & +\tfrac{37}{3}n^2 & -150m & -75n & & (4)\\
(5) & +2m^3 & +12m^2n & +8mn^2 & +n^3 & +\tfrac{181}{3}m^2 & +84mn & +\tfrac{229}{6}n^2 & -382m & -403n & & (5)
\end{array}
$$

$$
\begin{array}{lrrrrrrrrrrrr}
(1) & +\alpha(& & & 3m^3 & +12mn & +3n^3 & -61m & -69n & +582) & +\alpha^2(& -9)\\
(2) & +\alpha(& \tfrac{1}{3}m^3 & +m^2n & +mn^2 & +\tfrac{1}{3}n^3 & -\tfrac{13}{3}m^2 & -26mn & -\tfrac{13}{3}n^2 & +\tfrac{38}{3}m+\tfrac{38}{3}n-960) & +\alpha^2(& -\tfrac{2}{3}m-\tfrac{2}{3}n+28)\\
(3) & +\alpha(& & -m^2 & -\tfrac{3}{2}mn & +\tfrac{1}{2}n^2 & +19m & +\tfrac{5}{2}n & -54) & & & \\
(4) & +\alpha(& -\tfrac{1}{3}m^3 & -m^2n & -mn^2 & -\tfrac{1}{3}n^3 & -\tfrac{1}{2}m^2 & +\tfrac{7}{3}mn & +n^2 & +\tfrac{50}{3}m+\tfrac{50}{3}n+75) & +\alpha^2(& \tfrac{3}{2}m+\tfrac{3}{2}n-\tfrac{39}{2})\\
(5) & +\alpha(& & & 6m^2 & +12mn & +3n^2 & -86m & -55n & +357) & +\alpha^2(& -\tfrac{9}{2}).
\end{array}
$$

Verification is

$$(\tfrac{1}{2}+1)8+(-\tfrac{7}{2}-5-\tfrac{3}{2})4+(-20-5+\tfrac{39}{2})2+80+26+3\big((-1-\tfrac{3}{2}+\tfrac{1}{2})4+(19+\tfrac{5}{2})2-54\big)=0.$$

121. Eighth equation:

$$
\begin{array}{l|l}
=0= & \\
5(5) & -75m-75n+\alpha(45)\\
+(4,1) & -8m^2-20mn-8n^2+104m+104n+\alpha(6m+6n-66)\\
+\{\text{Supp.}\,(\overline{1},4)-\varkappa(\overline{1},4)\} & \\
\quad\{-(m-\tfrac{4}{3})(2(\cdot\,4)-(/4))\} & +1m-4n+\alpha(1)\\
+(-n+\tfrac{8}{3})(\cdot\,4) & 10mn+8n^2-\tfrac{50}{3}m-\tfrac{40}{3}n+\alpha(-6n+10)\\
+(-m+\tfrac{5}{3})(/4) & 8m^2+10mn-\tfrac{40}{3}m-\tfrac{50}{3}n+\alpha(-6m+10)\\
+5n(\overline{5}) & +5n.
\end{array}
$$

Verification is $1-4+3.1=0$.

122. Ninth equation:

$$
\begin{array}{l|l}
=0= & \\
4(4,1) & -32m^2-80mn-32n^2 \quad (1)\\
+2(3,2) & \quad (2)\\
+2(3,1,1) & -3m^3-20m^2n-20mn^2-3n^3+109m^2+232mn+109n^2 \quad (3)\\
+\{\text{Supp.}\,(\overline{1},1,3)-\varkappa(\overline{1},1,3)\} & \\
\quad\{-(m-2)(2(\cdot\,1,3)-(/\,1,3))\} & -2m^2+12mn+2n^2 \quad (4)\\
+(-n+2)(\cdot\,1,3) & +8m^2n+12mn^2+3n^3-16m^2-80mn-59n^2 \quad (5)\\
+(-m+2)(/1,3) & +3m^3+12m^2n+8mn^2-59m^2-80mn-16n^2 \quad (6)\\
+n\big(4(\overline{4},1)+2(\overline{2},3)\big) & -4mn-4n^2 \quad (7)
\end{array}
$$

(1) $+416m+416n+\alpha($ $+24m+24n-264)$

(2) $+240m+240n+\alpha($ $-8m-8n-156)+6\alpha^2$

(3) $-868m-868n+\alpha(3m^2+12mn+3n^2-69m-69n+582)-9\alpha^2$

(4) $-6m+30n+\alpha($ $-4m-10n-6)+3\alpha^2$

(5) $+112m+106n+\alpha(-6mn-3n^2+12m+45n-78)$

(6) $+106m+112n+\alpha(-3m^2-6mn+45m+12n-78)$

(7) $-36n+\alpha($ $+6n$).

Verification is $(-2+12+2)2-6+30+3\big((-4-10)2-6\big)+3.9.2=0.$

123. Tenth equation:

$=0=$

$3(3, 2)$	$+360m+360n$	(1)
$+(2, 2, 1)$	$24m^2+54mn+24n^2-468m-468n$	(2)
$+\{\text{Supp.}(\bar{1}, 2, 2)-\varkappa(\bar{1}, 2, 2)$ $-(m-2)\big(2(\cdot 2, 2)-(/2, 2)\big)\}$	$+6m+33n$	(3)
$+(-n+2)(\cdot 2, 2)$	$-27mn-24n^2+54m+48n$	(4)
$+(-m+2)(/2, 2)$	$-24m^2-27mn+48m+54n$	(5)
$+3n(\bar{3}, 2)$	$-27n$	(6)

(1) $+\alpha(-12m-12n-234)+\alpha^2($ $+9)$

(2) $+\alpha(-8m-8n+327)+\alpha^2(\frac{1}{2}m+\frac{1}{2}n-12)$

(3) $+\alpha($ $-3n-13)+\alpha^2($ $+1)$

(4) $+\alpha($ $+20n-40)+\alpha^2(-\frac{1}{2}n+1)$

(5) $+\alpha(20m-40)+\alpha^2(\frac{1}{2}m+1)$

(6) $+\alpha($ $+3n$).

Verification is $+33+3(-3.2-13)+9.2=0.$

124. Eleventh equation:

$=0=$

$3(3, 1, 1)$	$-\frac{8}{3}m^3-30m^2n-30mn^2-\frac{8}{3}n^3$	(1)
$+4(2, 2, 1)$		(2)
$+3(2, 1, 1, 1)$	$+18m^3+90m^2n+90mn^2+18n^3$	(3)
$+\{\text{Supp.}(\bar{1}, 1, 1, 2)-\varkappa(\bar{1}, 1, 1, 2)$ $-(m-\frac{7}{3})\big(2(\cdot 1, 1, 2)-(/1, 1, 2)\big)\}$	$-\frac{8}{3}m^3-\frac{8}{3}m^2n-6mn^2-3n^3$	(4)
$+(-n+\frac{7}{3})(\cdot 1, 1, 2)$	$-24m^2n-36mn^2-12n^3$	(5)
$+(-m+\frac{7}{3})(/1, 1, 2)$	$-12m^3-36m^2n-24mn^2$	(6)
$+n\big(3(\bar{3}, 1\ 1)+2(\bar{2}, 2, 1)\big)$	$+\frac{8}{3}m^2n+6mn^2+\frac{8}{3}n^3$	(7)

$$
\begin{array}{lll}
(1) & +\tfrac{3}{4}\tfrac{1}{7}m^3+348mn+\tfrac{3}{4}\tfrac{1}{7}n^3-1302m-1302n+\alpha(& (1)\\
(2) & +96m^3+216mn+96n^3-1872m-1872n+\alpha(& (2)\\
(3) & -522m^3-1044mn-522n^3+3960m+3960n+\alpha(\ \tfrac{1}{3}m^3+3m^2n+3mn^2+\tfrac{1}{3}n^3 & (3)\\
(4) & +\tfrac{3}{3}\tfrac{1}{1}m^3-\tfrac{3}{3}\tfrac{3}{3}mn+18n^3-2m-191n+\alpha(& (4)\\
(5) & +56m^3+252mn+196n^3-392m-392n+\alpha(\ -m^2n-2mn^2-\tfrac{1}{3}n^3 & (5)\\
(6) & +196m^3+252mn+56n^3-392m-392n+\alpha(-\tfrac{1}{3}m^3-2m^2n-mn^2 & (6)\\
(7) & -\tfrac{1}{3}\tfrac{5}{3}mn-\tfrac{1}{3}\tfrac{5}{3}n^3+189n+\alpha(& (7)
\end{array}
$$

$$
\begin{array}{lll}
(1) & \tfrac{8}{3}m^3+18mn+\tfrac{8}{3}n^3-\tfrac{2}{3}\tfrac{0}{3}\tfrac{7}{3}m-\tfrac{2}{3}\tfrac{0}{3}\tfrac{7}{3}n+873)+\alpha^2(\ -\tfrac{2}{3}\tfrac{7}{3}) & (1)\\
(2) & -32m-32n+1308)+\alpha^2(\ 2m+2n-48) & (2)\\
(3) & -\tfrac{4}{3}\tfrac{5}{3}m^3-78mn-\tfrac{4}{3}\tfrac{5}{3}n^3+358m+358n-2880)+\alpha^2(-\tfrac{8}{3}m-\tfrac{8}{3}n+84) & (3)\\
(4) & -\tfrac{4}{3}mn-2n^2+\tfrac{3}{3}\tfrac{3}{3}m+\tfrac{2}{6}\tfrac{3}{3}n+55)+\alpha^2(\ m+n-\tfrac{3}{4}\tfrac{1}{1}) & (4)\\
(5) & +\tfrac{7}{3}m^3+\tfrac{8}{3}\tfrac{9}{3}mn+\tfrac{4}{3}\tfrac{7}{5}n^3-\tfrac{1}{3}\tfrac{7}{3}\tfrac{5}{3}m-\tfrac{1}{6}\tfrac{0}{3}\tfrac{3}{1}n+322)+\alpha^2(\ \tfrac{8}{3}m-\tfrac{7}{4}) & (5)\\
(6) & +\tfrac{4}{3}\tfrac{7}{7}m^3+\tfrac{8}{3}\tfrac{9}{3}mn+\tfrac{7}{4}n^3-\tfrac{1}{6}\tfrac{0}{3}\tfrac{3}{1}m-\tfrac{1}{3}\tfrac{7}{3}\tfrac{5}{3}n+322)+\alpha^2(\ +\tfrac{8}{3}n-\tfrac{7}{4}) & (6)\\
(7) & 2mn+2n^2-\tfrac{6}{3}\tfrac{0}{3}n\ ). & (7)
\end{array}
$$

Verification is $4(-\tfrac{8}{3}-\tfrac{8}{3}-6-3)+2(\tfrac{3}{3}\tfrac{1}{1}-\tfrac{3}{3}\tfrac{3}{3}+18)-2-191$

$$+3\big((-\tfrac{4}{3}-2)4+(\tfrac{3}{3}\tfrac{3}{3}+\tfrac{2}{6}\tfrac{3}{3})2+55\big)+9\big((1+1)4-\tfrac{3}{3}\tfrac{1}{1}.2\big)=0.$$

125. Twelfth equation:

$$
\begin{array}{l|l}
2(\bar{2},1,1,1) & =0= \quad (1)\\
+5(1,1,1,1,1) & \tfrac{1}{3}\tfrac{1}{4}m^3+\tfrac{5}{1}\tfrac{1}{3}m^2n+\tfrac{5}{3}m^2n^2+\tfrac{5}{3}mn^2+\tfrac{1}{1}\tfrac{5}{3}mn^3+\tfrac{1}{3}\tfrac{1}{4}n^3 \quad (2)\\
+\left\{\begin{array}{l}\text{Supp. }(\bar{1},1,1,1,1)-\kappa(\bar{1},1,1,1,1)\\ -(m-\tfrac{8}{3})(2(\cdot 1,1,1,1)-(/1,1,1,1))\end{array}\right\} & (3)\\
+(-n+\tfrac{8}{3})(\cdot 1,1,1,1) & -\tfrac{1}{1}\tfrac{1}{3}m^2n-\tfrac{8}{3}m^2n^2-mn^3-\tfrac{1}{3}mn^3-\tfrac{1}{3\,4}n^4 \quad (4)\\
+(-m+\tfrac{8}{3})(/1,1,1,1) & -\tfrac{1}{3\,4}m^3-\tfrac{1}{3}m^2n-mn^2-\tfrac{8}{3}m^2n^2-\tfrac{1}{1}\tfrac{1}{3}mn^3 \quad (5)\\
+2\kappa(\bar{2},1,1,1) & \quad (6)
\end{array}
$$

$$
\begin{array}{lll}
(1) & +12m^3+60m^2n+60mn^2+12n^3 & (1)\\
(2) & -\tfrac{5}{1}\tfrac{5}{3}m^3-\tfrac{3}{6}\tfrac{5}{3}m^2n-10m^2n^2-\tfrac{3}{6}\tfrac{5}{3}mn^3-\tfrac{5}{1}\tfrac{5}{3}n^3-\tfrac{5}{4}\tfrac{6}{4}\tfrac{5}{5}m^3-\tfrac{1}{1}\tfrac{0}{3}\tfrac{4}{3}\tfrac{5}{3}m^2n-\tfrac{1}{1}\tfrac{0}{3}\tfrac{4}{3}\tfrac{5}{3}mn^2-\tfrac{5}{3}\tfrac{6}{4}\tfrac{5}{5}n^3 & (2)\\
(3) & -\tfrac{1}{6}m^3-\tfrac{3}{4}m^2n-\tfrac{1}{3}\tfrac{0}{3}m^2n^2-3mn^2-\tfrac{1}{3}n^3+4m^2+\tfrac{3}{3}\tfrac{3}{3}m^2n+\tfrac{6}{3}\tfrac{1}{3}mn^2+9n^3 & (3)\\
(4) & +\tfrac{3}{6}m^3+\tfrac{6}{1}\tfrac{1}{3}m^2n+\tfrac{1}{3}\tfrac{7}{3}m^2n^2+\tfrac{3}{3}\tfrac{6}{3}mn^2+\tfrac{1}{3}\tfrac{3}{3}n^3-\tfrac{4}{3}m^2+\tfrac{6}{1}\tfrac{5}{3}m^2n+\tfrac{4}{3}\tfrac{7}{3}mn^2+\tfrac{3}{1}\tfrac{1}{4}\tfrac{3}{3}n^3 & (4)\\
(5) & +\tfrac{1}{3}\tfrac{3}{6}m^3+\tfrac{3}{6}\tfrac{6}{3}m^2n+\tfrac{1}{3}\tfrac{7}{3}m^2n^2+\tfrac{1}{1}\tfrac{6}{3}mn^2+\tfrac{3}{3}n^3+\tfrac{2}{3}\tfrac{1}{1}\tfrac{3}{4}m^2+\tfrac{4}{3}\tfrac{7}{3}m^2n+\tfrac{8}{1}\tfrac{5}{3}mn^2-\tfrac{3}{4}n^3 & (5)\\
(6) & \tfrac{1}{3}m^2n+2m^2n^2+2mn^2+\tfrac{1}{3}n^3-5m^2n-16mn^2-5n^3 & (6)
\end{array}
$$

$$
\begin{array}{lll}
(1) & -348m^3-696mn-348n^3+2640m+2640n+\alpha(\tfrac{1}{8}m^3+2m^2n+2mn^2+\tfrac{1}{8}n^3 & (1)\\
(2) & +\tfrac{6433}{128}m^3+\tfrac{2985}{8}mn+\tfrac{6433}{128}n^3-3159m-3159n+\alpha(-\tfrac{5}{4}m^3-\tfrac{15}{8}m^2n-\tfrac{15}{8}mn^2-\tfrac{5}{4}n^3 & (2)\\
(3) & -\tfrac{27}{2}m^3-14mn-\tfrac{239}{6}n^3-\tfrac{13}{3}m+\tfrac{437}{3}n+\alpha(\tfrac{1}{6}m^3+m^2n+mn^2+\tfrac{1}{6}n^2 & (3)\\
(4) & -\tfrac{361}{9}m^3-\tfrac{202}{3}mn-\tfrac{4343}{36}n^3+\tfrac{764}{3}m+\tfrac{806}{3}n+\alpha(\tfrac{3}{4}m^2n+3mn^2+\tfrac{3}{4}n^3 & (4)\\
(5) & -\tfrac{4343}{36}m^3-\tfrac{202}{3}mn-\tfrac{361}{9}n^3+\tfrac{806}{3}m+\tfrac{764}{3}n+\alpha(\tfrac{3}{4}m^3+3m^2n+\tfrac{3}{2}mn^2 & (5)\\
(6) & +\tfrac{74}{3}mn+\tfrac{74}{3}n^2-150n+\alpha(& (6)
\end{array}
$$

$$
\begin{array}{lll}
(1) & -15m^2-52mn-15n^2+\tfrac{716}{3}m+\tfrac{716}{3}n-1920)+\alpha^2(-3m-3n+56) & \\
(2) & +\tfrac{1445}{4}m^2+115mn+\tfrac{1445}{4}n^2-\tfrac{16885}{4}m-\tfrac{16885}{4}n+2430)+\alpha^2(\tfrac{45}{8}m+\tfrac{45}{8}n-75) & \\
(3) & -\tfrac{3}{2}m^2-mn+\tfrac{3}{2}n^2-\tfrac{3}{2}m-\tfrac{89}{3}n-34)+\alpha^2(-\tfrac{3}{2}m-\tfrac{3}{2}n+13) & \\
(4) & -4m^2-\tfrac{52}{3}mn-\tfrac{643}{4}n^2+\tfrac{172}{3}m+\tfrac{1511}{12}n-238)+\alpha^2(-\tfrac{9}{2}n+3) & \\
(5) & -\tfrac{643}{4}m^2-\tfrac{52}{3}mn-4n^2+\tfrac{1501}{12}m+\tfrac{172}{3}n-238)+\alpha^2(-\tfrac{9}{2}m+3) & \\
(6) & -3mn-3n^2+29n). &
\end{array}
$$

Verification is

$$(-\tfrac{1}{6}-\tfrac{4}{3}-\tfrac{10}{3}-3-\tfrac{1}{2})8+(4+\tfrac{28}{3}+\tfrac{61}{3}+9)4+(-\tfrac{27}{3}-14-\tfrac{239}{6})2+(-\tfrac{13}{3}+\tfrac{437}{3})$$
$$+3\big((\tfrac{1}{6}+1+1+\tfrac{1}{6})8+(-\tfrac{3}{2}-1+\tfrac{3}{2})4+(-\tfrac{3}{2}-\tfrac{89}{3})2-34\big)+9\big((-\tfrac{3}{2}-\tfrac{3}{2})4+13.2\big)=0.$$

126. It will be observed that in the eighth and following equations, viz. those wherein the expression of the Supplement contains the symbol $(\overline{1})$, I have included along with the Supplement within the { }, the terms $-(m-\frac{5}{3}\{2(\cdot 4)-(/4)\}$ &c., viz. these are $-(m-\frac{5}{3})$ into number of point-pairs (4), &c.: this is for convenience only; it simplifies the calculation, both from the symmetrical form under which the remaining terms present themselves in the several equations, and because the expressions of the terms in question, (these terms being mere multiples of a number of point-pairs) are by ZEUTHEN's theory known in terms of the Capitals. It is to be noticed that for any equation, to find the system to which the Capitals belong, we diminish by unity the barred number and then remove the bar; thus for the seventh equation, where we have Supp. $(\overline{2}, 1, 1, 1)$, the Capitals belong to the system (1, 1, 1, 1).

127. Referring to Nos. 41 to 47 of the First Memoir, for convenience I collect the capitals which belong to a single curve, giving the values in terms of m, n, α as follows.

(1, 1, 1, 1)

(1) $A = \frac{1}{2}\delta(\delta-1) = \frac{1}{2}m^4 - \frac{1}{2}m^3 + 2m^2n - \frac{1}{2}m^3 - 2mn + 8n^2 + \frac{1}{2}m - 2n + \alpha(-\frac{3}{2}m^2 + \frac{3}{4}m - 6n + \frac{3}{4}) + \frac{3}{2}\alpha^2;$

(2) $B = \delta(n-4)(m-4) = \frac{1}{2}m^3n - 2m^3 - \frac{5}{2}m^2n + 4mn^2 + 10m^2 - 14mn - 16n^2 - 8m + 64n + \alpha(-\frac{3}{2}mn + 6m + 6n - 24);$

(4) $C = \tau.\frac{1}{2}(m-4)(m-5) = \frac{1}{4}m^2n^2 + 2m^2 - \frac{1}{4}m^2n - \frac{9}{4}mn^2 - 18m^2 + \frac{9}{4}mn + 5n^2 + 40m - 5n + \alpha(-\frac{3}{4}m^2 + \frac{27}{4}m - 15);$

(3) $D = \iota.\frac{1}{2}(m-3)(m-4) = -\frac{3}{2}m^3 + \frac{21}{2}m^2 - 18m + \alpha(\frac{1}{2}m^2 - \frac{7}{2}m + 6).$

(2, 1, 1)

(3) $E = \delta(n-4) = \frac{1}{2}m^2n - 2m^2 - \frac{1}{2}mn + 4n^2 + 2m - 16n + \alpha(-\frac{3}{2}n + 6);$

(3) $F = 2\delta(m-3) = m^3 - 4m^2 + 8mn + 3m - 24n + \alpha(-3m + 9);$

(6) $G = 2\tau(m-4) = mn^2 + 8m^2 - mn - 4n^2 - 32m + 4n + \alpha(-3m + 12);$

(2) $D[=\iota.\frac{1}{2}(m-3)(m-4)\ \textit{suprà}];$

(1) $H = \delta\varkappa = -\frac{3}{2}m^2n + \frac{3}{2}mn - 12n^2 + \alpha(\frac{1}{2}m^2 - \frac{1}{2}m + \frac{17}{2}n) - \frac{3}{2}\alpha^2;$

(2) $I = \varkappa(n-3)(m-4) = -3mn^2 + 9mn + 12n^2 - 36n + \alpha(mn - 3m - 4n + 12);$

(5) $J = \iota(m-3) = -3m^2 + 9m + \alpha(m-3).$

(2, 2)

(9) $K = \tau = \frac{1}{2}n^2 + 4m - \frac{1}{2}n + \alpha(-\frac{3}{2});$

(3) $L = \varkappa(n-3) = -3n^2 + 9n + \alpha(n-3);$

(1) $M = \frac{1}{2}\varkappa(\varkappa-1) = \frac{9}{2}n^2 + \frac{3}{2}n + \alpha(-3n - \frac{1}{2}) + \frac{1}{2}\alpha^2;$

(2) $N = \iota = -3m + \alpha(1);$

(1) $O = \varkappa = -3n + \alpha(1).$

(3, 1)

(2) | P $=2\delta$ $= m^2 - m + 8n + \alpha(-3)$;

(2) | Q $=2\tau$ $= n^2 + 8m - n + \alpha(-3)$;

(5) | J [$=\iota(m-3)$ *suprà*];

(4) | R $=\varkappa(m-3)$ $= -3mn + 9n + \alpha(m-3)$.

(4)

(4) | N[$=\iota$ *suprà*];

(2) | O[$=\varkappa$ *suprà*].

128. I make the following calculations, serving to express in terms of ZEUTHEN's Capitals, the terms in { } contained in the twelve equations respectively.

$$\begin{array}{l} \mathrm{N} = -3m + \alpha \\ \hline -3m + \alpha \end{array}$$ (first equation).

$$\begin{array}{l} 2\mathrm{J} = -6m^2 + 18m + \alpha(2m-6) \\ +\mathrm{R} = -3mn + 9n + \alpha(m-3) \\ \hline -6m^2 - 3mn + 18m + 9n + \alpha(3m-9) \end{array}$$ (second equation).

$$\begin{array}{l} 6\mathrm{K} = 3n^2 + 24m - 3n + \alpha(-9) \\ +\mathrm{L} = -3n^2 + 9n + \alpha(n-3) \\ +3\mathrm{N} = -9m + \alpha(3) \\ +2\mathrm{O} = -6n + \alpha(2) \\ \hline 15m + \alpha(n-7) \end{array}$$ (third equation).

$$\begin{array}{l} \mathrm{E} = \frac{1}{2}m^2n - 2m^2 - \frac{1}{2}mn + 4n^2 + 2m - 16n + \alpha(-\frac{3}{2}n + 6) \\ +\mathrm{F} = m^2 - 4m^2 + 8mn + 3m - 24n + \alpha(-3m + 9) \\ +2\mathrm{G} = 2mn^2 + 16m^2 - 2mn - 8n^2 - 64m + 8n + \alpha(-6m + 24) \\ +\mathrm{D} = -\frac{3}{2}m^2 + \frac{21}{2}m^2 - 18m + \alpha(\frac{1}{2}m^2 - \frac{7}{2}m + 6) \\ +3\mathrm{J} = -9m^2 + 27m + \alpha(3m - 9) \\ +\mathrm{J}' = -3n^2 + 9n + \alpha(n - 3) \\ \hline -\frac{1}{2}m^2 + \frac{1}{2}m^2n + 2mn^2 + \frac{23}{2}m^2 + \frac{11}{2}mn - 7n^2 - 50m - 23n + \alpha(\frac{1}{2}m^2 - \frac{19}{2}m - \frac{1}{2}n + 33) \end{array}$$

(fourth equation).

$$Q = n^3 + 8m - n - 3\alpha$$
$$n^3 + 8m - n - 3\alpha \text{ (fifth equation).}$$

$$\begin{aligned}
3G = {} & 3mn^2 + 24m^2 - 3mn - 12n^2 - 96m + 12n + \alpha(\quad -9m \quad +36) \\
+\ I = {} & -3mn^2 \quad +9mn + 12n^2 \quad -36n + \alpha(mn - 3m - 4n + 12) \\
+4J = {} & \quad -12m^2 \quad +36m \quad +\alpha(\quad 4m \quad -12) \\
+2J' = {} & \quad -6n^2 \quad +18n + \alpha(\quad 2n - 6) \\
\hline
& 12m^2 + 6mn - 6n^2 - 60m - 6n + \alpha(mn - 8m - 2n + 30)
\end{aligned}$$

(sixth equation).

$$\begin{aligned}
B = {} & \tfrac{1}{3}m^3n \quad -2m^3 - \tfrac{8}{3}m^2n + 4mn^2 \quad +10m^3 - 14mn - 16n^2 - \quad 8m + 64n \qquad (1) \\
+4C = {} & \quad m^3n^2 + 8m^3 - \quad m^2n - 9mn^2 \quad -72m^2 + \ 9mn + 20n^2 + 160m - 20n \qquad (2) \\
+4D = {} & \quad -6m^3 \quad +42m^2 \quad - \ 72m \qquad (3) \\
+\ D' = {} & \quad -\tfrac{8}{3}n^3 \quad +\tfrac{21}{3}n^2 \quad -18n \qquad (4) \\
\hline
& \tfrac{1}{3}m^3n + m^2n^2 \quad -\tfrac{7}{3}m^2n - 5mn^2 - \tfrac{8}{3}n^3 - 20m^2 - \ 5mn + \tfrac{29}{3}n^2 + \ 80m + 26n \qquad (5)
\end{aligned}$$

$$\begin{aligned}
(1) \quad & +\alpha(\quad -\tfrac{8}{3}mn \quad + \ 6m + 6n - 24) \\
(2) \quad & +\alpha(-3m^2 \quad +27m \quad -60) \\
(3) \quad & +\alpha(\ 2m^2 \quad -14m \quad +24) \\
(4) \quad & +\alpha(\quad \tfrac{1}{3}n^2 \quad -\tfrac{7}{3}n + \ 6) \\
\hline
(5) \quad & +\alpha(- \ m^2 - \tfrac{8}{3}mn + \tfrac{1}{3}n^2 + 19m + \tfrac{5}{3}n - 54) \text{ (seventh equation).}
\end{aligned}$$

$$\begin{aligned}
-\tfrac{1}{3}N = {} & m \quad -\tfrac{1}{3}\alpha \\
+\tfrac{4}{3}O = {} & \quad -4n + \tfrac{4}{3}\alpha \\
\hline
& m - 4n + \alpha \text{ (eighth equation).}
\end{aligned}$$

$$\begin{aligned}
(\bar{2}, 3) = {} & -4m - 4n - 6 + 3\alpha \\
+2(\bar{4}, 1) = {} & \ 2m + 2n - 12 \\
\hline
& -2m - 2n - 18 + 3\alpha \text{ (used } \textit{infrà}\text{)}
\end{aligned}$$

$$\begin{aligned}
-2P = {} & -2m^2 \quad +2m - 16n + \alpha(\quad 6) \\
-\ Q = {} & \quad -n^2 - 8m + \ n + \alpha(\quad 3) \\
-2R = {} & \quad 6mn \quad -18n + \alpha(-2m \quad +6) \\
+\ J' = {} & \quad -3n^2 \quad +9n + \alpha(\quad n - 3) \\
\varkappa((\bar{2}, 3) + 2(\bar{4}, 1)) = {} & \quad 6mn + 6n^2 \quad +54n + \alpha(-2m - 11n - 18) + 3\alpha^2 \\
\hline
& -2m^2 + 12mn + 2n^2 - 6m + 30n + \alpha(-4m - 10n - 6) + 3\alpha^2
\end{aligned}$$

(ninth equation).

$$
\begin{aligned}
3\mathrm{L} &= -9n^2 + 27n + \alpha(3n - 9)\\
+2\mathrm{M} &= 9n^2 + 3n + \alpha(-6n - 1) + \alpha^2\\
-2\mathrm{N} &= +6m + \alpha(-2)\\
-\mathrm{O} &= 3n + \alpha(-1)\\
&\overline{\qquad\qquad}\\
&6m + 33n + \alpha(-3n - 13) + \alpha^2 \text{ (tenth equation).}
\end{aligned}
$$

$$
\begin{aligned}
\varkappa(\bar{2}, 2, 1) &= -18mn - 18n^2 - 162n && (1)\\
-\tfrac{1}{3}\mathrm{D} &= \tfrac{1}{3}m^2 - \tfrac{7}{3}m^2 + 6m && (2)\\
-2\mathrm{E} &= -m^2n + 4m^2 + 1mn - 8n^2 - 4m + 32n && (3)\\
-2\mathrm{F} &= -2m^2 + 8m^2 - 16mn - 6m + 48n && (4)\\
-\mathrm{G} &= -mn^2 - 8m^2 + 1mn + 4n^2 + 32m - 4n && (5)\\
+\tfrac{1}{3}\mathrm{H} &= -\tfrac{1}{2}m^2n + \tfrac{1}{3}mn - 4n^2 && (6)\\
+\tfrac{5}{3}\mathrm{I} &= -5mn^2 + 15mn + 20n^2 - 60n && (7)\\
-\tfrac{10}{3}\mathrm{J} &= +10m^2 - 30m && (8)\\
+2\mathrm{D}' &= -3n^2 + 21n^2 - 36n && (9)\\
-\mathrm{J}' &= 3n^2 - 9n && (10)\\
&\overline{\qquad\qquad}\\
&-\tfrac{5}{3}m^2 - \tfrac{5}{2}m^2n - 6mn^2 - 3n^2 + \tfrac{21}{3}m^2 - \tfrac{33}{3}mn + 18n^2 - 2m - 191n && (11)
\end{aligned}
$$

$$
\begin{aligned}
(1)\quad & +\alpha(-3mn - 3n^2 + 6m + 51n + 54) + \alpha^2(m + n - 15)\\
(2)\quad & +\alpha(-\tfrac{1}{3}m^2 + \tfrac{7}{3}m - 2)\\
(3)\quad & +\alpha(3n - 12)\\
(4)\quad & +\alpha(6m - 18)\\
(5)\quad & +\alpha(3m - 12)\\
(6)\quad & +\alpha(\tfrac{1}{3}m^2 - \tfrac{1}{3}m + \tfrac{17}{3}n) + \alpha^2(-\tfrac{1}{3})\\
(7)\quad & +\alpha(\tfrac{5}{3}mn - 5m - \tfrac{30}{3}n + 20)\\
(8)\quad & +\alpha(-\tfrac{10}{3}m + 10)\\
(9)\quad & +\alpha(1n^2 - 7n + 12)\\
(10)\quad & +\alpha(-n + 3)\\
(11)\quad & +\alpha(-\tfrac{4}{3}mn - 2n^2 + \tfrac{23}{3}m + \tfrac{215}{3}n + 55) + \alpha^2(m + n - \tfrac{31}{3})
\end{aligned}
$$

(eleventh equation).

$$\begin{aligned}
\varkappa(\overline{2},1,1,1) = {} & -\tfrac{1}{2}m^5n - 3m^3n^3 - 3mn^5 - \tfrac{1}{2}n^6 + \tfrac{15}{2}m^5n + 24mn^5 + \tfrac{15}{2}n^6 - 37mn - 37n^3 && (1)\\
-\tfrac{1}{2}A = {} & -\tfrac{1}{2}m^6 + \tfrac{1}{2}m^5 - \tfrac{5}{2}m^3n + \tfrac{1}{2}m^3 + \tfrac{5}{2}mn - \tfrac{3}{2}n^3 && (2)\\
-\tfrac{1}{2}B = {} & -\tfrac{5}{2}m^5n + \tfrac{10}{2}m^3 + \tfrac{25}{2}m^3n - \tfrac{30}{2}mn^3 - \tfrac{50}{2}m^3 + \tfrac{70}{2}mn + \tfrac{80}{2}n^3 && (3)\\
-\tfrac{1}{2}C = {} & -\tfrac{1}{2}m^3n^3 - \tfrac{5}{2}m^3 + \tfrac{1}{2}m^3n + 3mn^3 + 24m^3 - 3mn - \tfrac{30}{2}n^3 && (4)\\
-2D = {} & 3m^3 - 21m^3 && (5)\\
-D' = {} & \tfrac{3}{2}n^3 - \tfrac{21}{2}n^3 && (6)\\
\hline
& -\tfrac{1}{2}m^6 - \tfrac{1}{2}m^5n - \tfrac{10}{2}m^3n^3 - 3mn^5 - \tfrac{1}{2}n^6 + 4m^5 + \tfrac{25}{2}m^3n + \tfrac{61}{2}mn^3 + 9n^3 - \tfrac{37}{2}m^3 - 14mn - \tfrac{229}{6}n^3 && (7)
\end{aligned}$$

$$\begin{aligned}
(1)\quad & +225n + \alpha(\tfrac{1}{2}m^3 + m^2n + mn^2 + \tfrac{1}{6}n^3 \quad -\tfrac{5}{2}m^2 - \tfrac{7}{2}mn + 2n^2 + \tfrac{37}{3}m - \tfrac{187}{6}n - 75) + \alpha^2(-\tfrac{3}{2}m - \tfrac{3}{2}n + \tfrac{29}{2})\\
(2)\quad & -\tfrac{1}{2}m + \tfrac{3}{2}n + \alpha(\qquad m^2 \qquad - m + 8n - 1) + \alpha^2(\qquad - \tfrac{3}{2})\\
(3)\quad & +\tfrac{40}{3}m - \tfrac{233}{3}n + \alpha(\qquad + \tfrac{5}{2}mn \quad -10m - 10n + 40)\\
(4)\quad & -\tfrac{118}{3}m + \tfrac{20}{3}n + \alpha(\qquad m^2 \qquad - 9m \qquad +20)\\
(5)\quad & +36m \qquad +\alpha(\qquad -m^2 \qquad + 7m \qquad -12)\\
(6)\quad & +18n + \alpha(\qquad -\tfrac{1}{2}n^2 \qquad + \tfrac{7}{2}n - 6)\\
\hline
(7)\quad & -\tfrac{13}{3}m + \tfrac{487}{3}n + \alpha(\tfrac{1}{2}m^2 + m^2n + mn^2 + \tfrac{1}{6}n^3 - \tfrac{5}{2} \quad m^2 - mn + \tfrac{3}{2}n^2 - \tfrac{3}{2}m - \tfrac{29}{3}n - 34) + \alpha^2(-\tfrac{3}{2}m - \tfrac{3}{2} + 13)
\end{aligned}$$

(twelfth equation)

129. We have consequently, by means of the results just obtained,

Supp. $(\overline{5}) = \varkappa(\overline{5}) + N$ . . . . . (first equation)

Supp. $(\overline{4}, 1) = \varkappa(4, 1) + 2J + R$ . . . . . (second equation)

Supp. $(\overline{3}, 2) = \varkappa(\overline{3}, 2) + 6K + L + 3N + 2O$ . . . . . (third equation)

Supp. $(\overline{3}, 1, 1) = \varkappa(\overline{3}, 1, 1) + D + E + F + 2G + 3J + J'$ . . . . . (fourth equation)

Supp. $(\overline{2}, 3) = \varkappa(\overline{2}, 3) + Q$ . . . . . (fifth equation)

Supp. $(\overline{2}, 2, 1) = \varkappa(\overline{2}, 2, 1) + 3G + I + 4J + 2J'$ . . . . . (sixth equation)

Supp. $(\overline{2}, 1, 1, 1) = \varkappa(\overline{2}, 1, 1, 1) + B + 4C + 4D + D'$ . . . . . (seventh equation)

$$\text{Supp. } (\overline{1}, 4) = \varkappa(\overline{1}, 4) + (m-\tfrac{8}{3})(4N+2O) - \tfrac{1}{3}N+\tfrac{4}{3}O \quad \text{(eighth equation)}$$

$$\text{Supp. } (\overline{1}, 1, 3) = \varkappa(\overline{1}, 1, 3)+\varkappa(\overline{2}, 3)+\varkappa.2(\overline{4}, 1) + (m-2)(2P+2Q+5J+4R) - 2P-Q-2R+J' \quad \text{(ninth equation)}$$

$$\text{Supp. } (\overline{1}, 2, 2) = \varkappa(\overline{1}, 2, 2) + (m-2)(9K+3L+ M+2N+O) + 3L+2M-2N-O \quad \text{(tenth equation)}$$

$$\text{Supp. } (\overline{1}, 1, 1, 2) = \varkappa(\overline{1}, 1, 1, 2)+\varkappa(\overline{2}, 2, 1) + (m-\tfrac{7}{3})(3E+3F+6G+2D+ H+2I+ 5J) - 2E-2F- G-\tfrac{1}{3}D+\tfrac{1}{3}H+\tfrac{8}{3}I-\tfrac{10}{3}J+2D'-J' \quad \text{(eleventh equation)}$$

Observe that

$$G-2E'=0, \quad G'-2E=0,$$

and

$$3G+I+8J=3G'+I'+8J',$$

relations which may be used to modify the form of the last preceding result.

$$\text{Supp. } (\overline{1}, 1, 1, 1, 1)= \varkappa(\overline{1}, 1, 1, 1, 1)+\varkappa(\overline{2}, 1, 1, 1) + (m-\tfrac{5}{3})(A+2B+4C+3D) - \tfrac{4}{3}A-\tfrac{5}{3}B-\tfrac{4}{3}C- D' \quad \text{(twelfth equation)}$$

130. We may in these equations introduce on the right-hand sides in place of a symbol such as $\overline{p}$ the symbol $\overline{p\varkappa 1}$: for example, in the fifth equation, writing

$$(\overline{2}, 3)=(\overline{2\varkappa 1}, 3)+[(\overline{2}, 3)-(\overline{2\varkappa 1}, 3)],$$

and therefore also

$$\varkappa(\overline{2}, 3)=\varkappa(\overline{2\varkappa 1}, 3)+\varkappa[(\overline{2}, 3)-(\overline{2\varkappa 1}, 3)],$$

the second term $\varkappa[(\overline{2}, 3)-(\overline{2\varkappa 1}, 3)]$ can be expressed in terms of ZEUTHEN's Capitals. The remark applies to all the twelve equations; only as regards the first four of them, inasmuch as $(\overline{5\varkappa 1})=0, \ldots (\overline{3\varkappa 1}, 1, 1)=0$, it is the whole original terms $\varkappa(\overline{5}) \ldots \varkappa(\overline{3}, 1, 1)$ which are thus expressible by means of ZEUTHEN's Capitals. By the assistance of the formulæ (First Memoir, Nos. 69 and 73) we readily obtain

Referring to

$$\varkappa(\overline{5}) = \varkappa = O. \quad \text{(first equation)}$$

$$\varkappa(\overline{4}, 1) = \varkappa(m+n-6) = R+J' \quad \text{(second equation)}$$

$$\varkappa(\overline{3}, 2) = \varkappa(-9+\alpha) = \varkappa(3(n-3)+\varkappa\overline{-1}+1) \qquad \text{Referring to}$$
$$= 3\mathrm{L}+2\mathrm{M}+\mathrm{O}. \qquad \text{(third equation)}$$

$$\varkappa(\overline{3}, 1, 1) = \varkappa(\tfrac{1}{2}m^3+2mn+\tfrac{1}{2}n^3-\tfrac{1}{2}^3m-\tfrac{1}{2}^3n+27-\tfrac{3}{2}\alpha)$$
$$= \mathrm{H}+2\mathrm{I}+\mathrm{D}'+\mathrm{J}'. \qquad \text{(fourth equation)}$$

viz.
$$\begin{aligned} \varkappa^{-1}\mathrm{H} &= \tfrac{1}{2}m^3 \quad - \tfrac{1}{2}m + 4n - \tfrac{3}{2}\alpha \\ \varkappa^{-1}.2\mathrm{I} &= 2mn - 6m - 8n + 24 \\ \varkappa^{-1}.\mathrm{D}' &= \tfrac{1}{2}n^3 - \tfrac{7}{2}n + 6 \\ \varkappa^{-1}\mathrm{J}' &= \cdot \quad n - 3 \\ & \tfrac{1}{2}m^3+2mn+\tfrac{1}{2}n^3-\tfrac{1}{2}^3m-\tfrac{1}{2}^3n+27-\tfrac{3}{2}\alpha. \end{aligned}$$

$$\varkappa(\overline{2}, 3) = \varkappa(\overline{2\varkappa 1}, 3). \qquad \text{(fifth equation)}$$

$$\varkappa(\overline{2}, 2, 1) = \varkappa(\overline{2\varkappa 1}, 2, 1)+\varkappa(n-3)$$
$$= \varkappa(\overline{2\varkappa 1}, 2, 1)+\mathrm{J}'. \qquad \text{(sixth equation)}$$

$$\varkappa(\overline{2}, 1, 1, 1) = \varkappa(\overline{2\varkappa 1}, 1, 1, 1)+\varkappa.\tfrac{1}{2}(n-3)(n-4)$$
$$= \varkappa(\overline{2\varkappa 1}, 1, 1, 1)+\mathrm{D}'. \qquad \text{(seventh equation)}$$

$$\varkappa(\overline{1}, 4) = \varkappa(\overline{1\varkappa 1}, 4)+\varkappa$$
$$= \varkappa(\overline{1\varkappa 1}, 4)+\mathrm{O}. \qquad \text{(eighth equation)}$$

$$\varkappa(\overline{1}, 1, 3)+ \varkappa(\overline{2}, 3)+\varkappa 2(\overline{4}, 1)$$
$$= \varkappa(\overline{1\varkappa 1}, 1, 3)+\varkappa(\overline{2\varkappa 1}, 3) + \varkappa(n-3)$$
$$+\varkappa(2\varkappa 1, 3)$$
$$+ \quad \varkappa(2m+2n-6)$$
$$= \varkappa(\overline{1\varkappa 1}, 1, 3)+2\varkappa(\overline{2\varkappa 1}, 3) + 2\mathrm{R}+3\mathrm{J}' \qquad \text{(ninth equation)}$$

$$\varkappa(\overline{1}, 2, 2) = \varkappa(\overline{1\varkappa 1}, 2, 2)+\varkappa\{3(n-3)+\varkappa-1\}$$
$$= \varkappa(\overline{1\varkappa 1}, 2, 2)+3\mathrm{L}+2\mathrm{M}) \ldots \qquad \text{(tenth equation)}$$

$$\varkappa(\overline{1}, 1, 1, 2)+\varkappa(\overline{2}, 2, 1)$$
$$= \varkappa(\overline{1\varkappa 1}, 1, 1, 2)+\varkappa(\overline{2\varkappa 1}, 2, 1)+\varkappa\{\tfrac{1}{2}(n-3)(n-4)+\delta+2\,\overline{n-3}\,\overline{m-4}\}$$
$$+\varkappa(\overline{2\varkappa 1}, 2, 1)+\varkappa(n-3)$$
$$= \varkappa(\overline{1\varkappa 1}, 1, 1, 2)+2\varkappa(\overline{2\varkappa 1}, 1, 2)$$
$$+\mathrm{D}'+\mathrm{H}+2\mathrm{I}+\mathrm{J}'. \qquad \text{(eleventh equation)}$$

$\varkappa(\overline{1}, 1, 1, 1, 1)+\varkappa(\overline{2}, 1, 1, 1)$ Referring to

$=\varkappa(\overline{1\varkappa 1}, 1, 1, 1, 1)+\varkappa(\overline{2\varkappa 1}, 1, 1, 1)$

$+\varkappa(\overline{2\varkappa 1}, 1, 1, 1)+\varkappa.\frac{1}{2}(n-3)(n-4)$

$=\varkappa(\overline{1\varkappa 1}, 1, 1, 1, 1)+2\varkappa(\overline{2\varkappa 1}, 1, 1, 1)+D'$ . . . . (twelfth equation)

131. Hence, substituting in the expressions of the several Supplements, we have

Supp. $(\overline{5})$ $= O$

$+N$ . . . . . (first equation)

Supp. $(4, 1)$ $= R+J'$

$+2J+R$ . . . . . (second equation

Supp. $(\overline{3}, 2)$ $= 3L+2M+O$

$+6K+L+3N+2O$ . . . . . (third equation)

Supp. $(\overline{3}, 1, 1)$ $= H+2I+D'+J'$

$+E+F+2G+D+3J+J'$ . . . . (fourth equation)

Supp. $(\overline{2}, 3)$ $= \varkappa(\overline{2\varkappa 1}, 3)$

$+Q$ . . . . . (fifth equation)

Supp. $(\overline{2}, 2, 1)$ $= \varkappa(\overline{2\varkappa 1}, 1, 1)+J'$

$+3G+I+4J+2J'$ . . . . . (sixth equation)

Supp. $(\overline{2}, 1, 1, 1)$ $= \varkappa(\overline{2\varkappa 1}, 1, 1, 1,)+D'$

$+B+4C+4D+D'$ . . . . . (seventh equation)

Supp. $(\overline{1}, 4)$ $= \varkappa(\overline{1\varkappa 1}, 4)+O$

$+(m-\frac{4}{3})(4N+O)$

$-\frac{4}{3}N+\frac{4}{3}O$ . . . . . (eighth equation)

Supp. $(\overline{1}, 1, 3)$ $= \varkappa(\overline{1\varkappa 1}, 1, 3)+2\varkappa(\overline{2\varkappa 1}, 3)+2R+3J'$

$+(m-2)(2P+2Q+5J+4R)$

$-2P-Q-2R+J'$ . . . (ninth equation).

Supp. $(\overline{1}, 2, 2)$ $= \varkappa(\overline{1\varkappa 1}, 2, 2)+3L+2M$

$+(m-2)(9K+3L+M+2N+O)$

$+3L+2M-2N-O$. (tenth equation).

$$\begin{aligned}\text{Supp.}(\overline{1}, 1, 1, 2) \;=\; & \alpha(\overline{1\alpha}1, 1, 1, 2)+2\alpha(\overline{2\alpha}1, 1, 2)\\ & +\mathrm{H}+2\mathrm{I}+\mathrm{D}'+\mathrm{J}'\\ & +(m-\tfrac{7}{3})(3\mathrm{E}+3\mathrm{F}+6\mathrm{G}+2\mathrm{D}+\mathrm{H}+2\mathrm{I}+5\mathrm{J})\\ & \quad -2\mathrm{E}-2\mathrm{F}-\mathrm{G}-\tfrac{1}{3}\mathrm{D}+\tfrac{1}{3}\mathrm{H}+\tfrac{5}{3}\mathrm{I}-\tfrac{10}{3}\mathrm{J}+2\mathrm{D}'-\mathrm{J}'.\end{aligned}$$

(eleventh equation)

$$\begin{aligned}\text{Supp.}(\overline{1}, 1, 1, 1, 1, 1) = \; & \alpha(\overline{1\alpha}1, 1, 1, 1, 1)+2\alpha(\overline{2\alpha}1, 1, 1, 1)+\mathrm{D}'\\ & +(m-\tfrac{8}{3})(\mathrm{A}+2\mathrm{B}+4\mathrm{C}+3\mathrm{D})\\ & \quad -\tfrac{4}{3}\mathrm{A}-\tfrac{5}{3}\mathrm{B}-\tfrac{4}{3}\mathrm{C}-2\mathrm{D}-\mathrm{D}'.\end{aligned}$$

(twelfth equation)

132. Hence finally, merely collecting the terms, we have the following expressions of the Supplements in the twelve equations respectively.

Supp. $(\overline{5})$ $=\mathrm{N}+\mathrm{O}$ . . . . . (first equation)

Supp. $(\overline{4}, 1)$ $=2\mathrm{J}+2\mathrm{R}+\mathrm{J}'$ . . . . . (second equation)

Supp. $(3, 2)$ $=6\mathrm{K}+4\mathrm{L}+2\mathrm{M}+3\mathrm{N}+3\mathrm{O}$ . . . . . (third equation)

Supp. $(\overline{3}, 1, 1)$ $=\mathrm{D}+\mathrm{E}+\mathrm{F}+2\mathrm{G}+\mathrm{H}+2\mathrm{I}+3\mathrm{J}+\mathrm{D}'+2\mathrm{J}'$ . . . . . (fourth equation)

Supp. $(\overline{2}, 3)$ $=\alpha(\overline{2\alpha}1, 3)+\mathrm{Q}$ . . . . . (fifth equation)

Supp. $(\overline{2}, 2, 1)$ $=\alpha(\overline{2\alpha}1, 2, 1)+3\mathrm{G}+\mathrm{I}+4\mathrm{J}+3\mathrm{J}'$ . . . . . (sixth equation)

Supp. $(\overline{2}, 1, 1, 1)$ $=(\alpha\overline{2\alpha}1, 1, 1, 1)+\mathrm{B}+4\mathrm{C}+4\mathrm{D}+2\mathrm{D}'$ . . . . . (seventh equation)

Supp. $(\overline{1}, 4)$ $=\alpha(\overline{1\alpha}1, 4)\quad+(4m-7)\mathrm{N}+(2m-1)\mathrm{O}$ . . . . . (eighth equation)

Supp. $(\overline{1}, 1, 3)$ $=\alpha(\overline{1\alpha}1, 1, 3)+2\alpha(\overline{2\alpha}1, 3)$
$+(2m-6)\mathrm{P}+(2m-5)\mathrm{Q}+(5m-10)\mathrm{J}+(4m-8)\mathrm{R}+4\mathrm{J}'$. (ninth equation)

Supp. $(\overline{1}, 2, 2)$ $=\alpha(\overline{1\alpha}1, 2, 2)$
$+(9m-18)\mathrm{K}+3m\mathrm{L}+(m+2)\mathrm{M}+(2m-6)\mathrm{N}+(m-3)\mathrm{O}$. (tenth equation)

Supp. $(\overline{1}, 1, 1, 2)$ $=\alpha(\overline{1\alpha}1, 1, 1, 2)+2\alpha(\overline{2\alpha}1, 1, 2)$
$+(2m-5)\mathrm{D}+(3m-9)\mathrm{E}+(3m-9)\mathrm{F}+(6m-15)\mathrm{G}$
$+(m-1)\mathrm{H}+(2m-1)\mathrm{I}+(5m-15)\mathrm{J}+3\mathrm{D}'$ . . . . (eleventh equation)

Supp. $(\overline{1}, 1, 1, 1, 1, 1)=\alpha(\overline{1\alpha}1, 1, 1, 1, 1)+2\alpha(\overline{2\alpha}1, 1, 1, 1)$
$+(m-4)\mathrm{A}+(2m-7)\mathrm{B}+(4m-12)\mathrm{C}+(3m-10)\mathrm{D}$, . (twelfth equation)

where I recall the remark, *ante*, No. 126, that in each equation the Capitals belong to the system obtained by diminishing the barred number by unity and removing the bar; (4) for the first equation, (3, 1) for the second, and so on.

133. These are, I think, the true theoretical forms of the Supplements, viz. (attending to the signification of the Capitals) the expressions actually exhibit how the Supplement arises, whether from proper conics passing through or touching at a cusp, or from point-pairs (coincident line-pairs) or line-pairs (including of course in these terms line-pair-points). Thus, for instance, Supp. $(\overline{5})=N+O$. Referring to the explanations, First Memoir, Nos. 41 to 47, $N(=\iota)$ is the number of the line-pair-points described as "inflexion tangent terminated each way at inflexion," and $O(=\varkappa)$ the number of the line-pair-points described as "cuspidal tangent terminated each way at cusp," or in what is here the appropriate point of view, we have as a coincident line-pair each inflexion tangent and each cuspidal tangent. Reverting to the generation of the first equation, when the point P is a point in general of the given curve, the curve Θ is the conic (5), having with the curve 5 intersections at P, and besides meeting it in the $2m-5$ points P'. When the point P is at an inflexion, the curve Θ becomes the coincident line-pair formed by the tangent taken twice, the number of intersections at P is therefore $=6$, and the inflexion is therefore (specially) a united point. Similarly, when the point P is at a cusp, the curve Θ becomes the coincident line-pair formed by the tangent taken twice, the number of intersections at P is therefore $=6$, and the cusp is thus (specially) a united point: we have thus the total number of special united points $=\varkappa+\iota$, agreeing with the foregoing *à posteriori* result, Supp. $(\overline{5})=N+O$.

134. Or to take another example; for the fifth equation we have

$$\text{Supp.}\,(\overline{2},\,3)=\varkappa(\overline{2\varkappa 1},\,3)+Q;$$

$Q(=2\tau)$ is the number of the line-pair-points described as "double tangent terminated each way at point of contact," or, in the point of view appropriate for the present purpose, we have each double tangent as a coincident line-pair in respect to the one of its points of contact, and also as a coincident line-pair in respect to the other of its points of contact. Reverting to the generation of the equation, when the point P is a point in general on the given curve, the curve Θ is the system of conics $(\overline{2},\,3)$ touching the curve at P, and having besides with it a contact of the third order; since for each conic the number of intersections at P is $=2$, the total number of intersections at P is $=2(\overline{2},\,3)$, and the remaining $(2m-2)(\overline{2},\,3)$ intersections are the points P'. Suppose that the point P is taken at the point of contact of a double tangent; of the $(\overline{2},\,3)$ conics, 1 (I assume this is so) becomes the coincident line-pair formed by the double tangent taken twice, and gives therefore 4 intersections at P, the remaining $(\overline{2},\,3)-1$ conics are proper conics, giving therefore $2(\overline{2},\,3)-2$ intersections at P, or the total number of intersections at P is $2(\overline{2},\,3)+2$ intersections; or there is a gain of 2 intersections. As remarked (No. 96), this does not of necessity imply that the point in question is to be considered as being (specially) 2 united points; I do not know how to decide *à priori* whether it is to be regarded as being 2 united points or as 1 united point, but it is in fact to be regarded as being (specially) only 1 united point; and as the points in question are the 2τ points of contact of the double tangents, we have thus the number 2τ of special united points.

Again, when the point P is at a cusp, all the $(\overline{2}, 3)$ conics remain proper conics $((2\varkappa\overline{1}, 3)=(\overline{2}, 3)$, First Memoir, No. 73), but each of these (*quà* conic touching the cuspidal tangent) has with the given curve at the cusp not 2 but 3 intersections, so that the total number of intersections at P is $3(2\varkappa\overline{1}, 3)$, $=3\,(\overline{2}, 3)$, and there is a gain of $(\overline{2}\ 3)=(2\varkappa\overline{1}, 3)$ intersections. Each cusp counts (specially) as $(2\varkappa\overline{1}, 3)$ united points, and together the cusps count as $\varkappa(2\varkappa\overline{1}, 3)$ united points; we have thus the total number $\varkappa(2\varkappa\overline{1}, 3)+2\tau$ of special united points, agreeing with the expression Supp. $(\overline{2}, 3)=\varkappa(2\varkappa\overline{1}, 3)+Q$.

135. As appears from the preceding example, or generally from the remark, *ante*, No. 96, I have not at present any *à priori* method of determining the proper numerical multipliers of the Capitals contained in the expressions of the several Supplements. I will only further remark, that the reason is obvious why (while in the first seven equations the multipliers are mere numbers) in the eighth and following equations the multipliers are linear functions of m; in fact in these last equations the barred symbol is $\overline{1}$, that is, when P is a point in general on the given curve, each of the conics which make up the curve Θ has with the given curve not a contact of any order, but an ordinary intersection at P. Imagine a position of P for which one of these conics becomes a coincident line-pair; this regarded as a single line has with the given curve $(m-\alpha)$ ordinary intersections (α a number, $=4$ at most, depending on the contacts which the line may have with the curve); for each of the $m-\alpha$ points, taken as a position of P, one of the conics which make up the curve Θ becomes the coincident line-pair, and there are in respect of this conic two intersections at P instead of one intersection only. We have thus in respect of the particular coincident line-pair a group of $(m-\alpha)$ special united points, viz. these are the $m-\alpha$ ordinary intersections of the coincident line-pair regarded as a single line with the given curve, and we thus understand in a general way how it is that the order m of the given curve enters into the expressions of the multipliers of the several Capitals in the last five equations. The object of the present Memoir was, however, the *à posteriori* derivation of the expressions (*ante*, No. 132) of the twelve Supplements; and having accomplished this, but being unable to discuss the results with any degree of completeness, I abstain from a further discussion of them.

VI. *Addition to Memoir on the Resultant of a System of two Equations.*

By Professor CAYLEY, *F.R.S.*

Received August 6,—Read November 21, 1867.

THE elimination tables in the Memoir on the Resultant of a System of two Equations (Phil. Trans. 1857, pp. 703–715), relate to equations of the form $(a, b \ldots)(x, y)^m = 0$, *without* numerical coefficients; but it is, I think, desirable to give the corresponding tables for equations in the form $(a, b, \ldots)(x, y)^m = 0$ *with* numerical coefficients, which is the standard form in quantics. The transformation can of course be effected without difficulty, and the results are as here given. It is easy to see *à priori* that the sum of the numerical coefficients in each table ought to vanish; these sums do in fact vanish, and we have thus a verification as well of the tables of the present Addition as of the tables of the original memoir, by means whereof the present tables were calculated.

Table (2, 2).

Resultant of
$(a, b, c)(x, y)^2$,
$(p, q, r)(x, y)^2$.

Table (3, 2).

Resultant of
$(a, b, c, d)(x, y)^3$,
$(p, q, r)(x, y)^2$.

Table (4, 2).

Resultant of
$(a, b, c, d, e)(x, y)^4$,
$(p, q, r)(x, y)^2$.

Table (3, 3)*.

Resultant of
$(a, b, c, d)(x, y)^3$
$(p, q, r, s)(x, y)^3$.

* N.B. In the corresponding table of the memoir, there is an error in the signs of the last two terms; they should be

Table (4, 3).

Resultant of
$(a, b, c, d, e)(x, y)^4$,
$(p, q, r, s)(x, y)^3$.

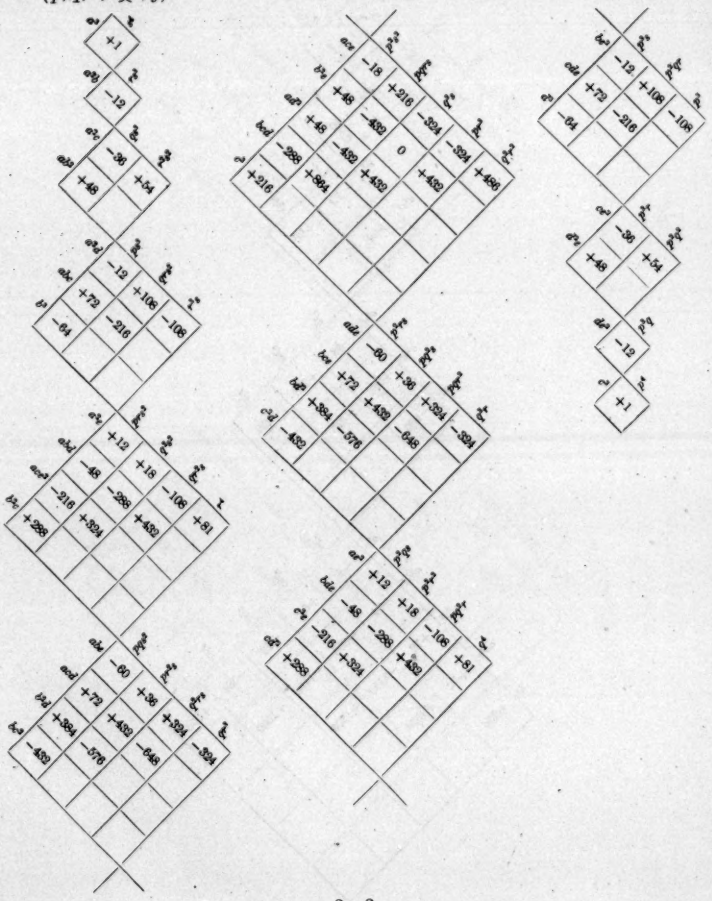
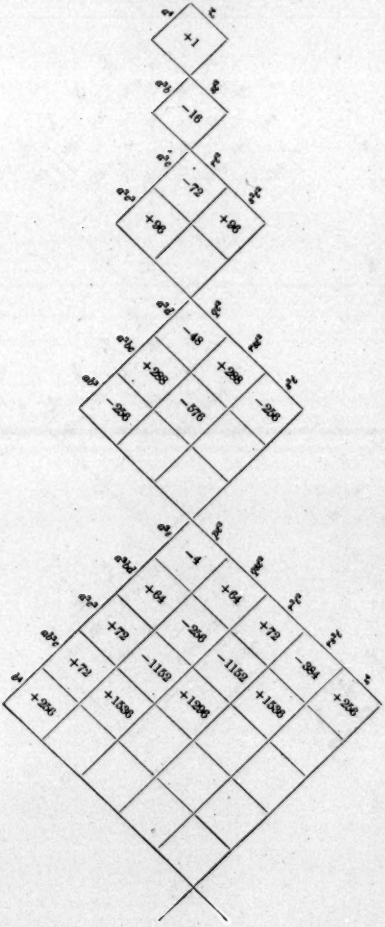


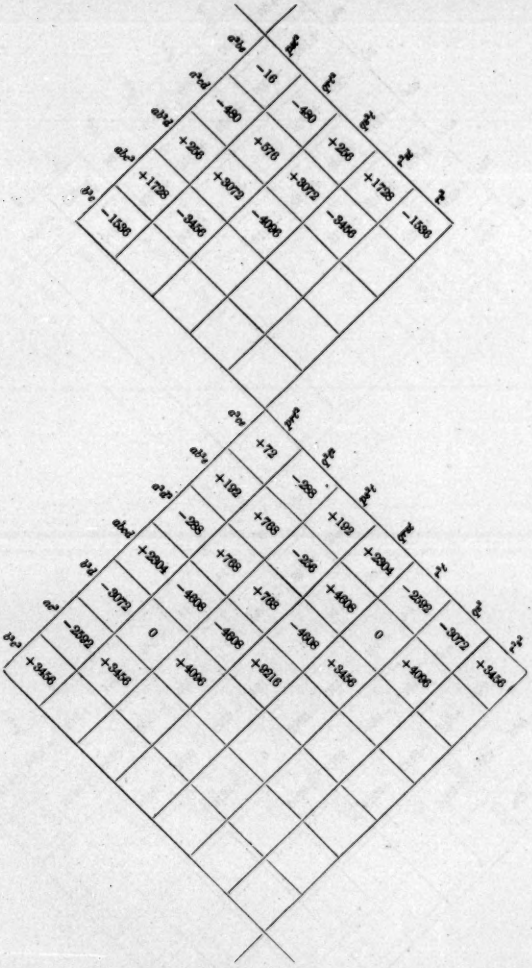
Table (4, 4).

Resultant of

$(a, b, c, d, e)(x, y)^4$,

$(p, q, r, s, t)(x, y)^4$.

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+256 +576 +256
+1728 +3072 +3072 +1728
−1536 −3456 −4096 −3456 −1536
+72
+192 −288 +192
−288 +768 −256 +2304
+2304 +768 −4096 +768 −2592
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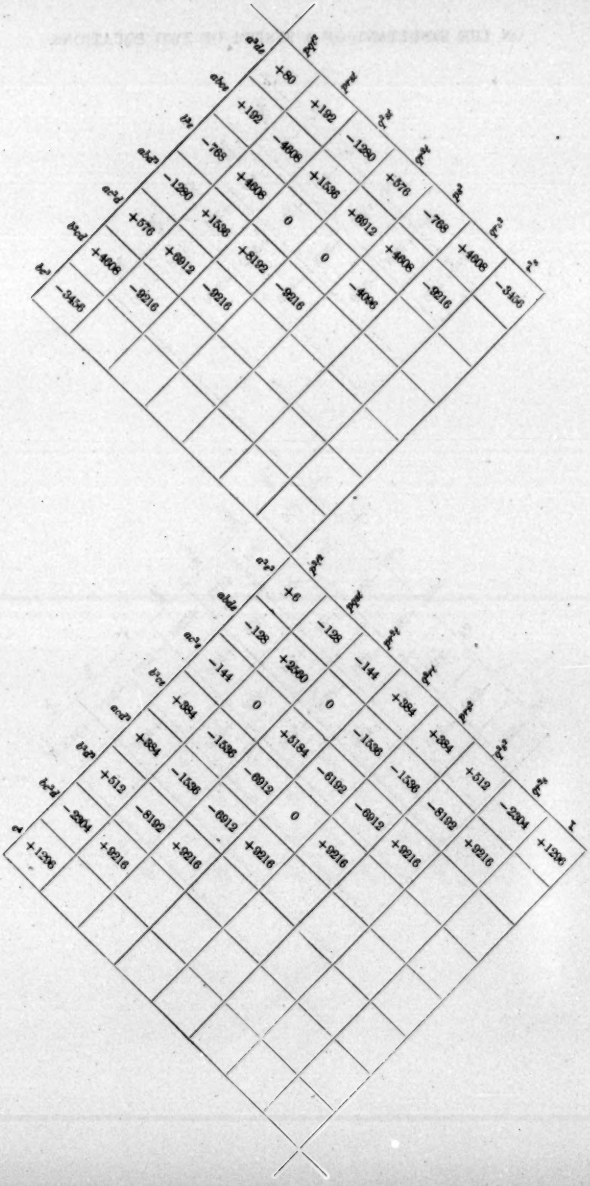
+3456 +4096 +9216 +3456 +3456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−768 +4608 0 +6912 +4608 −9216
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−1280 +6912 +4608 −9216
+576 +4608 −9216
−9216
+6 −126 −126 −144 +384 +384 +512 −2304 +1536
−126 +2560 −144 +384 −1536 −1536 −8192 +9216
−144 0 0 +5184 −6192 +9216
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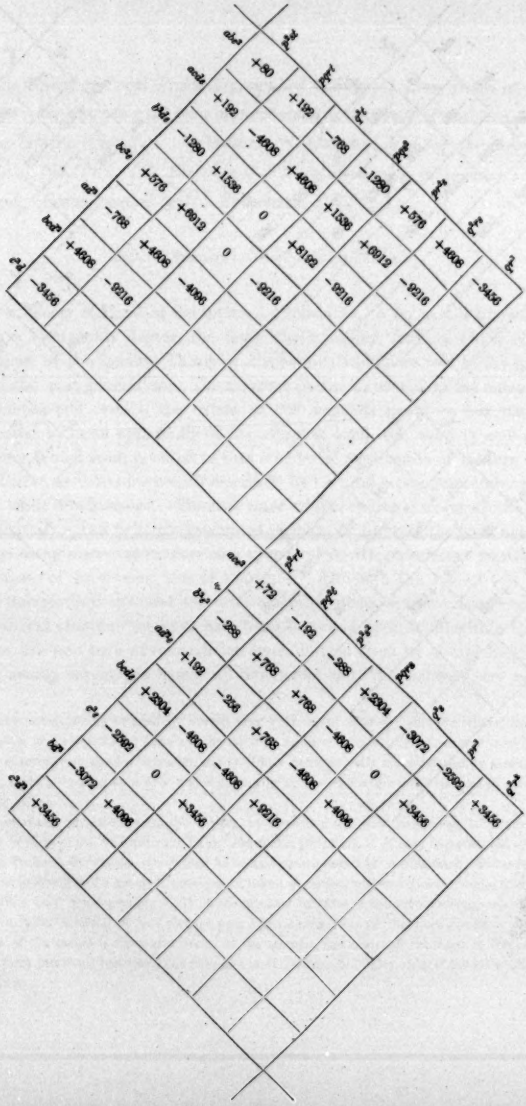
+512 −8192 −6912 +9216
−2304 +9216 +9216
+1536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+192
+192
−768
−1280
+576
−768
+4608
+4608
−9216
−3456
+1536
+6912
0
+8192
+4608
−4096
−9216
+4608
−9216
−3456
+72
−192
−288
+192
+768
+2304
−288
−256
+768
+2304
−3072
−2592
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−4608
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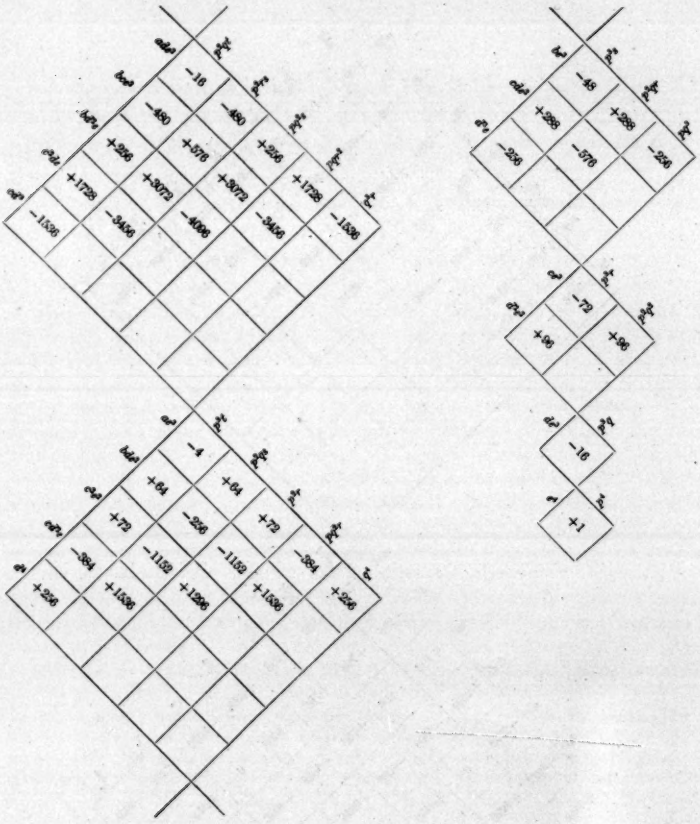
+3456
+3456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a⁴d³b +64 +576 −3072 −9456
a⁴d²b² −256 −3072 +4096
a⁴db³ +256 +1728
a⁴b⁴ −1536
a⁴d³ −4 −64 +256 −384 +256
a⁴d²b +64 +72 −256 −1152 +1536
a⁴db² −256 −1152 +1296
a⁴b³ +256 +1536
a⁴d² −48 −288 +256
a⁴db +288 −576
a⁴b² −256
a⁴d −72 +96
a⁴b +96
a⁴ −16
a³ +1

VII. *On the Theory of Local Probability, applied to Straight Lines drawn at random in a plane; the methods used being also extended to the proof of certain new Theorems in the Integral Calculus.* *By* MORGAN W. CROFTON, *B.A., of the Royal Military Academy, Woolwich; late Professor of Natural Philosophy in the Queen's University, Ireland. Communicated by* J. J. SYLVESTER, *F.R.S.*

Received February 5,—Read February 27, 1868.

1. THE new Theory of Local or Geometrical Probability, so far as it is known, seems to present, in a remarkable degree, the same distinguishing features which characterize those portions of the general Theory of Probability which we owe to the great philosophers of the past generation. The rigorous precision, as well as the extreme beauty of the methods and results, the extent of the demands made on our mathematical resources, even by cases apparently of the simplest kind, the subtlety and delicacy of the reasoning, which seem peculiar to that wonderful application of modern analysis—*ce calcul délicat*, as it has been aptly described by LAPLACE—reappear, under new forms, in this, its latest development. The first trace which we can discover of the Theory of Local Probability seems to be the celebrated problem of BUFFON, the great naturalist*—a given rod being placed at random on a space ruled with equidistant parallel lines, to find the chance of its crossing one of the lines. Although the subject was noticed so early, and though BUFFON's and one or two similar questions have been considered by LAPLACE, no real attention seems to have been bestowed upon it till within the last few years, when this new field of research has been entered upon by several English mathematicians, among whom the names of SYLVESTER and WOOLHOUSE† are particularly

* The mathematical ability evinced by BUFFON may well excite surprise; that one whose life was devoted to other branches of science should have had the sagacity to discern the true mathematical principles involved in a question of so entirely novel a character, and to reduce them correctly to calculation by means of the integral calculus, thereby opening up a new region of inquiry to his successors, must move us to admiration for a mind so rarely gifted.

† Many remarkable propositions on the subject, by these eminent mathematicians, have appeared in the mathematical columns of the 'Educational Times' and other periodicals. A very important principle has been introduced by Professor SYLVESTER, which may be termed *decomposition of probabilities*. For instance, he has shown that the probability of a group of three points, taken at random within a given triangle, fulfilling a given *intrinsic* condition (*i. e.* one depending solely on the internal relations of the points among each other), may be expressed as a linear function of two simpler probabilities; viz. that of the same condition being fulfilled (1) when one of the points is fixed at a vertex of the triangle, and a second restricted to the opposite side; (2) when all three points are restricted, one to each side of the triangle. The order of the integrations required

distinguished. It is true that in a few cases differences of opinion have arisen as to the principles, and discordant results have been arrived at, as in the now celebrated *three-point* problem, by Mr. WOOLHOUSE, and the *four-point* problem of Professor SYLVESTER; but all feel that this arises, not from any inherent ambiguity in the subject matter, but from the weakness of the instrument employed; our undisciplined conceptions of a novel subject requiring to be repeatedly and patiently reviewed, tested, and corrected by the light of experience and comparison, before they are purged from all latent error.

The object of the present paper is, principally, the application of the Theory of Probability to straight lines drawn at random in a plane; a branch of the subject which has not yet been investigated. It will be necessary to begin by some remarks on the general principles of Local Probability. Some portion of what follows I have already given elsewhere*.

2. The expression "*at random*" has in common language a very clear and definite meaning; one which cannot be better conveyed than by Mr. WILSON'S expression "*according to no law*." It is thus of very wide application, being often used in cases altogether beyond the province of mathematical measurement or calculation.

In Mathematical Probability, which consists essentially in arithmetical calculation, when we speak of a thing of any kind taken at random, there is always a direct reference to the *assemblage of things* to which it belongs and from which it is taken, at random,—which here comes to the same thing as saying that any one is as likely to be taken as any other. When we have a clear conception of what the assemblage is, from which we take, and not till then, we can proceed to sum up the favourable cases.

In many problems on probability there is no difficulty in forming a clear conception of the total number of cases. Thus if balls are drawn from an urn, the number of cases is the number of balls, or of certain combinations of them; and if the number of balls be supposed infinite, no obscurity arises from this. But there are several classes of questions in which the totality of cases is not merely infinite, but of an inconceivable nature. Thus if we try to imagine how to determine completely by experiment the probability of a hemisphere thrown into the air falling on its base, we may suppose an infinite number of persons to make one trial each; afterwards we may suppose each person to make two, three, or an infinite number of trials; again, we may suppose for every trial that has taken place an infinite number of others, varying, for instance, in the substance, size, &c. of the body employed; and so on. We can thus continually suppose variations of the experiment, each variation giving a new infinity of cases. Now problems of this nature are treated by means of the following principle:—

In any question of probability regarding an infinite number of cases, all equally pro-

is thus reduced by *three*. The same method applies to any polygon, and also to the points taken in space within a tetrahedron. It is to be hoped that Professor SYLVESTER will give these remarkable results to the public in a detailed form: a general account of them was given to the British Association at Birmingham in 1865.

* Educational Times, May 1867.

bable, the result will be unaltered if we take, instead of these cases, *any lesser infinity of cases, chosen at random from among them**.

3. The case of a point or straight line taken at random in a plane or in space is a problem of the above description. Thus, if a point be taken at random in a plane, the total number of cases is of an inconceivable nature, inasmuch as a plane cannot be *filled* with mathematical points, any infinitesimal element of the plane containing an unlimited number of points. We see, however, by means of the above principle, that we may consider the assemblage we are dealing with, as *an infinity of points all taken at random in the plane.*

Let us examine the nature of this assemblage. As the points continue to be scattered at random over the plane, their density tends to become uniform. It is evident, in fact, that a random point is as likely to be in any element dS of the surface, as in any equal element dS′; and therefore by continuing to multiply points, the number in dS will be equal (or *subequal*, to use a term of Professor De Morgan's) to that in dS′. Of course, though the density tends to become uniform, the disposition of the points does not tend to become symmetrical; those within any element dS will be dispersed in the most irregular manner over that element†. However, it is important to remark that, *for all purposes of calculation*, the ultimate disposition may be supposed symmetrical; for as the position of any point is determined by that of the element dS, within which it falls, it matters not what arbitrary arrangement we assume for the points within the element.

* This proposition, of which, in a somewhat different form, a mathematical demonstration is given by Laplace (Théorie Analytique des Probabilités, chap. 3), may be regarded as almost axiomatic. Thus, suppose an urn to contain an infinite number of black and white balls, in the proportion of 2 to 3; if any lesser infinite number of balls be drawn from it, the black ones among them will be to the white as 2 to 3. For, imagine all the balls ranged in a row ACB, the black from A to C, the white from C to B; if we now select an infinite number at random from among them, it appears self-evident that, if the line be divided into five equal parts, the numbers of balls taken from each part will be the same, or rather, will *tend* to equality on being increased indefinitely. Hence the black balls selected will be to the white as AC to CB, or as 2 to 3. When the numbers are *large*, but not infinite, this principle is approximately true, and forms, as is well known, the basis of most of the practical applications of Probability. Thus the chance of an infant living to the age of twenty is as truly found from, say, 1,000,000 of observed cases, as it would be from the total number.

In its strict mathematical form, the proposition may be thus stated:—In any unlimited number of cases, divided into favourable and unfavourable, if p be the ratio of the favourable to the whole number of cases, and if we select any infinite number of cases at random from among them, *the probability is infinitely small, that the same ratio, as determined from the selected cases, shall differ from* p *by a finite quantity.*

† Order thus results from disorder, the uniform density of the aggregate being unaffected by the disorder and irregularity of arrangement of its ultimate constituents; much as a nebula of uniform brightness is related to the stars which compose it. This remarkable law is to be traced, under one form or another, in most of the applications of the Theory of Probability.

"Elle mérite l'attention des philosophes, en faisant voir comment la regularité finit par s'établir dans les choses même qui nous paraissent entièrement livrées au hasard."—*Laplace.*

A familiar illustration of the tendency to uniform density in the random points may be derived by observing the drops of rain on a pavement at the commencement of a shower: as the drops multiply, it will be evident to the eye that their density tends more and more to uniformity.

Hence we may, if we please, assume that, when a point is taken at random in a plane, those from which it is taken are an infinite number symmetrically disposed over the plane.

Likewise, points taken at random in a line may be supposed equidistant. And if random values be taken for any *quantity*, they may be supposed to form an arithmetical series, with an infinitesimal difference.

Let us now consider the case of a straight line drawn at random in an infinite plane: the assemblage from which we select it is, as before, *an infinity of lines drawn at random in the plane*. What is the nature of this aggregate? First, since any direction is as likely as any other, as many of the lines are parallel to any given direction as to any other. Consider one of these systems of parallels; let them be cut by any infinite perpendicular. As this infinite system of parallels is drawn at random, they are as thickly disposed along any part of the perpendicular as along any other; the intersections being in fact random points on the perpendicular. Hence it is easily seen that, for all purposes of calculation, the assemblage of lines may be thus conceived. Divide the angular space round any point into a number of equal angles $\partial\theta$, and for every direction let the plane be ruled with an infinity of equidistant parallel lines, the common infinitesimal distance being the same for every set of parallels. Or we may suppose one such system of parallels drawn, and then turned through an angle $\partial\theta$, then through another equal angle, and so on, till they have returned to their former direction.

If we take any fixed axes in the plane, a random line may be represented by the equation

$$x\cos\theta+y\sin\theta=p,$$

where p and θ are constants taken at random.

There is no difficulty in extending now our conceptions to points, straight lines, and planes, taken at random in *space*.

4. We may take any plane area as the *measure* of the number of random points within it: in the case of random lines, I proceed to prove the following important principle:—

The measure of the number of random lines which meet a given closed convex plane boundary, is the length of the boundary.

Draw any system of parallels meeting the boundary, their common infinitesimal distance being ∂p. *If we take this distance as unity*, the number of these parallels is AB, a line cutting them at right angles. Let AB$=\zeta$, and let θ be its inclination to any fixed direction in the plane; conceive now a consecutive system of parallels inclined to the former at an angle $\partial\theta$, then a third, and so on, till the parallels return to the direction in the figure; then the total number of lines will be

Fig. 1.

$$\frac{1}{\partial\theta}\int_0^{\pi}\zeta d\theta;$$

or, if O be any fixed pole inside the boundary, and OV$=p$, the perpendicular on the

tangent to the boundary, θ its inclination to a fixed axis, the measure of the number of lines* is

$$N=\int_0^{2\pi} p\,d\theta.$$

Now the integral $\int p\,d\theta$ extended through four right angles gives the *whole length of the boundary*, whatever be its nature, provided it be convex†.

Hence if L be the length of the boundary,

$$N=L.$$

This result may be obtained also as follows. It may be shown very simply by the above principles that the measure of the number of random lines which meet any finite straight line of length a, is $2a$ (it may indeed be assumed as self-evident that the number is proportional to a). Conceiving now the boundary L as consisting of straight elements, the number of lines meeting any element ds, is $2ds$; so that the whole number which meet the boundary would be 2L; but as each line cuts the boundary in *two* points, we should thus count each line twice over; hence the true number is L.

Hence if L be the length of any convex boundary, and l that of another, lying wholly inside the former, the probability that a line drawn at random across L shall also intersect l, is

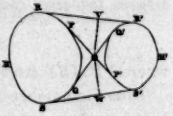
$$p=\frac{l}{L}.$$

It is important to observe that the measure of the number of lines which meet any *non-convex* boundary is *the length of a string drawn tightly round it*; as is obvious on consideration. The same is true for a boundary which is not closed.

5. Let there be any two boundaries external to each other: let X be the length of an endless band passing round both, and crossing between them, and Y the length of another endless band also enveloping both, but not crossing; then *the measure of the number of random lines which meet both boundaries is* X—Y.

It will be easily found from the principles explained above, that the number required will be the integral $\int p\,d\theta$ (referred to O as pole), taken for the left-hand curve from the position RR′ of its tangent, to the position PO; then for the right-hand one from the position P′O of its tangent, to the position S′S; then for the left-hand one, from SS′ to QO; then for the right-hand one, from Q′O to R′R. Now the values of these integrals are, drawing the perpendiculars OV, OW to RR′, SS′,

Fig. 2.



* It will be well to remember that this *measure* of the number of lines, N, means *the actual number multiplied by the constant factor* $\delta\theta$. Our notation is thus simplified, and no confusion need arise from sometimes saying "the number of lines," for shortness, instead of "the measure of the number of lines." As $\delta\theta$ remains constant throughout our investigations, henceforth we will denote it by δ.

† As $L=\int_0^{\pi} \zeta\,d\theta$, we see that *the mean breadth of any convex area is equal to the diameter of a circle whose circumference equals the length of the boundary*. By *breadth* is meant the distance between two parallel tangents, whose direction is supposed to alter by uniform increments.

1. the mixed line RPO $-$ RV,
2. „ „ S′P′O $-$ S′W,
3. „ „ SQO $-$ SW,
4. „ „ R′Q′O $-$ R′V,

and the sum of these is evidently equal to $X-Y$.

I will add a different proof of this proposition, deduced from art. 4, as it is interesting to see our results verified.

For shortness, I will use the symbol N(S) for "the number of random lines meeting the space S;" and N(S, S′) for the number meeting both S and S′.

The number of lines meeting both boundaries is evidently identical with the number meeting both the mixtilinear figures OPHQ, OP′H′Q′. These two figures together form the mixtilinear reentrant figure HPP′H′Q′Q, and by art. 4, $N(HPP'H'Q'Q)=Y$.

Now $N(OPHQ)+N(OP'H'Q')=N(HPP'H'Q'Q)+N(OPHQ, OP'H'Q')$. But OPHQ, OP′H′Q′ being convex figures, the number of lines meeting each is represented by its length; therefore

$$X=Y+N(HPQ, H'P'Q').$$

The probability that a line drawn at random across a given convex boundary of length L shall also meet a given *external* boundary is therefore

$$p=\frac{X-Y}{L}.$$

6. If two convex boundaries L, L′ intersect each other, in two or more points, it may be proved in a similar manner that the number of random lines which meet both is represented by $L+L'-Y$, where Y is the length of an endless band passing round both. Hence the probability that a line which meets L shall also meet L′, is

$$p=\frac{L+L'-Y}{L}.$$

7. It may easily be proved that the measure of *the number of random lines which pass between two given convex boundaries is*

$$N=PP'+QQ'-\text{arc } PQ-\text{arc } P'Q',$$

where PP′, QQ′ are the two common tangents which cross each other.

Thus the number of random lines which pass between the two branches of an hyperbola is represented by Δ, the difference between the whole length of the hyperbola and that of its asymptotes. This difference, as is known, is given by the definite integral

$$\Delta=4a\int_0^{\alpha}\sqrt{1-e^2\sin^2\theta}\,.\,d\theta,$$

where $\sin\alpha=\frac{1}{e}$.

8. Two lines are drawn at random across a given convex area: to find the probability of their intersection lying within the area.

Let AB be the internal portion of any random line crossing the area: the number of its intersections with all the random lines in the area is the number of those lines which meet it. Now this number is $\frac{2AB}{\delta}$ (art. 4); hence the number of intersections of the system of parallels to AB with all the random lines in the area, is twice the sum of the lengths of all these parallel chords divided by δ. But this sum is the area of the figure (we have taken the common distance δp of the chords as unity).

Fig. 3.

Let Ω be the area, L the length of the boundary. As, then, $\frac{2\Omega}{\delta}$ is the number of intersections for any system of parallels, and the number of those systems is $\frac{\pi}{\delta}$, the total number of intersections is $\frac{2\pi\Omega}{\delta^2}$. But we have thus counted each intersection twice; so that the real number of intersections which fall inside the area Ω is $\frac{\pi\Omega}{\delta^2}$.

Hence the required probability is

$$p=\frac{2\pi\Omega}{L^2},$$

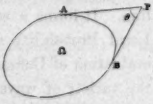
since the whole number of intersections is $\frac{1}{2}\left(\frac{L}{\delta}\right)^2$.

Thus it is an even chance that two random chords of a circle intersect within the circle; for any other figure the chance is less than $\frac{1}{2}$.

If an infinity of lines are drawn at random in an infinite plane, *the density of their intersections* (*i. e.* the measure of the number* of intersections in any given space, divided by the space) is uniform, and equal to π.

9. If an infinity of random lines meet a given area, *the density of their intersections, at any external point* P, is

Fig. 4.

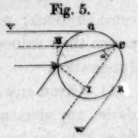


$$\rho=\theta-\sin\theta,$$

where θ is the apparent angular magnitude of the area from that point.

Conceive an infinitely small circle, or other figure (whose dimensions, however, infinitely exceed δp), at P, and let us calculate the number of the said intersections which fall inside this circle. Let the figure represent this circle, *magnified* as it were; QV, RW being the tangents PA, PB. Draw one of the random lines CD, which meet both the circle and the area Ω, the actual number of intersections which lie on CD will be $\frac{1}{\delta}$N(Ω, CD), which is found from art. 5 to be

Fig. 5.



$$\frac{1}{\delta}(2CD-CH-CI),$$

* We take for this measure the actual number multiplied by $\delta\delta'$, or δ^2 (see note, art. 4).

or

$$\frac{CD}{\delta}\Big(2-\cos\alpha-\cos(\theta-\alpha)\Big).$$

Hence the actual number of intersections on all the chords parallel to CD is

$$\frac{1}{\delta}(\text{area of circle})\Big(2-\cos\alpha-\cos(\theta-\alpha)\Big).$$

Therefore the measure* of the whole number of intersections lying within the circle is

$$\tfrac{1}{\pi}(\text{area})\int_0^\theta\Big(2-\cos\alpha-\cos(\theta-\alpha)\Big)d\alpha=(\text{area})(\theta-\sin\theta),$$

which proves the theorem.

10. The number of the intersections external to the given area is, then, measured by the integral

$$\iint(\theta-\sin\theta)dS$$

extended over the whole plane outside Ω; dS being the element of the area. Now the number of internal intersections is $\pi\Omega$ (art. 8), and the sum of both is $\frac{1}{2}L^2$. We obtain thus, in a singular manner, the following remarkable theorem in Definite Integrals:—

If θ be the angle between the tangents drawn from any external point (x, y) *to any given convex boundary, of length* L, *enclosing an area* Ω, *then*

$$\iint(\theta-\sin\theta)dxdy=\tfrac{1}{2}L^2-\pi\Omega,$$

the integration extending over the whole space outside Ω.

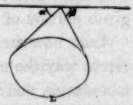
It does not seem easy to deduce this integral, in its generality, by any other method. It may be verified by direct integration for the cases of a circle, and of a finite straight line. It forms a striking example of what will doubtless be found, as the study of Local Probability advances, to be one of its most remarkable applications, viz. the evaluation of Definite Integrals. All who have studied the subject must have remarked the variety of ways in which almost every problem may be considered; now it often happens that a question in which we are baffled by the difficulties of the integration, when we attempt it in a particular way, may be solved with comparative ease by other considerations: we can then return to the integrals which we were unable to solve, and assign their values. I proceed to give some further applications of the above theory to Integration.

11. Given any infinite straight line outside a given convex boundary of length L, let dx be any element of this line; α, β the inclinations of dx to the two tangents drawn from it to the boundary, then

$$\int_{-\infty}^{\infty}(\cos\alpha+\cos\beta)dx=L.$$

* We take for this measure the actual number multiplied by $\delta\delta'$, or δ^2 (see note, art. 4).

Fig. 6.



It is easy to see from art. 5 that the number of random lines cutting L, which also meet dx, is $dx(\cos\alpha+\cos\beta)$; now the sum of all such elements gives the number of lines cutting both L and the given infinite straight line; that is, L (art. 4). This integral may be otherwise verified.

If the boundary L be enclosed within any outer convex boundary, let ds be the differential of the length of the latter, α, β the inclinations of ds to the tangents from it to L, then we find in the same manner,

$$\int(\cos\alpha+\cos\beta)ds=2L,$$

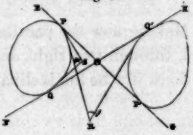
the integral extending all round the outer curve.

I mention this merely as an illustration; it is in fact easy to show independently that

$$L=\int\cos\alpha\,ds=\int\cos\beta\,ds.$$

12. If an infinite number of random lines pass between two convex areas, the density of their intersections will be (as in art. 9) at any point R in the angle FOG, or in EOH,

Fig. 7.



$$\varrho=\theta-\sin\theta;$$

and at any point S in the spaces POQ, P′OQ′,

$$\varrho=\pi-\varphi-\sin\varphi;$$

now the whole number of intersections is (art. 7) measured by

$$\tfrac{1}{2}(PP'+QQ'-PQ-P'Q')^2.$$

Hence

$$\iint(\theta-\sin\theta)dS+\iint(\pi-\varphi-\sin\varphi)dS=\tfrac{1}{2}(PP'+QQ'-PQ-P'Q')^2,$$

the first integral extending over the infinite spaces FOG, EOH, and the second over the spaces POQ, P′OQ′.

Thus if θ be the angle between the tangents drawn from any external point to an hyperbola,

$$\iint(\theta-\sin\theta)dxdy=\tfrac{1}{2}\Delta^2,$$

where Δ is the difference between the hyperbola and its asymptotes, and θ means the *external* angle of the tangents, in the cases where they touch the same branch of the curve, the integral extending over the whole space outside the hyperbola.

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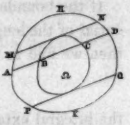
13. If we consider a system of random lines disposed over the whole surface of an infinite plane, and a second system all of which meet a given convex area Ω within the plane, and then fix our attention on the infinite system of points in which the latter system cuts the former, it will be seen that *the density of these intersections, at any point* (x, y) *exterior to* Ω, *is equal to* 2θ, θ being the angle which Ω subtends at the point

(x, y); hence $2\iint\theta dxdy$ represents the number of these intersections which lie on any given portion of the plane outside Ω.

Take now an arbitrary convex boundary surrounding Ω; we will calculate in a different way the number of intersections which lie on the annular space between the two boundaries, and thus arrive at a value for the above definite integral, extended over the same annulus.

Fig. 8.



Let AD be a random line of the second system, meeting Ω; the number (within the annulus) of its intersections with the first system will be measured by (art. 4) $2AB+2CD$; and hence the total number of intersections of all parallels to AD (between the tangents MN, PQ), with the first system, will be measured by double the area, cut from the annulus, between MN and PQ. Hence if Θ represent the annulus, the actual number of intersections which lie on those random lines of the second system which are parallel to those in the figure, is

$$n_0=\tfrac{1}{\pi}(2\Theta-2 \text{ segment MHN}-2 \text{ segment PKQ}).$$

Making now the parallel tangents MN, PQ revolve by constant changes of inclination, δ, through two right angles, we have for the *measure* of the total number of intersections, if φ be the inclination of MN to a fixed line,

$$N=\int_0^{\pi}(2\Theta-2\text{MHN}-2\text{PKQ})d\varphi.$$

But if we make the tangent MN revolve through 4 right angles instead of 2, it will occupy all the positions of PQ; denoting then the segment MHN by Σ, we have

$$N=2\pi\Theta-2\int_0^{2\pi}\Sigma d\varphi;$$

therefore

$$\iint\theta dxdy=\pi\Theta-\int_0^{2\pi}\Sigma d\varphi.$$

The mean or average value of the segment Σ, as the tangent alters by uniform changes of inclination, is

$$A=\frac{1}{2\pi}\int_0^{2\pi}\Sigma d\varphi;$$

we have, then, the following theorem:—

If θ be the angle subtended at any point (x, y) by a given convex area Ω, then

$$\iint\theta dxdy=\pi(\Theta-2A),$$

the integration extending over the annulus between Ω and any given exterior convex boundary; Θ standing for the area of that annulus, and A denoting the average area of the segments cut from the annulus by the tangents to the boundary of Ω.

This theorem gives the value of the integral in those cases where we are able to calculate the value of A: if Σ is constant, we have the theorem:—

Let there be any two convex boundaries so related that a tangent to the inner cuts off a constant area from the outer. Let θ be the angle subtended by the inner boundary at any external point (x, y); and let Δ be the difference of the parts into which the annular space between the two is divided by any tangent to the inner, then

$$\iint \theta dx dy = \pi\Delta,$$

the integration extending over the whole of the annulus.

For instance, we may apply the theorem to two similar coaxial ellipses. We may deduce thus the following definite integral,

$$\iint \tan^{-1}\left(\frac{2\sqrt{a^2y^2+b^2x^2-a^2b^2}}{x^2+y^2-a^2-b^2}\right)dxdy = \pi ab k^2(\pi \sin^2\tfrac{1}{2}\alpha - \alpha + \sin\alpha),$$

the limits being given by $1 < \frac{x^2}{a^2}+\frac{y^2}{b^2} < k^2$; putting $\cos\frac{1}{2}\alpha = \frac{1}{k}$.

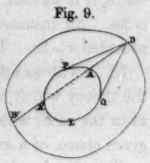
In the case where $k^2=2$, the value of the integral is $2\pi ab$; that is, the area of the outer ellipse.

14. If we suppose an infinite plane covered with random lines, and then imagine these divided into two systems, the first comprising all those lines which meet a given convex boundary, the second all those which do not meet it, and if we now consider the assemblage of points in which the first system intersects the second, we shall find (as in art. 9) that *the density of these intersections, at any point outside the boundary*, is $2\sin\theta$, θ being, as before, the apparent angular magnitude of the boundary.

Hence the number of intersections which lie on any given space is represented by the integral $2\iint \sin\theta dS$.

If we now suppose an endless string (of length Y) passed round the given boundary (whose perimeter we call L), and if this string be kept stretched by a moving point which thus traces out a new contour enclosing the given one (as the outer of any two confocal ellipses may be generated from the inner), we may estimate in a different manner the number of intersections which lie on the intermediate annular space, and thus obtain the following value for the above integral extended over that space,

Fig. 9.



$$\iint \sin\theta dS = 1(Y-L).$$

Let AB be a line of the first system meeting the two boundaries in A, B; the number of points in which AB is cut by a system of random lines *covering the whole plane* is (art. 4)

$$\frac{2AB}{\delta}.$$

If we subtract from this the number of intersections of AB with those lines which

meet the boundary L, the remainder will be the number of intersections of AB with the *second* system of lines above, viz. (art. 6)

$$\frac{2AB}{\delta}-\frac{1}{\delta}(2AB+L-Y),$$

that is,

$$\frac{1}{\delta}(Y-L).$$

This is constant for every position of AB; hence the number of intersections lying on the annulus will be the above constant, multiplied by the number of positions of AB; now this number is $\frac{2L}{\delta}$ (art. 4) (remembering that for every line AB, there is also one A'B', forming a portion of the same straight line). Hence the total number of intersections is

$$\frac{2}{\delta^2}L(Y-L).$$

If, then, the integration extend over the annulus,

$$\iint \sin\theta . dS = L(Y-L).$$

This theorem will apply to an ellipse, the outer boundary being a confocal ellipse. A particular case, which admits of verification by using elliptic coordinates, will be:—

If θ be the angle which two fixed points F, F' subtend at the element dS,

$$\iint \sin\theta dS = 8c(a-c);$$

the integration extending over an ellipse whose foci are F, F', $2a$ being the axis of the ellipse, and $2c$=FF'.

The above method will also show that in this case the integral remains unchanged in value, if it extend over any Cartesian oval whose *internal* foci are FF', and whose axis is $2a$. An instance of such a Cartesian is a circle from F as centre with a as radius, provided $a>2c$. The same will appear by means of elliptic coordinates*.

15. I will mention the following integral here, as, though strictly not derived from the theory which forms the subject of this paper, the principle used in obtaining it is, as in the cases which precede, the calculation of the number of intersections lying on a given space, of a given reticulation of straight lines.

Given a closed convex boundary without salient points; if we draw an infinity of tangents to it, each making an infinitesimal angle (δ) with the preceding, and consider the *intersections* of all these tangents with each other, it will not be difficult to show (as in art. 9) that the number of intersections lying on any element dS will be

$$\frac{1}{\delta^2}\cdot\frac{\sin\theta}{TT'}\,dS,$$

* The general integral above admits also of being established by means of a certain generalization of elliptic coordinates, which defines the position of a point by the sum and difference of two strings, each of which is attached to a fixed point on a given oval curve; they are then wrapped round the curve in opposite directions, and leave it as two tangents, meeting and terminating at the proposed point.

where T, T′ *are the tangents from* d*S to the boundary, and* θ *their mutual inclination.*

Now the whole number of tangents is $\frac{2\pi}{\delta}$, and that of intersections $\frac{2\pi^2}{\delta^2}$. We infer therefore that

$$\iint \frac{\sin\theta}{TT'}\,dS = 2\pi^2,$$

the integral extending over the whole external surface.

If the integral extend over *the annulus between the given boundary and an outer line along which* θ *has a constant value* (α), then

$$\iint \frac{\sin\theta}{TT'}\,dS = 2\pi(\pi-\alpha).$$

If the same integral extend over *the space between the given boundary and two fixed tangents, including an angle* α, its value will be $\frac{1}{2}(\pi-\alpha)^2$. If it extend over *the infinite angle formed by those tangents produced*, its value will be $\frac{1}{2}\alpha^2$.

If the given boundary contain salient points, then for every such point, where the bounding line changes direction abruptly through an angle A, a number of the tangents, equal to $\frac{A}{\delta}$, meet at that point; hence a number $\left(\frac{1}{2}\frac{A^2}{\delta^2}\right)$ of intersections coincide there, and consequently we must subtract $\frac{1}{2}A^2$ from each of the above integrals. Hence if there are any number of salient points A A′ A″, &c. in the boundary, the first integral becomes

$$\iint \frac{\sin\theta}{TT'}\,dS = 2\pi^2 - \tfrac{1}{2}\Sigma A^2,$$

and likewise for the second.

Thus for a regular polygon of (n) sides, the value is

$$2\pi^2\left(1-\frac{1}{n}\right).$$

If instead of drawing tangents to the given boundary at uniform angular intervals, we draw a system of tangents whose points of contact are distant from each other by a common infinitesimal interval, we shall find that the density of the intersections in this case varies as

$$\frac{\rho\rho'}{TT'}\sin\theta,$$

where $\rho\rho'$ *are the radii of curvature of the boundary at the points of contact of* TT′: this gives us the integral

$$\iint \frac{\rho\rho'}{TT'}\sin\theta\,dS = \tfrac{1}{2}L^2,$$

L being the whole perimeter of the boundary, the integral extending over the whole plane.

Many analytical definite integrals may be deduced by expressing the general theorems now given, in the language of different systems of coordinates, for various particular

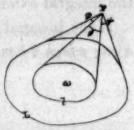
cases. Thus the first theorem in this article, applied to the ellipse, gives

$$\iint \frac{a^4y^2+b^2x^2}{\{(x^2+y^2+c^2)^2-4c^2x^2\}\sqrt{a^4y^2+b^2x^2-a^2b^2}}\,dxdy=\pi^2;$$

the equation of limits being $\frac{x^2}{a^2}+\frac{y^2}{b^2}>1$.

16. Let there be a closed convex area ω, length of boundary l, enclosed within another of length L; let θ be the apparent magnitude of ω at any external point; by considering two systems of random lines, one crossing the boundary L, and the other l, and examining the law of the density of the intersections of the former with the latter, we arrive at the theorem:—if we put for shortness

Fig. 10.



$$\alpha-\sin\alpha=u_\alpha,$$

$$\iint(u_{\theta+\phi}+u_{\theta+\psi}-u_\theta-u_\phi)dS+2\iint\theta dS=Ll-2\pi\omega;$$

the first integral extending over the whole space outside L, the second over the space between L and l.

17. But few problems on random straight lines admit of such simple results and of such generality as those we have been discussing. In general they can only be solved for particular forms of the boundaries. However, the above principles, applied to each particular question, generally suffice to reduce it at least to a problem of the Integral Calculus. I will give one or two examples.

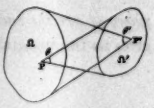
If two random lines cross a given convex area, the chance of their intersection falling on any *internal* portion of the area ω, is evidently (art. 8)

$$p=\frac{2\pi\omega}{L^2}.$$

But the chance of the intersection falling on any *external* area is less easy to find; it depends on the integral $\iint(\theta-\sin\theta)dS$ extended over that area. Could we succeed in finding the required probability by any different method, we could give the value of this integral for any external area.

A line is drawn at random across each of two given convex areas Ω, Ω', external to each other, lengths of boundaries L, L'; to find the chance of their intersection being outside both areas.

Fig. 11.



The density of the intersections of the system of random lines crossing Ω with those crossing Ω', at any point P within Ω, is 2θ, where θ means the apparent magnitude of Ω' at P. Within Ω', the density is $2\theta'$. Hence it is easy to see that, as the whole number of intersections is LL', the required probability is

$$p=1-\frac{2}{LL'}\left(\iint\theta dS+\iint\theta' dS'\right),$$

the integrals extending over Ω and Ω' respectively. It is evident that these integrals, however, can only be evaluated for particular forms of the areas.

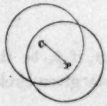
18. The following problems relate to a circular boundary:—

1. A random point falls within a given circle and a random straight line is drawn across the circle; to find the chance of the line passing within a given distance of the point.

As the general solution is somewhat complicated, I will take the particular case where the given distance is the radius of the circle, which will serve equally well as an example of the application of the foregoing principles.

Let C be the centre of the given circle, P any position of the random point, r the radius of the given circle; draw an equal circle with P as centre; then the number of random lines meeting the given circle and passing within a distance r of the point P, is the same as the number of random lines cutting *both* circles; this number is measured (art. 6) by the excess of the two circumferences over an endless band wrapped round them; that is, putting $CP=\rho$,

Fig. 12.



$$2\pi r-2\rho.$$

If dS be an element of the surface at P, the sum of the favourable cases will be

$$F=\iint(2\pi r-2\rho)dS=2\int_0^r(\pi r-\rho).2\pi\rho d\rho;$$

$$\therefore F=(\pi-\tfrac{2}{3})2\pi r^3.$$

But the whole number of cases is $2\pi r\times\pi r^2$; hence the required chance is

$$p=1-\frac{2}{3\pi}.$$

I will give another solution of this problem:—Let AB be a position of the random line; take $MN=r$, then all the favourable positions of the random point are within the segment EHF; the number of favourable points is therefore

Fig. 13.

$$r^2(\pi-\varphi+\sin\varphi\cos\varphi).$$

We have to multiply this by the differential of CM, and integrate from $CM=r$ to $CM=0$, which will give the favourable combinations for all random lines parallel to AB, passing between C and H; doubling this, we have the result for *all* lines parallel to AB; that is,

$$F_0=2r^3\int_r^0(\pi-\varphi+\sin\varphi\cos\varphi)d.r\cos\varphi$$

$$=2r^3\int_0^{\frac{\pi}{2}}(\pi-\varphi+\sin\varphi\cos\varphi)\sin\varphi\,d\varphi;$$

$$\therefore F_0=2r^3(\pi-\tfrac{2}{3}).$$

Now if the system of lines parallel to AB revolve through two right angles, we have for the *measure* of the whole number of favourable cases

$$F=2\pi r^3(\pi-\tfrac{2}{3});$$

hence, as before,

$$p=1-\frac{2}{3\pi}.$$

The problem in its general form can be solved without any great difficulty by the same methods. The result may be expressed in this form:—Let D be the given maximum distance; draw a circle of radius D with its centre on the given circumference; let Y be a band enveloping both circles, and 2θ the inclination of the two straight portions of this band; then the probability of the line passing within a distance D of the point will be

$$p=\frac{2\pi r+2\pi D-Y}{2\pi r}+\frac{\cos^2\theta}{3\pi};$$

or, if p_0 be the probability when the point is taken anywhere on the *circumference* of the given circle, then the general value of the probability is

$$p=p_0+\frac{\cos^2\theta}{3\pi}.$$

If a random point and a random straight line be taken within any convex boundary of length L, the chance that the line shall pass within a distance D of the point, D being small, is approximately,

$$p=\frac{2\pi D}{L}.$$

2. If three lines are drawn at random across a given circle, to determine the probability that their three intersections shall lie within the circle.

Let AB be one of the random lines. The total number of favourable triads of random lines, each triad of which includes AB, is the same as the *number of intersections, which fall within the circle*, of all random lines which cross AB. For every such intersection which lies within the circle, gives a pair of lines meeting AB, forming a triad whose intersections all lie within the circle. Now if θ be the angle which AB subtends at any internal point P, the number of these intersections will be measured by (art. 9)

Fig. 14.

$$N=\iint(\theta-\sin\theta)dS,$$

extended over the whole circle.

To integrate this, conceive the circle divided into an infinite number of elementary crescents, by segments of circles on AB; let O be the centre of the segment APB, ρ its radius; then the area of the segment APB is, putting AB$=2a$,

$$\text{segment}=(\pi-\theta)\rho^2+a\rho\cos\theta, \text{ or as } \rho\sin\theta=a$$

$$=a^2\left(\frac{\pi-\theta}{\sin^2\theta}+\cot\theta\right).$$

Differentiating this for θ, we obtain for the area of the crescent between APB and the consecutive arc on AB,

$$\text{crescent} = \frac{2a^2}{\sin^2\theta}\left(1+(\pi-\theta)\cot\theta\right)d\theta.$$

Hence the number of intersections *above* AB will be

$$N=2a^2\int_\alpha^\pi \frac{\theta-\sin\theta}{\sin^2\theta}\left(1+(\pi-\theta)\cot\theta\right)d\theta;$$

$$\therefore\ \frac{N}{2a^2}=\int_\alpha^\pi\left\{\frac{\theta d\theta}{\sin^2\theta}+\pi\frac{\theta\cos\theta d\theta}{\sin^3\theta}-\frac{\theta^2\cos\theta}{\sin^3\theta}d\theta-\frac{d\theta}{\sin\theta}-\pi\frac{\cos\theta d\theta}{\sin^2\theta}+\frac{\theta\cos\theta}{\sin^2\theta}d\theta\right\}.$$

All these are elementary integrals, and give (reducing the indeterminate forms by the usual methods)

$$\frac{N}{2a^2}=\frac{3}{2}-\frac{\alpha^2}{\sin^3\alpha}-\frac{\pi-\alpha}{\sin\alpha}+\frac{\pi}{2}\cot\alpha+\frac{\pi\alpha}{2\sin^2\alpha}.$$

To find the number of intersections *below* AB, change α into $\pi-\alpha$; this gives for the whole number of favourable triads (including AB),

$$N=2a^2\left(3-\frac{\pi}{\sin\alpha}+\frac{\alpha\pi-\alpha^2}{\sin^2\alpha}\right);$$

or if c be the radius of the given circle, $a=c\sin\alpha$;

$$\therefore\ N=2c^2(3\sin^2\alpha-\pi\sin\alpha+\alpha\pi-\alpha^2).$$

Multiply this by the differential of CM, and integrate from c to $-c$, and we have the sum of all favourable triads, each of which includes *any one* of the random lines parallel to AB,

$$F=2c^3\int_0^\pi(3\sin^2\alpha-\pi\sin\alpha+\alpha\pi-\alpha^2)\sin\alpha d\alpha$$

$$=2c^3\left(8-\frac{\pi^2}{2}\right).$$

Multiply this by π, and we have the measure* of the total number of favourable triads: however, this must be divided by 3, as it is clear we should thus count each triad thrice; hence total value of

$$F=\frac{\pi c^3}{3}(16-\pi^2);$$

and the whole number of cases being $\frac{1}{6}(2\pi c)^3$, we find for the probability sought,

$$P=\frac{4}{\pi^2}-\frac{1}{4}\dagger.$$

19. An interesting inquiry, though of a more difficult nature than that which has occupied us in this Paper, would be the extension of the foregoing principles to straight lines and planes drawn at random in space. It involves several intricate and curious points relating to the general theory of surfaces. With regard to the *measure of the number of random straight lines which meet a given closed convex surface*, it is easy to show that this measure is *the surface itself*.

* i. e. the actual number multiplied by δ^2 (as art. 8).

† It is not unlikely that this result may be obtained in some simpler manner.

It may be assumed as self-evident that if space be filled with an infinity of random straight lines, and they be cut by any infinite plane, the points in which it cuts them are distributed with uniform density over the plane; and this density will be the same for any other plane. Hence the number of the random lines which meet any plane area is proportional to that area. Hence the number meeting any plane element $d\mathrm{S}$ of the surface is proportional to $d\mathrm{S}$; the same is true for every other element; and each random line cuts two elements and only two; hence the whole number of lines is proportional to S.

We might view the question as follows. The entire body of random lines may be considered (as in art. 3) as a system of parallels disposed uniformly and symmetrically in space, which is afterwards turned round by infinitely small angular displacements, into every possible position. Let the figure represent one of these systems of parallels meeting the surface S, and of course bounded by the cylinder, enveloping S, whose generatrix is parallel to these lines. Let Ω be the area of the perpendicular section of this cylinder, then Ω is the measure of the number of these parallels. Let θ, φ be the angular coordinates of the direction of these parallels, and let them now pass into every possible angular position; the whole number of lines which meet S will be proportional to

Fig. 15.

$$\iint \Omega \sin\theta d\theta d\varphi,$$

extended through half the solid angular space round a point. We infer from this that

$$\int_0^{\pi}\int_0^{\pi} \Omega \sin\theta d\theta d\varphi = k\mathrm{S}.$$

To determine the constant k, we may apply the theorem to any particular case, as a sphere; this gives $k=\frac{\pi}{2}$. We may accept this manner of viewing a system of random lines, then, as a proof of the theorem in surfaces:—

If Ω be the area of the section of a cylinder enveloping a convex surface S; θ, φ the angular coordinates of the generatrix of the cylinder,

$$\int_0^{\pi}\int_0^{\pi} \Omega \sin\theta d\theta d\varphi = \frac{\pi}{2}\mathrm{S}.$$

The measure of *the number of random planes* which meet a given surface is easily seen to be (as in art. 4)

$$\mathrm{N} = \int_0^{\pi}\int_0^{\pi} p \sin\theta d\theta d\varphi,$$

where p is the perpendicular from any internal point on the tangent plane, and θ, φ the angular coordinates of p. I am not aware that this integral has ever been considered. It is probable that it admits of some simple geometrical representation, which possibly will be found to be the length of some closed curve, traced upon the given surface, and bearing some remarkable relation to the general curvature of the surface.

It is well to notice, with regard to the applications to integration of the theory laid down in this Paper, that the theorems thereby deduced in no way depend for their truth upon the doctrine of Probability, although it has been the occasion which has led to them. The apparatus of a system of equidistant parallels, revolving through constant angular displacements, which has been used in establishing their truth, is a strictly geometrical conception, and which, as here employed, may be viewed as a method in the Integral Calculus. A simpler species of reticulation, consisting of *two* systems of parallels, crossing at a finite angle, has already been used by EISENSTEIN and others in the Theory of Numbers and in Elliptic Functions.

It will be borne in mind also that this apparatus of lines is used only as a correct and convenient *representation* of an infinite system of random lines, for the purposes of calculation. Of course it is not asserted that all those random lines which are parallel to a given direction will be equidistant, or that there will be none of the random lines intermediate in direction between θ and $\theta+\delta\theta$. Just as an infinity of points arranged in horizontal rows and vertical columns will faithfully represent, for the purposes of calculation, an infinity of random points, so will the above apparatus represent the lines. Other arrangements, in either case, may easily be conceived which will represent them equally correctly, and which possibly will be found, in certain cases, more convenient. Thus if an infinite plane be covered with points arranged symmetrically, the system of lines obtained by joining each pair of points will, undoubtedly, truly represent a system of random lines.

It is unnecessary to point out, that if we can succeed in the difficult inquiry involved in extending the above methods to space, not only will the theory of probability be advanced, but various remarkable results in the Integral Calculus may be expected.

VIII. *On the Anatomy of* Balænoptera rostrata. *By* Alexander Carte, *A.M., M.D. Univ. Dubl., F.R.C.S.I., M.R.I.A., F.L.S. Lond., Hon. Mem. Imp. Zool. and Bot. Soc. Vienna, &c., and* Alexander Macalister, *M.D., L.R.C.S.I., Demonstrator of Anatomy R.C.S.I. Communicated by* W. H. Flower, *F.R.S.*

Received June 20,—Read June 20, 1867.

Opportunities for the careful dissection of the larger Cetaceans are so rare that the memoirs on their structure are still very imperfect. Hitherto the greater part of our information on the softer tissues of these animals has been due to the investigations of Hunter, Knox, Jacob, Eschricht, Reinhardt, and Lilljeborg; and even these distinguished anatomists have left their results more or less incomplete, partly owing to the infrequency of the captures, and partly to the numerous difficulties attendant on making a minute and careful investigation of creatures so unwieldy.

The capture of a small cetacean on the Irish coast off Clogher Head, a fishing-station near Drogheda, about 30 miles north of Dublin, having afforded a favourable opportunity of making some further examinations into the anatomical details of one of the species, we willingly took advantage of it; but owing unfortunately to the length of time that elapsed from the period of the capture of the animal before it came into our possession, we were not able to make an examination of the brain and spinal cord, which we regret, as it would have added to the completeness of this communication.

The animal was captured on the 8th of May, 1863, by a fisherman, in whose nets it had become entangled, and after having been exhibited for a few days to the public at Drogheda, it was for a similar purpose brought to Dublin, where, after the lapse of a fortnight, we were enabled to procure it for anatomical examination, which we commenced on the 20th, after first having obtained, through the kindness of Mr. William H. Baily, of the Geological Survey, an accurate drawing of the external form of the animal, which was made on a scale of one inch to the foot. This drawing is reproduced on one of our Plates, an explanation of which will be found at the end of the paper.

On examination the animal proved to be the Piked Whale of Pennant, *Balæna rostrata* of O. Fabricius, or that described by Lacépède and Cuvier as *B. acuto-rostrata* or *Baleine museau-pointu*, and identical with the *Balæna minimus borealis* or *Rorqualus minor* of Knox, *Pterobalæna minor* of Eschricht, *Balænoptera rostrata* of Gray, *B. Eschrichtii* of Rasch.

Our specimen was considerably larger than that described by Knox, but in most other respects agreed with the description he has given of that individual.

Its body, though robust, was not quite so large in proportion to its length as in the animal caught at Cherbourg, whose girth, as recorded by Lacépède, was 10′ 2″, its length

being 15′ 10″. The proportion existing between these two measurements in the present specimen was as 9 to 18⅝, which will be seen to approximate closely to the proportionate measurements of Knox's specimen, in which the girth was to the length as 9 to 17; and it differed but little from the same relative dimensions of the fœtus described by Eschricht, which he states was 1′ 6¾″ in circumference and 3′ in length; while, on the other hand, it differed considerably in these proportional measurements from the specimen described by Hunter, in which the girth of the animal was less than a third of its length.

External Form.

From the central line of its greatest circumference, which was about 2 feet behind the posterior border of the anterior extremity or paddle, the body gradually tapered forwards to the level of the posterior canthus of the eyelids, from whence it narrowed much more rapidly to the anterior extremity of the snout, without any indication of the frontal convexity which has been represented on the forehead of a fœtus of this species by Eschricht, and figured in the first volume of the 'Untersuchungen über die Nordischen Wallthiere,' Tab. VI. fig. 1.

The outline of the head was much more pointed in front, when seen in profile, than when viewed from the dorsal aspect; the difference being due to the great lateral width of the interval between the rami of the lower jaw at their point of greatest convexity, which caused the transverse diameter of this portion of the head to be greater than the vertical. The contour lines of these aspects likewise differed, as when viewed from above they were curved, whereas when seen on its lateral aspect the head appeared more decidedly wedge-shaped with straighter borders. The point of greatest lateral convexity in the inferior maxillæ was situated about 21 inches behind the symphysis.

Posterior to the before-mentioned point of greatest width the body commenced to taper backwards, and as it merged into the tail was altered from its cylindrical form, and became in section somewhat lozenge-shaped, with its major axis directed from the dorsal to the ventral surface.

This alteration in the figure of the body at its caudal extremity was due to the presence of a slight dorsal and ventral ridge, the former of which was continued from behind the posterior edge of the dorsal fin, and extended down to the notch that separated the lobes of the tail; this ridge at its origin was rather obscure, but as it approached the tail became somewhat more distinct.

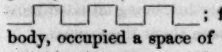
As in all the species of *Balænoptera* and its allied genera, a dorsal protuberance or fin existed, which in outline was the shape of a scalene triangle, its apex being directed backwards, its anterior edge convex, and its posterior falcate.

Hunter conjectured the use of this appendage to be to prevent the animal rotating in the water*.

* But little value is to be attached most probably to the precise position of the dorsal fin, as a means of distinguishing species, as from the descriptions of the different individuals recorded it would appear to vary considerably. Dr. Gray states that it is placed about two-thirds of the length of the entire body from the point

No parasites were discoverable on the external surface of the animal.

The integument on the geniothoracic surface of the body was arranged in a series of longitudinal folds or plicæ, which at their origins, immediately beneath the rami of the lower jaw, were flat and inconspicuous, but as they extended down towards the thoracic region they became enlarged and much more numerous; subsequently, as they approached the abdominal parietes, they decreased in number but increased in width, being finally lost in the neighbouring skin.

The arrangement of these folds was by HUNTER very appropriately compared to the bands of a ribbed stocking. There were fifty-four of them on the thoracic surface; the average depth of the furrow or interval between each of them measured about three-eighths of an inch, and on stretching the skin, which this peculiar arrangement greatly facilitated, the outline of each rib or band was somewhat rectangular, thus  ; five of them, on the anterior or geniothoracic region of the body, occupied a space of 3 inches, whereas the same extent of surface posteriorly, or towards the pudendal outlet, contained but three such folds. The subcutaneous tissue being highly elastic, rendered the skin in this region very distensible

No bristles were found at the extremity of the snout, as described by KNOX in the specimen of the same sex examined by him in 1839.

The epidermis was extremely thin and easily separable, coming off in delicate flakes.

The colour of the animal on the back was of a dark bluish black, merging gradually into a pale pinkish white on the thorax and abdomen. The pectoral extremities or flippers were of the same bluish-black colour, with a pinkish-white band crossing the central part of each of them. The tail was of a uniform dark blackish hue on both surfaces.

The eye was situated above and a little in front of the commissure of the lips; it was comparatively small and oval in shape, with the major axis running parallel to that of the body of the animal.

The external auditory foramen was a very minute, oval, slit-like pore situated on a line with the commissure of the lips, and about $7\frac{1}{2}$ inches behind the posterior angle of the eye.

The external nares or blowholes occupied the summit of the head, but were not elevated above its general surface; they were two, curved, linear, slit-like outlets, each of them about $4\frac{1}{2}$ inches long, running in the direction of the long axis of the body, with their convexities turned towards the mesial line; these semilunar orifices were separated

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of the nose; according to the measurements given by KNOX it would appear that in his specimen it was situated nearer to the tail, and in the animal figured by HUNTER it is represented as being placed still further back; in the present instance, on the other hand, this protuberance was situated anterior to the position mentioned by Dr. GRAY.

Upon the situation of this fin ESCHRICHT founds the distinction between his two varieties, *Grœnlandica* and *Bergensis*, with the latter of which our specimen seems to agree, not in this point alone, but likewise in another character of that variety, viz. the non-coalition of the lateral processes of the fifth and sixth cervical vertebræ.

Dr. GRAY also refers to the position of this fin as being situated posterior to a line drawn perpendicularly upwards from the pudendal fissure. In the present case it was placed vertically over this outlet.

from each other by a straight median sulcus, which commenced about 1 inch in front of the anterior commissures of the semilunar nasal outlets, and fell short of their posterior commissures by about the same distance, thus ; it was about $\frac{1}{2}$ an inch in depth, and its use was evidently to facilitate the approximation of the lips of the nares.

No hairs were discernible in the immediate vicinity of these outlets, as existed in the specimen described by Eschricht and Reinhardt; on the contrary, the skin in this situation was smooth and polished.

The pudendal fissure was situated a little posterior to where the abdominal folds terminated, at a point on the ventral surface coincident with a perpendicular line let fall from the dorsal fin; it opened by a longitudinal slit or sulcus, which was bounded on each side by thick and rounded labia on the central line of the abdomen. Into this the orifices of the urethra, vagina, and rectum opened; the urethra being situated most anteriorly, the rectum at some distance posteriorly, and between them the vagina.

The umbilicus also lay in a small slit-like furrow, at the distance of about $6\frac{1}{2}$ feet anterior to the fork or commissure of the tail.

External and parallel to the pudendal fissure were two other sulci, in which the nipples of the mammary glands were concealed, but owing to the immaturity of the animal these glands were not well developed.

The mouth and fauces were lined by mucous membrane of a pale pinkish colour, which was arranged in longitudinal folds similar to those on the geniothoracic region of the animal; this membrane, together with the mylohyoid muscle and skin, formed the wall of the great submaxillary pouch, which from the elastic nature of its component parts, was capable of great distention; this enabled the animal to take into its mouth a large quantity of water containing such substances as constituted its food; the latter was retained in the mouth by the straining mechanism of the baleen plates, while the superfluous water was expelled by the contraction of the pouch.

The number of baleen plates was about 280 in each upper jaw, or 560 in all. The longest blades were placed at the posterior third of the long axis of the jaw, and measured $3\frac{1}{2}$ inches in length on their outer border; from this point they gradually decreased, both anteriorly and posteriorly, to the length of about 1 inch, finally passing into coarse bristles about $\frac{1}{4}$ inch in length. Each plate was somewhat triangular in outline, and was attached to the jaw by its base or shortest side, which was imbedded in the basal pulp, and measured $1\frac{3}{4}$ inch in length. The outer margins were long and curved, their concavity directed outwards; this curve was greatest at the centre and continued very well marked to the anterior extremity, but diminished towards the posterior end, where the outer margins of the plates became comparatively straight from above downwards; the inner borders of the plates were fringed by bristles, which were coarse towards the apex of each plate, but became much shorter and finer towards the internal margin of the wreath. Each plate was curved transversely, the concavity being directed backwards; the inner margin of each lamina, just where it came in contact with the subsidiary plates, had, however, a slight inclination forwards.

The subsidiary plates varied in number from one to six or seven for each pulp ridge, and occupied an extent of from $\frac{3}{4}$ in the centre to $\frac{1}{4}$ of an inch in front, and towards the pharyngeal end they disappeared by merging into a series of short and fine bristles. The matrix or intermediate pulp was thick, tough, fibrous, and vascular.

In colour the blades were pale yellow, becoming slightly bluish at the base; on removing the pulp from the base of each lamina, the latter was divisible into an external horny whitish yellow cortical substance, and an inner fibrous or bristly portion of a purplish-blue colour; this layer, by protruding beyond the external lamella, formed the terminal fringe of bristles.

On removing the pulp from the attached margin of the baleen plates, it became obvious that just at the point of contact between the larger laminæ and the subsidiary plates, the bases of both were transversely united for about half a line in extent, and a similar transverse union existed at every interval between the different blades.

The following Table contains the measurements of the animal as compared with those of Hunter and Knox:—

	Specimen.		Hunter's.		Knox's.	
	ft.	in.	ft.	in.	ft.	in.
Total length of the animal from point of snout to the bifurcation of the tail	13	11	17	0	9	11
Total length from point of snout to extreme point of fluke of tail	14	4				
Width of tail	2	10	5	0	2	8½
Length of upper jaw from point of snout to the commissure of the lips	2	4½	3	3		
Length of lower jaw from the symphysis to the commissure of the lips	2	9	.....		2	0
Distance from point of snout to the anterior angle of the eyelids	2	4½	3	4		
Length of the long axis of the eye	0	2½	0	2½		
Distance from the point of the snout to the anterior boundary of the flippers or pectoral extremities	4	4				
Length of flippers, external to integuments	1	11	2	4	1	3
Greatest width of flippers	0	6½	0	9		
Distance between the flippers measured across the thorax	2	3				
Distance from the point of the snout to anus	8	10				
Height of dorsal fin or fluke	0	6				
Length of dorsal fin	0	9	1	0		
Distance from the bifurcation of the tail to the posterior edge of dorsal fin or fluke	4	4	.....		2	10
Distance from point of snout to the anterior commissure of cloaca	8	4				
Length of the meatus of cloaca	1	2				
Distance from the bifurcation of tail to the posterior commissure of cloaca	4	4	4	4		
Length of mammary sulci	0	3				
Length of blowholes or external nares	0	4½				
Length of median sulcus between external nares	0	5				
Distance between the posterior extremities of the blowholes	0	2½				
Distance between the anterior extremities of the blowholes	0	3¼				
Girth of the body, taken 2 feet behind the flippers, the largest part	6	9	5	0	5	2
Girth at the junction of the tail with the body	1	6½	1	0		
Length of the cranium, taken on palate-surface	3	1	.....		2	11
Breadth of the cranium, taken at the articulation of the lower jaw	1	5	.....		1	3
Length from the hamular process of the sphenoid bone to the foramen magnum	0	6	.....		0	6½

TABLE (continued).

	Specimen.	Hunter's.	Knox's.
	ft. in.	ft. in.	ft. in.
Length of lower jaw	2 11	.....	2 8
Length of longest plate of baleen	0 9	.....	0 4
Length of transverse processes of nineteenth vertebra from tip to tip	1 2½	.....	0 11
Length of first rib	1 3		
Length of third rib	2 2	.....	1 10
Length of fourth rib, the longest	2 3½		
Length of eleventh rib, the last rib	1 6		
Length of scapula, transverse diameter	0 7½	.....	0 6½
Length of scapula, longitudinal diameter	0 11½		
Length of the humerus	0 5½	.....	0 4½
Breadth of the humerus at central part	0 3		
Length of radius	0 8½	.....	0 7¼
Length of ulna	0 7½		
Length of carpus, metacarpus, and longest digit	.....	.....	0 9
Number of cervical vertebræ	7	7	7
Number of dorsal	11	.....	11
Number of lumbar and caudal	28	.....	30
Number of ribs	11	.....	11
Number of chevron or V-shaped bones	7		
Chevron bones commenced at	32 33	.....	31

Skeleton.

The cranium, viewed from above, was considerably flattened; its shape was somewhat that of an elongated isosceles triangle, having its apex at the extremity of the intermaxillary bones, and its base corresponding to a line drawn from the anterior border of one glenoid process to that of the other; its length was a little more than twice its breadth, and its component segments, with the exceptions of the petrous and tympanic, were extremely soft and spongy in structure.

The occipital bone was made up of the combined interparietal, supraoccipital, exoccipital, and basioccipital elements; it was the largest of all the cranial bones, and consisted, first, of a broadly expanded superior portion, irregularly triangular in outline, which formed almost the entire roof of the skull; its somewhat rounded apex extended forwards to articulate with the posterior edge of the frontal: the external margins overlapped those of the parietals and frontals, and the basal line, which extended from the extremity of one exoccipital to that of its fellow of the opposite side, formed the posterior boundary of the cranium; this margin was irregular in outline and lay on a plane posterior to the foramen magnum, which latter was situated at the junction of the basilar and calvarial portions; it was oval in shape, and measured 1¾ inch in its vertical by 2½ inches in its transverse diameter; it was bounded laterally by the condyles, which approximated inferiorly, where they were separated only by a sulcus whose breadth varied from half an inch at its widest, to one-eighth of an inch at its inferior and narrowest part. The condyles were ovoid in shape, and extended backwards and outwards; their superior rounded extremities formed the boundaries to the inferior and lateral halves of the foramen magnum, and were thickly covered with incrusting cartilage, but

no synovial membranes intervened between them and the atlas. There were no condyloid foramina. The calvarial portion was convex posteriorly towards the foramen magnum, and was marked by an obscure central protuberance, from which very faintly marked ridges were traceable running forwards and outwards. More anteriorly a blunt median ridge existed, which was bounded on either side by a superficial concavity for muscular attachments. Its inner or cerebral surface was concave generally, but presented a prominent median ridge, which was deeply grooved on the right side for the superior longitudinal sinus.

The basioccipital portion of the bone projected forwards to meet the posterior border of the basisphenoid; it was marked by a ridge on each side, where it articulated with the bullate tympanic bone; its inferior surface was also rough and concave, for the attachment of the anterior cervical muscles; its cerebral surface was slightly concave in the antero-posterior direction, and presented a transverse ridge, where it united with the body of the sphenoid bone.

Each exoccipital element extended outwards from the external border of the foramen magnum and terminated by articulating with the mastoid or squamous bone; it presented a rough surface posteriorly and inferiorly for muscular attachments, and was separated from the condyles behind by a deep notch, which formed the posterior boundary of the jugular foramen.

The squamous bone was irregular in shape, and was composed of, first, a large, somewhat crescentic glenoidal process, the inferior surface of which was concave, and formed a large but shallow cavity for the articulation of the lower jaw; its anterior extremity was nearly in contact with the orbital plate of the frontal, and the posterior extended backwards on a level with the anterior border of the occipital condyles; to this the sterno-mastoid and several other muscles were attached. A deep sulcus, perhaps the homologue of the digastric groove, existed internal and posterior to the process, giving origin to the depressor muscle of the lower jaw; this sulcus ran backwards and outwards, and was separated from a second groove, which existed on its outer side for the reception of the posterior and external process of the periotic bone, by a sharp ridge which ran in the same direction; the posterior border united with the lateral process of the occipital bone by an extensive surface, which extended as far as the posterior superior angle of the parietal. The lateral aspect of this process was convex in front and concave posteriorly and superiorly; inwards or towards the temporal fossa it was deeply hollowed, where it was continuous in front and above with the squamous plate, from which it was separated below and anteriorly by a large fissure.

The squamous plate was slightly convex, its temporal surface looked upwards and outwards; it was a narrow slip of bone and united anteriorly, by suture, with the posterior border of the parietal, and a small portion of the basisphenoid, which was here to be seen intermediate between the other bones forming the temporal fossa. Inferiorly and in front the squamous plate was continued into a pyramidal mass, which was directed forwards to meet the posterior border of the pterygoid bone, the outer edge of which it overlapped. The temporal surface was smooth, and its inferior aspect pre-

sented an oval aperture which transmitted the Eustachian tube inferiorly, and the tensor tympani muscle superiorly; the former passed into the upper part of the tympanic cavity, the latter hooked round a spur of bone which lay immediately external to the border of the former. On its inner surface the squamous bone was deeply excavated for the accommodation of the petrous element, and its cerebral surface was smooth and concave*.

The alisphenoid or pterygoid bone exhibited three aspects. First, an inferior or pterygoid, which consisted of two osseous plates, external and internal, separated by a deep ovoid cavity, the pterygoid fossa; the external plate was united with the petrous or periotic and mastoid bones by suture, the latter of which overlapped it; and the anterior part of the former was received into a pyramidal depression formed by the meeting of the mastoid and pterygoid bones. The internal plate was thick behind, extended further back than the external, and met the lateral ridge on the basioccipital; this plate formed the inner boundary of the posterior nares, and was convex from before backwards on its inner aspect; above and internally it met the outer edge of the vomer, into a groove in which it sent a sharp vaginal ridge forming a schindylesis articulation. The pterygoid plates united below to form a round blunt hamular† process, about an inch and a half in length, which was directed backwards and slightly outwards. In front of the pterygoid bone a series of lamellæ projected, which interlocked with a similar series of plates on the palate-bone. The second or temporal surface was somewhat triangular in outline and rounded along its upper edge; this surface was concave outwards, it articulated with the parietal in front, with the mastoid posteriorly and above, and with the temporal aspect of the basisphenoid anteriorly. The inner or cerebral surface was smooth, and appeared concave in front, where it formed the floor of the sphenoid fissure.

The basisphenoid presented on its upper or cerebral surface a central hollow or groove, sella turcica, in which appeared four small foramina; this fossa was bounded in front by a very small roundish eminence, the rudimentary middle clinoid, or, more correctly, olivary process; and posteriorly by a similarly diminutive posterior clinoid; the posterior edge of the bone presented an elevated transverse ridge, which marked its union with the basioccipital bone. Midway between the external edge and the sella turcica, on each side, a foramen was seen, from which a narrow canal was traceable downwards, backwards, and outwards, and opened on a small triangular surface of this bone externally between the basioccipital and the alisphenoid. This vascular channel represents the canal of Vesalius. In front of the sella turcica a transverse suture was situated, which separated the anterior or presphenoid element from the basisphenoid. Still more anteriorly the former was grooved for the optic nerves and commissure; as this groove extended outwards and forwards on each side, it was formed into a foramen by a thin scale of bone which completed it superiorly and posteriorly, and appeared to

* For a description of the petrous bone, vide "The Organ of Hearing."

† Eschricht and Reinhardt mention that this process is scarcely distinguishable, which description does not accord with the present specimen.

be the representative of the anterior clinoid process. The optic foramen was situated on a plane anterior to the sphenoidal fissure, which latter was formed by the frontal and sphenoid bones. Anterior to the optic foramina and groove a slight ridge extended transversely, in front of which the ethmoidal process was continued forwards to the posterior border of the cribriform plate of the ethmoid bone. The inferior surface of the basisphenoid was connected behind with the basilar process of the occipital; in front it was overlapped by the posterior expanded part of the vomer, and the alisphenoid covered it on each side; laterally it was concealed by the mastoid and alisphenoid bones, with the exception of a small irregular-shaped portion situated in the temporal fossa between these latter and the parietal bones.

The parietal bone was irregularly quadrilateral in outline, having two surfaces and four edges. The external or temporal surface was concave posteriorly and convex in front, and formed the inner wall of the temporal fossa. The superior surface was convex and the most extensive; it articulated with the occipital, the external table of which overlapped it, but its internal table formed with the latter a serrate suture; more anteriorly this edge became thickened into a small wedge-shaped triangular process, and lay between the frontal bone in front and the occipital behind; from this a flat, thin scale of bone was continued forwards lying on the outer side of the frontal, on which it gradually attenuated to its anterior edge. Some of the small lamellæ of this bone that indigitated with similar plates on the side of the occipital, anteriorly approached their fellows of the opposide side, but did not come into actual contact. The posterior edge was thicker and almost straight, and was situated at the posterior part of the temporal fossa; it extended downwards and forwards, and united by a plane edge with the anterior margin of the squamous plate of the temporal bone. The inferior edge was the shortest; it ran forwards to articulate with the small lateral part of the basisphenoid (before described) that formed a portion of the inner wall of the temporal fossa; from this the margin was continued downwards and forwards to articulate with the upper edge of the great wing of the alisphenoid, which was overlapped by it posteriorly, but in turn was more superficial in front. The anterior inferior border was deeply hollowed for the reception of the orbital plate of the frontal, and was continued forward as the thin scale of bone before mentioned, that ended in the blunt anterior angle. The inner or cerebral surface was concave, and formed a large part of the superior lateral boundary of the cranial cavity.

The ethmoid bone consisted, first, of a cerebral surface, or cribriform plate, situated between the frontal and sphenoid bones; it presented a median ridge (crista-galli), on each side of which existed three or four deep depressions perforated by foramina for the exit of the olfactory nerves. Secondly, in the interior of the nares it presented four irregularly shaped turbinated bones that were situated internal and superior to the nasal canals; these bones were soft and spongy, and were separated from each other by three meatuses posteriorly, but anteriorly they communicated in common with the air-passages. Thirdly, the ethmoid presented an orbital surface or os planum situated beneath the orbital plate

of the frontal and extending back as far as the anterior border of the pterygoid bone; the groove for the nasal nerve lay between this bone and the base of the vomer, with which latter this plate was closely united. Fourthly, the large nasal lamella, situated in the median line; it was thicker below than above, and was fitted into the deep longitudinal channel excavated in the vomer.

The superior maxillary bone in shape resembled a ploughshare, having its apex somewhat rounded in front, and its base behind. It presented four surfaces, one external or facial, a second internal or nasal, a third inferior or palatine, and a fourth posterior. The external or facial surface was triangular in outline, slightly convex and smoothish; it was prolonged upwards and backwards into a nasal process, which articulated by foliaceous indigitations with a similar arrangement of osseous plates on the frontal bone. This surface was marked by eight or nine irregularly placed foramina, from which channels passed outwards and downwards for the transmission of blood-vessels, &c. to the baleen plates; the infraorbital foramen was large and situated at the junction of the nasal process with the body of the bone; from this a canal passed, at first downwards and outwards, then backwards and inwards; through this canal the superior maxillary nerve and vessels made their exit to supply the integument and muscles on the side of the snout. The inner or nasal surface was slightly concave, and was directed upwards and inwards; it was furrowed for the intermaxillary bone above, and presented a groove for the upper edge of the vomer, which was overhung by a sharp bony ridge: two oblique foramina, which passed inwards and forwards into the base of the bone, were visible here; these were for the transmission of the nutrient vessels. The inferior or palatine surface was deeply concave, and exhibited fifteen oblique apertures, the orifices of channels for the transmission of blood-vessels which passed in various directions, radiating and grooving the surface of the bone.

The posterior temporo-frontal surface or base was irregular, and divided into two unequal surfaces by large excavated vascular foramina communicating with the apertures already described on the various other surfaces; the upper plane articulated with the frontal and formed the anterior boundary of the temporal fossa; the lower was free, and passed under the supraorbital plate of the frontal bone without coming into actual contact with it. The upper or nasal edge was marked by a shallow groove for the intermaxillary bone; the posterior border, for about one-sixth of its extent, was sharp and harmonized with the outer edge of the nasal bone. The inner or palatine edge was concave, and its anterior four-fifths overlapped the vomer; its posterior fifth was deeply hollowed, and was covered by the palate-bone. The outer edge was sharp and convex, and was prolonged into a blunt zygomatic spur, behind which it extended downwards and backwards below the level of the bone for 3½ inches, where it formed the anterior border of the orbit; this spur was separated from the outer edge by a deep groove, and above internally its continuation formed the anterior boundary of the temporal fossa.

The vomer was a long deeply channelled or spout-like bone, concave on its upper aspect, narrowing anteriorly, and convex beneath, forming a blunt median ridge that

lay between and separated the superior maxillæ, where it assisted in forming the roof of the mouth. This ridge was sharp behind, rounded in front, and partially covered by the palate-bones posteriorly; in front it was overlapped by the superior maxillaries: the external edge of each lateral plate fitted into a groove which was situated on the nasal aspect of that bone. Posteriorly it became very thin and flattened, and was separated from the basisphenoid by a small interval, into which the rudimentary rostrum was projected. On either side it united by schindylesis with the base of the internal pterygoid process, where it formed the upper and internal wall of each posterior naris; more anteriorly it united with the frontal, sending a small process between that bone and the pterygoid, which entered into the suture between the palate, alisphenoid, and frontal bones; still further forwards its outer edge united with the ethmoid by harmonia. The deep channel on the upper surface was occupied posteriorly by the nasal lamella of the ethmoid bone, more anteriorly, in the recent state, a thick firm fibrocartilage completed the septum narium, and occupied this sulcus as far forwards as the extremity of the bone.

The palate-bone was quadrilateral in outline and exhibited two surfaces. First, a superior or vomerine, which presented a ridge that fitted into a groove on the vomer; and second, an inferior or palatine surface, which was smooth and concave, and formed the posterior superior wall of the palate. Of the edges of the palate-bone, the posterior was convex and presented a series of plates, by which it was articulated to the anterior surface of the alisphenoid by a similar arrangement of lamellæ. The inner border was smooth and rounded behind, where with its fellow of the opposite side it formed the inferior boundary of the posterior nares; its outer edge was sharp, and was adapted to the contiguous surface of the superior maxilla, and the anterior border was short and flat.

The intermaxillary bone was long, narrow, and somewhat twisted; anteriorly it was triquetrous, but its posterior third was flattened and presented but two surfaces. Its external surface was convex anteriorly, but as it passed backwards, the bone being, as it were, twisted on its long axis, this surface finally looked directly inwards, where it also became concave to accommodate the opening of the anterior nares. For the anterior two-thirds of its length its internal surface was concave from side to side, broad in the centre, gradually narrowing anteriorly to the point of the upper jaw, and behind to a sharp ridge, which represented this surface on the posterior third of the bone. The inferior surface was also laterally concave and channelled deeply behind for the reception of the upper edge of the superior maxilla; one vascular foramen, looking downwards and forwards, was found on the outer surface of the left bone only.

The three edges bounding these surfaces were sharp, convex, and irregular; posteriorly the bone was flattened and formed a single lamina, which was fitted into a sulcus in the frontal, internal to the nasal process of the superior maxilla, and external to the nasal bone; its anterior part or apex projected for nearly 3 inches in front of the superior maxilla.

Each nasal bone was a five-sided mass of light, spongy, osseous tissue, resembling a blunt wedge with the narrowest end posteriorly; its posterior surface was furrowed into a number of deep narrow longitudinal sulci for the accommodation of the articular lamellæ of the frontal bone; the inferior surface was grooved along its posterior border to articulate with the nasal ridge of the frontal, which projected forwards as a shelf to support this extremity; it united externally by a flat surface with the intermaxillary, and internally by a similar facet with its fellow of the opposite side; the anterior and upper surface was convex, and was broader below than above.

The inferior maxillary was a curved elongated bone composed of a condyle, coronoid process, and ramus. The condyle was divided into two unequal segments by a transverse sulcus; the upper portion was much the larger, and articulated on a plane anterior to the occipital condyles with the glenoid cavity of the squamous bone, through the interposition of a large fibrocartilaginous interarticular cushion; but, as stated by Hunter, no true synovial membrane was found in this articulation, although the existence of a double synovial sac has been demonstrated in *B. mysticetus* by Eschricht and Reinhardt. The lower portion formed about one-fourth of the entire condyle, and gave insertion to the posterior set of fibres of the depressor muscle of the lower jaw. A shallow sigmoid notch, 6 inches wide, separated the condyle from the coronoid process, which latter was prominent and directed upwards and outwards, its posterior margin being $2\frac{3}{4}$ inches and the anterior 2 inches in height, where it gave insertion to the tendon of the temporal muscle.

The projection of this bony process is one of the distinctive characters of this species.

Below the centre of the sigmoid notch and on the inner surface of the bone the orifice of the inferior maxillary canal, for the transmission of the nerves and vessels, was situated; this canal continued through the ramus, sending off numerous branches which opened ultimately by seven oblique foramina in a regular series along the upper border of the external surface of the bone. A series of similar but smaller canals, about twenty in number, with their orifices directed forwards, perforated the internal surface close to its upper edge. The rami were firmly united by a dense fibrocartilage at the symphysis. The lower jaw projected beyond the intermaxillary bones for about 2 inches, and the transverse width between the rami, taken at the point of their greatest convexity, which was about 1 foot 7 inches from the symphysis, exceeded the transverse width of the superior maxillaries, measured from the same place, by about 9 inches; and each ramus towards its anterior extremity exhibited a slight tendency to that torsion which is so distinctly marked in the lower jaw of *B. mysticetus*; and at the symphysial end there was a distinct fissure, the remains of the channel for the distal extremity of the Meckelian cartilage.

The lachrymal bone was in outline a scalene triangle with rounded angles; its external border was the thickest and somewhat concave, its upper margin being raised into a well-defined ridge, the other sides being thinned off like a wedge; the inferior surface was

nearly flat, corresponding in shape to the superior surface of the external angular process of the superior maxilla, and its superior surface, which was rough and slightly convex internally, underlay the anterior portion of the orbital plate of the frontal, where the lachrymal was wedged in between these two bones.

The malar or zygomatic bone was slender and somewhat rib-like or falcate; it formed the inferior boundary of the orbit, and was broader behind than in front; its anterior border articulated with the external orbital or zygomatic spine of the superior maxilla; its wider or posterior extremity was flattened, and fitted in between the anterior border of the glenoid process of the squamous bone and the posterior angular process of the frontal, where a digital depression existed for the reception of the former. In outline the bone presented a curve which was much sharper in front than behind.

The external aperture of the bony orbit which was formed by the frontal, squamous, malar, and lachrymal bones, measured $4\frac{1}{2}$ inches in length by $3\frac{1}{4}$ in breadth.

The cervical vertebræ were seven in number.

The first, or atlas, was an irregular bony ring, somewhat oval in outline, $5\frac{3}{4}$ inches in height, having two surfaces, the anterior of which presented two lateral, reniform, shallow concavities, for the reception of the condyles of the occipital bone; these were separated from each other by a shallow groove with slightly elevated edges; this in the recent state was occupied by the anterior atlanto-occipital ligament. The neural arch was angular in shape, with its apex terminating in a very short neural spine, directed backwards; the neurapophyses were separated from the articular surfaces by a deep semicircular channel for the transmission of the suboccipital artery. The transverse processes, short and stout, stood out on each side about the centre of the transverse axis of the bone external to the articular depressions, their upper edges being directed forwards and upwards; the entire width of the vertebra across these processes was $7\frac{3}{4}$ inches.

The posterior surface presented two reniform convex articular facets, which corresponded in outline with the articular cavities on the anterior aspect; these articulated with corresponding depressions on the anterior surface of the axis, and were separated posteriorly from the neurapophyses by a very shallow groove or depression, for the transmission of the second pair of cervical nerves. The neurapophyses arched backwards and coalesced to form a very rudimental neural spine.

On each side of the external surface of this vertebra an oblique ridge ran backwards and upwards from the superior and external angle of the transverse process to a small tubercle (zygapophysis) which overhung the groove that gave exit to the second cervical nerves. This ridge gave attachment to the rectus capitis lateralis. There was no anterior tubercle on the ventral aspect of the body of this bone.

The second, or axis, consisted of a very narrow body irregularly elliptical in outline with two surfaces, the upper of which presented a central hemispherical elevation (odontoid process), and which, when the bone was placed *in situ*, occupied the anterior semicircular space between the articular facets of the atlas. This eminence was partially separated from the body of the axis by a well-defined fissure, situated transversely in the posterior wall of the body of the vertebra, where the osseous incrustations or epi-

physial disks of two vertebræ were distinctly visible, separated from each other by a rudimental articular surface, thus bearing out the theory promulgated by Professor Owen, that the central elevation of the axis vertebra is in reality the homologue of the centrum or body, not of this segment, to which it would now seem to belong by osseous coalescence, but of the atlas. This theory is further evidenced by the fact that the odontoid process appears to have three points of ossification; first, a small irregular detached piece of bone imbedded in the apex of the elevation, and which represents the upper epiphysial disk of the atlas; secondly, a point of ossification, but much larger, in fact forming the greater part of the odontoid process, and coalescing anteriorly and laterally with the upper surface of the axis; and thirdly, a small flat thin plate which formed the upper boundary of the transverse fissure, before noticed as existing in the posterior wall of the body or centrum of the bone.

Surrounding the hemispherical eminence, there was a crescentic-shaped depression, which received the posterior articular surfaces of the atlas.

The posterior surface of the body of the axis was concave, and in outline irregularly elliptical; it was slightly grooved posteriorly. The neurapophyses arched backwards and terminated in a bifid neural spine, which was more robust than that of the atlas. The oblique or articular processes (zygapophyses) were small, with nearly circular facets situated towards the bases of the neurapophyses, with their articular surfaces directed backwards, downwards, and inwards. The diapophyses were large and robust, and projected from the sides of the centrum. The pleurapophyses were also well developed, and, together with the diapophyses, formed nearly three-fourths of an oval, which in the recent state was completed by interposed cartilage; the spaces thus enclosed in this vertebra measured about $1\frac{3}{8}$ inch in diameter.

The remainder of the cervical vertebræ were in outline somewhat similar to the axis, but their bodies were much more compressed in the antero-posterior direction. Their parapophyses extended from the sides of the anterior portions of each centrum forwards and outwards, diminishing in length to the seventh or last cervical, where they were merely rudimental. The lateral processes (diapophyses) were much longer but less robust than the parapophyses, and those of the third, fourth, and fifth were directed backwards, while the other two arched forward from the bases of the neurapophyses; these, in conjunction with the parapophyses anteriorly and sides of the centrum, internally formed an oval-shaped space, which in the recent state was completed by fibrocartilage uniting the extremities of the parapophyses to those of the diapophyses. The zygapophyses resembled both in shape and position those of the axis. In consequence of the lateral extension of these processes, motion in the cervical region was greatly restricted; this, however, when we take into consideration the economy of the animal, was of little moment as compared to the protection afforded by these bony arches to the important vascular sinuses, which in the recent state were lodged in these cavities on each side of the neck, and which were formed by the interlacement of numerous vessels from the vertebral and carotid arteries, inosculating freely with each other and producing a beautiful example of a rete mirabile.

There were eleven dorsal vertebræ. The first closely resembled the last cervical, except that its centrum was thicker, and was furnished with longer and more robust and compressed diapophyses.

The remainder of the dorsal vertebræ had their lateral processes more horizontally placed and more elongated, those of the anterior six being directed forwards, while those of the eighth to the eleventh pointed backwards, and those of the seventh projected at right angles with the axis of the body of the vertebra; these processes were thickened at their free extremities, where they articulated with the ribs; their neural spines were flattened and somewhat quadrilateral in outline, with their free extremities directed, those of the second and third upwards, and the others backwards and upwards, each of them becoming gradually more decumbent and elongated as they descended towards the lumbar region. The zygapophyses were prominent and directed forwards; rudimentary parapophyses were developed in the first and second dorsals, similar to those described by Lilljeborg in a female skeleton preserved in the Museum at Bergen. The bodies became more robust as they descended towards the lumbar region, and were all obtusely carinated on their inferior surfaces*.

The neural canal, which in the cervical and upper dorsal regions had its greatest diameter in the transverse direction, in the lower dorsal region gradually diminished in calibre, and its major axis changed its direction from transverse to vertical.

The bodies of the lumbosacral and caudal vertebræ were mostly solid and massive with flat and thin lateral processes, which gradually became shorter and thicker as they approached the tail. Those of the first lumbosacral vertebræ were directed transversely outwards, those from the second to the twelfth were inclined forwards, those of the thirteenth and fourteenth looked directly outwards, and the remainder were turned forwards; at the nineteenth the lateral processes were entirely suppressed. Their neural spines gradually increased in length to the middle of this region, and then rather rapidly diminished towards the tail, the seventh caudal being the last that possessed a spine. The centrums presented on their anterior surface a bluntish keel, which gradually increased in prominence to the thirteenth; those of the twelve lower caudal were perforated on each side of their dorsal aspect by a canal which opened on the ventral surface in a depression situated external to the articular surfaces for the chevron bones. The crest on the anterior aspect of the caudal vertebræ was bifid, the lateral ridges being separated by a central longitudinal groove, which was pierced about the central part of each centrum by two foramina.

The articular processes or zygapophyses of all the lumbo-sacral vertebræ articulated with each other, while those of the caudals were free.

The foramina on the dorsal aspect of the sixteenth vertebra was situated at the root of the transverse process, but when these processes ceased to exist, these apertures perforated the sides of the centrum on the dorsal aspect.

* This differs from the description of Lilljeborg, who says that none of the dorsal vertebræ are ridged on the lower side of the corpus.

The chevron or inferior spinous bones were seven in number, each being composed of the two lateral branches which inferiorly coalesced and formed the hæmal spine. Of these segments the second was somewhat larger than the first, in which latter, the hæmal spine, though less prominent inferiorly, was more elongated posteriorly, with its inferior or free edge, in outline, more elliptical; the hæmal spines of the remainder, with the exception of the last, in which it was entirely suppressed, were more or less circular, rounded, and decreased in prominence as they receded from the second. These bones commenced to be articulated at the intervertebral space between the thirty-second and thirty-third vertebræ, and ceased at the intervertebral space between the thirty-ninth and fortieth.

The eight terminal caudal vertebræ were destitute of a spinal canal, which was merely represented by a shallow groove, and their centrums were mere bony cubes with their angles rounded off.

The ribs, eleven in number, were bony arcs, thicker, flatter, and somewhat more spongy in texture at their sternal than at their vertebral extremities. The heads were articulated with the transverse processes of the dorsal vertebræ through intervening cartilage, and from each capitulum a bony ridge passed outwards to the angle or tubercle on each.

The head of the first rib was simple and undivided, and its shaft was flatter and wider than the others. The fourth was the longest, measuring along its convex border 2 feet 3 inches, and also the most curved. The heads of all the posterior ribs were marked by a slight sulcus, and the fourth and fifth had a distinct tubercle placed about 2 inches external to their heads, but this process ceased to exist on the four posterior ribs.

The sternum was a light flat porous bone, irregularly heart-shaped; it was notched in front and prolonged posteriorly into a bluntish extremity. Mr. Flower describes the shape of this bone, in the animal examined by him, as being that of an elongated cross; this difference may possibly be due to the different age of the two animals. The first ribs alone were articulated to this segment.

The ischiatic bones existed as mere rudimentary fibrocartilaginous bodies about 6 inches long, of an arcuated or sigmoid shape, narrowing to their inferior extremities; they were imbedded in the muscles of the abdomen, and were placed perpendicularly beneath the first chevron bones.

The scapula was triangular and flattened, slightly concave on its subscapular aspect, where it presented four very superficial diverging furrows; the dorsum was likewise slightly concave and perforated by a few vascular foramina towards the neck; the external border was rounded and concave, blunt towards the glenoid cavity, becoming sharper towards the posterior angle; the posterior or vertebral edge was curved and porous, being, in the recent state, bordered with cartilage; the anterior edge was nearly straight and bevelled off obliquely towards the dorsum, forming an extremely narrow marginal surface, separated from the proper dorsal surface by a slightly raised edge that was bounded at its external third by a prominent flattened recurved acromion process,

which was about 3 inches long by 1½ inch in breadth on its outside. The glenoid cavity was shallow, rough, and ovoid in outline, with its larger extremity towards the external edge, straighter along the subscapular than the dorsal border; its margins were rounded and covered with fibrocartilage. Anteriorly this cavity was bounded by a blunt coracoid eminence ⅝ of an inch long, tipped with cartilage, which received the insertion of the suprascapular ligament. In structure this bone was cancellous and soft towards the superior border, and denser towards the glenoid cavity.

The humerus was a flattened irregularly club-shaped bone; its head, roughly rounded, was united to its shaft by cartilage, its area being much greater than that of the glenoid cavity, and its long axis lying in the antero-posterior direction. The shaft was compressed in front and convex behind, presenting several rough facets for muscular attachments; in those on the posterior surface were inserted the supraspinatus, infraspinatus, and deltoid above, the first occupying the most external position, the second placed internally, and the third below and intermediate. From the posterior border of this surface the external head of the triceps arose, and lower down on the shaft the humerus was slightly hollowed to give origin to the extensor digitorum communis. The anterior surface presented internally a tubercle for the subscapularis, behind which extended an obscure ridge for the teres major; above and externally the humeromastoid was inserted. The lower extremity presented two elliptical articular surfaces, so placed as to produce an obtuse angle with each other, the radial or larger surface looking downwards and outwards, the ulnar upwards and inwards. These surfaces were not covered by synovial membrane, as described in *B. mysticetus* by Eschricht and Reinhardt.

The radius was a flattened, curved, elongated bone; its humeral head was oval, with its long axis placed transversely; the lower extremity was slightly dilated to correspond with the carpal bones.

The ulna was likewise an elongated slightly curved bone, flattish, and separated from the radius by a small interosseous space. These bones at the proximal and distal extremities were in contact, and connected in the recent state by fibrocartilage.

The humeral extremity of the ulna was larger than its carpal, and presented a prominence on its internal side, the olecranon process, which in the recent state supported a large triangular mass of cartilage from which the three flexor muscles arose, and into which the triceps was inserted.

Of the carpal bones there were only four perfectly ossified, and in shape these were spherical, varying in diameter from ½ to a ¼ of an inch. The metacarpals were four in number, and ranged from 1 to 1½ inch in length, the second being the longest. The phalanges were, as well as could be ascertained, four in each of the central digits, and three in each of the lateral ones.

Eschricht and other authors have enumerated a larger number of phalanges as belonging to individuals of this species, but as in the present specimen the terminal extremities of the digits were purely cartilaginous, it was difficult to determine the precise number of rudimentary phalanges in each. According to Lilljeborg the typical

numbers are in the first digit four, in the second seven, in the third six, and in the fourth three; the fifth digit, and the hooked-like carpal bone connected to it, mentioned by Eschricht as being present in the fœtus of this species, could not be detected.

Muscles.

On removing the integument from the anterior and lateral part of the body, the following muscles were exposed to view.

Pectoralis major was a flat, triangular, or fan-shaped muscle covering the anterior surface of the wall of the thorax. It arose from the anterior border and inferior surface of the sternum, from the sternal cartilaginous extremities of the five anterior ribs, and from the dense fibrous aponeurosis which covered the muscles of the abdomen; its line of origin extended backwards for about 3 feet posterior to the anterior or cervical border of the sternum, to within about a foot of the umbilicus. The anterior fibres passed directly outwards, the middle and posterior ran forwards and outwards, all converging to form a very broad flat tendon, which, crossing the anterior wall of the axilla, was inserted into the anterior portion of the head of the humerus; some of the lower fibres, especially those arising from the abdominal aponeurosis, were connected with the integuments and fascia which covered and formed the outer boundary of the axilla; the tendon of this muscle was separated from the front and inner part of the head of the humerus by a small synovial bursa, and its external margin covered the long respiratory nerve, which, accompanied by a large artery and vein, passed to the serratus muscle; this nerve, a branch of the brachial plexus, with its accompanying long thoracic vessels, lay between the anterior border of the serratus magnus and the greater pectoral; the artery and vein ran backwards on the side of the thorax and abdomen, the vein emptying itself into the subclavian by turning over the edge of the first rib.

The lessor pectoral was absent, but beneath the pectoralis major a cellular lamina existed which separated this muscle from the following.

The intercostales, twenty on each side, were arranged in two sets, external and internal; the former ran downwards and outwards, the latter in the opposite direction, or upwards and inwards.

On removing the dense integument and thick cervical fascia from the front of the neck, the platysma myoides could be seen only in the median line.

Sternomastoid.—This muscle arose by two fleshy heads, one about $2\frac{1}{2}$ inches wide, from the middle line of the sternum, and from the cartilaginous extremities of the first two ribs. The second head, somewhat rounded in shape, took its origin from the external part of the first rib; these two heads soon coalesced, and their united fibres ran upwards, and converging, formed a tendon which passed over the external pterygoid muscle, and was inserted into the mastoid process of the squamous bone.

Omohyoid.—A large monogastric flat muscle lay beneath the last named, and arose tendinous from the upper surface of the coracoid process and from the anterior border of the scapula; it passed forwards and inwards to be inserted into the lower surface of

the posterior cornu of the body of the os hyoides, crossing over the middle portion of the carotid artery and jugular vein.

Scalenus anticus.—A round muscle; it arose about 4 inches from the sternal end of the first rib; its fibres ran upwards and outwards to be inserted into the anterior surface of the transverse process of the upper cervical vertebræ; in its course it covered the axillary vessels, and the cervicalis ascendens passed along its external border.

In *Globiocephalus Svineval*, Gray, the scalenus anticus was attached to the first and second ribs in front of their angles, and was inserted into the transverse process of the first cervical vertebra.

Mastohumeral was pyramidal in shape; it arose from the transverse process of the anterior cervical vertebræ, and from the paramastoid process; its fibres passed downwards and outwards, converging to form a long round tendon that, towards the head of the humerus, became flattened prior to its insertion, which was into the anterior and internal part of that bone; from this point a tendinous expansion was continued in front of the humerus: this muscle probably represents the trachelo-acromial of quadrumana and other mammals, somewhat similar to the levator claviculæ which is often developed as an anomaly in Man; a branch of the cervical plexus of nerves pierced the belly of this muscle.

The sternohyoid muscle was long and flat, measuring 6 inches in length by 2 inches wide; it arose from the upper border of the sternum, and passing upwards and outwards was inserted into the lower border of the body of the os hyoides.

No sternothyroid muscle was found, although it exists in other Cetaceans. In the *Globiocephalus Svineval* it arose from the back of the inner part of the cartilage of the first rib, and was inserted into an angular line on the ala of the thyroid cartilage, the direction of its fibres being upwards and inwards, and in its course overlapping the carotid vessels.

Thyrohyoid was a flat quadrilateral muscle; it arose from the external and superior border of the thyroid cartilage, passing forwards and outwards to be inserted into the internal half of the superior cornua of the os hyoides; in *G. Svineval* the origin of this muscle was from an oblique line on the ala of the thyroid cartilage, and its insertion was into the posterior border of the body of the os hyoides.

The thyroid body was a large thin somewhat triangular or, rather, V-shaped mass enclosed in a fibrous capsule and about 6 inches in length, from before backwards, having its apex directed downwards and backwards towards the thorax, almost reaching to the bifurcation of the trachea; its upper border or base corresponded to the inferior portion of the thyroid cartilage, and concealed a part of the lateral edge of the cricoid, being itself overlapped by the sternohyoid muscles. Large superior thyroid branches of the carotid artery were traceable into it, but no inferior thyroids were seen. In *Globiocephalus Svineval* these vessels exist and attain a large size.

Superior to the depressor muscle of the lower jaw and posterior to the articulation of that bone with the glenoid cavity, the portio dura of the seventh pair was situated; this

nerve, after passing through a large oval foramen in the pterygoid bone, ran directly outwards and divided into three branches. First, the anterior or motor division wound round the superficial surface of the fibrocartilage of the lower jaw, where it bifurcated, sending one branch to the depressor maxillæ inferioris, and the second to the temporal and masseter muscles. Second, the posterior branch; this ran backwards for about 1 inch and divided into two, one of which pierced the mastohumeral muscle and joined some filaments of the cervical nerves; the other branch supplied the muscles about the shoulder-joint. The third, or internal branch of the portio dura, passed upwards and inwards to supply the muscles of the neck, some filaments from it being distributed to the integuments situated around the fibrocartilaginous ear-pedicle.

From the integument surrounding the external orifice of the ear there passed inwards a round dense fibrocartilaginous cord about $\frac{1}{8}$ an inch in diameter and $4\frac{1}{2}$ long; this structure was surrounded by a fibrous investment, and was traceable winding round and behind the mastoid process to the tympanic cavity, to the wall of which its deep-seated extremity was firmly attached.

Occupying the inferior or superficial part of the interspace between the rami of the lower jaw in the anterior part of the middle line was a condensed fibrous expansion, which extended forwards as far as the symphysis, and was bifurcated posteriorly at the middle point of the lower jaw, giving attachment to the following muscle.

Mylohyoid or compressor of the inframaxillary pouch.—This muscle, broad, thin, and flat, arose from the inner edge of the inferior border of the lower maxilla as far back as its angle, and anteriorly from the aponeurosis just described; the fibres passed inwards, converging and interlacing towards the mesial line, where they met, and were inserted in common with the corresponding portion of the muscle of the opposite side.

In structure this muscle resembled a platysma myoides or panniculus carnosus, or, more correctly speaking, was of a dartoid character, being composed of fine muscular fibres and areolar tissue permeated by numerous blood-vessels. Its use was by its contraction to compress and thereby empty the submaxillary pouch. The inferior or posterior fibres of this muscle ran downwards as far as the lower part of the pouch, and some of them were traceable backwards in the median line, forming a kind of subcutaneous muscular expansion on the anterior surface of the abdomen; this, however, did not expand laterally in the cervical region, and hence that portion of the neck external to the inner margin of the sternomastoid muscle had no superficial muscular investment.

Longus colli.—On the anterior surface of the spine, after removing the trachea, œsophagus, and inframaxillary pouch, there existed an enormous muscular mass corresponding to the longus colli, longus atlantis, and rectus capitis anticus muscles, and in part also to the scalenus posticus and medius and supracostalis muscles. This mass arose by three heads, the most internal from the anterior or, more correctly, inferior surfaces of the bodies of the five posterior cervical and two anterior dorsal vertebræ; this portion represented the longus colli muscle. The middle portion arose from the outer part of the first rib by a short fleshy mass and by a longer and flatter portion, which, descending

over the thorax, was connected below and behind with the external oblique and rectus abdominis muscles posteriorly, and appeared to be the combined equivalent of the scalenus medius, rectus sternalis, and supracostalis muscles. The most external or superficial fibres arose from the posterior portion of the three anterior ribs. The fibres of these three portions converged, and uniting formed a single inseparable musculotendinous mass which was inserted into the basilar process of the occipital bone extending as far forwards as the posterior orifices of the nares, and by some tendinous slips into the inferior and anterior cervical transverse processes.

On removing this muscular mass a very large plexus of blood-vessels, principally venous, was seen extending from the first cervical to the fourth dorsal vertebra, lying on the heads of the ribs, and passing also upwards into the large spaces formed by the transverse processes of the cervical vertebræ; a portion of this plexus likewise passed in the interval between the two heads of the trachelomastoid muscle, and in the cup-like cavities or interspaces between the necks of the ribs. In the interval between the transverse processes of the second and third dorsal vertebræ it received a large vein which passed out of the spinal canal; this vessel and its fellow of the opposite side were traceable upwards to the interior of the cranial cavity, one lying on each side of the medulla spinalis external to the dura mater, and its origin appeared to be from a venous sinus formed within the cranial dura mater. Arteries derived from the internal carotid accompanied these veins, and were arranged in a somewhat similar, though more numerous, finer, and more intricate retiform manner.

At the posterior or thoracic part of this rete mirabile the veins gradually coalesced, and thus formed several large trunks on each side of the spine; these again uniting ultimately formed two large veins, which were distinctly traceable into the superior vena cava immediately prior to its entrance into the right auricle. Two large spinal arteries, after traversing this rete, were traceable downwards along the spinal canal, one on each side of the medulla spinalis on the external surface of the arachnoid, but within the dura mater, gradually converging and becoming smaller in calibre until they reached a point opposite to the articulation of the first or most anterior chevron bone, where they united and formed a single vessel, which was from thence traceable to near the termination of the spinal canal in the vertebræ of the tail. These arteries were accompanied anteriorly by a somewhat similar arrangement of veins, branches passing into the internal jugular, which were perhaps less complex in their anastomosis than were the arteries, although the latter constituted a larger part of the plexuses.

The Depressor maxillæ inferioris, situated behind the angle of the jaw, which it clothed and almost completely concealed, was a powerful deltoid-shaped muscle, and arose from the posterior surface of the mastoid process of the squamous bone, and the sulcus behind and internal to it; the fibres converging, ran downwards and forwards to be inserted into the posterior and inferior surface of the angle of the maxilla, and also into a portion of its ramus for the extent of about 4 inches anterior to that point. This muscle appeared to be homologous with the digastric of other mammals, from its posi-

tion as well as action, which latter was twofold, first to depress, and second to retract the inferior maxilla.

On removing a strong temporal aponeurosis and a quantity of dense cellular tissue the Temporal was exposed; this muscle arose fleshy from the entire of the temporal fossa, which latter was formed by the union of the squamous, frontal, parietal, basisphenoid and pterygoid bones, extending as far forwards as the posterior angle of the orbit, and backwards to the anterior edge of the glenoid cavity and interarticular fibrocartilage; the greater part of the fibres ran downwards and forwards, converging to be inserted by a very strong tendon into the prominent coronoid process of the inferior maxilla, which its tendinous insertion embraced; this muscle was of great thickness, and composed of fine and firm muscular fibres. Posterior to the tendon of this muscle, and in front of the interarticular cartilage, was placed a large oval, probably glandular body, about 1½ inch thick and 2 inches long. A large plexus of arteries and veins surrounded its outer and posterior part, but no duct was detected in connexion with this otherwise seemingly glandular body. Probably it was a rudimentary salivary gland, although the existence of an organ of this nature has been generally denied to cetaceans.

The Pterygoid muscle was small and flat; it arose fleshy from the external surface of the pterygoid plate, which formed the outer wall of the posterior nares; the muscle ran downwards and backwards, and was inserted into the internal border of the lower jaw near its angle, sending some of its posterior fibres to be inserted into the interarticular fibrocartilage. This muscle was evidently the representative of the external pterygoid; no muscle corresponding to the internal pterygoid was found.

Superior and external to this muscle was placed a very remarkable plexus of arteries and veins which lay in a distinct cavity, bounded internally by the pterygoid muscle, and externally by the angle of the inferior maxilla and fibrocartilage; the cavity was lined by an extremely delicate glistening membranous structure, similar in texture and appearance to the seroid lining membrane of the veins. The vascular plexus itself extended from the coronoid process of the lower jaw to a point midway between the angle of the latter bone and the upper border of the sternum. The venous ramifications that partly formed the plexus gradually united as they passed backwards, and ultimately formed one trunk, the jugular vein. The arterial portion of it was derived from the trunk of the internal carotid. The common carotid artery ascended on the anterior surface of the longus colli muscle, lying under the omohyoid and external to the great cornua of the os hyoides, and midway between the top of the sternum and the angle of the jaw; it divided into four branches.

First, the external maxillary or facial, which passed upwards to the angle of the lower jaw, lying external to the vascular plexus just described, and ramified on the surface of the pterygoid muscle and maxilla. Secondly, the occipital, which ran parallel with the longus colli muscle, crossing superficial to the pneumogastric nerves, and terminated on the posterior portion of the occipital bone among the deep muscles in this situation. The third branch or internal maxillary was the largest; it passed forwards and outwards,

and was lost among the deep structures at the base of the skull; and fourthly, the internal carotid, which formed part of the before-mentioned plexus, continuing its course into the cranial cavity to supply the brain through the medium of its rete mirabile, where its inosculating branches interlaced with those of the venous plexus at the base of the skull.

The Masseter was a quadrilateral muscle consisting, as usual, of two planes of fibres, superficial and deep. The former arose by a flat tendon from the central portion of the inferior border of the orbit, from whence it ran downwards and backwards to be inserted into the posterior part of the angle of the lower jaw in front of the depressor maxillæ inferioris; a few of its fibres were inserted into the interarticular fibrocartilage. The latter or deeper set of fibres arose tendinous from the margin of the glenoid cavity, extending as far forwards as the posterior edge of the orbit; the fibres of this plane ran downwards and a little forwards, and were inserted into the base of the lower jaw about 3 inches in front of its angle, and occupied by its insertion about 3 inches of the outer surface of this bone.

In a triangular space, bounded by the masto-humeral externally, the sternomastoid internally, and the pectoralis major below, corresponding to the posterior inferior triangle of the neck in the human subject, lay a considerable number of lymphatic glands, and also a very remarkable oval glanduloid body which, under the microscope, was found to consist of numerous cells or cavities imbedded in a matrix of loose fibro-areolar tissue, and into which very few blood-vessels were traceable; these are probably the representatives of the glandulæ concatenatæ.

On removing the integument and fascia from the abdominal wall, the following muscles were disclosed.

External oblique.—This muscle arose by fleshy slips from the eight inferior ribs, and its origin was connected to the before-described muscular mass situated on the anterior part of the thorax; its fibres ran downwards and inwards, forming a large tendinous expansion which covered the anterior part of the abdomen, and was inserted into the linea alba as far backwards as the anterior commissure of the cloacal sulcus; a thin fascia was interposed between this and the following muscle.

Internal oblique, which was situated beneath the foregoing, arose tendinous from the lumbar fascia as far backwards as the vulva; its fibres ran forwards and inwards, and were inserted into the sternal cartilaginous extremities of the eight or nine inferior ribs, and into the linea alba as far forwards as the lower border of the sternum, and backwards to the anterior commissure of the vulval fissure.

Transversalis abdominis.—This muscle lay deeper than the last described, and arose fleshy from the lumbar fascia, extending as far back as the posterior limit of the origin of the internal oblique, and continuing as far forwards as the tenth rib; the fibres ran inwards, and were inserted tendinous into the linea alba, the entire insertion being situated on the deep-seated surface of the rectus abdominis.

Rectus abdominis arose by a central tendon from the three or four anterior chevron

bones, from whence its fibres passed forwards and outwards, expanding, arranged in a penniform manner, and were inserted into the second, third, fourth, fifth, and sixth ribs, external to their cartilaginous or inner extremities, by a flat tendinous expansion. This muscle covered, and was connected to, the rudimentary pelvic bones; over it were expanded the tendons of the external oblique and internal oblique, and behind it lay the transversalis muscle.

On removing the integument and fascia from the back of the animal, the following muscles were disclosed.

Latissimus dorsi.—A flat thin muscle, which arose by a broad aponeurotic expansion from the spines of all the dorsal and a few of the anterior lumbar vertebræ, behind the attachment of the rhomboid muscle; the fibres passed downwards and outwards, and were inserted into the internal edge of the inferior surface of the humerus.

Rhomboideus.—This muscle was thin and weak; it took its origin from the spinous processes of all the dorsal and one or two of the anterior lumbar vertebræ by a thin aponeurosis, which was partly united to the origin of the latissimus dorsi; the fibres passed forwards and outwards to be inserted into the inferior angle of the scapula anterior to the following muscle; there was no sign of segmentation into major or minor portions.

Serratus magnus vel depressor anguli scapulæ.—This muscle arose from the aponeurosis investing the abdomen and the eight inferior ribs; the fibres ran upwards and forwards almost parallel to the ribs, and were inserted into the inferior edge of the scapula 4 inches above its inferior angle, where it united with the posterior edge of the origin of the teres: another flat slip of the serratus muscle arose from the second rib and lay above the inferior and posterior part of the muscle. The long respiratory nerve of Bell supplied its inferior portion.

Levator anguli scapulæ.—A small pyramidal muscle; arose from the transverse process of the last cervical vertebra, from whence it passed directly outwards, and was inserted into the posterior superior or cervical angle of the scapula.

Trachelomastoid was a short thick fleshy muscle; it arose by two heads, one from the transverse process of the first dorsal vertebra, and the second from the sides of the bodies of the three or four posterior cervical; the fibres passed upwards and outwards to be inserted into the posterior surface of the mastoid process of the squamous bone behind the articulation of the lower jaw. The two heads of this muscle were separated by an offshoot of the great vascular plexus, as before described.

Longissimus dorsi.—A large and somewhat pyramidal-shaped muscle; arose by a broad flat tendon from the neural arches of the caudal vertebræ as far back as the extremity of the tail; the fibres passed forwards, being connected to its fellow of the opposite side by means of a strong tendon which passed up to within 3 feet of the posterior part of the skull, at which point they diverged and were inserted into the osseous crest on the external edge of the occipital bone as far forwards and outwards as the level of the temporomaxillary articulation.

External to but connected with this muscle, a distinct slip of muscular fibres passed from the same origin to the posterior portion of the mastoid process, the lower fibres of which seemed to represent the sacrolumbalis muscle, while the anterior portion was probably the slightly displaced homologue and representative of the splenius capitis.

Levator caudæ, a very powerful muscle, was overlapped by the last described, with which some of its fibres were continuous; it arose by strong fleshy fibres from the dorsal aspect of the transverse processes of the lumbosacral and caudal vertebræ about 5 feet anterior to the root of the tail; it was inserted by eight strong flattened tendons, each of which measured from $\frac{1}{2}$ to $\frac{3}{4}$ of an inch in thickness, into the sides of the neural spines of the lower caudal vertebræ, expanding posteriorly into the fibrocartilaginous tissue which formed the substance of the lateral flukes of the tail.

Depressor caudæ major, an enormous muscular mass which formed the entire bulk of the posterior and inferior part of the body from the pudendal fissure to the base of the tail, and arose from the anterior surfaces of the bodies and transverse processes of all the lumbosacral and all the caudal vertebræ, and also from the ventral and lateral surfaces of the chevron bones. This muscle was divided into about nine subordinate bundles, each of which terminated in a round powerful tendon, which, at its origin from the muscle, measured about 1 inch in diameter; all these tendons passed backwards, and expanding were inserted into the sides and anterior surfaces of the bodies of the caudal vertebræ, the last tendons expanding into the under surface of the substance of the tail.

Depressor caudæ minor.—This muscle was similar to but smaller than the last described; it arose below and external to it, from the anterior surfaces of the caudal vertebræ, extending as far forwards as midway between the last rib and the posterior extremity of the spinal column, and was inserted by roundish tendons into the substance of the tail, sending a few of the most anterior fibres to the most posterior of the caudal vertebræ immediately connected with that organ.

The anterior or pectoral extremity, although it externally appeared but a solid single lancet-shaped organ, yet on examination was found to contain the rudiments of the various segments of the typical vertebrate upper extremity as found in the other orders of Mammalia, viz. scapula, humerus, radius and ulna, carpus, metacarpus, and phalanges.

The muscles, as might be expected, were small, rudimentary, and undeveloped; nevertheless it was not difficult to determine the position and homologies of those that were present.

Among those of the shoulder.—The Deltoid was small, fleshy, and triangular in shape; it arose from the upper half of the external flat surface of the scapula above the infraspinatus; the fibres passed outwards and backwards, converging towards the head of the humerus, into the anterior and internal surface of which they were inserted by a strong flat tendon; its action was probably to advance the paddle upon the scapula.

Teres major.—A flattish muscle, arose tendinous and fleshy from the inferior border

of the scapula; the fibres passed downwards, forwards, and outwards, and were inserted into the anterior and internal surface of the neck of the humerus; some of the fibres were inseparably connected at their origin with the insertion of the serratus magnus.

Supraspinatus.—This muscle lay immediately beneath the deltoid; it arose from the upper border of the dorsum of the scapula and acromion process. The fibres ran downwards and backwards, converging to form a large triangular muscle, about half the size of the deltoid, and soon ended in a tendon, which was inserted into a ridge on the anterior inferior and external surface of the head of the humerus.

Infraspinatus.—A large flat triangular muscle, arose fleshy from the lower half of the dorsum of the scapula; its fibres passed upwards and outwards, and were inserted into the superior and external part of the head of the humerus.

Subscapularis, also large and triangular, with its base towards the vertebral border of the scapula; it arose from the entire subscapular fossa, and its fibres passed downwards, converging to form a flat tendon, which was inserted into the front of the head of the humerus superior to the humeromastoid muscle, which latter at this point formed a round tendon that lay in a groove in the head of the humerus. This muscle exhibited very few and indistinct tendinous intersections, thereby contrasting in a striking manner with the corresponding muscle in *Globiocephalus Svineval*, in which there were eight radiating tendinous plates running nearly parallel and partitioning the muscle into nine fleshy slips, which ended in a tendon that did not pierce the capsule of the scapulo-humeral articulation.

The axillary vessels and nerves passed in front of the subscapularis tendon; and the artery, which in this situation was of about the size of the human popliteal, had two venæ comites accompanying it, and bifurcated at the lower border of the tendon, where it sent one branch, about the calibre of the human radial, down along the humerus, and the other (the subscapular) ran along the inferior border of the scapula. The brachial plexus of nerves was composed of very small branches, which broke up into muscular and cutaneous filaments. The plexus itself, prior to its division, lay on a plane superior and external to the vessels.

At the upper part of the axillary space two remarkable series of veins were situated, one set being placed anterior and horizontal, the second set posterior and vertical; the former was contained in a flat and membranous expansion, and was somewhat quadrilateral in outline; the vessels composing it, with the exception of a few intercommunicating branches, ran parallel to each other, and were held together by a strong cellular investing membrane. This plexus was situated above the subscapularis and posterior to the origin of the omohyoid muscle.

The second, or vertical plexus, lay about 2 inches posterior to that just described; it was flat and triangular in outline, with its apex, which was thin and membranous, directed towards the vertebral column; its base or inferior border, enclosing the second series of vessels, was formed of dense cellular tissue, and presented a well-defined border, which extended from the first rib, internally, to the apex of the shoulder on the external or pectoral

side of the omohyoid muscle. The free border of this structure may be considered as homologous to the ligamentum bicorne or costocoracoid membrane of other mammals*.

Coracobrachialis, a short round muscle placed anterior to the above-described venous plexus; it arose from the apex and anterior surface of the coracoid process, from whence it passed downwards and outwards to be inserted, by a flat tendon, into the anterior and internal part of the head of the humerus on the scapular side of the groove for the humeromastoid muscle. This muscle had no connexion with the capsular fibrocartilage, but in *Globiocephalus Svineval*, in which a synovial capsule existed, it was inserted into the ligament, thus forming a coracocapsular muscle similar to that which exists in several other animals, and which has been lately described by Mr. Wood as an abnormal condition in Man†.

No flexors of the fore arm were developed, but external to the coracobrachialis there were a few tendinous bands which appeared to be the sole representatives of this group of muscles, being probably the rudiments of the biceps flexor cubiti.

The triceps was a flat irregular mass, and consisted, first, of a long head which arose from the inferior surface of the neck of the scapula, from whence it passed backwards to be inserted into the extremity of the cartilaginous condylo-olecranal apophysis; this division of the muscle crossed over the teres major, forming, with it, the humerus and scapula, a quadrilateral space through which the circumflex artery and nerve passed, winding round the neck of the humerus. The second, or external head, arose tendinous from the middle part of the upper and posterior edge of the humerus, and passing towards the long head, was inserted into the upper surface of the cartilaginous process anterior to, but separate from, the long head; this division of the muscle ran posterior to the elbow-joint. The third, internal or shortest head, arose tendinous from the posterior edge of the humerus, about 1½ inch below its head; the fibres ran backwards and inwards, and were inserted into the highest or most posterior point of the cartilaginous olecranon, about 3 inches above the insertion of the long head; this division of the muscle was about 2 inches long.

This muscle could have had but a very restricted action. In *Globiocephalus Svineval* it was represented merely by interlacing fibrous bands without a trace of muscularity.

The rudimentary muscles of the fore arm were the following‡: on the dorsal surface

* Vascular *retia* or reservoirs, similar to those recorded as existing in other Cetacea, were, in the present specimen, situated in three localities; of these reservoirs the first and largest was placed in the thorax, and corresponded to that described by Hunter and others, but did not, in the present instance, extend so far outwards on the ribs as in the Porpoise and Dolphin. The second occupied the pterygomaxillary region, and has not been mentioned by previous authors; and the third, as above described, lay in the axilla. There was no plexiform arrangement of vessels on the abdominal surface of the depressores caudæ (psoæ) muscles as noticed in other Cetacea by Tyson, von Baer, and Breschet.

† Royal Society's Proceedings, June 1867.

‡ No muscles passing from the fore arm to the manus have been observed in any of the Toothed Whales. Their presence in the large Fin-whale (*Physalus antiquorum*, Gray) was noticed by Mr. Flower (Proc. Zool. Soc. Dec. 1865), but without detailed description.

there was but one, the extensor digitorum communis, which was a mere slender fasciculus of muscular fibres having its origin from the interosseous space and from the fibro-cartilaginous capsule surrounding the heads of the radius and ulna, a few fibres being traceable downwards from the interosseous space between the bones of the fore arm; it soon formed a flat tendon which, opposite to the first row of carpal bones, divided into four tendinous slips that were inserted into the terminal phalanges of each digit, sending a slight expansion into the sides of each of the proximal phalanges.

On the anterior aspect of the fore arm the following flexor muscles were found, which though small were much better developed than the extensors.

Flexor carpi radialis.—This muscle arose fleshy from the anterior surface of the humeral extremity of the radius; it soon formed a round tendon, which running downwards, was inserted into the base of the metacarpal bone of the first digit. To its inner side lay

The Flexor digitorum communis, which arose by short tendinous fibres from the internal surface of the cartilaginous olecranon process, and by a few fibres from the inner edge of the humerus; its fibres, after a short course, terminated in a round tendon, which passed downwards to the level of the first row of the carpal ossicles, where it divided into four tendons, one of which was inserted into the distal phalanx of each digit, and in its course sent a slip to each of the intermediate phalanges.

Palmaris longus, an extremely delicate muscle, arose from the cartilaginous olecranon, from whence, soon becoming tendinous, it passed downwards and was inserted into the metacarpal bone of the fourth digit and into a rudimental palmar fascia, which was expanded over the carpal bones and cartilages, and covered the tendons of the flexor muscles.

Flexor carpi ulnaris was the most internal and the strongest of these muscles; it arose from the internal surface of the olecranon, and soon became tendinous; it ran downwards to be inserted into the inner side of the metacarpal bone of the fourth digit, posterior to the last-described muscle.

The scapulo-humeral articulation was surrounded by a distinct capsular ligament, lined by a thin distinct synovial membrane, and was perforated internally by the tendon of the subscapularis muscle, which lay in contact with the synovial capsule; in this it differed from the shoulder-joint of *Globiocephalus Svineval*, in which latter animal the tendon of the subscapularis did not pierce this capsule. In general the capsular ligament was thin, except inferiorly, where it was strengthened by several fibrous bands, forming the inferior scapulo-humeral accessory ligament. These fibrous bands are not present on the capsule of this joint in *Globiocephalus Svineval*.

The glenoid cavity of the scapula was deepened by a well-marked cotyloid ligament, and a distinct synovial membrane lined the interior of this articulation.

The ligament of the notch of the scapula was a strong fibrous band flattened at its attachments and rounded in the centre; it passed from the coracoid process to the superior border of the scapula, and the suprascapular nerve and artery passed under it.

The articulations existing between the radius, ulna, and humerus, as well as those

between the former bones and the carpus, were amphiarthrodial in character, the several osseous surfaces being united by a very soft, fatty fibrocartilaginous substance. This character of articulation also obtained between the carpals, metacarpals, and phalanges, without the slightest trace of synovial investments.

Viscera.

The mucous membrane on each side of the cavity of the mouth, which lined the space intervening between the lateral border of the tongue and the internal or gingival surface of the ramus of the lower jaw, was disposed in longitudinal folds corresponding to those already described as existing on the geniothoracic region of the animal. These folds were arranged in two sets, a superficial or primary, and a deeper or secondary.

The former series were strongly marked, with deep furrows between them, and ran parallel to the ramus of the lower jaw. The latter, which were much shallower and finer, were situated in the furrows or interspaces between the former, and their course, instead of being parallel to the primary folds, ran in an oblique direction forwards and outwards across their interstices.

This peculiar arrangement of the mucous membrane and integuments greatly facilitated the expansibility of the submaxillary pouch.

The primary folds in the immediate vicinity of the tongue were coarse and irregularly disposed, but as they approached the internal surface of the inferior maxilla they became smooth, fine, and more unequally scattered over the surface.

The roof of the mouth represented in outline an elongated isosceles triangle, and was divided longitudinally into two smaller and more acute triangles by a prominent keel, formed by the inner border of the palate-plate of the superior maxillary as it overlapped the vomer; this keel extended along the central line of the palate, from the meeting of the superior maxillaries anteriorly to the isthmus faucium posteriorly. In front the keel was rather obscurely marked, but as it extended backwards it became much more prominent, and by its downward projection divided the palatine surface into two lateral elongated triangular spout-like concavities, each of which was deeper posteriorly than anteriorly, and more excavated along its inner or vomerine border, from the depth of the median crest in this situation, than near to its outer or labial edge, which latter was formed by the margin of the superior maxillary bone, where it met and was concealed by the formative nidus or wreath of the baleen plates. The mucous membrane lining the roof of the mouth was dark in colour, rugose, and highly vascular.

The free edge of the upper lips was thick and rounded, but, although they nearly covered the wreath, did not project downwards as far as the level of the median palatine keel. The lower lips were thicker and larger. The capacity of the mouth was much diminished by the encroachment of the baleen plates, as well as by the median crest; however, this seemed to be compensated for in a considerable measure by the great expansibility of the submaxillary pouch.

Eschricht and Reinhardt, in comparing this species with the Greenland Whale, *Balæna mysticetus*, allude to and explain the great difference in the capacity of the bony framework of the mouth of these two animals, the enormous size of that of the latter being due to the immense vertical antero-posterior arch of the palatine surface, while in the *B. rostrata* the roof of the mouth has scarcely any appreciable curvature in this direction; there is, however, no provision in the soft parts of the Mysticete for buccal or submaxillary expansion so characteristically developed in the Rorqual; and although in the transverse direction the great outward lateral concavity of the anterior part of the lower jaw in *Balæna rostrata* is not sufficient to enlarge greatly the mouth from side to side, as it is counteracted by the posterior inward convexity of the bone, yet it seems to us that the great expansibility of the submaxillary pouch in *B. rostrata* compensates for the greater proportionate capacity of the bony framework in *B. mysticetus*.

There was no appearance of any of those pits and fissures on the mucous surface of the palate described in *B. mysticetus* by Eschricht and Reinhardt, and supposed by them to be the rudiments of the Stenonian ducts, although in consequence of the presence of the peculiar glandular mass, already noticed as existing between the muscles of the lower jaw, a very careful search was made, but in vain, for even the slightest trace of a duct.

The mucous membrane of the roof of the mouth was extremely vascular, especially towards the wreath of the baleen plates; eighteen or nineteen large blood-vessels, varying from the size of a goosequill to that of a crowquill, traversed the superior maxillary bone obliquely forwards and outwards, and were distributed to the formative nidus of the whalebone; of these vessels the anterior ran obliquely forwards, the middle outwards, and the posterior backwards.

The tongue was oblong and flattened, somewhat elliptical in outline; it measured 1 foot 9 inches from base to tip, and 10 inches in breadth at its widest part, which was about the centre of the organ. It was completely fixed to the floor of the mouth*; the frænum wide and short, extending to its tip, which was rounded and obtuse. On each side the mucous membrane was reflected from its edges to become continuous with that lining the general buccal cavity. The borders of the tongue were more distinctly defined anteriorly than towards its pharyngeal end, but in no situation could the organ be called free. Its dorsal surface was covered with a thick yellowish scaly epithelium, transversely rugose, and easily detached; this coat invested the small fungiform and filiform papillæ, which were irregularly distributed all over the surface of the organ. The basement layer of mucous membrane displayed also considerable rugosity, the folds being small and placed transversely towards the centre of the tongue, but larger and more obliquely situated near its edges. No calyciform papillæ were observed, and of the other two series the fungiform were the most numerous, and were better marked laterally than in the mesial line.

* In a specimen of *Globiocephalus intermedius*? recently (April 1867) captured off Newhaven the tongue was free and moveable both at its tip and edges.—A. M.

Proceeding from within outwards the muscles of the tongue were arranged in the following order, on either side of the well-developed, central, cellular raphe.

First. The Genioglossus, which was irregularly quadrilateral in outline and arose from the internal border of the inferior maxilla near its anterior or symphysial extremity; the fibres ran upwards and inwards, forming a thin flat belly to be inserted into the deep surface of the mucous membrane in the centre of the tongue.

Second. The Lingualis, or intrinsic muscle of the tongue, consisted of several series of interlacing muscular fibres running both in a transverse and longitudinal direction, which together contributed to form a considerable portion of the bulk of the organ; the longitudinal part lay between the genioglossus and the hyoglossus muscles; the transverse fibres decussated with the fibres of the other muscles of the tongue.

A vertical series of lingualis fibres were found interposed between the lingual artery and the hyoglossus muscle, separating the latter from the other sets of intrinsic fibres.

Third. The Hyoglossus arose by a round fleshy mass from the great cornu of the hyoid bone, from whence it ran forwards expanding in its course, and was inserted into the entire length of the side of the tongue from base to tip.

Fourth. The Palatoglossus formed the orbicular sphincter surrounding the isthmus faucium, and was exposed by removing the mucous membrane from the buccal surface of the lateral boundaries of this opening; it was a strong crescentic muscle $\frac{5}{8}$ of an inch in thickness, 2 inches in breadth, and about $3\frac{1}{2}$ inches in length, measured from the tongue to the middle line of the velum palati; it arose from the median line of the soft palate, and its fibres passed downwards and outwards in an arched direction to be inserted into the dorsum of the tongue, on a plane external to the insertion of the last-described muscle.

The Palatoglossus, from its position, was evidently of great importance in the economy of the animal, as by its contraction it completely closed the isthmus faucium and so prevented the entrance of water into the œsophagus during the prehension of food; it likewise cut off the passage of air through the mouth during the act of respiration.

Fifth. A thin expanded muscular plane external to the hyoglossus existed as a representative of the Styloglossus; it arose from the outer side of the fibrocartilage, which united the styloid bone or cornu of the os hyoides to the base of the skull, lying external to all the other muscles of the organ; its insertion extended forwards into the posterior half of the lateral margin of the tongue.

From the arrangement of the preceding muscles, the following conclusions may, we think, be deduced with respect to the motions of the tongue. Although fixed to the floor of the mouth, its sides could most probably be elevated by the contraction of styloglossus and palatoglossus muscles, and centrally depressed by the genioglossus and hyoglossus, thus rendering it transversely concave on the upper surface, especially towards the base; its apex might likewise be depressed, as in prehension, by the genioglossus, and its base, during the act of deglutition, might be lowered by the hyoglossus and depressor muscles of the hyoid bone. Its body might be shortened by the lingualis

either in its vertical or transverse direction; but from the fixity of the apex this portion would scarcely admit of any retraction, consequently any shortening in the antero-posterior direction could only be accomplished by the drawing forwards of the hyoid bone by the action of the stylohyoid, basiokeratic, and the other advancers of this bone.

The nerves supplying the tongue were:—First, the lingual or motor nerve, which was distributed to the muscles, and lay internal to the lingual artery upon the lingualis muscle and beneath the hyoglossus.

Second, the gustatory or sentient nerve, a branch of the fifth pair, lay on the outside of the genioglossus muscle, and sent branches upwards to be distributed to the papillæ on its mucous membrane.

Third, a small twig of the glossopharyngeal nerve from the eighth pair was distributed to the mucous membrane of the back of the organ.

The lingual artery, a considerable branch of the external carotid at the base of the tongue, gave off numerous branches which ramified among the muscles; its continued or ranine trunk ran along the outer border of the hyoglossus muscle as far forwards as the tip of the tongue.

The veins of the tongue lay nearer to the median line than the arteries, and formed a plexus in the raphe near its base which terminated in the great jugular vein.

At the inferior portion of the middle third of the tongue there existed a large closed cellular pouch, lined by a dense smooth layer of areolar tissue; this cavity was interposed between the genioglossi muscles, and was about 8 inches long by 3 inches wide; it extended forwards beneath the organ to within 4 inches of its tip, and was of sufficient capacity to contain the extended hand. Whether this cavity were a normal arrangement of parts, or merely a want of lateral coalescence depending upon the immaturity of the individual, it is difficult to decide; the latter theory seems to be the more probable, as there was no apparent function which such a cavity could fulfil.

Pharynx.

The pharynx was separated from the cavity of the mouth by the palatoglossus muscle already described.

From the anterior edge of this sphincter, the mucous folds which formed the great submaxillary pouch commenced and passed forwards on each side of the tongue, as before mentioned; the mucous membrane lining this sphincter was completely devoid of rugæ, but was thickly studded with numerous secreting crypts opening by pores on its surface, from which, by pressure, was expelled a quantity of viscid mucus. The velum had a firm feel and was destitute of a uvula; this probably is a normal arrangement in Cetacea, as there was no trace in *Globiocephalus Svineval*.

On the anterior portion of the pharyngeal cavity, behind the palatoglossus muscle, the surface of the mucous membrane was marked with slight crescentic folds, concave backwards; these folds bounded anteriorly and laterally a semicircular space that extended backwards as far as a curved fold of the mucous membrane, hereafter to be

described. This space was deeply alveolated, and its mucous surface presented a reticulated appearance, somewhat resembling the carneæ columnæ of the heart; its central portion, however, for about the width of 1½ inch was smooth, being perfectly free from all such alveolations. Posteriorly and laterally the alveoli were arranged in five or six transverse series, gradually diminishing in size until they entirely disappeared towards the middle of the pharynx, where the mucous membrane became perfectly smooth, and so continued until it reached the commencement of the œsophagus.

Two inches in front of the root of the epiglottis, and surrounded by the alveolated surface just described, there was a strong crescentic hood-like fold of mucous membrane, the concavity of which was directed backwards; this fold measured along its free posterior border 9 inches, and the lateral cornua of this edge were prolonged on each side of the opening of the glottis in the form of frœna, which were gradually lost in the mucous membrane of the upper part of the œsophagus; its convex margin anteriorly was continuous with the before-mentioned alveolated space; the depth of the fold at its central part, which corresponded to the opening of the larynx, was about 2½ inches; its inferior surface was smoother than its superior, and was studded with the orifices of numerous minute mucous pores. This hood-like fold formed the covering of a cavity capable of containing half a large orange. Beneath the mucous membrane of this fold were curved muscular fibres passing from side to side, whose use appeared to be for the contraction of the subjacent pouch.

This remarkable hood-like fold was capable of being drawn over the orifice of the glottis when the margins of that opening were approximated, and its use appeared to be to cover and protect the superior opening of the larynx during the act of deglutition, a purpose which could be completely accomplished by the contraction of the muscular fibres between its laminæ, which, when the arytenoid bodies and epiglottis were closely approximated, would suffice to cover and keep together the sides of the opening. Immediately posterior to the crescentic or hood-like fold was placed the upper opening of the larynx or glottis, on each side of which was situated a deep channel, which was directed backwards and downwards towards the œsophagus, being separated only from the glottis by that portion of the mucous membrane which was prominently elevated on each side to form the aryteno-epiglottidean folds. Numerous rugæ and mucous pores were situated on the outer wall of this channel and continued down to the œsophagus.

The orifice of the glottis was triangular in outline, the base being placed at the epiglottis, and the apex between the arytenoid bodies, the lateral boundaries being formed by the aryteno-epiglottidean folds.

The epiglottis was tongue-shaped and extremely flexible; its base was attached posteriorly to the central part of the upper surface of the thyroid cartilage, and its edges laterally became continuous with the aryteno-epiglottidean folds which embraced the bases of the arytenoid bodies. It was attached anteriorly in the median line by a well-developed frœnum, which extended into the floor of the pouch or sac before noticed; and

beneath this fold of mucous membrane there was a strong muscular band, the hyo-epiglottideus muscle, which ran to the anterior surface of the epiglottis from the centre of the posterior aspect of the body of the os hyoides. A prominent ridge extended along the middle of the epiglottis on its posterior superior surface, descending into the larynx, becoming more prominent as it passed downwards, and ceasing at the insertion of its base into the thyroid cartilage.

The anterior extremities of the arytenoid bodies were thick, fleshy, and somewhat tongue-shaped; they were united to each other in the mesial line from the top of the cricoid cartilage to within 2 inches of their apices, where they separated and formed two free linguate extremities, whose inner surfaces were directed towards each other, and could be readily approximated by the contraction of the arytenoideus proprius muscle; their outer borders were thick and fleshy, and were continued down as prominent ridges into the larynx.

There is an important difference existing between the glottis of *Balænoptera* and that of *Delphinus* and *Globiocephalus*; that the constituent parts of the free upward tubular prolongation of the larynx are closely united to a much greater extent in the latter animals, whereas in the former the lateral boundaries of this opening are quite separate, and only capable of being formed into a tubular elongation by the approximative action of their appropriate muscles.

The cartilaginous portion of the epiglottis formed but a small part of the centre of the entire organ, and beneath its mucous investment its lateral parts were constituted of a layer of interlacing muscular fibres, which were continued into the loose fold of mucous membrane that enveloped the base of the arytenoid bodies.

The larynx was firmly connected by muscles and ligaments to the os hyoides, which, as before described, consisted of a body and four cornua; the former occupied the median space about 4 inches anterior to the thyroid cartilage; its shape was that of a flattened triangle, whose rounded and emarginate apex was directed forwards, and whose lateral angles or posterior cornua were continuous with it, being prolonged backwards on each side. Its inferior surface was convex, and presented externally an oblique line that commenced about 2 inches from the apex and ran backwards and outwards to the tip of the external angle, to which was attached the mylohyoid muscle; anteriorly there was a rough surface, about $2\frac{1}{2}$ inches in extent, into which the basiokeratic muscle was inserted. The superior surface was concave, and was occupied in the middle line by the attachment of strong hyo-epiglottic ligaments and muscles; external to these the deep-seated basiokeratic muscle had its origin, and extended outwards to the end of the external angle. The anterior border terminated in front in the apex, which latter was bounded on each side by a concave space, to the anterior portion of which the base of the anterior cornu or styloid bone was closely united by strong fibrous tissue, the remainder of the border being merely occupied by cellular membrane. The posterior margin was rounded and concave; to it the sternohyoid and thyrohyoid muscles were attached, the former occupying a space extending outwards about 3 inches from the

median line, the latter lying on a plane superior was inserted into a line leading from the centre of the margin for about 1 inch in extent.

The cornua or ossa styloidea were distinct, curved, cylindrical bones, and were articulated internally by short and firm fibrous and fibrocartilaginous bands to the sides of the anterior portion of the body, as before described. In *Globiocephalus Svineval* the union is accomplished by a prolonged fibrous rounded band. ESCHRICHT and REINHARDT picture a similar elongated intervening ligament in the Mysticete.

At their external extremities these bones were somewhat club-shaped, and were closely united to the base of the skull by the interposition of a strong fibrocartilage; from their anterior or concave edge they gave attachment to the muscles of the tongue, viz. the hyoglossus and genioglossus; posteriorly three muscles arose from each of them; first, the superficial hyokeratic, which occupied the inner third; second, the deep hyokeratic, which had its origin from the middle and part of the inner third of the bone; this muscle was prolonged posteriorly and superiorly to the last described; and third, the keratopharyngeus, which took its origin from the remaining outer third of the cornu. No muscles covered their fibrocartilaginous cranial articulation; but in *Globiocephalus Svineval* the craniostyloid articulation was covered by a short flat muscle, the squamostyloid, which, arising from the squamous bone above the articulation, and crossing over the fibrocartilage, went to be inserted into the styloid bone at its cranial third. Another muscle in this animal, the stylohyoid, of a pyriform shape, arose from the squamous bone behind the styloid articulation, and was inserted into the extremity of the posterior cornua or lateral angle of the body of the hyoid bone, in its course crossing the external carotid artery and the lingual nerves.

Larynx.

The framework of the larynx was composed of four true cartilages, viz. thyroid, cricoid, and two arytenoid.

The thyroid formed the lateral and inferior boundary of the laryngeal cavity, and was composed of two symmetrical halves or alæ united by a central isthmus; each ala exhibited four edges bounding two surfaces. The anterior edge was concave forwards and smooth, it measured 4 inches in length; the posterior edge was 8½ inches in length, and was deeply excavated; it terminated internally in a triangular prolongation 2 inches in length, with its apex directed backwards.

The external or lateral border was convex and slightly prolonged forwards to form the anterior cornu, from whence the thyrohyoid muscle had its origin; posteriorly this border was prolonged into a round digital process which formed the posterior cornu, whose extremity was connected by fibrous tissue to a well-marked tubercle situated on the posterior and lateral aspect of the cricoid cartilage. The entire extent of this external margin, measured from its anterior to its posterior cornu, was 7 inches, and to it the inferior constrictor of the pharynx was attached. The inner border was formed of two distinct parts, an anterior and posterior; the former, about an inch in extent, was

united to its fellow of the opposite side, forming the median isthmus before mentioned, while the latter, which measured about 2 inches in length, was free, smooth, and separated from the corresponding portion of the opposite side by a deep triangular notch, which at its posterior extremity measured $1\frac{1}{2}$ inch in breadth. The inferior or superficial surface was flat, and exhibited the attachments of three muscles along its outer margin; of these the thyrohyoid was most anterior, the cricothyroid most posterior, and the sternothyroid intermediate.

The upper or deep-seated surface corresponded to and covered the upper third of the inferior surface of the great laryngeal pouch, to be hereafter noticed, and posteriorly it came into contact with the outer surface of the arytenoid cartilage; its central portion in front gave origin to the thyro-arytenoid muscle, and posteriorly to a few of the anterior fibres of the constrictor of the laryngeal pouch.

The cricoid cartilage, in outline, was irregularly rhomboidal, representing about two-fifths of a circle, and formed the inferior boundary of the posterior three-fourths of the larynx. It presented two surfaces, which were bounded by four irregularly shaped edges and angles. The superior surface presented on each side a slight concavity, from which the crico-arytenoideus posticus took its origin, and in the central line it was transversely concave to accommodate the passage of the œsophagus; the inferior surface was the reverse shape of the superior, *i. e.* concave on each side and convex on the central line; to this latter part the mucous membrane of the posterior three-fourths of the larynx was closely adherent. The centre of the anterior edge presented a slight concavity, to which the posterior edge of the arytenoideus proprius was attached. The posterior margin was extremely irregular in outline, presenting in the median line a tongue-shaped process, which was directed backwards towards the first ring of the trachea. The outer edge was slightly convex, where it gave origin to the cricothyroid muscle. The anterior and external angle formed a facet for articulating with the arytenoid cartilage; the posterior angle extended backwards, terminating in a flat process; and about 1 inch internal to this a small tubercle was situated, to which the inferior cornu of the thyroid cartilage was articulated by the interposition of strong fibrous tissue.

The arytenoid cartilages were extremely irregular in shape; each of them consisted of a body or central part and two processes; the body had three surfaces, an external, internal, and posterior; the first was enlarged by the projection of a prominent shelf-like process along its lower and outer border, from this the upper fibres of the constrictor of the laryngeal pouch arose.

The internal surface of the cartilage was convex, presenting on the middle of its posterior border an oval concave articular facet, which measured $1\frac{1}{4}$ inch in length by $\frac{1}{2}$ an inch in width; this was articulated with a convex surface placed on the outer edge of the posterior and lateral aspect of the cricoid cartilage, and was lined by synovial membrane and surrounded by a capsular ligament. The superior surface was somewhat triangular, and of much less extent than either of the other two; it lay in front and above the articular surface just described, and was covered by the crico-arytenoideus posticus

muscle. The posterior process was pyramidal, and prolonged downwards and inwards towards the median line, where it was united to its fellow of the opposite side by a strong round fibrous cord, about two lines thick and 1 inch in length, the transverse ligament of the arytenoid cartilage. The anterior process was flattened; it curved upwards and inwards, and terminated in a fibrous band, which was placed as a kind of support in the centre of the free tongue-shaped arytenoid body, this latter forming the posterior boundary to the rima glottidis. The entire length of this cartilage was 8 inches, its greatest width in the centre being $3\frac{1}{2}$ inches.

The muscles of the larynx were seventeen in number, viz. on the anterior aspect there were, first, the thyrohyoid, a flat muscle, 4 inches long by 1 inch broad; it arose fleshy from a ridge on the anterior and outer portion of the thyroid cartilage behind the base of the anterior cornu; from this the fibres ran forwards and inwards to be inserted tendinous into the posterior part of the body of the os hyoides, extending outwards 1 inch from the median line. Second, the crico-thyroid, a strong and flattish muscle, measuring 4 inches long by $1\frac{3}{4}$ inch wide; it arose fleshy from the outer surface of the anterior extremity of the cricoid cartilage, passed forwards and upwards to be inserted into the lower and external edge of the inferior cornu of the thyroid cartilage; it lay behind, and on a plane deeper than the sterno-thyroid. The muscles situated on the posterior aspect were, first, the crico-arytenoideus posticus, a large fleshy mass, which took its origin from the outer third of the posterior surface of the cricoid cartilage; its fibres ran forwards and outwards, and embraced the posterior and external angle of the arytenoid cartilage into which they were inserted. Second, the arytenoideus proprius, a flat, azygos muscle, which consisted of slightly decussating fibres running obliquely from the inner edge of one arytenoid cartilage to the corresponding border of its fellow; its lower fibres ran transversely, while the middle and upper were slightly curved. This muscle extended $1\frac{1}{2}$ inch above the superior border of the cricoid cartilage, and terminated $2\frac{3}{4}$ inches behind the commissure of the arytenoid bodies.

Between the layers of mucous membrane that formed the arytenoepiglottidean folds, on each side the superior aryteno-epiglottideus muscle was enclosed; it passed from the outer and anterior part of the arytenoid cartilage to be inserted into the external margin of the epiglottis. On the same plane, but posterior to it, the inferior aryteno-epiglottideus muscle was situated, whose attachments corresponded to those of the last described, the posterior border of the superior being in contact with the anterior edge of the inferior muscle.

The hyoepiglottideus muscle was triangular in shape, the base being directed towards the os hyoides and the apex attached superiorly and posteriorly to the epiglottis. It arose from a vertical median ridge on the upper surface of the body of the os hyoides, ran backwards and upwards to be inserted into the anterior surface of the epiglottis; this muscle was separated from its fellow of the opposite side by a narrow cellular interval, and lay beneath the frænum epiglottidis. The superior laryngeal nerve, which entered above and in front of the thyroid cartilage, ran enclosed in the aryteno-epiglot-

tidean folds of mucous membrane, and was distributed to the mucous membrane lining the larynx, apparently giving off no muscular branches.

On raising and reflecting the ala of the thyroid cartilage an accessory aryteno-epiglottideus muscle was exposed; it was somewhat triangular in shape, and arose from the anterior and external angle of the arytenoid cartilage; from this the fibres passed inwards, diverging, and were inserted into the side of the fibrocartilaginous pedicle of the epiglottis, posterior to which its fibres coalesced with those of its fellow of the opposite side.

The thyroarytenoid muscle was united to the posterior border of that last described, in fact both appeared to be but parts of one large muscular expansion; it arose from the vertical median ridge on the superior surface of the thyroid cartilage, where its fibres were distinctly separate from the foregoing muscle; it passed forwards and inwards, and was inserted into the anterior and external margins of the arytenoid cartilage, and into the contiguous margin of the cricoid.

There were no vestiges of chordæ vocales or of a ventricle, or a lateral laryngeal sacculus; but situated in front of the cavity of the larynx, and opening by a wide orifice immediately at the root of the epiglottis, there was a remarkable large musculo-membranous mesial sac or laryngeal pouch, which extended downwards and backwards in front of the trachea; its walls, which were thick, were almost entirely composed of circular muscular fibres, which anteriorly or nearer the glottis took their origin from the upper surface of the thyroid cartilage, and posteriorly formed a series of circles with no definite origin or insertion. The interior of this cul-de-sac communicated directly with the central portion of the laryngeal cavity by a wide orifice, and was lined by a continuation of the mucous membrane of the former.

From the situation and structure of this pouch its probable use was to assist, by the contraction of its walls, in the act of expiration, increasing the expulsive force of the respiratory apparatus, by the sudden contraction of its circular fibres expelling the contained air, and so not only considerably augmenting this power, but also sustaining the expiratory current, more especially towards the termination of this act.

It might also be suggested as an organ for the production or modulation of sound, if these animals possessed such a faculty, which must be considered as very doubtful. The great size of its laryngeal aperture, and the absence of all constricting bands or apparatus, militates considerably against the latter use of this organ.

A dissection of the soft part of the upper surface of the head exhibited the relative anatomy of the

External nares.

These were two semilunar slit-like orifices situated on the superior surface of the cranium acted on by three separate planes of muscular fibres. The lips of these apertures, which have already been described, would permit a complete approximation, as also of considerable divarication.

The first or most superficial plane of muscular fibres consisted of the dilator naris,

which was exposed by raising off the dark-coloured integument and subjacent fascia; it consisted of a fan-shaped series of converging fibres, which arose from the outer border of the superior maxillary bone as far forwards as the anterior extremity of the intermaxillary and the tip of the cartilaginous septum or primordial vomer, extending backwards to the anterior extremity of the malar bone, and to that portion of the superior maxilla which formed the front wall of the temporal fossa. It was also connected with the dense fascia investing its superficial surface; the fibres of the muscle passed inwards, converging, and were inserted, the most anterior, into the cartilaginous median raphe that extended from the extreme point of the snout to the anterior extremity of the median sulcus or groove interposed between the lips of the external nares. The middle and posterior fibres ran to be inserted into the outer lip of the nares by a flat expanded tendon. From the direction of the fibres of this muscle its use appeared to be to divaricate the alæ or lips of the nares, and by so doing to give free access to the air either during inspiration or expiration.

The second or middle plane of muscular fibres consisted of two parts; first, a frontal portion, retractor alæ nasi, which was probably a representative of the pyramidalis nasi; it arose fleshy from the depression on the anterior and external portion of the frontal bone immediately in front of the anterior occipital crest; the fibres of this portion ran directly forwards, forming a fleshy belly about $3\frac{1}{2}$ inches in length, which was inserted into the cartilaginous part that formed the posterior and lateral part of the lip or boundary of the nares. The use of this portion of the muscle appeared to be to retract and fix the posterior and external lips of the nares, and in conjunction with the following muscle to close these orifices. The second portion of this muscle, or constrictor naris, arose from the anterior edge of the temporal fossa, and from the upper edge of the superior maxillary bone; its fibres passed, in a curved direction, forwards and inwards to be inserted into the anterior superior surface of the median cartilaginous raphe, some of the fibres passing into the outer lip of the nares, and others into the upper edge of the superior maxilla. The fibres of this muscle in passing forwards took an arched course, the concavity of which was directed inwards and upwards, and with the corresponding muscle on the opposite side embraced the nasal outlets like a sphincter; consequently, when these muscles contracted simultaneously, their action would be to compress the external alæ of these outlets and so close the nasal apertures. These muscles are probably the homologues of the compressores nasi.

The third, or deepest plane, was formed of a single muscle on each side, which arose from the whole of the upper and inner surfaces of the intermaxillary bone, and from the inferior and external aspect of the cartilaginous mass or primordial vomer, which occupied the fossa between these bones; the fibres curved outwards and backwards, and were inserted into the side of the cartilaginous body forming the outer boundary of the naris. This muscle, probably the representative of the depressor alæ nasi, seemed available to open the nostril, its action being to rotate the alar cartilage, and thus to withdraw these moveable nasal pads.

On dissecting off the muscles, a large solid central cartilaginous crest or primordial vomer was disclosed which extended anteriorly from the point of the snout, where it passed 7 inches in advance of the bony vomer and completed the median portion of the roof of the mouth, to the divergence of the intermaxillary bones posteriorly, where it divided into three; the two lateral parts passed forwards and gave support to the constrictor naris; the central division anteriorly, where it joined the primordial vomer, was cartilaginous, but posteriorly and superiorly it became fibrous.

The external or alar fibrocartilages were oval in shape, and so arranged as to be capable of completely stopping the nasal apertures; they were composed of a soft fibrocartilage permeated, especially on the external side, with fat-cells, which rendered them highly elastic; their inner or nasal surface was lined with a thick mucous membrane, arranged in longitudinal folds and of a deep bluish-black colour; on raising this a strong muscle was exposed, which arose from the outer side of the median cartilaginous crest, from whence it ran backwards and was inserted into the outer side of the nasal fibrocartilage; this muscle by its contraction would draw forwards and inwards the anterior extremity of the fibrocartilage, and so remove it from the nasal aperture, leaving this orifice perfectly free. The closing of these orifices appeared to depend, in a great measure, on the relaxation of this muscle.

On raising the posterior border of the second plane of muscles, a large nerve, the infra-orbital branch of the fifth pair, was seen emerging from the foramen in the superior maxilla; it divided into three branches, which were traceable into the muscles and integuments of the nares.

As the blowholes are the only orifices through which the atmospheric air is allowed to pass during the act of respiration, we find accordingly that these outlets are amply provided with the means of carrying on this important function. Their alæ can be divaricated by the simultaneous action of the superficial and deep muscular planes, and, on the other hand, they could be brought together by the middle plane of fibres, which from their fasiculi running in an elliptical direction on each side of the blowholes, formed a regular sphincter and approximate the external lips of the orifices; at the same time the central portion of the muscle, from its deep-seated position, would press the external or alar cartilages against the septum, and so completely close these passages; the pressure of water from the outside may account for the fact that the muscles for opening are much more powerful than those for closing these canals.

There are some points worthy of notice respecting the mechanism by which the functions of respiration and deglutition are performed, in this and other Cetaceans, which we think have not hitherto been clearly explained.

As regards the first or *respiratory* function, it is obvious that when the animal approaches the surface of the ocean its first act will necessarily be that of expiration, in order to expel from its lungs the deoxidized air contained therein after the preceding inspiratory act; expiration is accomplished in a great measure, as in other mammals, by the contraction of the abdominal muscles drawing in the ribs, and propelling upwards

the musculo-tendinous diaphragm, and thus forcibly diminishing in every direction the capacity of the thorax; this action, from the great mobility of the ribs, their non-attachment anteriorly, and the obliquity of their position, can in these animals be carried to an extent proportionally greater than in any other mammals. Whilst this is being accomplished the entire larynx is drawn forwards and elevated, and the posterior and internal lips of the arytenoid bodies being at the same time brought into contact, form by being overlapped and covered by the flexible margins of the epiglottis a protrusible tube; in this state the upper opening of the glottis is extended into the pharyngeal orifice of the nares. The elevation of these parts appears to be accomplished in the following way:—First, the styloid bones or great cornua of the os hyoides are drawn upwards and fixed by the action of the stylohyoid muscles; next, the body of the hyoid bone is raised and carried forwards by the hyokeratic and kerato-pharyngeal muscles, whose combined action is to approximate the body to the cornua; and finally, the stylopharyngeus, constrictor and hyoepiglottic muscles draw forwards the glottis, whose tubular shape is preserved by the arytenoepiglottideus muscles approximating the lateral and free margins of the arytenoid bodies on each side to the epiglottis in front of them, the edges of the latter overlapping those of the former. This being accomplished, the palatine muscular fibres situated round the nasopharyngeal orifices contract and grasp the intruded glottis, and so by this beautiful and simple contrivance a free and direct channel of communication is formed between the lungs and the external orifices of the blowholes. While these actions are in progress the palatoglossus by its contraction closes the isthmus faucium, and shuts off all communication with the mouth. The lateral prolongations of the peculiar hood-like fold, before described as existing between the root of the tongue and epiglottis, would seem by their contraction to engirdle the anterior and lateral parts of the base of the tubular prolongation of the glottis, and so fix and bind together the organs forming this aperture. The anterior nares or blowholes being now opened by the action of the muscles before described for this purpose, and the fibrocartilaginous pads being at the same time withdrawn from the nasal passages by the appropriate muscles, the contained air is expelled, and this by its expiratory momentum may likewise assist in freeing the channel.

This act of blowing or spouting has been represented by G. Cuvier and others as having the appearance as if the animal forcibly ejected from its nostrils a *large mass* of water, and it has therefore been conjectured that the creature for this purpose must necessarily be furnished with some peculiar anatomical contrivance to retain such fluid. But this phenomenon may be accounted for in a different way, and without the gratuitous assumption of the animal having any such reservoir, which, so far as the anatomical structure of the present specimen bears on the point, is an assumption absolutely groundless.

Expiration is most probably commenced when the animal's head is near, but as yet beneath the surface of the water, and when but a slight stratum of fluid is lying above the outer orifices of the blowholes, which as the creature expires, combined with the

condensed halitus from the lungs being suddenly and forcibly driven upwards, may simulate a column of water; but that a large quantity of ingested fluid is got rid of by the act of expiration through the blowholes is, at least in the present instance, not only improbable, but in fact impossible, as from the mechanism of the organs engaged there was no contrivance by which the entrance of the fluid from the larynx, if present, into the lungs or digestive canal could be prevented; the only appearance of a receptive cavity in connexion with the respiratory tract was the large laryngeal pouch before described, but from the position of the opening of this sac into the cavity of the larynx, any fluid admitted into it must necessarily enter the lungs and so suffocate the animal.

Expiration having been accomplished, the animal now protrudes its blowholes above the surface of the water; the abdominal muscles are relaxed, and the diaphragm descends; the nares at the same time being opened, a stream of air enters the lungs through the larynx, the apex of which is still retained in the orifice of the posterior nares; the thorax is likewise expanded by the action of the serrati, intercostales, and pectoral muscles, and when the act of inspiration is accomplished the larynx descends, the middle plane of muscles closes the nares, and aëration of the blood, by means of the inhaled air, is carried on. In other cetaceans the process of respiration is effected in a manner somewhat similar, though the anatomical arrangements admit of individual varieties. In *Globiocephalus Svineval* there was developed a special muscle, the basio-thyrohyoid, which arose tendinous from a longitudinal antero-posterior line on the basilar process of the occipital bone, and was inserted into the posterior border of the thyroid cartilage, and into the posterior cornu on the basihyoid bone; this muscle seems to be developed for the purpose of drawing the larynx upwards, and so compressing the pharynx against the spine, and preventing the possibility of the entrance of air into the digestive canal. In this animal likewise the superior constrictor of the pharynx arose from the anterior edge of the internal pterygoid plate, about 3 inches within the openings of the posterior nares, from whence the fibres ran inwards and backwards to the median line of the pharynx, which it forcibly draws forwards. These additional arrangements facilitated the drawing upwards of the larynx.

The act of *deglutition* appears to be more simply performed; the animal, according to Lilljeborg and others, subsists chiefly on fish, together with mollusca and medusæ; in the stomach of our specimen nothing was found but the crystalline lens of the eye of some small fish.

The mode of taking food appears to be as follows:—the mouth is opened under water, and with the relaxed submaxillary pouch encloses a quantity of animals and fluid together, while at the same time the palatoglossus being contracted, shuts off all communication with the pharynx; the mouth is now closed, and the pouch contracted by the action of the mylohyoid or constrictor muscle aided by the elasticity of the pouch itself; these together exert a force sufficient to expel all the superfluous water, while the baleen plates prevent the escape of all solid matters, which being strained are left in a condition fit to be swallowed: the palatoglossus now relaxes, and the tongue being

pressed upwards the contents of the mouth are passed into the pharynx; the glottis is simultaneously closed by the approximation of the arytenoid bodies, which are brought together by the arytenoideus proprius and drawn upwards and forwards by the arytenoepiglottidis muscle, when they meet and are overlapped by the tip of the epiglottis, the whole being covered by the hood-like fold of mucous membrane, which seems to be especially adapted for this purpose. It will be seen that by this admirable contrivance all extraneous matter is strictly excluded from the larynx while the food is passing through the œsophagus to the stomach.

The Trachea and Lungs.

The trachea was continued backwards from the posterior and inferior extremity of the larynx for about $5\frac{3}{4}$ inches, and terminated opposite the superior aspect of the arch of the aorta by bifurcating into the right and left bronchi; it measured in diameter $3\frac{3}{8}$ inches, and its cartilaginous rings, which were irregular, often bifurcating and coalescing, formed almost complete circles; the last was peculiar in shape, and consisted of two arches united in the centre so as to form the commencement of the two bronchial tubes; this union was much more extensive in front than behind, and the pair of rings immediately above were joined to it on the anterior aspect by means of a cartilaginous plate continued up from the vertex of the angle of union of the lateral halves of the last ring.

The tracheal mucous membrane had a peculiar folded arrangement, being traversed by a number of longitudinal sulci, apparently to permit of its being distended. The right bronchus bifurcated immediately after its origin, which gave to the termination of the trachea the appearance of its being trifurcate: both these bronchi on the right side entered the root of the lung at a considerable distance from each other. The upper, or what may be termed the accessory division, was much smaller than the inferior, measuring only 1 inch in diameter, whereas the lower division was nearly $2\frac{1}{4}$ inches in the same direction. On the left side there was but a single bronchus, whose diameter was 3 inches.

The right lung measured $15\frac{1}{4}$ inches in length and 7 inches in width; the left was 17 inches long and $6\frac{1}{2}$ inches broad, and passed further forwards than the right. The bases of both lungs were much hollowed where they lay in contact with the diaphragm, and measured 13 inches in breadth from their anterior to their posterior borders.

In the root of the right lung the pulmonary artery lay above the bronchus, and the two principal pulmonary veins lay below and on a plane anterior to it; at the top of the root a small accessory pulmonary vein passed outwards, lying 3 inches below the truncated apex of the lung, and $3\frac{1}{4}$ inches above the pulmonary artery.

On the left side the pulmonary artery lay also above and behind the bronchus, and the veins, two in number, were situated one on a level with the bronchus, the other about 3 inches more posteriorly. The lining membrane of the pulmonary veins displayed a series of longitudinal plicæ similar to those seen in the trachea. The roots of

the lungs were bounded anteriorly and posteriorly by the pulmonary plexuses formed by the pneumogastric and phrenic nerves. The divisions of the bronchi were traceable into the substance of the lung for a very considerable distance, retaining their cartilages to their seventh division, when the tubes measured about one line in diameter. Nothing remarkable was found in the minute anatomy of the lung; its parenchyma was highly elastic, and contained a quantity of yellow fibrous tissue.

Pericardium and Heart.

The pericardium was conical in shape, with its expanded base attached posteriorly to the central portion of the muscular and tendinous diaphragm; it exhibited no peculiarities in its arrangement or in the sheaths accompanying the vessels that arose from the heart, which were similar to those of mammals in general.

The crescentic fold described by Marshall, which corresponded to the obliterated left superior vena cava, was not perceptible; this fold, we may observe, does not seem to bear in its development any relative proportion to the size of the animal in which it is found, as, for example, we have not seen it nearly so well developed in the horse as in the human subject.

The heart in its external configuration displayed little worthy of special remark; it was rather flatter than the hearts of most mammals, and had no white spot of Baillie; its apex was rounded, and its auricular appendices were short and blunt; the coronary arteries were large, forming a vascular collar round the circumference of the organ, and sent off branches which, as usual, marked the divisions between its cavities.

The wall of the right auricle was about the fifth of an inch thick, and its endocardiac aspect exhibited very strong musculi pectinati; the greater eustachian valve was but slightly marked, but the lesser fold was more distinct, and the superior cava had no protecting valve.

The foramen ovale was obliterated, but the annulus Vieussensii surrounding it was well defined, prominent, and muscular. The right auriculoventricular opening was guarded by the usual tricuspid valve, which was extremely strong and well marked.

The wall of the right ventricle was about $\frac{3}{8}$ of an inch thick, and its cavity was crossed by a strong trabecula, the moderator band of King, which was attached to its anterior and septal walls. The conus arteriosus was short and dilated, and, as usual, gave off the pulmonary artery.

The left auricle was rather smaller than the right; its wall was also smoother and thinner, measuring only $\frac{1}{10}$ of an inch in thickness; five pulmonary veins opened into it, two on the left and three on the right side.

The wall of the left ventricle, at the base of the heart, measured $1\frac{3}{4}$ inch in thickness and $\frac{3}{4}$ of an inch at its apex; its muscular bundles were extremely powerful, and its mitral valve large and strong.

The aortic and pulmonary valves displayed no novel anatomical points worthy of remark.

The aorta, after its origin from the left ventricle, was directed upwards and forwards, and at the sinus of Morgagni, which was well marked, it measured 3 inches in diameter; its coats were a line and a half in thickness and highly elastic. The entire length of the arch was 8 inches; it curved to the left side, immediately beneath the thymus gland and in front of the trachea. The arteria innominata arose from its first curve, corresponding to the upper border of the sinus of Morgagni; it then ran forwards, outwards, and towards the right side, being crossed in front by the left vena innominata just as it was about to terminate by dividing into the right carotid and right subclavian arteries; its entire length was 4 inches, and its diameter nearly $1\frac{1}{2}$ inch; its point of division was situated about 4 inches above the first rib. The left carotid arose about 5 inches to the left of the arteria innominata; it then ran upwards by the side of the thyroid body, where it measured about 1 inch in diameter, and continuing its course forwards for about 5 inches, terminated in the usual manner by dividing into the external and deep-seated branches. $1\frac{1}{2}$ inch still further to the left of the carotid the subclavian arose, and after a course of $\frac{3}{4}$ of an inch it gave off three branches, viz. first, the vertebral, which ran directly forwards and outwards to the transverse process of the seventh cervical vertebra; second, the transversalis colli, which passed forwards and outwards; and third, the axillary, a small branch that passed outwards to supply the rudimentary anterior extremity.

The descending aorta passed backwards into the posterior mediastinum, where it measured only 1 inch in diameter and gave off intercostal branches, twenty-two in number; it then passed through the diaphragm into the abdomen, in which cavity it was continued backwards, finally terminating as caudal by being distributed to the muscles in this region. In this part of its course its branches presented no variation from the ordinary arrangement, with the exception that the superior mesenteric arose from the cæliac axis, a variety occasionally met with in the human subject.

The main trunk of the pulmonary artery was $2\frac{3}{4}$ inches in diameter, its coats being about $\frac{1}{4}$ of an inch thick; it passed to the left of the aorta, and after getting behind that vessel divided into the right and left branches. The large cord-like ligamentous remains of the ductus arteriosus was crossed by a cardiac branch of the pneumogastric nerve, which passed downwards in front of the aorta, between it and the pulmonary artery, to join the cardiac plexus.

Digestive Apparatus.

No distinct superior pharyngeal constrictor was visible; but a few scattered muscular fibres seemed to be substituted for it, having no very determinate course or attachment. In other Cetaceans, however, this muscle is present, as, for example, we found it in *Globiocephalus Svineval.*

The middle constrictor was a flat fan-shaped expanded muscle; it arose fleshy from the entire length of the posterior border of the great cornu of the os hyoides; the fibres ran in a radiating manner, the anterior forwards and inwards, the middle directly inwards and the posterior backwards, all being inserted into the median raphe on the

superior aspect of the pharynx, along which it stretched for about 9 inches, passing forwards as far as the base of the skull.

The stylopharyngeus was a strong round fleshy band; it arose external to the basilar process of the occipital bone, crossed over the middle of the anterior fibres of the last muscle, and was inserted into the median raphe.

The inferior constrictor took its origin from the whole length of the posterior border of the thyroid cartilage; its fibres passed inwards, and were inserted into the posterior portion of the median raphe for an extent of about 11 inches.

These two constrictors were separated by the glossopharyngeal nerve and the keratopharyngeal muscle.

From the lower border of the pharynx, and on a level with the posterior margin of the cricoid cartilage, the œsophagus was continued through the posterior mediastinum for the length of about 18 inches, where it terminated at the cardiac orifice of the stomach.

The œsophagus measured 5 inches in circumferenee, and presented the usual number of coats, viz. cellular, longitudinal, circular muscular, and mucous; the last, or mucous, was covered with a thick, softish, easily detached epithelium, which was only connected closely to the subjacent basement layer by the processes which it sent in through the numerous mucous crypts; these latter were arranged in longitudinal rows extending the entire length of the tube. Intervening between these series of apertures were rugæ whose direction was principally longitudinal, their use being to facilitate distension; the rugæ became smaller and more irregularly placed as they approached the cardiac orifice, where their direction and appearance became more assimilated to those on the mucous surface of the first stomach. The entire thickness of the wall of the œsophagus measured $\frac{1}{4}$ of an inch.

The gastric cavities were five in number, well covered with peritoneum; of these the second was the largest, and the third much the smallest.

The first stomach was somewhat oblong in shape; it measured $12\frac{3}{4}$ inches in length and 20 inches in circumference at its central part; the spleen was closely connected to its fundus by a short gastrosplenic omentum. The wall of this cavity was $\frac{3}{4}$ of an inch thick, was strong and firm, and divisible into the following layers, viz. first, peritoneal; second, longitudinal muscular, very well developed; third, circular muscular; fourth, fibrous, firm, in colour white, and about half a line thick; fifth, a thin pulpy layer of fatty material of a light yellowish colour; sixth, a white soft areolar lamina containing nerves; seventh, a second bright yellow fatty layer similar to the fifth; eighth, basement vascular layer of mucous membrane; and ninth, epithelial or cuticular coat. The only thing found in this cavity was the crystalline lens of a small fish, which corroborates the statement of Lilljeborg that this animal feeds occasionally on fish.

On opening the stomach its lining membrane presented a very remarkable rugose appearance, the long axis of the rugæ running transversely from right to left, each fold being about a line in depth; from these primary transverse ridges secondary spurs were continued at unequal intervals, thus giving the whole surface a minutely convoluted cerebriform appearance.

A number of wide and rather shallow furrows ran from the cardiac orifice in a radiating manner over the wall of the cavity, passing towards its fundus, crossing but not interrupting the epithelial rugæ just described.

The orifice of the œsophagus measured in its transverse diameter about $1\frac{3}{4}$ inch; it was thrown into plicæ in the direction of the long axis of the tube, and presented a small valvular fold on the anterior border of the opening, which measured about $\frac{1}{2}$ an inch in depth and rather more than $\frac{3}{4}$ of an inch in length; numerous pore-like orifices studded the entire epithelial surface, both on and between the above-described furrows.

The aperture of communication between the first and second stomachs was defined by the sudden cessation of the cuticular lining of the former in a fringe-like border; at this point the diameter of the viscus measured $3\frac{1}{4}$ inches.

The second stomach was much larger than the first; it measured 27 inches in length and 22 inches in circumference, cylindrical in shape with rounded extremities, and in contour slightly sigmoid. The coats were much thinner and softer than those of the last-described cavity, their entire thickness being only $\frac{3}{8}$ of an inch, and consisted of a peritoneal, circular, a fibrous, a submucous, which was very vascular, and an epithelial layer. The rugæ on the mucous surface were much thinner, sharper, and more prominent than those in the first stomach, and towards the superior and posterior or concave margin they became very small, and interlacing with each other, produced quite a reticulated appearance in this situation. The inferior border was thrown into transverse folds, about twenty in number, which ran in an antero-posterior direction parallel to each other; each fold at its origin was small and flat above, but towards the convex edge it became large, thick, and prominent. These folds extended over the surface for about the length of 10 inches, with an average depth between them of 1 inch; in the centre, at the distal extremity of the second stomach, these folds attained their greatest development, becoming smaller as they approached the orifice leading from the second to the first cavity. They were crossed by many smaller longitudinal rugæ, which varied both in size and direction.

The mucous membrane lining the orifice leading from the second to the third stomach was perfectly smooth; the opening measured 5 inches in circumference, and was surrounded by a well-defined muscular annulus or sphincter, whose free margin was mainly directed towards the cavity of the second stomach, and measured in depth about 1 inch.

The communication between the third and fourth stomachs was through a remarkably constricted oblique and valvular aperture which measured in circumference 5 inches; it was guarded on the distal side, or that towards the fourth cavity, by a crescentic fold of mucous membrane which was very thin, and gradually shaded away on the wall of the cavity on both sides; its extremities were about 1 inch apart, and its depth in the centre measured about $1\frac{1}{2}$ inch.

Within this orifice there was a pouch about $3\frac{1}{4}$ inches long, 3 inches broad, and $3\frac{1}{2}$ inches deep; it opened backwards into the third stomach by a large orifice, which was surrounded by a muscular annulus whose fibres were better marked below than above;

this pouch, with the exception of one transverse fold of mucous membrane that traversed its fundus, was perfectly smooth.

The fourth stomach in shape was broadly pyriform; its length was about 14½ inches, its circumference varying from 24 inches at its widest or proximal to 19 inches at its narrow or distal extremity; its mucous surface was smooth, being destitute of rugæ; its coats were extremely thin, especially at its inferior margin, where they measured only one line in thickness; superiorly the thickness increased to 1½ or 2 lines. Immediately below the opening from the third stomach, the fourth dilated and afterwards narrowed gradually towards its distal extremity; in this situation a few longitudinal rugæ were irregularly distributed, and some glands were detected, more particularly round the orifice leading into the fifth stomach; this orifice presented a crescentic-shaped appearance on its proximal, but was oval in shape on its distal side, the difference in shape being caused by a semilunar fold or valve-like process which partially surrounded the aperture on the side corresponding to the fourth cavity; this fold measured along the chord of its arc 1¼ inch; the distal side, or that corresponding to the orifice of the fifth stomach, was even narrower, measuring only 1 inch in diameter, and was surrounded with a few longitudinal rugæ and the orifices of some mucous glands.

The fifth or last stomach was small in comparison to the others; in length it measured only 9½ inches, and in circumference about 20 inches; its coats were slightly thicker than those of the fourth cavity; its walls were studded with numerous glandular follicles, and it communicated with the duodenum through the medium of a small pylorus, which was guarded by a muscular sphincter and marked with numerous rugosities, both longitudinal and transverse; the circumference of the pyloric orifice measured 4½ inches. This cavity seemed to be more especially the seat of gastric secretion, and so probably of digestion. In other Cetaceans the second or third stomachs are often supposed to be the true digestive cavities.

On the outer or peritoneal surface and inferior border of these cavities the omentum was attached, not quite ½ an inch in thickness, as in the case described by Hunter, but thicker at its left than at its right extremity; however, nowhere did it exceed ¼ of an inch in thickness. The pneumogastric nerves ramified over the surface of all the stomachs, as also did the gastric arteries, but more especially on the coats of the second stomach; the long descending branch of the gastric artery ran in a groove which separated the first from the second stomach. Immediately below the pyloric extremity of the fifth stomach the duodenum commenced and passed on to join the jejunum; its entire course was superficial in the abdomen, only overlapped by the colon and omentum, and its length, measured to where it passed through the root of the mesentery, was about 1 foot. The hepatic and pancreatic ducts lay on its posterior aspect; these ducts joined each other about ⅓ an inch before they entered the outer or peritoneal covering of the intestine. The conjoined duct ran very obliquely for 2 inches between the coats of the intestine, opening in a valvular manner under a very peculiar hood-like fold of mucous membrane into the interior of the duodenum, at the distance of about 6⅝ inches below the pyloric orifice.

The remaining portion of the small intestine extended from the duodenum to the cæcum; it measured in length 80 feet, and in circumference from 4 to 5½ inches; in its course four remarkable dilatations were observable placed at variable distances from each other; some of these measured 10 inches in circumference and about 4 inches in length. Of the three divisions of the small intestines the duodenum was the largest in calibre, the jejunum the smallest, and the ilium intermediate in size.

The mesenteric vessels were very large and numerous, and many lacteals and glands clustered about the root of the mesentery, more particularly at the point of junction between the ilium and cæcum; a chain of glands lay in the concavity of the terminal coil of the former, enclosed in the layers of the mesocæcal fold of peritoneum, each of which was about the shape of a kidney bean; similar glands were traceable upwards, but less distinctly, and at greater intervals along the entire length of the ilium.

The intestinal wall consisted of four coats. First, a peritoneal; second, a longitudinal series of muscular fibres; third, a series of circular fibres, which in the regions of the dilatations or pouches before mentioned were thinner and more expanded than elsewhere; and fourth and lastly, the mucous lining, which was thrown into five or six folds or plications, which in the upper portion of the intestine were arranged principally in a longitudinal direction at irregular intervals round its wall; these longitudinal folds were here and there connected by transverse ones, which were best marked along the concave or attached margin of the gut, and projected from its mucous surface for the depth of about 3 lines.

The depth of the primary or longitudinal plicæ varied from 6 to 9 lines; sometimes, but rarely, they joined each other at an acute angle, and wherever this occurred they enclosed a long irregular lozenge-shaped space; in the intervals of these folds the mucous membrane was perfectly smooth, particularly on the convex wall of the intestine; in the other portions, especially on the sides of the jejunum, numerous patches of mucous follicles were distributed. In the ilium transverse folds (valvulæ conniventes) took the place of the longitudinal ones and were increased in size. The glands of Peyer occurred, both solitary and in scattered patches, all through the ilium.

On laying open one of the before-mentioned dilatations, the mucous membrane lining it was seen to be arranged in the form of crescentic folds at each of its extremities, the concavities of the folds being placed in opposite directions, and the portion of the wall of the gut intervening between them was sacculated and extremely thin, being but scantily covered by the muscular coat; this was most observable at the fundus of each pouch, where the wall of the intestine seemed only to consist of the serous and mucous coats.

In the parietes of the jejunum, at its gastric extremity, a number of hard tubercular bodies were situated, which to the touch conveyed the sensation of their being lymphatic glands; these, on opening into the intestine, presented on its mucous surface the appearance of somewhat bluntish conical papillæ, which were perforated at their apices by small orifices, one on each eminence, capable of admitting the head of a small pin. These apertures led into tortuous canals enclosed in the coats of the intestine, and were

for the most part occupied by the bodies of entozoa, one in each nidus, their heads being firmly attached by hooks imbedded in the fundus of each canal.

On examination these animals proved to be the *Echinorhynchus porrigens* of RUDOLPHI, by whom they were discovered in the same species of whale, and described in his 'Synopsis Entozoorum.' They are also mentioned by HUNTER in the second volume of his 'Essays and Observations'*.

The large intestine extended for the length of 5 feet 8 inches, and communicated with a cæcum about 8 inches long, which latter exhibited no vermiform appendix; the blind or cæcal extremity of this intestine was situated on the right side, and pointed downwards and backwards, with the small intestine lying along its inner margin enclosed in the layers of the mesocæcum for the distance of about 7½ inches. This portion of the large intestine was cylindrical, and measured 7 inches in circumference; from it the colon was continued forwards and to the left side, forming a few slight curves for about 1 foot of its extent and terminated in the rectum, which was 4 feet long, and passed directly backwards to the cloaca. The circumference of this intestine varied from 5 at its centre to about 2 inches at its cloacal or anal termination; this latter was surrounded by circular muscular fibres that formed a well-defined and strong sphincter. The colon was imperfectly sacculated, and its coats were nearly the same as those of the small intestines. The whole intestinal tract was thus about six times the length of the body of the animal.

The liver was a large brownish-coloured gland, irregular in outline, and presenting two surfaces and two edges. The upper surface measured 22 inches across from side to side, and was divided into two lobes right and left; of these the former was the smaller in area; it was convex, and measured 16 inches in depth from its vertebral or thick to its ventral or thin edge. The left lobe was thinner, flatter, and more pointed in front; it measured 16½ inches in breadth.

The anterior edge was thin and excavated in the middle into an angular notch, 6 inches in depth, for the reception of the umbilical vein. The posterior edge was thick and nearly straight, corresponding to a very short coronary ligament, whose layers were several inches apart. The falciform ligament separated these lobes, and was readily separable into two layers; it ran the usual course from the umbilical vein in front to the vena cava behind.

The under surface was divided into three lobes by two intervening fissures. The right lobe below was smaller than the corresponding portion on its anterior surface, and in shape was somewhat crescentic, being, however, much wider anteriorly than along its vertebral margin; to the left it was separated from the Spigelian lobe by the vena cava, whilst its right edge corresponded to the right lateral ligament. The lower surface of the left lobe was concave, to accommodate the anterior aspect of the stomach; and its left margin presented two small notches and terminated anteriorly in a rather acute point.

* Specimens from this Whale have been described by Drs. JOHN BARKER and A. MACALISTER in the 'Proceedings of the Dublin Natural-History Society' for 1865.

The Spigelian or central lobe was elongated, and extended from the vertebral edge behind to the transverse fissure in front, which its free anterior tongue-shaped extremity bounded to the left; its right edge was thin and overlapped the vena cava partially behind, while it completely covered it in front, and became continuous with the right lobe by a thin bridge of hepatic substance; to the left a superficial branch of the vena portæ formed its boundary.

On the lower surface there were only two fissures, a transverse situated between the right and Spigelian lobes for the transmission of the vena portæ, hepatic artery and duct, and a second for the reception of the vena cava. The portal vein, which measured 4 inches in circumference, entered at the posterior and right side of the fissure, where it formed a dilatation or sinus, and then divided into two branches, of which the left was the larger. The hepatic artery, which was an inch in circumference, entered the fissure midway between the duct and the vena portæ, but on a plane inferior to both, and after dipping into the hepatic substance it divided into its right and left branches; the duct which lay to the left side of the fissure was formed by the union of the right and left branches, of which the latter was much the larger; its coats were highly elastic, and it measured about three-quarters of an inch in circumference. The umbilical vein entered the free border of the falciform ligament, and for about the distance of 5 inches from the left end of the transverse fissure was pervious. At 3 inches from its point of entrance it measured in circumference 2½ inches, but beyond this point it suddenly narrowed and soon after became obliterated; one of its branches united with a radicle of the left branch of the vena portæ, but no distinct ductus venosus could be seen, as is the case in the horse. The vena cava grooved the posterior surface of the gland for about 10 inches of its course, and formed a curve whose convexity was directed to the left, where it entered this channel; its circumference measured 5 inches, and at its point of emergence it had increased to 6 inches; its coats were comparatively thin, but were strengthened by a thin layer of the capsule of Glisson, and also by the liver substance that crossed and partly surrounded it: no gall-bladder existed. A transverse section through the gland displayed nothing but the orifices of extremely large vessels and coarse parenchymatous tissue connecting the granules.

The kidneys were elongated, prismatically shaped organs, presenting three surfaces bounded by three blunt edges; they lay one on each side of the spine in the anterior part of the lumbar region, and measured 15 inches in length by 5 inches in breadth, and 2½ inches in thickness in the centre, but diminishing towards their extremities by tapering into bluntish points. They were made up of numerous small lobules, some being round, others polygonal in shape, averaging about half an inch in diameter. The renal arteries entered their inner edge about 4 inches from the anterior extremity. The veins made their exit immediately above the arteries; the ureters commenced in the centre of the gland by the direct union of the tubes from the several lobules, there being no regular pelvis to the organ, and passed out from the inferior angle. Numerous large

filaments of the sympathetic nerve formed a plexus in front of the renal artery and vein on each side.

The spleen was single, extremely small, oval in shape, of a brownish-blue colour and attached to the left, or bulging of the first stomach, by a very short gastrosplenic omentum.

Ear.

The external auditory meatus opened on the surface by a very minute oval slit, whose longest diameter was about one line; it was concealed at the bottom of a furrow in the integuments, which was situated about $7\frac{1}{2}$ inches posterior to the external canthus of the eyelid, and on a line continued backwards horizontally from the commissure of the lips. From this aperture the auditory canal was continued backwards and inwards, winding round the posterior border of the squamous bone to reach the membrana tympani, where it terminated. The tube was readily separable from the deep-seated layers of the integuments, and was accompanied for a part of its extent by a fibrocartilaginous cord, which latter ceased about 3 inches from the superficial or tegumentary end of the meatus; internally it was united to the posterior and external angle of the periotic bone.

Immediately within the auditory meatus the calibre of the acoustic tube became slightly enlarged, but soon again resumed its original diameter; its whole extent was filled with a dark, greyish, sebaceous substance, the product of a very distinct series of ceruminous glands, the orifices of whose ducts were visible on its lining membrane. The ear-tube itself was composed of three coats. First, an external or fibrocellular one, which, about $1\frac{1}{2}$ inch from the external orifice, exhibited a thin stratum of circular muscular constrictor fibres enclosed in its layers; second, a middle or fibrous coat, which was much the thickest and strongest; and third, an internal or pseudomucous lining membrane, which was formed by an involution of modified cuticle, and arranged in three longitudinal folds that commenced about three-quarters of an inch from the external meatus, and terminated at the dilated portion of the canal.

The petrous or periotic bone was irregular in shape, and was lodged in a deep cavity between the squamous, pterygoid, basioccipital, and exoccipital bones; it was separated from the last-named by the foramen lacerum posterius, which gave exit to the jugular vein. In this cavity it was loosely placed, being merely fitted in, but not united by suture to the surrounding cranial bones, and consequently could be removed from the base of the skull without much difficulty. It presented four parts for description. First, a posterior or opisthotic; second, an anterior or prootic; third, a central or labyrinthic; and fourth, an inferior or tympanic. The opisthotic portion was prolonged backwards and outwards, being somewhat curved, with the concavity directed forwards; it lay in a deep groove in the squamous bone external and anterior to the sulcus for the depressor maxillæ inferioris, and was elongated, laterally compressed, and marked with irregular longitudinal ridges and furrows; where its lower edge was joined to the labyrinthic portion, it presented a prominent bony pedicle for the attachment of the tympanic element.

The prootic portion resembled a three-sided pyramid, having its apex pointing forwards and outwards; it projected into a cavity in the squamous bone, which was traversed by the Eustachian tube and tensor tympani muscle; its surfaces were rough and somewhat tuberculated, pierced by numerous small irregular foramina. On its inferior surface it presented a flattened pedicle, the processus anterior mallei, by which it was united to the anterior extremity of the tympanic bone. The tympanic portion was united to the petrous by the two flattened osseous pedicles just described; it measured $3\frac{1}{2}$ inches in length by 2 inches in breadth, and $1\frac{1}{4}$ inch in depth; in consistence it was extremely dense and brittle, and in shape it resembled a Cypræa or Bulla shell*: its upper and outer edge formed an extremely irregular extended lip and presented three notches, one situated in front of the anterior osseous pedicle of the tympanic bone, which was hollowed into a groove, and transmitted the tensor tympani muscle. The second notch was placed between the anterior pillar of the tympanic bone in front and a large promontory or ridge behind, which was continued upwards into a bony spur, and was continuous with the handle of the malleus. This notch was for the entrance of the eustachian tube. The third, or posterior notch, was situated behind the prominent ridge before mentioned, and in front of the posterior pillar of the tympanic bone; this was the auditory meatus, and towards it the fibro-cartilaginous ear-pedicle ran; in the recent state it was somewhat quadrilateral in shape and looked directly outwards. The inner lip of the tympanic bone was bullate and rugose; it did not come into contact with the petrous bone, but was separated from it by a small interspace, which in the recent condition was filled up by the continuation of the fibromucous membrane that lined the cavity of the tympanum.

The labyrinthic segment, which contained the internal ear, formed the roof of the tympanic cavity; it presented four surfaces, viz. superior, inferior, external, and a posterior. The superior surface was concave, rough, and porous; it exhibited internally a large oval aperture, the internal auditory meatus, whose long axis extended from below and without upwards and inwards, it presented internally on its floor two digital foramina, one small and obliquely placed, the aqueductus Fallopii, which transmitted the portio dura; the other, or larger foramen, was placed posterior and inferior, and terminated in a conical fossa, which was perforated by an irregular series of foramina (*Macula cribrosa*) for the transmission of the portio mollis to the internal ear: these foramina were separated from each other by an osseous ridge.

Posterior and external to the internal meatus there was a deep funnel-shaped depression which terminated in a cul-de-sac, internal to which, and directly posterior to the internal auditory meatus, was a small oval tube, which opened into the cochlea, the aqueductus cochleæ; the lower or tympanic surface of this portion of the bone was convex, hard, and smooth, and its anterior external border was perforated by the aperture of exit of the portio dura nerve. The posterior surface, which was continuous with

* A good representation of this segment is given in the 'Catalogue of Seals and Whales in the British Museum' by Dr. Gray, 1866.

the inferior, was marked below by a large smooth round aperture that led forwards and outwards to the inferior scala of the cochlea, the fenestra rotunda, above which was a sharp ridge that separated it from a large oval foramen, which was partly closed at its upper and anterior part by a thick scale of bone; this aperture was directed into the jugular foramen behind, and into the highest part of the tympanic cavity in front. The external surface was deeply hollowed, and formed part of the superior and internal boundary of the cavity of the tympanum; anteriorly it was marked by an opening for the transmission of the corda tympani nerve which communicated with the aqueduct of Fallopius; posterior and internal to this the fenestra ovalis was situated, which latter was closed accurately by the base of the stapes with its narrow end directed forwards, and its long axis pointing downwards and backwards; it communicated with the vestibular cavity, a small space that presented on its superior, posterior, and internal part extremely diminutive rudimentary semicircular canals. At the upper and outer part of the labyrinthic segment the aqueduct of the vestibule wound obliquely to the lower part of this cavity.

The inferior posterior angle of the labyrinthic segment was prolonged backwards and almost touched the opisthotic portion, where it formed the inferior boundary of the large irregular foramen before noticed; on the lower surface of this portion of the bone, the cochlea was exposed on removal of the superficial osseous substance, which was extremely dense.

The cochlea consisted of two turns, with the apex of the modiolus directed downwards and outwards; the upper and inner part, or scala tympani, was somewhat larger than the lower and outer, the scala vestibuli. The lamina spiralis was deeply grooved for the reception of the filaments of the auditory nerve.

The separate ossicles of the ear were but two in number, as the malleus was, correctly speaking, a process of the tympanic bone from which it projected upwards from the centre of its margin. It consisted of a short, stout handle, which was continued as a ridge of bone into the inner concave side of the tympanic cavity; its upper or free portion presented a rounded process (or capitulum) projecting outwards, surrounded by a slight groove that separated it from the articular surfaces for the incus; these latter surfaces were placed on the posterior aspect, and were twofold; one larger surface, ovoid in shape, looked backwards, the second, directed upwards, was convex, and situated immediately below and at right angles to the former; a minute canal traversed this bone, piercing it in a small triangular interval that separated the rounded extremity from the articular surfaces, and opened into the fossa beneath the rounded process above described.

The incus in shape somewhat resembled one of the phalanges of a diminutive great toe; it presented two concave articular facets on its posterior extremity, which corresponded perfectly with the surfaces on the malleus; the distal end was rounded and bituberculate, but non-articular. On the upper surface a small oval facet was placed, covered with a discoid articular crust to articulate with the narrow extremity of the following ossicle,—

The stapes, which was short, stout, and triangular in shape; it consisted of two rami,

a base, and an apex; the latter was covered with an epiphysary lamina, and articulated with a corresponding facet on the upper surface of the incus. The base was ovoid, and exactly fitted into the fenestra ovalis; the two rami were short and thick, cylindrical, and not channelled, and the intervening foramen small and ovoid.

Eye.

The eye was situated about 28½ inches from the point of the upper jaws, 2½ inches above, and directly in a line perpendicular to the commissure of the lips, and about 7½ inches in front of the external auditory meatus.

The eyelids measured 2½ inches along their free margin, and the depth of each of them, taken at its centre, was about thirteen lines from above downwards. The palpebræ were surrounded and defined by two crescentic furrows in the integuments, one of which was placed an inch above the margin of the upper, the other 1⅝ inch below the margin of the lower lid; neither Meibomian follicles, ciliæ, nor tarsal cartilages could be detected at the border of the lids.

The thickness of the palpebræ varied from three lines and a half at the base or orbital attachment to one line at the free margin, the cuticle being three-fourths of a line in depth. The basement layer exhibited a large number of elliptically disposed tendinous fibres intermingled with light-coloured granular fat; on removing this structure a strong orbicularis palpebrarum was exposed, which was half an inch thick at the anterior canthus, but gradually became thin towards its posterior angle, where it measured only one line in thickness. This muscle arose from the inner angle of the orbit; its fibres ran in a semicircular direction, coursing round the upper lid and became slightly attached externally to a strong tendinous band, that ran from the external canthus to the outer wall of the bony orbit; passing this the fibres of the muscles curved downwards, and were attached to the conjunctiva in the centre of the lower lid, where they were gradually lost.

On removing the orbicularis palpebrarum, a large quantity of dark-brown fat was disclosed, in which, at the lower and outer part of the orbit, the lacrymal gland lay imbedded; this latter appeared as a small bilobed granular body, of a yellowish-brown colour, and irregular in outline; it measured 2 inches in its long diameter, which extended from before backwards, and 1 inch transversely; its external or superficial lobe was somewhat square in shape, and was covered by the orbicularis palpebrarum; the internal or deeper-seated lobe was round, and lay within the orbit. Its ducts, about eleven in number, opened on the palpebral conjunctiva by oblique pores. A detached and lighter-coloured glandular body (HARDER'S gland?) lay above and external to the gland proper, and its ducts opened by numerous fine pores into the superior palpebral sinus. The lacrymal nerve lay inferior to the gland, and was traceable through it to the conjunctival surface of the eyeball.

Three folds of conjunctiva passed, almost horizontally, from the globe of the eye to the anterior canthus, or commissure of the lids; above and below the central fold two small

culs-de-sac were thus formed, which seemed as though they were rudimentary puncta; each of these extended to the depth of nearly a line, and was protected superiorly by a similar fold of mucous membrane.

The muscles of the orbit were ten in number*.

The superior rectus arose by two heads, one from the common tendinous sheath of the optic nerve, or ligament of Zinn; the second head had its origin from the superior margin of the optic foramen; these origins soon united and passed forwards to be inserted into the sclerotic coat, about $1\frac{3}{4}$ inch posterior to the edge of the cornea. This muscle was crossed superficially by the frontal branch of the ophthalmic division of the fifth nerve, and its tendon was split by the following muscle.

The superior oblique, which arose from the ligament of Zinn, ran forwards to a point about 3 inches behind the cornea, where the direction of its fibres was changed by passing through an opening formed by the splitting of the tendon of the superior rectus. From this pulley-like contrivance, which was lined by a small synovial bursa, the fibres of the superior oblique took a direction backwards, outwards, and slightly downwards, and were inserted tendinous into the sclerotic immediately in front of the insertion of the choanoid muscle.

The internal rectus arose from the inner side of the optic foramen; its fibres ran forwards on the posterior or external side of the globe of the eye to be inserted into the sclerotic about 1 inch behind the cornea.

The external rectus arose from the outer side of the optic foramen; its fibres ran forwards on the posterior or external side of the eyeball to be inserted into the sclerotic about 1 inch behind the cornea.

The inferior rectus had its origin, like the preceding, from the border of the optic foramen, and in addition from the ligament of Zinn; its fibres passed forwards and outwards, and in a manner precisely similar to the superior rectus. This muscle was perforated by the fibres of the inferior oblique; its tendon formed by the reunion of the separated fasciculi was inserted into the sclerotic at about $1\frac{1}{4}$ inch below and behind the cornea.

The inferior oblique was a short curved muscle; it arose from the inferior and anterior angle of the orbit; its fibres passed backwards and upwards, and having wound through the pulley-like contrivance formed for it in the inferior rectus, before referred to, was inserted into the posterior and inferior portion of the sclerotic.

The choanoid muscle was distinctly quadripartite, having its origin from the sheath of the optic nerve, and being inserted, as in other mammals, into the posterior aspect of the sclerotic coat.

The nerves seen in the dissection of the orbit were the third, which supplied the superior, inferior, and internal recti, the inferior oblique, and the choanoid; the fourth, which was distributed to the superior oblique; the sixth to the external rectus; and the fifth to the soft parts of the orbit.

* The levator palpebræ superioris was absent.

It is unnecessary to enter into any description of the globe of the eye, as it presented no points of departure from the frequently described normal cetacean type, in the great posterior thickness of the sclerotic, the comparatively small depth of the vitreous cavity, and the globular shape of the lens.

Conclusion.

The *Balænoptera rostrata* has already furnished a subject for several descriptions, each of them more or less complete. Of its osteology a most comprehensive and accurate account has been given by Mr. Flower in the Proceedings of the Zoological Society of London, 1864, with which, in most points, the osteological relations of the present specimen will be seen to agree, the main points of difference being that in our specimen the foramina between the lateral processes of the axis were not completed by bone, but remained as large oval-shaped notches with an interval of a little more than 1 inch between the free extremities of these processes.

Another difference in the present specimen was the existence of a small osseous tubercle on the left side of the centrum of the seventh cervical vertebra, the rudiment of a parapophysis. Mr. Flower notices the existence of a similar tubercle on both sides of the body of the same segment in a specimen in the Louvain Museum, but in the specimens described both by him and other authors this tubercle does not appear to have been present. Lilljeborg describes it as being developed in one individual more distinctly on the right side than on the left.

There was also a discrepancy in the number of vertebræ, which in our specimen amounted only to forty-six, whereas in most of the other recorded instances from forty-eight to fifty are stated as the number present*. It is, however, possible, as our animal was immature, that two or more cartilaginous terminal bodies might have been present, and from their rudimental state have escaped detection. The last caudal bony centrum measured three-quarters of an inch in its transverse, and five-eighths in its perpendicular diameter; in texture it was spongy and friable.

All the cervical vertebræ, as in the specimen described by Hunter, were free; whereas in the individual described by Dr. Gray in the "Zoology" of the 'Erebus' and 'Terror,' and also in one of the specimens in the Louvain Museum, the second and third were united by bone.

The shape of the sternum of the present specimen differed from those heretofore described; it had not the elongated cruciform outline of that described and figured by Mr. Flower, but, as before stated in this communication, it was somewhat heart-shaped; this difference, however, might possibly be due to the immaturity of the animal.

* A specimen from Bergen is described in the Scandinavisk Fauna, in which forty-seven vertebræ were stated to be the number present. This, however, Mr. Flower explains by the loss of the last one or two segments.

Explanation of Plates.

PLATE IV.

Fig. 1. Drawing of animal, made to the scale of 1 inch to the foot.

Fig. 2. Lateral view of myology of entire animal.

A. Pectoralis major.
B. Deltoid.
C. Infraspinatus.
D. Latissimus dorsi.
E. Dilator naris.
F. Humerus.
G. Masto-humeralis.
H. Sternomastoid.
I. Sternohyoid.
J. Mylohoid, covering maxillary pouch.
K. Depressor maxillæ inferioris.
L. Masseter, superficial fibres.
M. Temporal.
N. Rhomboideus.
O. Longissimus dorsi.
P. Serratus magnus.
Q. External oblique.
RR. Depressor caudæ minor.
S. Depressor caudæ major.
T. Levator caudæ.

PLATE V.

Fig. 1. Rete mirabile shown in the neck and in the axilla.

A. Masto-humeralis, cut across.
B. Upper axillary vascular plexus.
C. Lower axillary vascular plexus.
D. Cervical rete mirabile.

Fig. 2. Muscles of the paddle, front view.

A. Vertebral edge of scapula.
B. Subscapularis.
C. Acromion process.
D. Masto-humeralis, cut across.
E. Costo-coracoid ligament of the notch.
F. Supraspinatus.
G. Origin of teres major.
H. Insertion of teres major, cut and reflected.
I. Brachial artery and venæ comites.
J. Subscapular artery.
K. Long head of triceps.
L. Short or inner head of triceps.
M. Bifurcation of brachial artery.
N. Radial artery.
O. Ulnar artery.
P. Flexor carpi radialis.
Q. Space from which the palmaris longus had been removed.
R. Flexor digitorum communis.
S. Flexor carpi ulnaris.
T. Condylo-olecranon process.
U. Radius.
V. Ulna.
W. Carpus.
X. Metacarpus.
Y. Interposed fibrocartilages.
Z. Phalanges.

Fig. 3. Muscles of paddle, back view.

A. Supraspinatus.
B. Acromion process.
C. Ligament of the notch.
D. Origin of deltoid.
E. Infraspinatus.
F. Cartilaginous inferior angle of scapula.
G. Circumflex artery and vein.
H. Humerus.
I. Long head of triceps.
J. Teres major.
K. Latissimus dorsi, cut across.
L. External head of triceps.
M. Condylo-olecranon process.
N. Ulna.
O. Radius.
P. Extensor communis digitorum.
Q. Carpus.
R. Metacarpus.
S. Phalanges.

Fig. 4. Os hyoides.

A. Body.
B. Posterior cornua.
C. Anterior notch.
D. Articulation with great cornua.
E. Styloid bones, or great cornua.
F. Temporal extremities.

Fig. 5. Thyroid cartilage.

A. Anterior cornua.
B. Posterior cornua.
C. External margin.
D. Central notch.
E. Anterior margin.

Fig. 6. Skeleton of larynx, *in situ*, dorsal aspect.

A. Styloid bones, rotated outwards.
B. Articulation of styloid bones with the body of the os hyoides.
C. Os hyoides.
D. Cricoid cartilage.
E. Posterior border of thyroid cartilage.
F. Crico-arytenoid articulation.
G. Crico-thyroid articulation.
H. Arytenoid cartilage.
I. Transverse arytenoid ligament.
J. Epiglottis.

Fig. 7. Laryngeal pouch laid open, ventral aspect.

A. Os hyoides.
B. Wall of laryngeal pouch.
C. Arytenoid bodies.
D. Cavity of larynx.
E. Alveolations on mucous surface at apex of pouch.
F. Trachea.

PLATE VI.

Fig. 1. Sternum, showing its irregularly cordiform outline.

Fig. 2. Vertical section through the second vertebra or axis, showing the morphological relations of its different parts.

A. Anterior border of centrum.
B. Spinal canal.
C. Neurapophysis cut through.
D. Apical epiphysary crust or summit of odontoid process.
E. Central portion of the upper or second ossific part.
F. Upper epiphysary crust above the primordial fissure, which typically separates the odontoid segment from the centrum.
G. Inferior epiphysary boundary of the primordial fissure.
H. Cancellous tissue of centrum.
I. Inferior epiphysary crust.

There are four epiphysary crusts indicating the presence of two distinct vertebral bodies; the letters D and F indicating the upper and lower surfaces of the first or atlas-vertebra; G, I, the epiphysary laminæ of the second or axis-vertebra.

Fig. 3. Muscles situated on the front of the larynx.

A. Styloid bones.
B. Body of os hyoides.
C. Thyroid cartilage.
D. Laryngeal pouch.
E. A fold of mucous membrane reflected.
F. Kerato-pharyngeus.
G. Deep basio-keratic.
H. Superficial basio-keratic.
I. Sterno-hyoid, cut across.
J. Mylo-hyoid.
K, K. Thyro-hyoid.
L. Crico-thyroid.
M. Inferior constrictor.
O. Aryteno-epiglottideus.

Fig. 4. Laryngeal pouch laid open to show the thickness of its muscular wall.

Fig. 5. Front view of the tongue and pharynx.

A. The tongue.
B. Mucous folds extending along its border.
C. Isthmus faucium, through which is passed a rod.
D. Palato-glossus.
E. Opening of glottis.
F. Epiglottis.
G. Arytenoid bodies.
H. Alveolated surface of mucous membrane.
I. Peculiar pre-epiglottic hood-like fold of mucous membrane.
J. Posterior great cornua of os hyoides.
K. Trachea.
L. Œsophagus.

Fig. 6. Lateral view of tongue and pharynx. References similar to fig. 5.

Fig. 7. Glottis closed, as during the act of deglutition. The letters A, B, C, D, E, F, G, H, I, J, K, L refer to the same parts as in fig. 5.
M. Frena of hood-like fold of mucous membrane.

Fig. 8. Glottis with its lateral walls divaricated. The letters as before.

Fig. 9. Glottis open with the lips retracted. Similar references.

Fig. 10. Tympanic bone with malleus, incus, and stapes attached *in situ*.

Fig. 11. Periotic bone, lower surface.

a. The wall of the labyrinth.
b. Foramen ovale.
c. Pedicles by which the tympanic bone was attached.
d. Opisthotic portion.
e. Prootic portion.

PLATE VII.

Fig. 1. Cranium viewed from above, showing superficial anatomy of the blowholes.

A. Foramen magnum.
B. Occipital bone.
C, C. Temporal fossæ.
D. External nares.
E. Median sulcus.
F. Retractores naris.
G. Dilator naris.
H. Superior maxillary bone.
I. Primordial vomer.
J. Constrictor naris.

Fig. 2. Anatomy of the deep structures of the blowholes as viewed from above. The letters D, E, F, L, J, K refer to same parts as in fig. 1.

Fig. 3. Deep structures of blowholes as viewed from below.

A. Cavity of naris.
B. Black epithelial layer of mucous membrane.
C. Alar fibrocartilages.
D. Deep dilator of the nares.
E. Median crest.
F, I, J refer to same parts as in fig. 1.

Fig. 4. Stomach and liver. Front view.

A. Œsophagus.
B. First stomach.
C. Second stomach.
D. Third stomach.
E. Fourth stomach.
F. Fifth stomach.
G. Duodenum.
H. Liver.
I. Lesser omentum.
J. Spleen.

Fig. 5. Stomach, and outline of liver, back view. Letters A to G refer to same parts as in fig. 4.

Fig. 6. Tympanic bone, upper surface with the ossicles attached.

IX. *Researches on the Intimate Structure of the Brain.*—Second Series.

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CHAPTER I.

(1) Before I begin to describe the parts which form the subject of this communication, and to show how some of them are merely modified portions or developments of others that belong to the *medulla oblongata*, it will be advisable to recur to those morphological changes in the medulla, which I formerly pointed out as themselves arising from modifications of the *spinal cord*. And while in unravelling structures so extremely complex, such a course seems almost necessary to facilitate their comprehension, and convey to the reader a just notion of their morphological changes, in relation on the one hand, to the remaining parts of the encephalon, and on the other hand, to the spinal cord, it will afford me an opportunity of adding to this recapitulation some new facts that have been elicited by subsequent observation and a more extended experience.

It is gratifying to know that many of the results of my previous researches have been found to throw considerable light on certain diseases of the nervous system, especially on some forms of paralysis; and my own pathological investigations, as well as a close study of nervous disorders, have not only enabled me to shape my present researches as much as possible in accordance with the requirements of the pathologist, but, by pointing to the probability of certain anatomical connexions suggested by morbid symptoms, they have sometimes been the means of directing the course of my dissections in a very peculiar way*.

Some of the results comprised in this communication were obtained by adopting the method of dissection which I so strongly recommended in my former Memoir on the Medulla Oblongata. This plan consists in first making a number of delicate and careful dissections in the course of the fibres of the medulla, after it has been well hardened in strong spirit of wine, and then examining microscopically thin sections, made in different directions, and rendered transparent by the method which is known as my own. By this means we may often succeed in unravelling structures which are so complex that the investigation at first sight appears to be almost hopeless. Before the first dis-

* Dr. Hughlings Jackson (Physician to the Hospital for Paralysis and to the London Hospital) was the first to apply with great ability and industry the recent advances made in the minute anatomy of the nervous centres to the explanation of certain forms of paralysis. See especially 'The London Hospital Reports,' vols. i. and ii.; 'Ophthalmic Hospital Reports,' vol. v., &c.

section is begun, the hardened medulla should be macerated for a few days in strong spirit to which a small quantity of liquor potassæ has been added.

(2) At the highest point of the spinal cord, that is, at the first pair of cervical nerves, a transverse section of both the white and grey substances has the appearance represented in fig. 1, Plate VIII. Here the central grey substance (*t*) surrounding the canal, is of considerable size and of somewhat quadrangular shape. The neck of the posterior horn, *cervix cornu posterioris*, between *b'* and *o*, is very slender, and after stretching obliquely outward and backward from the middle line, terminates in a tuft or expanded extremity (*e*), which I have named the head or *caput cornu posterioris*. Between the central canal and the bottom of the anterior median fissure (*z*) are seen the decussating fibres of the *anterior commissure* extending on each side into the anterior white column; and in the deep and somewhat oval space (*d'*) of the lateral column (*d*) is a beautiful network of nerve-fibres and blood-vessels extending from the borders of the surrounding grey substance, and enclosing in its meshes a number of separate fasciculi which, a little higher up, in the medulla oblongata, cross over to the opposite side in front of the canal, and with the corresponding fibres of the anterior commissure, form the opposite anterior pyramids (see fig. 2). This network in the lateral columns contains a number of cells of different shapes and sizes, and which in animals are larger and more numerous than in Man.

(3) At the lower part of the medulla oblongata, that is, at the points of the anterior pyramids, a transverse section presents the appearances delineated in fig. 2. Here the central grey substance (*t*) has become narrower laterally, but has increased a little in depth from before backward in the middle line, and the posterior cornua (*b'*, *o*, *e*) are somewhat depressed or thrown more to the *side* of the medulla. The head or caput cornu posterioris (*e*) has increased in size, and begun to detach itself from the extremity of the cervix (*o*), being joined to it only by a small network containing slender bundles of longitudinal fibres, while the *cervix* itself has become a little thicker and more bulbous. Meanwhile, the bundles of longitudinal fibres contained in the meshes of the lateral columns cross over to the opposite side and plunge into the anterior columns to form the anterior pyramids at *z*. Ascending a little higher in the medulla (fig. 3), we find that while the *caput cornu posterioris* continues gradually to enlarge, it becomes more and more detached from the extremity of the cervix, or rather, is joined to it only by an increase of the intervening network already mentioned. At the same time the central grey substance (*t*) is encroached upon at the sides of the canal by the bundles of the lateral columns destined for the anterior pyramids; so that it has now a triangular shape, and together with the posterior cornua has some resemblance to a bird with both its wings expanded. From the root (*b'*) of the posterior cornu, on each side of the posterior median fissure, there now arises a grey network of fibres and blood-vessels containing nerve-cells and extending backwards through the posterior pyramid (*b*) to form the rudiment of its grey nucleus, or the post-pyramidal ganglion. Still further out, or nearer its extremity, the surface of the cervix cornu is raised into an eminence (*o*) which

projects into the restiform body (*c*) and constitutes its grey nucleus, or the restiform ganglion. In figs. 4 & 5, we find the same changes carried to a still greater extent. In the latter the decussating bundles of the anterior pyramids come only from the central grey substance and roots of the anterior cornua at the sides of the canal, and scarcely at all, as they did in fig. 4, from the lateral columns (*d'*), in which now the original network is again distinguishable; but it is larger, closer, more beautiful than we found it in fig. 1, and contains in its meshes bundles that have no share in the formation of the anterior pyramids*. The central grey substance, on each side of the median line, with the corresponding cornu, are now represented by *t' t b' o e* (compare figs. 1, 2, 3, 4 & 5). In fig. 6, which represents a transverse section a little higher up, just below the bulb of the posterior pyramids, we see that the *anterior* part (*t'*) of the central grey substance immediately surrounding the canal, is distinct from the posterior part (*t*), which covers it as by a kind of slanting roof. This latter is the vesicular tract or nucleus which gives origin to the upper roots of the spinal-accessory nerves. The anterior part (*t'*) is the vesicular tract or nucleus of the hypoglossal nerve, and is the analogue of a portion of the anterior or motor cornu of the spinal cord. These cornua are now scarcely distinguishable, but their places are traversed by roots of the hypoglossal nerves in their passage outward from their nuclei. Traversed also by these roots, and lying on the outer and back part of each anterior pyramid (*y*), is a layer of small cells which, higher up, form the anterior or inner part of the lamina of the olivary body (*h*, figs. 7 & 8). On the outer side of this *layer* is another distinct group of cells (*s*) connected by a network in the lateral column, and by nerve-fibres from the grey tubercle (*e*). This is the *antero-lateral* nucleus. Meanwhile the post-pyramidal and restiform nuclei (*b'*, *o*) have increased in size, and from these, especially the post pyramidal, proceed the chief portion of the decussating fibres which sweep round the front of the canal to enter the anterior pyramids.

(4) In ascending the medulla, the changes just described continue to increase in extent, and an important modification may be observed to take place. Fig. 7 represents a transverse section on a level with the lower part of the olivary bodies. Here the posterior pyramid (*b*) is nearly filled with its grey substance (*b'*); and that of the restiform body (*o*) has also greatly increased, although it is still covered by a considerable layer of white substance (*c*), by which it is separated, on the one side, from the postpyramidal nucleus (*b'*), and on the other side from the grey tubercle of ROLANDO (*e*). The spinal-accessory nucleus (*t*) and the hypoglossal nucleus (*t'*) have become enlarged and somewhat modified in shape. The latter (*t'*), on each side and in front of the elongated canal, now contains the large multipolar cells which give origin to the upper roots of the hypoglossal nerve (*x*). As in fig. 6, a number of fibres proceeding from the posterior pyramids (*b'*) sweep round in a direction forward and inward, and decussate in front of the hypoglossal nucleus; but instead of plunging into the opposite anterior *pyramid*, as they did in fig. 5, they terminate in the opposite *lateral and olivary column*. Further outward and forward, other arciform fibres, proceeding from the grey tubercle (*e*), and

* See my memoir on the Medulla Oblongata, fig. 19, Plate XV. Philosophical Transactions, Part I. 1858.

grey substance of the restiform body (*o*), sweep through and around the olivary bodies (*w*), some of them decussating in bundles across the back of the anterior pyramids (*y*), and assisting in forming the commissure of the olivary bodies.

(5) If we examine a transverse section of the medulla a little higher up, at the point of the calamus scriptorius, as in fig. 8, we find that the posterior pyramid (b') is not only thrown somewhat aside, but is a little more flattened, and more overlays the restiform body (*o*), from the grey substance of which it is separated only by a smaller quantity of white substance, as seen at b'' on the right side of the figure, where the dotted line indicates its outer boundary. At the same time the grey substance of the restiform body (*o*) has not only increased, but become more irregularly scattered in groups or patches through its white substance, as seen on the right side. It will be observed also that the spinal-accessory nucleus (t') has not only increased in size, but has begun to split into two lateral halves, by an antero-posterior fissure, at the bottom of which the central canal has still a distinct existence.

(6) On a level with the middle of the olivary bodies, the vesicular column which gave origin to the spinal-accessory nerve now gives origin to the vagus nerve, and may therefore be called the *vagal* nucleus. As shown in a transverse section (fig. 9, *g*), it is much larger than in the preceding figure, and has become completely divided by its median fissure into two lateral halves which are thrown widely apart at the floor of the fourth ventricle. It will be seen also that the increase in the size of the vagal nucleus takes place posteriorly, and apparently at the expense of the inner and under part of the posterior pyramid (*b*), upon which it encroaches; for it is evident that this pyramid, the limit of which is indicated by the dotted line (on the right side), is smaller here than in fig. 8. At the same time the grey substances of the posterior pyramid and restiform bodies have coalesced, the *white* portion of the restiform body which previously separated the two being now filled with its grey substance.

(7) A transverse section at a somewhat higher level presents the appearances seen on the left side of the medulla in fig. 10. Here t' is a section of the hypoglossal nucleus; *g* is a section of the upper end of the oval nucleus of the vagus nerve (*g*, fig. 11, Plate IX.), separating t' from *i*, which is a section of the auditory nucleus; *n* represents a longitudinal fasciculus of fibres of the lateral column between the two horns of *g*, the vagal nucleus; c' *o* is a section of the upper end of the pyriform grey substance of the restiform and postpyramidal bodies seen at *o*, fig. 12, Plate IX., and forming part of the auditory nucleus; and *p* is a section of the superficial grey substance (*p*, fig. 12) of the restiform body, covered by arciform fibres, as shown at A, fig. 13, which represents a lateral view of the medulla oblongata.

(8) Having thus briefly described these elementary structures of the medulla oblongata as they are seen in transverse sections, I will now exhibit them in longitudinal dissections, and employ, as far as possible, the same letters to indicate the same parts.

The posterior surface of the medulla oblongata, especially in the neighbourhood of the fourth ventricle, presents a greater diversity of appearance, amongst different indi-

viduals in Man, than in mammalia of the same species; but the general arrangement of the parts is nevertheless the same in all. Fig. 11, Plate IX. represents the posterior aspect of the medulla oblongata of a healthy middle-aged man who died in consequence of an accident: *a* is the cut surface of the lower end of the medulla, near the level of the points of the anterior pyramids. Adjoining the median sulcus, on each side, is the posterior pyramid (*b*), which, as it ascends to the point of the calamus scriptorius, expands into a thick bulbous mass (*b'*), and then diverges as a flattened band. External to the pyramid is the restiform body (*c*), which also enlarges as it ascends to the same level. Between this and the posterior edge of the antero-lateral column (*d*) is a superficial tract of grey substance (*e*), consisting of the expansion of the caput cornu, or dilated extremity of the posterior horn, and known as the grey tubercle of ROLANDO.

(9) At the point of the calamus scriptorius, on each side of the median line, is an oval or pyriform mass of ganglionic substance (*g*), of a bluish or pearly hue, constituting the superficial part of the nucleus of the vagus nerve, seen in section at *g*, fig. 9, Plate VIII.; and the small tract (*t'*) between this and the median fissure is the *upper* part of the nucleus of the hypoglossal nerve, seen in section in fig. 10, Plate VIII.; its *lower* end being covered in by the spinal-accessory nucleus and the posterior pyramids (*t*, *b'*, figs. 6, 7 & 8, Plate VIII.). In the angle between the outer side of this vagal nucleus (*g*, fig. 11, Plate IX.) and the upper divergent end of the posterior pyramid (*b'*), is the commencement of another and larger mass of grey substance (*i*), which is seen in section in fig. 10, Plate VIII., and forms the posterior nucleus of the auditory nerve, covered by epithelium.

(10) If the posterior pyramid *b b'* (*on the left side*) be carefully dissected from the restiform body (*c*), from below upwards, and be thrown forward, as shown in fig. 12, Plate IX., the exposed surface of the medulla will present the appearances delineated at the lower half of this figure; *t* is the downward continuation of the vagal nucleus or tract, constituting the nucleus of the spinal-accessory nerve, and previously covered in by the bulb of the posterior pyramid (*b'*), as shown at *t*, fig. 7, Plate VIII. in a transverse section. The oval mass *g*, marked off by the dotted line, is the inner and posterior portion of the *vagal* nucleus exposed at the point of the calamus scriptorius by the divergence of the posterior pyramid; *l* is its inner and more anterior portion, *covered*, like the spinal-accessory nucleus *t*, lower down, by the posterior pyramid, as seen in transverse sections, figs. 8 & 9, Plate VIII. Its upper point (*m*) forms the principal nucleus of the glossopharyngeal nerve. Along the outer and anterior part of this grey tract is a slender, longitudinal white column (*n*), which it lodges, as it were, in a groove (see *n*, fig. 9, Plate VIII.), and which tapers to a point as it descends obliquely inward along the base of the posterior pyramid to the mesial line (see *n*, fig. 12). In its course upward it ascends along the inner edge of the pyramid, and joins those fibres of the latter which pass into the anterior or outer auditory nucleus (see *n*, fig. 42, Plate XII., and fig. 58, Plate XIV.). On the outer side of this slender white column is a somewhat fusiform mass of grey substance, *o* (fig. 12, and *o o'*, fig. 24, Plate X.), imbedded

in the inner side of the restiform body, and exposed by the removal of the posterior pyramid. From the upper end of this fusiform grey mass a thin but broad layer of fibres, mixed with some grey substance (*p*), radiates upward and outward on the restiform body (see also *p*, fig. 24).

Fig. 14, Plate IX. is another careful dissection of some of these parts. The posterior pyramid (*b*) has been separated on the left side of the medulla from the inner side of the posterior column (*c*), and drawn aside, exposing nearly the whole of the continuous tract of grey substance constituting below, the *spinal-accessory* nucleus (*t*), and above, the *vagal* nucleus (*g*). This grey tract is intimately connected and continuous at the point of the calamus with the posterior pyramid (*b*), which sends forward into it a series of fine fibres. It is composed of round, oval, crescentic, or angular cells of large size, and a multitude of very fine fibres, of which a large number are longitudinal, and ascend with an inclination outward from *t* to the calamus scriptorius at *g*, where those on the outer side are connected with the posterior pyramid, from which again a bluish process runs forward to the oval or vagal mass left uncovered by the posterior pyramid at *g*, as shown in fig. 12.

(11) I will now briefly describe, in a series of transverse sections, the medulla oblongata of the Monkey, not only because it exhibits certain interesting deviations from the appearances observed in Man, but because it illustrates in a beautiful manner the highly important morphological changes which I have just been endeavouring to explain.

In the lower kind of Apes it differs considerably in shape from the human medulla, the antero-posterior diameter being less than the lateral, but proportionably greater than in other animals. The form and arrangement of its different parts may be considered to possess a kind of intermediate character between man and the higher mammalia. The anterior pyramids and olivary bodies are proportionably larger than in any inferior animal, and, as in Man, are the chief cause of the increase in the antero-posterior diameter of the medulla. The posterior and lateral white columns are largely developed and very distinctly marked off from each other by indentations, as in some of the mammalia, especially of the feline species. Fig. 15, Plate IX. represents the grey substance at the lower part of the medulla on a level with the points of the anterior pyramids. It has a greater resemblance to the corresponding portion in Man than in any other animal, but it differs from both in being pierced behind the canal by a greater number of longitudinal bundles of fibres, represented by the four dark dots*; *d''* is a close network with nerve-cells springing out of the side of the anterior cornu, and forming, higher up, the *lateral nucleus*. Fig. 16, Plate IX. is an exact representation of the grey substance at a level corresponding very nearly to the same part in fig. 4, Plate VIII. of the human medulla. Fig. 17, Plate IX. shows both the white and grey substances still higher up, at a point nearly corresponding to that of fig. 5, Plate VIII. Here it will be seen that the posterior pyramids (*b*, *b*), and especially the restiform bodies (*c*, *c*), are largely developed. The lateral columns (*d*, *d*) are particularly large, and the posterior

* The same letters indicate the same parts as in all the preceding figures of the human medulla.

parts of them are separated from the rest of the antero-lateral columns by a deep notch on each side. Fig. 18, Plate IX. represents the posterior part of the right lateral half of the medulla somewhat higher up. Here the posterior pyramid (*b*) has increased in dimensions and become completely filled with grey substance. The restiform body (*c*) has also enlarged and is singularly prominent. The grey substance differs in some respects from that of the human medulla. The part *o* of fig. 17 has now enlarged into a broad mass, from which a multitude of conspicuous fibres radiate like a brush to a remarkable group of cells situated near the surface of the column (*c*). These cells are much larger and more diversified in shape than those of the *inner* part (*o*) of the restiform nucleus, and are united together by their processes in a network. This grey mass exists likewise in Man, but is not so largely developed. It is seen to commence in fig. 5, Plate VIII. at *c' c'*, as a small offset from *o*, and to increase in size through the rest of the figures. In Man there are many more fibres constituting the network than in the Ape, and the cells are somewhat smaller and more elongated. Fig. 26, Plate X. represents some of the cells from the Ape, magnified 220 diameters. They are in close vicinity of bundles of longitudinal fibres. In fig. 19 this outer nucleus of the restiform body has increased in dimensions, and is united immediately, instead of by radiating fibres, to the inner restiform nucleus (*o*), which, like the posterior pyramid (*b*), is also larger than in the preceding figure. This inner nucleus (*o*) is now seen to consist of numerous oval, globular, crescentic, or irregular masses of granular substance, with small, and, for the most part, round, oval, or pyriform cells, which contrast strongly with the large, variously-shaped, and branched cells of the outer nucleus (*c'*). At each side of the canal, *t t'* has now taken the form of an oval mass, the lower part of which has become the nucleus of the hypoglossal nerve; while the upper or posterior part (*t*), which is joined to its fellow by a light commissure, has become the nucleus of the spinal-accessory nerve. At this upper or posterior part it is directly continuous with the posterior pyramid (*bb''*); but its outer side is separated from *o*, the inner restiform nucleus, by two sets of fibres, the one proceeding from that nucleus and from the posterior pyramid, to the remains of the anterior cornu, and the lower end (W) of the olivary body of the *same* side; while the other set (*n*) proceed from the lateral column at the side of the spinal-accessory nucleus (*t*) to the olivary body, and back of the anterior pyramid of the *opposite* side (*y'y'*), decussating with their fellows from the other half of the medulla. As usual, the caput cornu (*e*), or dilated extremity of the posterior horn, has been pushed forward from its cervix by the development of the post-pyramidal and restiform ganglia (*b* and *o*). The anterior pyramids (*y*, *y*) are much larger than in any of the mammalia (compare figs. 13, 14, Plate XII., and figs. 15, 16, Plate XIII. of my Memoir on the Medulla Oblongata, Phil. Trans. 1858). Fig. 20, Plate IX. represents a transverse section of the right lateral half of the medulla of the Monkey, at the point of the calamus scriptorius, where the connexions between the spinal-accessory nuclei and the posterior pyramids of opposite sides are broken through. The two oval masses (*t*, *t'*) are separated behind by the opening of the canal into the fourth ventricle, and resemble a pair of acorns joined at

their bases. In the lower or anterior part of each is seen the group of large cells (t') which form the hypoglossal nucleus; and in the upper or posterior part is seen another and oval group of smaller cells (t''), forming part of the nucleus of the spinal-accessory nerve. Behind this group of cells, the part t is finer in texture, consisting of granules, nuclei, small cells, and fine fibres. The posterior pyramid (b) is rather larger than in fig. 19, but it now begins to be bevelled off at its base, just below b; and between it and the spinal-accessory nucleus (t) there is a small triangular and paler mass (b''), which is in fact the inner side of the base of the pyramid, where it was joined to its fellow at the bottom of the median fissure, as seen at b'', fig. 19, enclosed in the dotted line on the right side. The outer restiform nucleus (c') has increased considerably in dimensions, while the inner nucleus (o) has somewhat diminished (compare figs. 7 & 8, Plate VIII. of human medulla on the right side). The decussating fibres (n, y') proceeding from the side of the spinal-accessory nucleus to the opposite olive and anterior pyramid are less numerous. In fig. 21, Plate IX. the outer restiform nucleus (c') is quite as large as in the preceding figure; while the inner nucleus (o) is much smaller, and is divided into several separate masses, which are partially blended with the outer nucleus (c'). The posterior pyramid (b) has considerably diminished, and the part b'' at its base has swollen into a larger eminence, which is now directly continuous with the posterior part (t) of the vagal nucleus, and seems to arise out of it (compare fig. 20), while its cells have increased somewhat in size. At the point between t and b'' there is an oval mass of epithelial cells and fibres (seen at t''' on the left side) which is continuous with the layer that covers the ventricle; it exists also in Man, but is much smaller than in the Ape. The hypoglossal nucleus (t') has increased considerably in dimensions, pushing, as it were, outward, the oval group of cells (t''), which now becomes the nucleus of the *vagus*, instead of the spinal-accessory nerve. The bundles of fibres (n, y') have ceased to proceed from the side of the spinal-accessory (now the vagal) nucleus to the opposite olive and anterior pyramid, the point whence they issued presenting now only the cut end of a slender column of longitudinal fibres (n) enclosed in a crescent between the horns of the nucleus. In fig. 22, Plate X. the outer restiform nucleus (c') and the scattered masses of the inner nucleus (o) have somewhat diminished and become still more intermixed. At the same time it will be observed that the posterior pyramid (b) is still further reduced, while the part b'' (which in fig. 20 belongs to the base of the pyramid) has extended in a corresponding proportion; so that now the posterior part (t) of the vagal nucleus, and the part b'' of the posterior pyramid form together a triangular mass (i) uniform in structure, and overlying the slender longitudinal column n and the oval group of cells (t'') which gives origin to the upper roots of the vagus nerve. The hypoglossal nucleus (t') reaches the surface of the ventricle, and completely separates i from its fellow of the opposite side. In fig. 23, Plate X. at p'' is seen a large bundle of arciform fibres coming from the outer part of the restiform body, and sweeping forward through the extremity of the caput cornu ($e\,e$) to the olivary body (W). The hypoglossal nucleus has considerably diminished, and retreated further backward and outward at the inner side of i,

and in the form of an uncircumscribed group of cells (t'); while the nucleus (t''), which now gives origin to the lowest roots of the glossopharyngeal nerve, instead of to the upper roots of the vagus, has been pushed, as it were, further outward, the bundles of the small longitudinal column (n) being now at the outer end of this nucleus, and separating the fibres of the nerve at their point of entrance. The posterior pyramid as a separate structure has wholly disappeared, while the triangular mass i, whose cells have been enlarging, is likewise increased in dimensions; and c' o of fig. 22 (the outer and inner nuclei of the restiform body) have now completely coalesced to form a beautiful network of cells and fibres (c' o), enclosing in its meshes longitudinal bundles of different shapes and sizes, and overlying, as in fig. 22, the extremity of the caput cornu posterioris (e e), through which transverse arciform fibres are sent across to the raphè to decussate with their fellows of the opposite side. Now the former of these two structures (i) gives origin (as I shall presently show) to the posterior root of the auditory nerve, and I have therefore termed it the *posterior* or *inner* auditory nucleus; while the latter ($c'o$) gives origin to the other division of the auditory nerve which passes underneath, or in front of the restiform body, and I have therefore termed it the *anterior* or *outer* auditory nucleus.

(12) By referring, then, to figs. 16, 17, 18, 19, 20, & 21, Plate IX. we find that (1°) the chief portion of the posterior pyramid with its grey substance, b', (2°) a large portion of the restiform body (c) with its grey substance (o) arising out of the posterior part of the cervix cornu posterioris (see especially figs. 17 & 18), and (3°) the posterior part of the central grey substance (t) which lies immediately beneath the pyramids, which gives origin to the grey substance of the pyramids, and which subsequently becomes itself the *posterior* part of the spinal-accessory and vagal nuclei (t t, figs. 20 & 21), are all by the histological and morphological changes above described transformed into the central organs of hearing, to say nothing at present of the similar relations of these to other parts of the brain.

Fig. 24, Plate X. is a longitudinal and horizontal section of the left half of the posterior part of the human medulla oblongata, presenting another view of the anatomical relations of the posterior pyramid, the restiform body, the vagal nucleus, and the auditory nucleus. At the lower end of the figure the section corresponds to the transverse horizontal line b'' b'' in fig. 6, Plate VIII., much below the point of the calamus scriptorius. b' is the grey substance of the pyramid, b is its white substance. Higher up the section takes off the surfaces of the inner and outer restiform nuclei (o c'), the former being still separated from the grey substance of the pyramid by its white substance (b). A little above this point the white substance of the posterior pyramid is lost, its grey substance being fused with that of the restiform body. At o' the section corresponds very nearly to the transverse line b'' b'' in fig. 9, Plate VIII., running through the middle of the vagal nucleus (g). Here the vagal nucleus has swollen into a large oval or pyriform body, while the posterior pyramid is much narrower and still more closely blended with the grey substance of the restiform body. Out of this united

mass several bundles of fibres with intervening layers of grey substance ascend longitudinally through the grey network with which they constitute the outer nucleus of the auditory nerve (*d' o*, fig. 23.). The inner nucleus (*i*, fig. 24) of the auditory nerve, although marked off from *g* by the dotted line to indicate its limits, is really directly continuous with it; neither can any alteration of structure be discovered, as the one passes into the other.

(13) I have been thus exact and particular in following out the gradual transitions and morphological changes which take place in ascending the medulla, not only on account of their extreme importance directly and indirectly in both a pathological and physiological point of view, but also on account of the necessity of understanding exactly the structure, connexions, and relations of the various elements of this part of the medulla, in order to interpret correctly the morphological and histological relations of the parts which are situated higher up. And indeed I believe that these changes which I have thus described, and which have cost me no small amount of thought and labour to follow out and confirm, will be the means of correcting many errors, and of leading to a more exact knowledge of other parts of the brain.

(14) But before I proceed to these parts it will be necessary to describe some new and important facts which I have recently ascertained with regard to the structure of the medulla oblongata.

In my former memoir on this subject, I gave the following composition of the anterior pyramids, according to my observations at the time:—

1. Decussating fibres of the anterior commissure, continuous with that of the spinal cord.
2. Decussating fibres from the opposite lateral columns, constituting their chief bulk.
3. Decussating fibres from the posterior grey substance.
4. Non-decussating fibres of the anterior columns, forming a small part on their outer side.

Now subsequent observations, while they have confirmed the truth of these statements, have enabled me to make some important additions. I had already shown that the decussating fibres from the posterior grey substance proceed out of the post-pyramidal nucleus, the restiform nucleus, and the posterior horn, near its extremity, as represented in figs. 3 & 4, Plate VIII. I have since ascertained, by means of longitudinal sections made at appropriate angles, that some of the decussating fibres of the anterior pyramids, which appear only to cross the antero-lateral *grey* substance, in their course from the opposite lateral column, do actually arise out of that substance at the points *d'*, *d'*, fig. 3, Plate VIII. I have further ascertained that the decussating fibres from both the posterior and the anterior grey substances *ascend toward the brain* after they have joined the pyramids. There is another exceedingly interesting structure, which, although it does not share in the formation of the anterior pyramid, I shall mention here, because it is mixed up with the decussating fibres of those bodies in the lateral column. In fig. 4, Plate VIII. on the right side, several curved bundles of fibres (*d'*) may be seen pro-

ceeding transversely outward to the lateral column, and crossing the fibres of that column which run to the opposite pyramid. These fibres take no share in the formation of the pyramids; and after running outward for a short distance they turn round and *descend* the cord obliquely across the longitudinal fibres of the lateral column. At the points where these two sets of fibres bend round (where the former or transverse set descend, and the lateral or longitudinal set become transverse as they cross to the opposite side) there is a very complicated and curved interlacement of bundles. Now the physiological importance of the former set is evident when we consider that the part of the posterior grey substance from which they arise is precisely that which forms the lower portion of the spinal-accessory and vagal nuclei, and with which the lower roots of the spinal-accessory nerves are connected. It is moreover interesting to observe that in ascending the medulla oblongata above the decussation of the pyramids, a similar system of fibres was found to proceed from the same respiratory centre, and to run down the lateral columns.

(15) Since the publication of my memoir on the medulla oblongata I have ascertained some further and important particulars regarding the structure and connexions of the respiratory centres. Fig. 25, Plate X. represents with great exactness a transverse section of the central canal with the left lateral half of the spinal-accessory and hypoglossal nuclei, at the level of the highest roots of the spinal-accessory nerve, as shown on a smaller scale in the entire transverse section of the medulla in fig. 8, Plate VIII. The spinal-accessory nucleus consists of two portions, an anterior and a posterior portion, which differ from each in structure, although they are closely connected. The posterior portion (A A, B, D, M, fig. 25, Plate X.) slopes outward and forward and is overlain by the posterior pyramid (see *b'*, fig. 8, Plate VIII.). It contains two or three thick longitudinal columns of grey substance, which in transverse section appear as oval or roundish masses (B, fig. 25, Plate X.), either wholly or only partially separated from each other. These masses consist of granules mixed with nuclei and small cells, and are enveloped by curved fibres and traversed by others in different directions. Some of the curved fibres at the outer borders of these masses run obliquely *outwards* and *downwards* along the medulla in straight lines to reach the base of the restiform body. Others from the anterior and posterior parts of the same or of different masses proceed *inward*, and, decussating across the middle line, connect these masses with their fellows of the opposite side (C C, fig. 25, Plate X.). This decussation of fibres is not observable in every section, and is most conspicuous a little lower in the medulla. Another set of fibres proceeding out of these masses run obliquely forward and outward (D) to the lateral column, and are joined by numerous others which issue from between the pyramids, at the bottom of the median fissure (E), and run on either side of the masses in the same direction. Those on the outer side (A A) curve forward and inward, and terminate in a plexus of bundles which decussate across the raphè (F) with their fellows in the opposite half of the medulla. They are interspersed with numerous small cells, of which some are darkened by pigment. The fibres on the inner side (G) run forward and

outward, partly into the lateral column (at *n*), accompanied by others from the masses B, but chiefly into the group of cells (H) forming the anterior portion of the nucleus, and are interspersed not only with small cells, but with numerous others of a larger kind, which are oval or fusiform, and elongated in the same oblique direction. They decussate with those in the opposite half of the nucleus across the central line at G. Many of them are evidently derived from a mass of epithelial cells situated at E between the bases of the posterior pyramids.

The anterior portion of the spinal-accessory nucleus consists of an oval or fusiform group of cells (H), sloping forward and outward from behind the canal. The cells are of various sizes, are mostly oval, and lie with their longer axes in both a transverse and longitudinal direction. Many of them are filled with dark pigment. From the outer and anterior extremity of this group fibres proceed to form the greater part of the spinal-accessory nerve *r*; and from its inner and posterior extremity another set of fibres extend obliquely inward and backward behind the canal, to the opposite group of cells. These commissural fibres, like the *others* that decussate between the opposite sides of the spinal-accessory nucleus, are not so distinct in Man as in the Ox, Sheep, Cat, Dog, or Rabbit, and are most conspicuous at a lower level of the medulla, especially at the inferior extremity of the olivary bodies, in sections corresponding to fig. 6, Plate VIII.

Between this group of cells in the anterior portion of the nucleus, and the curved outer fibres (A A) proceeding from the *posterior* portion, is a roundish or somewhat oval column of longitudinal bundles (*n*), either wholly or only partially enclosed on its posterior and inner sides by a band of fibres (M), which sweep forward and inward across the root of the spinal-accessory nerve *r*, and round the hypoglossal nucleus (J) to the median raphè (F), where they decussate with their fellows from the opposite half of the medulla. This little column is at intervals interspersed with small nucleated cells, sometimes united by a kind of network, and is, as I shall presently show, a structure of great importance.

I have already shown that at the lower end of the medulla oblongata, a network of fibres and cells is formed in the deep portion of the lateral column (*d'*, fig. 1, Plate VIII.) between the anterior and posterior horns; that from the meshes of this network proceed the bundles that form the chief proportion of the opposite anterior pyramid (figs. 2, 3, & 4); and that after these bundles have ceased to cross over, the original network reappears in a finer form (fig. 5, Plate VIII.) with an increase in the size and number of its cells, and containing in its meshes a multitude of bundles composed of finer fibres. Now the deepest or most internal and posterior of these longitudinal fibres (those lying immediately in front of the caput cornu or grey tubercle (*e*), at the angle formed by the lines L*n*, fig. 5, Plate VIII.) diverge from the rest as they ascend the medulla, and curving inward and backward, form the slender column *n* of fig. 8, Plate VIII. and fig. 25, Plate X. The derivation and course of this little column are well seen in a longitudinal section carefully made in a particular plane, that is, obliquely from behind forward in the direction of the line L*t*, fig. 8, Plate VIII., or almost in the course of

the spinal-accessory nerve, and carried downward through the parts of the medulla represented by figs. 7, 6, 5. From such a longitudinal section fig. 27, Plate X. was drawn. The lower edge (L t) of the figure corresponds to the line L t of fig. 5, Plate VIII. d is the superficial white substance of the lateral column, to the depth of the angle formed by the lines L O, fig. 5, Plate VIII. It consists of coarse longitudinal nerve-fibres, irregularly interlaced and crossed at right angles by some finer transverse fibres. d' is the posterior part of the network which reappears after the bundles in its meshes have ceased to supply the anterior pyramid, and corresponds to that part of the line L t in fig. 5, Plate VIII., which lies between O and n. It consists of finer fibres than those of the preceding or superficial white layer. These fibres are moreover collected into neat longitudinal bundles, which are very close together and parallel, but communicate with each other by very oblique lateral offsets, so as to enclose a multitude of fusiform, and often exceedingly elongated meshes, which are filled with groups and streaks of nucleated cells and nuclei, of a corresponding shape and size. Some of the processes of the cells extend along the longitudinal bundles, while others project across them in the plane of the section and in the direction of numerous fibres issuing from the spinal-accessory nucleus (t t).

As they ascend the medulla, the *deepest* longitudinal bundles (n', fig. 27, Plate X.) of this layer diverge inwards, and become much more widely separated from each other, but still communicate by lateral offsets. At the upper border of the figure this portion of the layer (n''') in the longitudinal section, occupies the part between N and n; and in a transverse section of the medulla at this level (fig. 8, Plate VIII.) it occupies that portion of the line L which also extends between N and n. Several of the innermost fibres, in ascending from n' (fig. 27, Plate X.), unite into bundles to form the slender longitudinal column n, which increases in thickness and curves inward and backward as it approaches the upper border of the figure. In fig. 8, Plate VIII., this little column (n) is seen in transverse section. In the transverse section (fig. 5, Plate VIII.), which corresponds to the lower border of the longitudinal section (fig. 27, Plate X.), it is at first separated from the spinal-accessory nucleus (t) by the numerous bundles of transverse fibres which sweep round from behind forward to decussate into the anterior pyramid. In the longitudinal section (fig. 27, Plate X.), the cut ends of these transverse bundles are represented at the lower border of the figure by the dark dots at n''. They are contained in the transversely elongated meshes of a network composed chiefly of fibres proceeding from the spinal-accessory nucleus (t), and connected with numerous nerve-cells. As the slender longitudinal column ascends the medulla to n, fig. 27, the transverse bundles (n'', dark dots) diminish in number until they are reduced to a comparatively few fibres, which are seen in the transverse section (fig. 25, M, Plate X.), arching round the inner border of n. But in proportion as these transverse bundles diminish on the *inner* side of the slender column, the fibres proceeding from the posterior portion (AA, fig. 25, Plate X.) of the spinal-accessory nucleus sweep forward round it on its *outer* side, in company with other fibres coming from the posterior pyramid. In fig. 8, Plate

VIII. these arciform fibres are seen between *n* and N. They pass between the separated longitudinal bundles n''' of fig. 27, Plate X., where their cut ends are represented by the dark dots, and between other transverse fibres proceeding outwards from the spinal-accessory nucleus; and this kind of plexus through which they pass contains numerous nerve-cells whose processes are continuous with the different sets of fibres.

The next or internal layer (*t t*) of fig. 27, Plate X. is the spinal-accessory nucleus or tract, the cells having been omitted to avoid obscuring the fibres. It has been already stated that a multitude of these fibres run longitudinally. In fig. 25, Plate X. it has been shown that a vast number run transversely outward, from G to the slender column *n* and to the lateral column beyond it. In the oblique longitudinal section (fig. 27) we see both sets of fibres together. The longitudinal are not quite parallel, but intersect each other at different angles, and bend round to be continuous with the transverse. The latter then run outward, between the *other* set of transverse *arciform* bundles (n'', fig. 27, dark dots) (and *n*, fig. 5, Plate VIII.), which cross them at right angles as they sweep round from behind forward to the anterior pyramid. On reaching the slender column (*n*, fig. 27, Plate X.) some of them run with it longitudinally; but by far the greater number, frequently in bundles, continue their course outward, and cross it more or less at right angles to enter the layer $n' n'''$, where they contribute to form the grey network between the longitudinal and the outer *transverse arciform* bundles. With the former of these bundles (n''', fig. 27, Plate X.) some of them are directly continuous; and with the latter (represented by the dots at n'') others appear to be connected through the medium of the intervening cells: the rest continue outward across the fine longitudinal fibres of the next layer, d', where similar connexions with the cells and longitudinal fibres appear to take place; some of them reaching the superficial white column *d*, in which they become lost.

The course and arrangement of the different sets of fibres in this extremely complicated structure may now perhaps be more completely understood, if we suppose the longitudinal section (fig. 27) to be applied with its plane perpendicular to the planes of the transverse sections (fig. 8 & fig. 5, Plate VIII.), so that the *upper* border (*t*) of fig. 27 shall coincide with the line L *t* of fig. 8, and its *lower* border (L *t*) with the same line in fig. 5.

In descending the medulla below fig. 5, Plate VIII., the innermost fibres (n') of the layer d' (fig. 27, Plate X.), in which the slender column *n* terminates, become still further removed from the central nucleus (*t*) of the spinal-accessory nerve, by the interposition of lateral grey substance, with which the lower roots of this nerve are connected, the cells of the central nucleus behind the canal being now small and sparing in number. Fig. 28, Plate X. represents the left lateral half of the grey substance of the medulla between figs. 4 & 5, Plate VIII. The central nuclei of the spinal-accessory and hypoglossal nerves (*t*, t') are only just marked out around the canal, while the lateral grey substance beyond them is crossed by portions of the remaining curved bundles proceeding from the restiform and postpyramidal nuclei to the decussation of the pyramids, which have ceased to derive their fibres from the *lateral* columns, that part of those columns

from which these fibres were derived being now occupied by the beautiful network d' d', fig. 28, Plate X., containing in its meshes the longitudinal bundles which are continuous with the layer d' n', at the lower border of fig. 27, Plate X. The innermost of these bundles (n'), in which the slender column (n) terminates, are, in fig. 28, Plate X., situated at the extremity of the line n; and at this point the lower rootlets of the spinal-accessory nerve (r) are connected with the border of the lateral grey substance. Some of the fibres of these rootlets turn backward round the little network of fibres (e), mixed with cells, joining the lateral grey substance to the grey tubercle or caput cornu posterioris, and seem to be connected with it; while others turn forward toward the remains of the anterior cornu (f'')*. Still further down these anterior roots of the spinal-accessory nerve may be traced to the anterior horn of the spinal cord, as far forward as the group of cells from which the anterior roots of the spinal nerves take their origin, as may be seen in fig. 29, Plate X., which represents a transverse section of the grey substance of the spinal cord at the lower part of the first cervical nerves†. r indicates three branches of the spinal-accessory nerve of the left side, proceeding to the lateral grey substance, and thence forward to the side of the group of cells in the anterior horn. e is the caput cornu or dilated extremity of the posterior horn; d'' indicates the posterior nerve-roots first passing transversely through the posterior white column immediately behind the caput cornu, and then crossing the cervix cornu to the lateral grey substance with which the spinal-accessory nerve is connected, and which they separate from the central grey substance (t). At the same part of the lateral grey substance we see the origin of some of the decussating bundles (f'') which form the point or termination of the anterior pyramids, and which frequently *seem* to be continuous with the posterior nerve-roots (d''). The spinal-accessory nerve appears to be connected with the lateral grey substance all along its border from before backward, within the curved line of the lateral column d', into which the greater number of the decussating fibres of the pyramids pass (see figs. 2, 3, & 4, Plate VIII.). The connexion of the anterior pyramids with this part of the grey substance is exceedingly interesting in reference to the influence of the will on the respiratory movements.

(16) The slender longitudinal column n (figs. 25 & 27, Plate X.) has the same kind of important connexions with the vagal and glossopharyngeal nuclei as those which it has been shown to form with the spinal-accessory; but it has moreover a direct and especial connexion with the glossopharyngeal *nerve*. While the spinal-accessory and vagus nerves enter their nuclei on the *inner side* of the slender column, although they are connected with it, many of the fibres of the glossopharyngeal nerve pass directly into it. Fig. 30, Plate X. represents an oblique longitudinal section of the human medulla in the plane of the glossopharyngeal nerve, and passing through the slender column n n and the glossopharyngeal nucleus t'. Some of the roots of the nerve (g) are distinctly seen to enter the slender column (n) and run with it *down* the medulla.

* See my Memoir "On the Medulla Oblongata," figs. 19 & 23, Phil. Trans. 1858.

† See my "Further Researches on the grey substance of the Spinal Cord," fig. 12, Phil. Trans. 1859.

(17) It has been shown that the *upper* roots of the spinal-accessory nerve arise from a special nucleus in the medulla oblongata, behind the canal, and that the *lower* roots have their origin in the lateral grey substance and anterior cornu of the spinal cord. It is an interesting fact that the hypoglossal nerve has a similar kind of double origin. Its *lower* roots arise from the upper remains of the anterior cornu, as I formerly showed in the Sheep*, and as may now be seen in Man (fig. 28 x, Plate X.). Here the central nucleus f', which gives origin to the *upper* roots, is only just marked out and contains only a few small cells; while the remains of the anterior cornu (f') contain two little groups of large multipolar cells in connexion with the *lower* roots x.

(18) In fig. 25, Plate X., J is an exact representation of the central nucleus of the *upper* roots of the hypoglossal nerve in Man, a little below the level of the calamus scriptorius, and on a level with the *upper* roots of the spinal-accessory nerve (r). It consists of a group of multipolar cells, which are larger, more varied in shape than those of the spinal-accessory nucleus, and resemble in every respect those of the anterior cornu of the spinal cord. These cells send their processes in different directions, and many of them are elongated longitudinally in connexion with fibres running in the same direction. There is no actual line of separation between this group and the oval group (H, fig. 25) belonging to the spinal-accessory nucleus, for each passes gradually, as it were, into the other by means of more sparingly scattered cells and nuclei which occupy the intervening space, exactly as is represented in the figure. In this intervening space some fine fibres may be seen passing from one nucleus to the other as well as from within outward; but at a higher level in the medulla I discovered a more intimate and important communication between the two nuclei, by means of nerve-processes and fibres, which will be described further on. At the anterior part of the hypoglossal nucleus, just where the nerves enter it, there is a little separate group composed of smaller cells than the others, and from which several large bands of commissural fibres proceed to the raphè, where they decussate with those of the opposite side (see J', fig. 25). This little group is most conspicuous at a higher level of the medulla, above the calamus scriptorius, where the other cells of the nucleus are also imperfectly divided into separate groups (see fig. 32 J', Plate XI.). There is also a remarkable fan-shaped set of commissure fibres (J'', fig. 25, Plate X.), which, arising from different parts of the hypoglossal nucleus, converge inward and forward to the raphè (F), where, like the others, they decussate their fellows of the opposite side. More externally, the roots of the nerves (x) are crossed, as already described, by numerous arciform fibres (M, A A) which proceed from the spinal-accessory nucleus and the slender column (n) adjoining. In animals these fibres are exceedingly numerous.

(19) In the Sheep, as I formerly stated, a few of the roots of the hypoglossal nerves (x) join these commissural fibres and cross the raphè; and in Man a few of the *higher* roots take the same course; but almost all of them enter the nucleus without decussating across the raphè. They traverse it from before backward and in other directions,

* See "Medulla Oblongata," Philosophical Transactions, 1858, fig. 13 x, Plate XII.

crossing the fan-shaped set of commissural fibres and reaching as far as the scattered cells intervening between it and the spinal-accessory nucleus. The lateral roots completely inclose the group of cells, those on the outer side curving inward, and those on the inner side taking an opposite course. The latter often approach the raphè at L′ so closely that they might easily be supposed to cross it; but they may always be traced outward and backward as far as a separate but small group of cells (K′) lying nearer the side and lower border of the canal. This little group or column contains numerous longitudinal fibres, and is composed of much smaller cells than those of the hypoglossal nucleus. At this level of the medulla it is in some sections scarcely perceptible, but it increases in size as it ascends, and is a structure of great importance; for I shall presently show that it forms the lower end of the column of cells and fibres which constitute the fasciculus teres, and part of the nucleus of the facial nerve. We have here, then, a most interesting communication between the facial nerve and even the *lower* roots of the hypoglossal. Still further back and nearer the canal is another very small group of nuclei, the use of which I have not ascertained.

(20) For some distance around the canal there is a dense layer of exceedingly fine nucleated fibres, which decussate each other both behind and in front (L′), and run down the raphè towards F. Further out from the side of the canal they are joined by similar fibres which proceed from the oval group of the spinal-accessory nucleus (H), and from the little group (K′) belonging to the facial nucleus. In the Ox, Sheep, Rabbit, &c., all these fibres are more conspicuous than in Man. They are crossed at right angles by a multitude of longitudinal fibres of a similar nature, which form a layer that is denser in proportion as it approaches the border of the canal.

(21) I have already stated that fine fibres run between the hypoglossal and spinal-accessory nuclei. At a higher level of the medulla I discovered a still more intimate and a most important connexion between these parts. It can only be properly seen in a longitudinal and horizontal section made at the point of the calamus scriptorius and carried up through the middle of the hypoglossal and spinal-accessory nuclei; for the fibres arch upward, and are therefore more or less divided in a transverse section. Fig. 31, Plate X. represents such a longitudinal section; and fig. 32, Plate XI. a transverse section of the parts of these nuclei left after the longitudinal section was made, so that the line of section J, H, H′ corresponds to the lower edge J, H, H′ of fig. 31, Plate X.; where J is the hypoglossal nucleus, and H the group of cells forming the anterior projecting horn of the spinal-accessory and vagus nucleus, on the inner side of the slender column *n*, where the nerve *r* enters. In the hypoglossal nucleus (J′, J″, fig. 31, Plate X.) a number of cells send their processes laterally outward in an arched direction to the inner or anterior horn (H) of the spinal-accessory nucleus; and from this, in return, elongated cells send their processes inward to the hypoglossal nucleus (J″). The majority of the cells of the anterior horn (H) of the spinal-accessory nucleus are elongated longitudinally with a multitude of fibres, and some of the transverse fibres coming from the hypoglossal nucleus (J″) cross these to the outer part of the spinal-accessory nucleus (H′), and thence to the slender column *n*.

(22) But besides this most intimate and important connexion between the two nuclei —the hypoglossal and the spinal-accessory—through the medium of their *cells*, the spinal-accessory *nerve*, as I formerly showed on several occasions, has a separate origin from the *hypoglossal* nucleus; and I also showed that both the vagus and glossopharyngeal nerves have each a separate origin from the same source. These interesting facts have been fully confirmed by Dr. JOHN DEAN, of Boston, U. S.*. In birds this separate origin of the vagus nerve from the hypoglossal nucleus is so striking, that in well-made preparations it may be seen almost at a glance†. In fishes I have recently discovered a beautiful and peculiarly interesting illustration of the same anatomical fact. In these animals there is no *separate* hypoglossal nerve, but the tongue is supplied by a branch of the *vagus*. Now I was very desirous of knowing what was the nature of the vagal nucleus in these animals, and of ascertaining whether it was a double or compound nucleus consisting of the vagal and hypoglossal nuclei joined together. On examination I found that the lower division of the vagus nerve, after proceeding transversely through the lateral part of the medulla, bifurcates into two distinct roots of considerable size; and that while one of these curves backward to spread into the grey substance behind the canal, the other bends forward into a round or oval nucleus, which, in regard both to the character of its cells and its position in front and at the side of the canal, corresponds exactly to the hypoglossal nucleus of the higher vertebrata. These appearances are so well marked,—this anterior nucleus is so distinct, and its connexion with the corresponding root of the vagus nerve is so evident,—that in well-made preparations they may be seen with the greatest facility. In fishes the roots of the vagus external to the medulla consist of two sets,—the one a little above the other. The lower set are on a level with the calamus scriptorius; the upper set are on a level with the large vagal eminence of grey substance which rests on the floor of the fourth ventricle beneath the cerebellum. Fig. 33, Plate XI. represents a transverse section of the medulla oblongata of the Cod at the point of the calamus scriptorius, just where the canal (6) opens into the fourth ventricle, and therefore on a level with the lower set of vagal nerves (14). The anterior portion of the medulla, 1, 1, 2, 3 (see left side of figure), on either side, consists of some nerve-cells, and a network of fibres and blood-vessels containing in its meshes a multitude of longitudinal bundles of different sizes. Many of these bundles are exceedingly large, particularly along the inner side next the raphè, —as represented by the round or oval spaces (1,1),—and in front at 2, where they appear to correspond to the anterior pyramids of the higher vertebrata. The nerve-cells are of considerable size, and chiefly collected into a small group (3) near the centre. Behind the two lateral halves of this anterior portion of the medulla, and lodged in a depression between them, are two other larger longitudinal bundles of fibres (4); and immediately behind these are two oval and still larger longitudinal columns of nerve-cells (5), which have precisely the same position at the side, and in front, of the canal

* On the Grey Substance of the Medulla Oblongata and Trapezium. By JOHN DEAN, M.D., 1864, pages 27, 28.

† See my Memoir on the Medulla Oblongata, Philosophical Transactions, 1858, Plate XIII. fig. 18.

as have the longitudinal columns of cells constituting the nuclei of the hypoglossal nerves in the higher vertebrata. Under a magnifying-power of about 100 diameters, a transverse section of one of these columns has the appearance represented in fig. 34, Plate XI. The cells are large and multipolar, resembling those of the hypoglossal nucleus. Between them are numerous longitudinal fibres represented by the dots; and from all parts of the nucleus a number of fibres radiate transversely toward the surface (see also fig. 33, 7, on left side), where they bend round and become lost, for there is no nerve attached to this part of the medulla. From this nucleus the *anterior* roots of the lower set of vagal nerves take their origin, and curving outward and backward in a thick compact bundle (8, fig. 33) join the *posterior* roots (9), which are somewhat more numerous, and arise from a large column of grey substance (10, 10) behind and at the side of the canal (6). This substance is finely granular, and contains a multitude of very small cells or nuclei. It sends numerous fibres into the anterior nucleus (5, see left side of figure), as well as others which, in company with those issuing from the nucleus, proceed outward to the surface at 7. On the outer side of the posterior nucleus (10) is the restiform body (11) containing some grey substance (12), from which numerous fibres converge into a large bundle (13) that runs forward and inward to decussate with its fellow across the raphè in front of the canal, and terminate on the opposite side in the anterior part of the medulla close to the group of cells (3). The correspondence of the anterior nucleus (5) and of the anterior vagus-roots (8) in the Fish, with the hypoglossal nucleus and hypoglossal nerve in the higher vertebrata, is very striking. The arrangement in the Fish is such that the fibres distributed to the tongue, instead of running forward across the medulla in the course of the dotted line x as a separate and special hypoglossal nerve, are thrown backward and aside at 8 to join the vagus and be distributed to the tongue as one of the branches of that nerve*.

(23) *Of the Olivary bodies.*—In my memoir on the Medulla, I gave a detailed account of the general appearance and structure of the olivary bodies in Man, in the Ape, in many of the Mammalia, and in Birds, and I have now only a few more observations to make on the subject. In fishes I have never been able to discover any structures to which I could point as the representatives of the olivary bodies of the higher animals. In Birds, however, they certainly exist, although their cells are not arranged in the form of a lamina, but are scattered about the column in which they are contained. In the higher Mammalia, however, they assume the form of a lamina, which, in the

* In *reptiles*, also, the vagus sends a considerable branch to the tongue, and indeed is the only nerve that supplies that organ. This branch must not be regarded as the glossopharyngeal, for it supplies not only the muscular tissue of the tongue, but the muscles which protrude and retract that organ. "La huitième paire," says Desmoulins, "est le seul nerf distribué aux muscles de la langue et de la glotte" (*Systèmes Nerveux*, tom. ii. p. 455). I have not examined the origin of the vagus in reptiles, but probably it is similar to that which I have discovered in fishes. The spinal-accessory, also, in reptiles and birds is only a part of the vagus, which, according to Bischoff, gives one branch, or several branches, to the neck. In reptiles and fishes, both the hypoglossal and spinal-accessory are only branches of the vagus, or rather the three nerves form one common system.

feline tribe especially, is bent into a simple curve or loop. Fig. 35, Plate XI. represents a transverse section of the right anterior pyramid and olivary body of the Cat. On the outer side of the pyramid (*y*) is the olivary body, which consists of three masses of cells, the one in the middle (W) being a *simple* loop. Now in ascending the animal scale, I found that in the common Ape the olivary bodies are not only much increased in size and more prominent on the surface of the medulla, but that the loops or folds of the lamina, which are *simple* in the highest Carnivora, as the Cat, become thrown into *secondary loops* or convolutions, as at *w*, fig. 23, Plate X. Still higher in the scale the convolutions of the lamina are more numerous, and the bulk of the entire organ is much greater, as may be seen in figs. 36 & 37, Plate XI. The latter figure represents the right anterior pyramid (*y*) and olivary body (W) of a young Orang Outang*. Fig. 36 was drawn from a transverse section of a piece of medulla given to me by the late Mr. Queckett, of the Royal College of Surgeons. From the strong resemblance of this medulla to that of Man, and the large size of the olivary bodies, it evidently belonged to one of the higher Simiadæ. The olivary body itself closely resembles that of the Orang Outang (fig. 37); but it seems to be imbedded in the anterior pyramid, instead of lying behind and on its outer side. Moreover, there is another peculiarity in the existence of a large separate column (*d*) forming part of the lateral column. In both this medulla and that of the Orang Outang (fig. 37), the convolutions of the olivary lamina are as numerous as in Man, and form a striking contrast to the *simple* loop of the Cat at W, fig. 35. On a former occasion I showed that in Mammalia generally the hypoglossal nerves are attached on the *outer* side of the olivary bodies, instead of on the *inner* side, as in Man. It is somewhat curious that in all the Apes, even in the Orang Outang and Chimpanzee, their point of attachment is intermediate, that is to say, they are attached to the olivary bodies themselves.

I shall now proceed to describe the structures forming the upper part of the fourth ventricle.

* For the medulla oblongata, pons Varolii, and part of the cerebellum and cerebral convolutions of the Orang Outang, I am indebted to the kindness of Dr. Muir, of the Zoological Gardens.

CHAPTER II.

(24) The little column of cells and longitudinal fibres seen at K', fig. 25, Plate X., lying at the side of the central canal and immediately behind the hypoglossal nucleus (J), increases in size as it ascends the medulla to constitute the so-called "*fasciculus teres.*" Above the *calamus scriptorius*, on the floor of the fourth ventricle, it increases most rapidly, and rather suddenly, just in proportion to the *decrease* of the *hypoglossal* nucleus. Fig. 38, Plate XI. represents a transverse section of the floor of the ventricle at this level, on the left of the median line. J is the hypoglossal nucleus considerably diminished in size. Behind and overlying it is the "*fasciculus teres*" (K'), much *increased* in size, in consequence of the greater number of both the fibres and cells which it contains, the fibres for the most part forming a layer nearer its posterior surface. It is an oval or pyriform mass, and lies with its longer axis transversely inward and outward. On its inner side it tapers over the back of the hypoglossal nucleus, to join a small oval group of closely aggregated cells (T), which now makes its appearance along each border of the raphè; and on the outer side it tapers under, or in front of, the oval group of cells (H) forming the inner portion of the vagal nucleus, which partly overlies it. Some of the fibres of the vagus *nerve* (*g*) appear to be connected with it, as they run along its lower border to the hypoglossal nucleus. The outer portion of the vagal nucleus is at *t*, and the slender longitudinal column at *n*.

(25) A transverse section of the floor of the fourth ventricle a little higher up (on a level with the roots of the glossopharyngeal nerve) is represented in fig. 39, Plate XI. Here we find that the hypoglossal nucleus has almost wholly disappeared, only a few small cells (J, J'), belonging apparently to the anterior group, remaining in company with a few commissural fibres and nerve-roots (*x*). The dark oval group of nuclei (T) is much larger than in the preceding section, and on its outer side, on the surface of the ventricle, is the cut end of one of the *striæ medullares* (T'). The "*fasciculus teres*" (K') has nearly doubled its size, and is connected by a curved band of fibres with the posterior nucleus (*i*) of the auditory nerve; *t* is the outer portion of the glossopharyngeal nucleus. It is much smaller than the corresponding part of the vagal nucleus (fig. 38), and has diminished in proportion as the auditory nucleus (*i*) has enlarged. On the right of *t*, the group of cells constituting the inner nucleus of the glossopharyngeal nerve has altered a little in form, being somewhat club-shaped instead of oval.

(26) In the central part of each lateral half of the medulla, and near the inner side of the caput cornu posterioris or grey tubercle (*e*, fig. 10, Plate VIII.), there is a peculiar group of large multipolar cells (U), precisely similar in appearance to those in the anterior cornu of the spinal cord. This is the lower part of a longitudinal column of cells, constituting the nucleus of the lesser root of the trigeminal or fifth cerebral nerve. Higher up in the medulla, on a level with the auditory nerve, it was pointed out by STILLING; but I have traced it much lower down—sometimes nearly as low as the level

of the inferior ends of the olivary body*. On a level with the glossopharyngeal nerve it forms a group of considerable size (U, fig. 39, Plate XI.). Its cells are lodged in the interspaces between the deep arciform and the longitudinal bundles of fibres already described (§ 15), and may frequently be seen to curve round or embrace the latter, with which some of their processes are continuous. By this continuity of their processes with the longitudinal bundles they probably establish connexions with other cells at different lengths of the column; and by a similar continuity with the transverse arciform fibres, which freely communicate with each other, and decussate their fellows on the opposite half of the medulla, they are brought into connexion with parts on either side. In some sections the cells are especially elongated forwards and backwards, as shown at U, fig. 39, and send their processes chiefly in both these directions. The processes which extend forward, but which are not shown in the figure, sometimes form a kind of tapering brush or tail of fibres in connexion with more scattered cells lying along their course, and may frequently be seen to communicate with the transverse bundles which traverse the *grey tubercle* (*e*, fig. 10, Plate VIII.), and the sensory roots of the fifth nerve contained therein. By means of a special set of commissural fibres (U', fig. 39, Plate XI.), the entire group has a very intimate connexion with other surrounding parts, but particularly with the cells on the right of *t*, belonging to the glossopharyngeal nucleus, and with the so-called "fasciculus teres." On proceeding from the group backward, some of these fibres (at U') turn round at different depths to end in the transverse arciform bundles; some turn round to be continuous with the longitudinal bundles; others pass further backward to the glossopharyngeal nucleus and "fasciculus teres;" while a few are seen to be continuous with returning fibres of the glossopharyngeal nerve which do not enter their nucleus.

(27) In 1857† I pointed out the fact that the descending portion of the posterior root of the fifth nerve, instead of traversing the *restiform body*, as it was previously believed to do, runs down through the front of the grey tubercle, or caput cornu posterioris. The transversely cut ends of its longitudinal fibres are represented by the dark spots in the grey tubercle at *e*, fig. 10, Plate VIII. Moreover, I formerly showed‡ that both the vagus and glossopharyngeal nerve, on their way to their nuclei, traverse the grey tubercle and pass through this portion of the fifth nerve; while I have since had reason to think that some of their roots terminate in the grey tubercle as one of their centres of origin.

* Schröder van der Kolk, who did not recognize this group of cells, says that Stilling has mistaken the antero-lateral nucleus for it. This is quite a mistake on the part of Schröder van der Kolk, who has himself mistaken the antero-lateral nucleus for what Stilling really means. Stilling was not acquainted with the antero-lateral nucleus, which I first pointed out in 1857, and which was subsequently more fully and well described by Dr. John Dean, who gave it that name. The two groups are perfectly distinct, as may be seen in fig. 10, Plate VIII., where *s* is the antero-lateral nucleus, and U the nucleus of the motor root of the fifth nerve.

† "Medulla Oblongata," read June 1857, Philosophical Transactions, 1858.

‡ Philosophical Transactions, 1858.

(28) This intimate connexion of the *sensory* division of the fifth nerve with the vagus and glossopharyngeal nerves in the grey tubercle, as well as the connexion, just shown, of its *motor* nucleus with the glossopharyngeal nucleus, with the returning fibres of the glossopharyngeal nerve, and with the "fasciculus teres," are facts of uncommon interest.

(29) The longitudinal fibres (V V', fig. 8, Plate VIII.) immediately in front, and on the outer side, of the hypoglossal nucleus (*t'*) have the same kind of anatomical connexion with that nucleus as the antero-lateral white columns of the spinal cord have with its anterior horn. In longitudinal sections carried through these columns and the hypoglossal nucleus the fibres along the border of the nucleus may be seen to enter or issue from it, just as the fibres of the antero-lateral columns do along the border of the grey substance in the cord. It has been seen that along the line L *t*, from the spinal-accessory nucleus outward, the longitudinal bundles between N and *n* are sparing and wide apart (see *n'''*, fig. 27, Plate X.). In front of this line, however, on the outer side of the hypoglossal nucleus, and of the root of the hypoglossal nerve, at V', fig. 8, on the opposite half the medulla, the longitudinal bundles are numerous and close together, but as in the former case, their fibres are finer than those of the white column at the surface. The smaller column (V) immediately *in front* of the hypoglossal nucleus, and between the raphè and the origin of the nerve, contains many coarser fibres than those just mentioned. Now this column, as far forward at least as the posterior part of the olivary body, is the remains of the anterior column of the spinal cord, left on the outer side of the decussating fibres of the anterior pyramid, and known as the non-decussating portion of that body. It is seen at V in figs. 2, 3, 4, 5 & 6, Plate VIII. In fig. 5, Plate VIII. this column on each side is pushed to the back of the pyramid (*y*), and separated from its fellow by the numerous decussating bundles (*n''*) which sweep round from the bases of the *posterior* pyramids and restiform bodies. In fig. 6, Plate VIII. the two columns (V) are brought more closely together, more in front of the hypoglossal nuclei; and the decussating bundles of arciform fibres (which are now wider apart), instead of separating them from these nuclei, cross them transversely and then run forward between them to the back of the *anterior* pyramids and lower ends of the olivary bodies. In fig. 8, Plate VIII. they have lost their definite outline, being fused with other longitudinal fibres at the back of the anterior pyramids, between the decussating commissure of the olivary bodies; and they are still closer together, being separated only by the central raphè, and bounded on their outer sides by the hypoglossal nerves. The anterior cornu (*f'*, fig. 4, Plate VIII.) of the spinal cord is in fig. 5 resolved into network enclosing longitudinal bundles, and blending with the network (*s*) further back, which is left after the lateral columns have crossed over to the anterior pyramids (see *n*, fig. 28, Plate X.). In its place at *f'*, fig. 5, Plate VIII. is developed the olivary body (W, fig. 6), which is moreover connected with the white substance of the anterior column at its surface. After the bundles from the lateral columns have crossed over to the anterior pyramids, the central part of each lateral half of the medulla (at the letter U, figs. 6, 9 & 10, Plate VIII.) contain, as I have already stated, a multitude of

longitudinal bundles. How many of these fibres belong to those portions of the lateral columns of the cord that have not contributed to the formation of the pyramids, it is not easy to determine; but it is certain that the greater number of them are new and finer fibres, arising from the grey substance of the medulla. Nearer the surface, however, and more anteriorly, the original and coarser fibres of the antero-lateral columns of the cord continue to ascend the medulla (*d*, figs. 4, 5, 6, 8, Plate VIII., and fig. 27, Plate X.). Schröder van der Kolk believed that some of the original fibres of the lateral columns of the cord ascend on the same side beyond the decussation, but that they terminate, at the vagal nucleus, in the slender longitudinal column *n*, figs. 25 & 27, Plate X.* This opinion, as will be readily seen, must be erroneous; for it is very uncertain whether the slender column (*n*) contains any of the original fibres of the lateral columns; and even if it does, it is merely an offset from the layer *d'* (fig. 27, Plate X.), which continues to ascend much beyond its point of divergence (*n'*), and, as I have shown, consists, chiefly, at least, of new and finer fibres; while at the same time the superficial and original fibres of the lateral columns (*d*, fig. 27, Plate X.), like the layer *d'*, continue their course upward much beyond the point of divergence (*n'*) of the slender longitudinal column.

(30) I have already shown that the little group of small cells (T, fig. 38, Plate XI.) which lies on the inner side of the hypoglossal nucleus (J), increases as it ascends the medulla. In fig. 39, Plate XI., just where the hypoglossal nucleus ends, it is much larger, is nearer the surface, and together with the outer column K', constitutes the "*fasciculus teres*," or longitudinal column which runs along the fourth ventricle on each side of the median line. Near the level of the auditory nerves this small and inner group (T) sometimes lies with its long axis more vertical, as in fig. 40, Plate XI., and therefore forms a slightly prominent column which borders the side of the median sulcus, while the outer column (K') is more divergent, and runs along the inner side of the auditory nucleus (*i*), to reach the digital fossa of the ventricle. It will be seen that in fig. 40 this outer portion (K') of the "*fasciculus teres*," as a separate column, has diminished in size, while it has become intimately blended with the auditory nucleus (*i*), from which, in fig. 38, it was completely separated by the interposition of the vagal nucleus (*t*, H). Amongst the commissural fibres (*y'*) which decussate across the raphè, there is a conspicuous group of large multipolar cells.

(31) *Of the Striæ Medullares.*—In the floor of the fourth ventricle of the human medulla, a variable number of white bundles, of different sizes, and composed of nerve-fibres, are seen running in several directions, but chiefly more or less transversely, from the median sulcus, over the "*fasciculus teres*" and the auditory nucleus toward the root of the auditory nerves. These bundles are known to anatomists by the name of *striæ medullares*, and have been made the subject of investigation by Piccolomini, Soëmmering, Santorini, Prochaska, Rudolphi, the Wenzels, Gall, Spurzheim, Cuvier, Treviranus, Boek, Langenbeck, Arnold, Serres, Burdach, Bergmann, Heusinger and

* Schröder van der Kolk, Medulla Oblongata.

others, but more particularly by Otto Fischer. According to Prochaska and others, these *striæ* are sometimes entirely absent from the human medulla, and when they are present are wholly unconnected with the auditory nerves*. On the other hand, Burdach, Bergmann, Heusinger, and Arnold consider that they are directly connected, either wholly or partially, with the auditory nerves. Treviranus thought that the delicacy and variety of the sense of hearing in Man are due to these fibres because they are absent in animals†. Arnold found that, as a rule, a large proportion of these striæ were always in connexion with the auditory nerves. According to Serres, the sense of hearing in the child is never very acute until the striæ make their appearance‡. Longet, Meckel, and Prochaska state that they are sometimes altogether absent. In one deaf and dumb subject examined by Schröder van der Kolk, they were scarcely to be found, while in another they were very fully developed.

(32) In the large number of medullæ that I have examined, I have always found these striæ present in a greater or less degree; but in every instance they differed to a certain extent both in size and direction. Sometimes they arose out of the *fasciculus teres*; sometimes out of the posterior nucleus of the auditory nerve, on the floor of the ventricle; but more frequently they sprang from the median sulcus, and from the *fasciculus teres* along its edge. Some of them crossed the ventricle transversely, either in straight lines or in curves with their convexities upward, and winding round the restiform body or the inner edge of the *flocculus*, terminated somewhat abruptly, either in the flocculus itself or in the auditory nerve. Commonly one thick bundle, below the auditory nerve, turned round the restiform body to its anterior surface, and running between it and the olive, was continued into the pons (see Z″, fig. 41, Plate XI.). This bundle is pierced by the roots of the glossopharyngeal nerve, and by the upper roots of the vagus in their course inward to their nuclei§. Some of the striæ, instead of passing transversely outward towards the auditory nerve, occasionally ran obliquely upward to the inner surface of the middle peduncle; and I have sometimes found them taking an almost longitudinal course along the floor of the ventricle, beneath the superior peduncle, toward the corpora quadrigemina.

(33) On examining a transverse section of the medulla made in the course of a large stria which ran transversely in a straight line from the auditory nerve to the median

* Prochaska. De structura nervorum.

† Untersuchungen über den Bau und die Funktionen des Gehirns.

‡ Serres, Anatomie Comparée du Cervesu, tom. i. p. 424.

§ This is probably one of the bundles described by Bergmann, Arnold, and Otto Fischer. Arnold thinks it is connected with the facial nerve: he says, "Gewöhnlich nimmt aber auch eine Partie der Streifen oder eine ziemlich starke *Tænia medullaris* (tab. viii. fig. 3, a) welche Bergmann den Klangstab nennt, ihre Richtung aus- und aufwärts gegen den Pedunculus cerebelli und verbindet sich mit ihm; nicht selten ziehen mehrere Streifen um die strangförmigen Korper nach aussen und unten, gehen in die Bruckenfasern, auch in die Wurzeln des *Nervus facialis* über, oder verlieren sich am obern Ende der Medulla Oblongata, in jener Grube zwischen der Brücke, den oliven und stranförmigen Korpern." Otto Fischer says only, "Spatium triangulare, quod inter restiforme olivareque corpus et pontem Varolii interest, iniebant."

sulcus of the ventricle, I found that on reaching the median sulcus its fibres sank into the medulla, and then ran at a right angle from *behind forward* down the corresponding lateral half of the raphè, as represented at R, fig. 42, Plate XII. On tracing the fibres from this point backward and outward, they were seen to pass through and on each side of the oval group of cells (T), constituting, as already described, the inner portion of the "*fasciculus teres*," and then to wind over the outer column (K′) of that fasciculus, and over the auditory nuclei (*i* and *c o′*) as far as Q Q′, where they were closely united to the auditory nerve (P). The restiform body (*c*) was enclosed by both the stria and the nerve. *g* represents the uppermost fibres of the glossopharyngeal nerve. Some of them in their passage beneath the restiform body (*c*) turn round to *e*, *e*, *e*, *e*, the cut ends of the descending portion of the fifth nerve in the grey tubercle or caput cornu posterioris; others go to the slender longitudinal column (*s*), and the rest to the group of cells (H), which constitutes the chief nucleus of the nerve, and is continuous with the column of cells that belongs to the vagal nucleus. U is the lower end of the column of cells forming the nucleus of the motor root of the trigeminus, and S is a network containing several large multipolar cells, with intervening longitudinal bundles of fibres, beneath the "*fasciculus teres*," and in the place where the hypoglossal nucleus ended. Fig. 40, Plate XI. represents another transverse section of the fourth ventricle a little lower down than the one just described, but passing through one of the striæ. In this case, however, the stria, instead of running in a *straight line* across the ventricle, curved downward as it approached the median sulcus, and was therefore divided obliquely at different parts of its course, as seen at Q Q′. On examining the section, and then comparing it with the cut surface of the ventricle from which it was taken, and which contained the remainder of the stria, it was not difficult to see that the fibres had different destinations. On reaching the median sulcus, the stria partially divided into two branches. One of these (at Q′) ran transversely forward through the oval group of cells (T), especially through its inner and outer sides, and then passed into the raphè as in the former case. The other branch turned longitudinally downward over and through the posterior part of the oval group (T). The cut ends of its fibres are represented by the dark spots at the extremity of the line (T) on the left side. It is most probable that these fibres, after running for different distances downward, pass, like the others, transversely forward through the vesicular column (T), and thence into the raphè.

(34) There can be no doubt whatever that many of the striæ medullares connect both the portio mollis and the auditory nucleus with other parts of the medulla and brain. Fig. 41, Plate XI. represents the side and under surface of one lateral half of the pons Varolii and medulla oblongata. W is the pons, *y* is the anterior pyramid; *w* the olivary body; X the portio dura or facial nerve; *c* the restiform body; Y the cut surface of the middle and inferior peduncles of the cerebellum; Z is the auditory nerve turned backward over the restiform body; Z′ the cut end of the fifth nerve. The large band of fibres proceeding out of the fourth ventricle, and turning round the restiform body at Z″, are seen to lie close against the roots of the auditory nerve (Z), as they pass

forward to the pons, where some of them, after running round the root of the fifth nerve (Z') plunge into its substance, while others are lost amongst its transverse fibres at W'. Both these sets of fibres are joined by others, which also spring from the fourth ventricle, but pass *beneath* the restiform body, and emerge at Y', decussating in their passage with the roots of the auditory nerve (Z), as represented in the figure. These fibres for the most part run toward W', where they are often continuous with a very prominent bundle of the pons. Above the auditory nerve a few more fibres (X') proceeding from the fourth ventricle curve round the restiform body as it turns upward, and are lost on the middle peduncle of the cerebellum. These fibres are joined by others proceeding from under the restiform body at its junction with the middle peduncle.

(35) The whole of the fourth ventricle and most of the parts that I have just been describing, are covered with a layer of columnar epithelium-cells which are shorter than those around the canal of the spinal cord, but, like them, they give off from their tapering ends a multitude of fine fibres which run in different directions through the subjacent tissue. At the lateral parts of the ventricle, the epithelium frequently assumes the form of a thicker membranous layer, which springs chiefly from the grey substance lying in the angle formed by the divergence of the vagal nucleus and the posterior pyramid, and sometimes from the vagal nucleus itself, upon which, as also shown in transverse section, there is often a small heap of epithelium-cells (fig. 21, Plate IX. *t''*). After winding outward round the posterior pyramid and restiform body, the membranous layer frequently spreads over the roots of the glossopharyngeal and vagus nerves, matting or joining them firmly together. Above, it is continuous with the general epithelial layer of the ventricle, and extends over the auditory nerve and side of the flocculus. When carefully peeled off from the side of the medulla, it is found to be firmly attached to the grey substance, between the vagal nucleus and posterior pyramid, by a multitude of fibres, which are the processes of the epithelial cells, and which on removal leave a corresponding number of minute perforations, giving to the surface a finely dotted appearance*.

Of the Auditory Nuclei and Nerves.

(36) I have already described the morphological changes that take place from below upward in the formation of the nervous centres which give origin to the auditory

* The membranous layer of epithelium above described is evidently the "membrane nerveuse" described by Foville as springing from the auditory nerve to spread over the whole of the ventricle and into the cerebellum. "C'est de cette membrane," he continues, "que se détache la lame nerveuse qui unit au nerf auditif le petit lobule suspendu au-dessus de lui (the flocculus); à cette membrane emanée du nerf auditif se rattache encore la lame grisâtre élevée de chaque côté du calamus scriptorius" (the vagal nucleus). He considers the *striæ medullares* as only thicker bundles of the fibres which spring from the auditory nerve and extend into this "nervous membrane." "Les tractus blancs, les tractus gris remarqués à la surface du plancher du ventricule cérébelleux et signalés comme des racines de l'auditif ou comme un ganglion de ce nerf, ne sont que des faisceaux plus forts parmi ceux qui du nerf auditif se portent à la membrane nerveuse pariétale du ventricule." Foville, Traité complet de l'Anatomie, &c. du système nerveux cérébro-spinal, pp. 503, 504.

nerves. These centres on each side consist of an inner and an outer nucleus. The inner nucleus (*i*, fig. 40, Plate XI.) forms a large convex mass at the surface on each side of the fourth ventricle. Lower down in the medulla it arises by a point or angle between the pyriform nucleus of the vagus (*g*, fig. 11, Plate XI.) and the upper extremity of the posterior pyramid (*b'*) in the way already described. In transverse section it is triangular, one of its angles projecting forward into the root of the caput cornu posterioris, or grey tubercle, *e*, at the side of the slender and longitudinal column, *n* (see figs. 42, 44, Plate XII.). At the lower part of the fourth ventricle it is separated from the hypoglossal nucleus, *t'* (fig. 11, Plate IX.), by the vagal nucleus (*g*); but higher up it is continuous laterally with the "fasciculus teres," K' (fig. 40, Plate XI.), the remains of the vagal nucleus (H, fig. 42, Plate XII.) having sunk away from between them to a deeper, more anterior, and lateral part of the medulla, at the root of the grey tubercle. This inner nucleus of the auditory nerve (*i*) is thickly interspersed with nerve-cells, which are round, oval, crescentic, triangular, or otherwise irregular in shape, and of every variety in size, the largest being nearly equal to those of the anterior cornu of the spinal cord.

(37) The *outer* nucleus of the auditory nerve (*c' o*, fig. 43, Plate XII.) is in contact with the outer side of that jus tdescribed. It consists chiefly of the outer and inner grey substance or nuclei of the restiform body (*c'o*, figs. 20 & 21, Plate IX., and figs. 22 & 23, Plate X.), blended together, and with what remains of the posterior pyramid after the complete formation of the *inner* auditory nucleus. This common mass now assumes the appearance of a remarkably beautiful network composed of nerve-fibres, and enclosing in its meshes a multitude of longitudinal fasciculi, the cut ends of which are represented by the dark spots at *c'o* in fig. 23, Plate X., and fig. 43, Plate XII. The manner in which these longitudinal bundles, with the grey substance between them, are derived from the restiform and post-pyramidal nuclei, is shown in the longitudinal section of the left half of the human medulla (fig. 24, Plate X.), where *c'o*, at the upper part, is seen to be formed out of *c'*, *o* blended together at *o'*. The transverse network enclosing these longitudinal fibres is composed chiefly of bundles which proceed out of the inner auditory nucleus (*i*, fig. 23, Plate X., and 43, Plate XII.), on the outer side of which they form, on leaving it, a beautifully serrated border. The cells are of different shapes and sizes, but are very much branched, or elongated on the one hand, in the direction of the transverse fibres of the network amongst which they lie, and which their processes contribute to form; and on the other hand, in the direction of the longitudinal bundles enclosed in its meshes.

(38) From both of these nuclei (the inner and the outer) the posterior division of the auditory nerve takes its origin. Near its exit from the inner nucleus (*i*, fig. 43, Plate XII.) there is generally a gentle swelling or elevation on the surface (*i'*), which contains a number of small cells dispersed amongst its fibres. From this point it winds, as a broad convex band (P), outwards and forwards over the restiform body (*p'*). In this course it contains at first a few small cells, but as it proceeds outward the cells become larger, elongated in the dirction of the fibres, and more numerous, until, at the

under or anterior border of the restiform body, they collect into a fusiform or a somewhat triangular group within a pyriform swelling of the nerve (P'). The inner layer of the nerve is whiter and more fibrous than the rest, and is continuous with transverse fibres (*p'*) which radiate from the restiform body around which it winds. These transverse radiating bundles (*p'*), which divide the longitudinal fibres of the restiform bodies into flattened bands, belong to a system of arciform fibres which sweep round in front of the grey tubercle to the antero-lateral nucleus and olivary body. They are seen also at *p' p''*, fig. 10, Plate VIII., and fig. 24, Plate X.

(39) A few sections higher up in the medulla, on the inner side of the ganglionic enlargement of the nerve, are several irregular clusters of fine nerve-cells (Z, fig. 44, Plate X.). They are separated from the grey tubercle (*e e*) only by the arciform fibres, just mentioned, which sweep round it from the restiform body, and they receive bands of fibres, which, like the upper roots of the glossopharyngeal nerve, arise from the slender longitudinal column (*n*), and proceed outward along the posterior surface of the grey tubercle (*e e*), in which the more or less circumscribed spaces represent the cut ends of the bundles in the descending portion of the sensitive division of the fifth nerve. At U is seen the *motor* nucleus of the same nerve, increased in dimensions and connected by fibres with the "fasciculus teres" (see fig. 46). Q Q' (fig. 44) are the cut ends of two *lineæ transversæ*, taking an oblique course over the inner auditory nucleus (*i*) and the "fasciculus teres." The network of the outer auditory nucleus (*d'o*) tapers forward and outward, and completely overlays the grey tubercle (*e e*), with which it is moreover intimately connected (see fig. 43). The ganglionic enlargement on the auditory nerve is seen at P'.

(40) A little above this level, in transverse sections carried somewhat obliquely upward into the fourth ventricle, the *anterior* division of the auditory nerve comes into view. It consists of two portions of unequal size. The smaller portion (P'', fig. 45, Plate XII.) runs backward along the upper edge of the restiform body (*c*), with an offset from which it then crosses transversely over the superior peduncle of the cerebellum to the inferior vermiform process. In this course it is accompanied by other fibres, which form with it a kind of band of white substance (*c''*, fig. 45, Plate XII.). This figure represents a side view of the human pons Varolii and medulla oblongata. *c* is the restiform body, bending backward to form a kind of fan-shaped expansion. The chief portion of it enters the lateral lobe of the cerebellum, grasping, as it were, the corpus dentatum; but a smaller portion turns inward, and arching over the superior peduncle of the cerebellum, enters the superior vermiform process, in company with the auditory roots just mentioned. This band of white substance (*c''*) is joined by some *twisted* fibres (*e'*) coming from beneath the fillet (*f'*) and the corpora quadrigemina (*q'*). Anteriorly it is continuous with the epithelium on the surface of the superior peduncle.

(41) The other portion of the anterior division of the auditory nerve (P'', fig. 46, Plate XII.) is much larger than the one just described. It traverses the medulla as a flattened band beneath the restiform body in a direction inward, backward, and a

little upward, and contains nerve-cells of considerable size, collected generally into two or three groups, and sparingly scattered at intervals with their longer axes in the direction of the fibres. In its course it immediately overlays the grey tubercle (*e*) enclosing the descending portion of the trigeminus, and is then inserted into the network of the outer auditory nucleus (*c' o*), where it diverges into several bundles. In careful dissections and longitudinal sections, the trunk of the nerve in its passage upward and inward is seen to wind round the root of the middle peduncle of the cerebellum, where it meets and overlays the descending root of the trigeminus contained in the grey tubercle, which is itself only the downward continuation of the grey nucleus of the trigeminus. first sight the two nerves seem to form a loop with each around the inner side of the middle peduncle; but on closer examination it is found that the auditory nerve passes over the descending portion of the trigeminus and enters its own nucleus a little behind and to the inner side of the conical mass which forms the nucleus of the large root of the fifth. In its passage, however, it is very intimately connected with this nerve and the grey tubercle. In fig. 46, Plate XII., there may be observed in the middle of the grey tubercle a circular mass or column (*e*) composed of small nerve-cells. In front of this and closely connected with it is a beautiful and delicate grey network containing small nerve-cells, and enclosing in its meshes the numerous bundles of the descending root of the fifth nerve, the cut ends of which are represented by the dark masses. This network is continuous behind with the coarser one which forms the outer auditory nucleus (*c' o*), and which gives origin to the anterior division of the auditory nerve (P''). On the inner side of the grey tubercle, at U, is the group of large multipolar cells composing the nucleus of the small root of the fifth nerve, and connected by a plexus of fibres with the *fasciculus teres* (K'); and in front of this nucleus and of the grey tubercle is a broad band of arciform fibres (*p'' p'''*) which proceed from the restiform body and middle peduncle at *p'*, to sweep round and diverge into the central parts of the medulla. Fig. 47, Plate XII. represents a dissection of the left side of the medulla and of the fourth ventricle, showing the anatomical relations of these and other parts. The dissection was made by first removing the epithelium on the posterior pyramid and peeling off the posterior root of the auditory nerve from without inward, including the *inner* nucleus in which it ends. The restiform body or inferior peduncle (*c c'*) of the cerebellum, and the middle peduncle (*c'''*), were then drawn gently away or aside from the central portion of the ventricle, and pressed a little downward, exposing a somewhat triangular space which is divided into two unequal parts (A', B') by the large root (C') of the fifth nerve in its passage to the conical eminence or nucleus (D') which projects from the side of the ventricle. The descending prolongation of this nucleus with the descending portion of the nerve, constituting together the grey tubercle (*e*) of fig. 46, are seen to be overlaid by the anterior division (P'') of the auditory nerve, as it passes obliquely upward and inward to its nucleus beneath *i*, which is the concavity left by the removal of the *inner* auditory nucleus with the *posterior* division of the nerve. The anterior and under portion of the trigeminal nucleus (D'), where it is connected with the point of the fillet

(f'), gives origin to the small or motor root (G') of the fifth nerve, which is separated from the large root (C'') by the smaller division (A') of the triangular space, transmitting a small bundle of the middle peduncle coming from the pons Varolii. The larger root of the fifth nerve (C'') is separated from the anterior auditory nerve (P'') by the larger division (B') of the triangular space, transmitting a corresponding portion of the middle peduncle, ascending also from the pons. At E' the *superior* peduncle of the cerebellum is rolled forward, exposing a thick and broad tract of grey substance (D'') which extends obliquely upward beneath it from the conical nucleus (D') of the fifth nerve. On the inner side of the auditory nucleus (*i*) is an oval mass (F') imbedded in the substance of the fasciculus teres, and connected with the facial nerve.

(42) In Mammalia generally the auditory nuclei and nerves present nearly the same appearances as those which I have described in Man; but in all those animals there are one or two points of difference, and in some there are certain peculiarities which deserve notice. In all the larger Mammalia, as the Ox, Sheep, &c., the nerve-cells of the outer nucleus which gives origin to the anterior or lower root of the auditory nerve, are much superior in size to those of the human medulla. Moreover the anterior root on entering the nucleus is generally somewhat more divergent than in Man, as may be seen at P'', fig. 48, Plate XII., which represents the left lateral half of the medulla and part of the inferior vermiform process of the cerebellum in the Rabbit. Amongst the divergent roots are many longitudinal fibres, which are especially numerous at the bottom (e'') of the grey tubercle (*e*). In the same animal the *posterior* root of the nerve is very large, and is especially remarkable for the enormous convex enlargement, or ganglion on its surface, at the side of the fourth ventricle just before it enters the inner nucleus. In the Cat this remarkable ganglion is absent; but as the nerve proceeds outward it gradually swells into a large club-shaped ganglionic mass (PP', fig. 49, Plate XIII.), which is crowded with nerve-cells. The inner nucleus (c' *o*) is largely developed, and its connexion with the inner layer (P''') of the nerve, which consists chiefly of *white* fibres, is very distinct. Below the inner nucleus ($c'o$) is the descending trigeminal root (e''), which here forms a broad and compact layer of fibres on the outer side of the grey tubercle (*e*). On the inner side of this tubercle is the large round *antero-lateral nucleus* (*s*), which is connected on the one hand with the arciform fibres sweeping round from the restiform body (p', p''), and on the other hand, with its opposite fellow, by means of a broad band of commissural fibres which decussate across the raphè with those of the opposite side*.

(43) Just above the posterior root of the auditory nerve the auditory nucleus becomes closely connected with the flocculus; and at the level of the anterior root (P'', fig. 48, Plate XII.) the nuclei (c' *o* and *i*) stretch backward and inward to join the side of the inferior vermiform process (I' I''). At this part the plexus of bundles from the outer

* In the Rabbit the glossopharyngeal nucleus is very large, and the fibres which cross it from the inner auditory nucleus (see t'', fig. 24, Plate X.) are unusually numerous. On a former occasion (Phil. Trans. 1859) I pointed out the very large size of the caput cornu and grey tubercle in all the Rodentia. The grey tubercle in the Rabbit is seen at *e*, fig. 48, Plate XII., surrounded by the descending root of the fifth nerve (e'').

nucleus (*c' o*) runs through the commencement of the superior peduncle, or *processus è cerebello ad testem* (E' E''), from which it extends into the vermiform process (I' I''). The light spaces between the bundles of the plexus at E'' are the longitudinal fasciculi of the superior peduncle. The same kind of appearances are observable in the human medulla. In fig. 46, Plate XII. a plexus of fibres containing nerve-cells is seen to extend backward from the outer nucleus (*c' o*), partly toward the corpus dentatum cerebelli (J'), but chiefly, in company with other fibres from the inner nucleus (*i*), over the fourth ventricle to the inferior vermiform process.

(44) A brief account of the formation of the auditory nuclei and of the course and connexions of the auditory nerves was given in my "Notes" published in the 'Proceedings of the Royal Society' for June 20, 1861, and has since been confirmed by Dr. JOHN DEAN, whose descriptions are much superior to those of either STILLING or SCHRÖDER VAN DER KOLK. STILLING, however, was the first anatomist that described the course and attachment of the auditory nerve within the medulla, and his descriptions, so far as they go, are very correct, and faithfully illustrated by very beautiful engravings. But he failed to trace the *posterior* auditory root to any nucleus within the medulla; that which I have shown to be its proper nucleus, he mistook for the nucleus of the glossopharyngeal nerve*; and although he was the first to trace the *anterior* auditory root to what I have called the *outer* auditory nucleus (*c' o*), he scarcely seems to consider this as its proper origin†. STILLING had evidently not discovered the morphological changes through which, as I have shown, the auditory nuclei are developed. Moreover the bundles of longitudinal fibres which run along the front of the grey tubercle, and which I have shown to belong to the descending root of the trigeminus, were mistaken by STILLING for the primitive fasciculi of the posterior columns of the spinal cord,—"pristina pars funiculorum alborum posteriorum." SCHRÖDER VAN DER KOLK'S account of the origin and connexions of the auditory nerves, although fuller than that of STILLING and in many respects superior, is nevertheless imperfect, and in some points erroneous. Like STILLING, he had evidently not detected the important morphological transformations through which, as I have already shown, the auditory nuclei are developed, and he was certainly not acquainted with the real nature and connexions of what I have termed the *inner* nucleus. Whether, like STILLING, he mistook this nucleus for that of the glossopharyngeal nerve, is not easy to say, as he has nowhere given us any account of the origin of that nerve. Certain it is that he had not discovered the connexion of the *inner* nucleus with the true posterior root, which he evidently confounded with the *striæ medullares*‡. He corrected, however, the error which STILLING had made in describing the longitudinal fibres in front of the grey tubercle as the primitive posterior columns of the spinal cord, and showed that they consist of the descending root of the fifth cerebral nerve§.

* Ueber die Medulla Oblongata, p. 40, tab. vii.

† Pons Varolii, pp. 158, 159.

‡ Spinal Cord and Medulla Oblongata, pp. 126, 127.

§ This important anatomical fact was discovered independently by SCHRÖDER VAN DER KOLK and myself at

Of the Facial Nucleus and Nerve.

(45) The peculiar course taken by the roots of this nerve, at their origin from the floor of the fourth ventricle, as well as the intimate communications which they form with the roots of the abducens nerve, have rendered their examination so exceedingly perplexing, that although they have been carefully investigated by several eminent histologists, it is acknowledged that the question has never yet been satisfactorily settled. My own account of the course pursued by this nerve will be found to differ considerably from that of preceding anatomists, in consequence, I believe, of the different and more varied plan of investigation which I have adopted, and which has enabled me to determine, with great accuracy, the points of peculiar difficulty. I shall first describe the minute structure of the facial nucleus. This nucleus occupies the greater portion of the convex longitudinal column which runs along the floor of the fourth ventricle on each side of the median furrow, and which is well known to anatomists as the "fasciculus teres." A transverse section of this column on the left side of the ventricle, and a little below the origin of the facial nerve, in Man, is accurately represented in fig. 50, Plate XIII. Like the rest of the ventricle, it is covered with a single stratum of columnar epithelium (f'''). Beneath the epithelium is a broad layer of grey substance, which is continuous with that of the opposite side, beneath the median fissure. It consists of nuclei, cells, and fibres. The nuclei, for the most part, appear to belong to the connective tissue of the layer, and are very numerous. The cells, on the contrary, are sparingly scattered, and of small size. They are larger and more numerous along the under surface of the layer, and at its lateral part, where they are frequently collected into a small group (g'). The fibres are of two kinds. Some of them, which are exceedingly fine, proceed from the tapering ends of the epithelial cells, and running through the layer in different directions, are connected in part with its nuclei. The others are connected with the nerve-cells and nerve-roots, and run both in a transverse and longitudinal direction. On the median side of the layer, at g''', the transverse fibres (which appear to consist of those derived from both the epithelium and nerve-cells) are particularly numerous, and some of them decussate in front of the median furrow (h''), with their fellows of the opposite side. Sometimes at the side of the furrow there is a very distinct oval or fusiform group of small cells, as shown on the right in the figure.

(46) Beneath the lateral and middle portions of this superficial layer of grey substance,

almost exactly the same time. My Essay on the Medulla Oblongata, in which the statement appears, was read before the Royal Society in June 1857, and published in Part I. of the Philosophical Transactions for 1858; and Schröder van der Kolk's Essay on the Medulla Oblongata, in which the same fact is stated, was also published in 1858 (see his translator's Preface). But there is some difference between Schröder van der Kolk's account and my own. He considers that the whole of the grey tubercle in the upper part of the medulla consists of the descending *root* of the trigeminus, and that it differs in structure from the grey tubercle below the olivary bodies; while I regard the grey tubercle throughout the medulla as the downward prolongation of the grey nucleus of the trigeminus, and the longitudinal bundles in front of the tubercle, as the downward prolongation of the fibres of the posterior root.

and partially blended with it, is a large and nearly cylindrical column of nerve-cells (Q'') constituting the principal nucleus of the facial nerve. The cells are much larger than those of the superjacent layer, but differ from each other considerably both in shape and size, and have a great resemblance to the cells of the hypoglossal nucleus. Their average size is smaller in Man than in the higher Mammalia, as the Sheep or Ox. From the lateral part of this nucleus bundles of curved fibres (R'') proceed outward in the direction of the facial nerve. From the middle of the nucleus other bundles of fibres (S'') extend forward to the motor nucleus of the trigeminal or fifth cerebral nerve; and on the *inner* side of the nucleus is another system of fibres (T'' T'''), extending, like the former, from behind forward, but differing among themselves with regard to their destination. Only a part of these transverse fibres, as we shall presently see, issue from the superficial grey layer (g'''); most of them radiate from a *longitudinal column*, which runs immediately beneath it, and which is composed of fibres of the facial and abducens nerves, but chiefly of the former. Some of them curve outward and enter the facial nucleus (Q''); while the rest extend forward, in company with those at S'' proceeding from the *middle* of the same nucleus, to reach the motor nucleus of the fifth nerve, and the *superior* olivary body*.

(47) Another set of transverse fibres (U'', U''') crossing at right angles the three sets of fibres just described, connect the column of cells (Q''), on the one hand with the auditory nucleus (at U''), and on the other hand with the opposite side of the medulla across the median raphè at U'''.

(48) In transverse sections made a little higher up, we see the oval cut-end of a longitudinal column, from which the third set of transverse fibres (T''), just described, radiate forward. This column is represented at T'', fig. 51, Plate XIII. It first makes its appearance, however, below the level at which the nerve enters the nucleus, in sections intermediate between fig. 50 and fig. 51. In fig. 51 may be observed a number of fibres converging from different parts of the superficial grey layer at g''', and turning round the oval column (T'') to decussate across the raphè, in company with others proceeding from the principal nucleus (Q''). Some of these converging fibres, however, instead of winding inward round the column to the raphè, run forward as roots of the abducens nerve (V').

(49) In the Mammalia the facial nuclei differ in some respects from those of Man. In the Rabbit the commissural fibres are remarkably well seen. Fig. 48, Plate XII., as before stated, represents a transverse section of the left half of the medulla. A little below the entrance of the facial nerve, near the median furrow, is the cut-end of the longitudinal column (T''), which is more cylindrical than in Man. On the outer side of this is the principal nucleus of the facial nerve, included within the crescentic and dotted line. The cells of this nucleus are very numerous, but smaller than those of Man, and somewhat different in shape. On its outer side are some much larger multipolar cells, sparingly scattered, but passing uninterruptedly into the denser group constituting the

* In the Orang Outang these fibres are uncommonly numerous.

outer auditory nucleus (c' o), to which they seem to belong. The superficial grey layer forming the floor of the fourth ventricle is deeper than in Man, and contains larger and more numerous cells. Fig. 52, Plate XIII. represents the same parts in the Rabbit more highly magnified. The longitudinal column (T'') is closely encircled by a dense layer of fibres, which, after reaching its inner side, run for the most part together across the raphè to decussate with their fellows immediately below the median furrow (h''). In this course they are joined by other fibres proceeding from the column itself. Some of them, however, instead of crossing the raphè appear to become longitudinal at the point (h'''). Adjoining this layer of fibres, on the left or outer side of T'', and forming the chief part of the same commissural system, is a broad fan-shaped expansion of fibres, which converge inward to the front of the longitudinal column (T''), where they interlace each other in an intricate plexus (k' k'), between a multitude of longitudinal bundles (represented by the dark masses) with intervening multipolar cells. The greater number make their way through this plexus to the raphè to decussate with their fellows of the opposite side; while the rest, instead of running to the raphè, bend forward precisely in the direction of the abducens nerve (k''), and joining in the general plexus between longitudinal bundles and nerve-cells advance towards the front of the medulla. It is probable, however, that many of these transverse fibres communicate their influence longitudinally by means of their connexion with the numerous nerve-cells, of which some of the processes run with the longitudinal bundles and form part of their constituent fibres; for such an arrangement I have discovered in other portions of the medulla having the same general appearance. If such be the case, the interlacement of the transverse fibres, and their communication with different longitudinal bundles, provide for a wonderfully extensive and complicated commissural connexion between parts at different levels. On considering the parts from which this fan-shaped system of transverse fibres is derived, we find that some of them issue from different portions of the superficial grey layer (g'' g''') with different degrees of curvature forward and inward; that others come from the neighbourhood of the auditory nucleus (c' o); and that many arise out of the large group of cells included in the crescentic dotted line, and constituting the principal nucleus of the facial nerve*. All the fibres derived from both g'' and c' o, c' o, run into this nucleus; but whether they all pass through it to reach the plexus or the raphè I cannot decide. It is probable that some of them, at least, are commissural between the facial nucleus and the parts from which they are derived.

Of the Facial Nerve.

(50) It was evident to me that the complicated course and connexions of the facial nerve could never be accurately ascertained by the method hitherto adopted of examining transverse sections alone. The plan pursued by myself was as follows. Having first ascertained the exact course and connexions of the nerve, as exhibited in *transverse* sections, I then carefully made a succession of longitudinal sections in planes inclining

* The cells have been omitted in order to show more plainly the fibres which pass through it.

inward in the direction of the line m', m'', fig. 51, Plate XIII. (examining by means of a lens the cut surface of the medulla after each section), until I found that I had exposed the *outer* fibres of the nerve. The remaining portion of the nerve, in its course inward to the "fasciculus teres," was then exposed by carefully taking off a succession of similar sections in planes inclining more and more horizontally inward as the "fasciculus teres" was approached, the last sections being quite horizontal, from l' to l'', fig. 56, Plate XIII., and therefore including the posterior portion of the longitudinal column (T'') lying at the side of the median furrow (h''). When this process was completed, it was found that the nerve, on reaching the fasciculus teres, bent longitudinally down the medulla, and after a very short course, again bent transversely forward, to form a loop along the side of the median furrow, as shown at q'' in the Sheep (fig. 53, Plate XIII.) and in Man (fig. 54). The summit of this curve constitutes the longitudinal bundle of fibres of which the oval cut-end in transverse sections is seen at T'', figs. 51, 52, & 56. In the Sheep the course of the nerve q'', fig. 53, through the medulla on its way to the "fasciculus teres" is nearly straight, and nearly at a right angle to the axis of the medulla; while in Man (fig. 54) its course through the medulla is not only rather serpentine or wavy, but inclines somewhat *upward*. Hence the differences of appearance between *transverse* sections in Man and those in animals. The loop which it forms reaches close up to the edge of the median furrow (h''), and encloses its nucleus (Q''), which together with it forms the oval swelling (r'') seen on the surface of the *fasciculus teres* of the opposite side. The lower arm of the loop in its course forward divides like a brush into separate fibres which plunge into the motor nucleus of the fifth nerve, and into the superior olivary body (s''). Fig. 55, Plate XIII., represents a thin section of this loop, made in the plane of its course, from Man, magnified about 100 diameters, and viewed as from the side of the medulla. $g''' g'''$ is the superficial grey layer on the surface of the *fasciculus teres*, and seen in transverse section at $l'' g'' g'''$, fig. 56, Plate XIII. The root of the facial nerve ($v''' q''$, fig. 55), just before it curves downward along the side of the median furrow, beneath the superficial grey layer, is divided into separate bundles by long but very narrow streaks of longitudinal fibres, represented by the perpendicular, dotted lines. Its uppermost fibres (at v''') run directly backward and slightly upward to a collection of small cells and nuclei (v'') lying beneath the superficial grey layer of the ventricle, which is here composed of fine longitudinal fibres and numerous oval cells. The rest of the nerve divides into two portions, composing a superficial and a deep stratum of fibres. Its superficial stratum, on reaching the grey layer, curves *downward*,—that is, to the right, and in the plane of the paper, as seen in fig. 55,—to form the loop already mentioned. Its deep stratum of fibres bends *forward*, that is, perpendicular to the plane of the paper, and not to the right, in looking at fig. 55. For the most part they directly enter the nucleus Q'', as shown at $q'' Q''$, fig. 56. The course of these two strata of fibres may be better understood, perhaps, by supposing fig. 56 to be turned halfway round to the right with its plane perpendicular instead of parallel to the plane of fig. 55, and so that its nerve q'' shall coincide with the nerve q''

of fig. 55. It must be understood that in fig. 55 the nucleus Q″ beneath the loop extends under the *trunk* of the nerve q''; and that in fig. 56 T″ is the loop cut transversely. Moreover, these two sets or strata of fibres decussate each other at the first bend of the loop; for some from the deep or anterior stratum (q', fig. 56) run first *transversely* and obliquely *backward* from W″, and then *longitudinally downward*, along the *posterior* portion of the loop at U; while others from the superficial or posterior stratum (l') run first *transversely* but obliquely *forward* from W‴, and then longitudinally downward along the *anterior* and *middle* portion of the loop at T″, U′, which represents a transverse section of it in the direction of the line Q‴, fig. 55. It is evident, then, that the descending fibres of the loop cannot be parallel, but must intersect each at different angles, as shown in fig. 55, Plate XIII., which represents exactly the appearances presented by a very thin slice of the loop made in the plane of its course, and magnified about 100 diameters. Some of the fibres of the superficial stratum are seen to extend *backward* to the superficial grey layer (q'', fig. 56); while many of those belonging to the deep stratum (q'') curve *forward* into the group of cells (Q″). But the proper fibres of the nerve are evidently joined and crossed by others which proceed from the smaller group of cells (v'', fig. 55), and from the superjacent grey layer of the ventricle.

(51) The *lower* arm of the loop (q''', fig. 55) is smaller, looser, or less sharply defined than the trunk of the nerve (q''), and hangs gracefully forward like a plume of feathers. Many of its fibres droop into the facial nucleus (Q′) over which the loop is formed, and which is traversed moreover by a large number of longitudinal bundles communicating with each other in the manner of a plexus.

(52) We shall now see what important light is thrown by this dissection on the otherwise inexplicable appearances presented in transverse sections. If we make a transverse section inclining somewhat upward through the medulla, in the direction of the line W″ W‴, fig 55, Plate XIII., and therefore in a plane passing through the *lower* arm of the loop (q'''), we have the appearances presented in fig. 50, Plate XIII., which, however, is more highly magnified. In this figure we see that the set of transverse fibres (T′, T‴), of which the origin was exceedingly obscure, belong to the lower arm of the loop (q''', fig. 55), and that their apparent abruptness of origin is due to the circumstance of their having been cut across at the point where they change their course from longitudinal to transverse. If we make another transverse section a little higher up, between the arms of the loop, in the direction Q″, Q‴, fig. 55, it is evident that instead of seeing the set of fibres (T″, fig. 50) which emanate abruptly from beneath the superficial grey layer (g'''), we shall see the transversely-cut surface of the longitudinal portion of the loop. In Man this cut surface is almost perfectly oval (T′, fig. 51, Plate XIII.), but in the Rabbit (fig. 52) and some other animals it is more circular. The positions of the cut-ends of its fibres evidently show that these fibres are not parallel; and this appearance is quite explained by the decussation along the summit of the loop, at Q‴.

(53) If a transverse section be now made along the lower side of the *trunk* of the nerve (q'', fig. 55, Plate XIII.), in the direction of the line g''', g''', we find that its fibres,

on reaching the commencement of the loop, spread out, and bend forward and inward, but, at the same time, obliquely downward, so as to enter different parts of the nucleus by a series of drooping bundles along the under surface of the loop, as seen at the end of the line (Q'''). A transverse section along this part of the nerve is represented in fig. 51, Plate XIII. If we turn this figure on its right side with the plane of the paper perpendicular to the plane of that on which fig. 55 is represented, we shall have a good idea of the course of these fibres, provided we recollect at the same time that they run *somewhat obliquely* downward or to the right. In consequence of this downward course, the fibres of the nerve, as they enter the nucleus Q'', fig. 51, Plate XIII., are observed, in transverse sections, to be shaved off obliquely at different lengths. Moreover, it will be seen that all these fibres belonging to the *lower* side of the nerve-trunk enter different parts of the nucleus from without inward, the innermost meeting and intimately mixing with those of the abducens nerve (V'), as they bend forward on the outer side of the oval column or loop (T''). It is evident, therefore, that none of them contribute to form the longitudinal column (T''), although a few pass in front of it to decussate across the raphè with their fellows of the opposite side.

(54) On examining a similar section of the medulla a little higher up, as at v''', for instance, fig. 55, Plate XIII., we find that the nerve divides into two portions, a superficial and deep portion, as already stated, and as represented at l' q'', fig. 56. The fibres belonging to its deep portion (q'') bend forward and inward, as in the former case, and enter the nucleus (Q''), which is now somewhat diminished, and where they meet the network of bundles proceeding from the abducens nerve (V'). The fibres of the other portion, on reaching the *fasciculus teres*, curve downward without entering the nucleus, and thus contribute to form the longitudinal part or summit of the loop, the transversely-cut end of which is shown at T''. In the transverse section, therefore (fig. 56), they are seen to enter the cut-end of this longitudinal part (T''). Before they enter it, however, their arrangement, as already stated, is very curious, and doubtless very important. Many of the fibres from the deep or *anterior layer* (q'') of the nerve are seen to run obliquely *backward* and inward to the *posterior* or upper layer (V) of the longitudinal column (T''), where they bend round and run downward; while the fibres forming the superficial or *posterior* layer (l') of the nerve run obliquely *forward* and inward to enter the middle and anterior layers (T'' V') of the longitudinal column, and run downward with it, like the others. Some of them, however, continue their course transversely in front of the column to decussate with their fellows of the opposite side, across the median raphè (at h). It is evident, therefore, that just before these two sets of fibres bend round and contribute to form the longitudinal portion of the loop, they decussate each other (as shown in fig. 56, W'' W''') immediately behind the facial nucleus (Q''), with which the fibres forming the deep or anterior layer of the nerve are connected*. This, however,

* This decussation is crossed nearly at right angles by roots of the abducens nerve proceeding to the superficial grey layer (g'', g'''), and by other fibres which proceed from this layer to the nucleus (Q''), and which are particularly numerous in the Rabbit, Sheep, and other animals.

although the principal, is not the only, nucleus of the facial nerve; for some of the superficial or posterior fibres, as represented in fig. 56, may be traced backward and inward to the superficial grey layer (g''', g'') of the *fasciculus teres*, and especially to a group of cells (g''') situated on its outer side. Behind this group are the cut-ends of three considerable blood-vessels enveloped in their sheaths. The trunk of the nerve (q'') is frequently separated into two portions by a long and broad streak of grey substance (q''') containing some nuclei or small cells and the branches of a blood-vessel, and behind these the fibres are again separated by long and narrow streaks of longitudinal fibres, represented by the dotted tracts.

(55) In transverse sections of the medulla made along the uppermost border of the facial nerve, almost all traces of the principal nucleus disappear, and almost all the fibres of the nerve may be seen to enter the longitudinal portion of the loop (T'', fig. 56, Plate XIII.). Of those that run inward along the front of this portion of the loop, some turn backward to enter it (at V'), while others pass onward to decussate across the raphè (at h), the rest appearing to stop at its inner side (h''', fig. 52), at the edge of the median furrow, where they seem to become longitudinal, amongst the cut-ends of a layer of longitudinal fibres. The fibres of the nerve which pass round the *posterior* surface (T'', fig. 56), all enter it in succession, none of them reaching the raphè, or even so far as the inner side of T'', but terminating in it about the point l''; so that it is not even enclosed by fibres of the nerve.

Some of these uppermost fibres of the facial nerve, however, instead of running down with the longitudinal portion of the loop, diverge somewhat upward and backward amongst an irregular group of smaller cells and nuclei (as represented at v'', fig. 55) lying beneath the surface of the fourth ventricle.

(56) Such is the remarkable and complicated course of the roots of this importamt nerve. The accounts which Stilling and Schröder van der Kolk have given of the origin of the facial are very imperfect, and contain serious errors. Stilling, however, has the merit of discovering that the lower fibres of the nerve arise from the large nucleus of multipolar cells. Higher up the fibres appeared to him to run inward to the raphè, without entering the nucleus, and to decussate with their fellows of the opposide side. In this course they enclose in front and behind, but do not *enter* the longitudinal column, which I have shown to be the longitudinal portion of the loop of the facial nerve itself. This column is considered by him to be what he calls "the *constant* root of the trigeminus" or fifth cerebral nerve. At the upper limit of the nerve he found that all traces of the nucleus had disappeared; but he thought that the fibres bend downward to the nucleus below, while the rest run through the raphè to the columns which he considers as prolongations of the anterior columns of the spinal cord. Upon these points, however, he speaks with some diffidence*.

(57) Schröder van der Kolk's account is exceedingly defective, being scarcely more than "a general assent to the description of the course of the facial nerve given by

* Stilling, Pons Varolii, pp. 37, 38, plates 3, 4 & 5.

Stilling"*, with this exception, however, that he fell into a still more extraordinary blunder than Stilling, by asserting that the longitudinal column is not, as Stilling believes, *a root of the trigeminus*, but "*one of the cut posterior roots of the auditory nerve*" (one of the striæ medullares)†. It is remarkable that it did not occur to Schröder that bundles of fibres so variable in their course and size as the striæ medullares could not always be found in exactly the same place and present exactly the same appearances. Frequently in transverse sections the cut ends of more than one of the striæ are seen at the surface of the ventricle, and one of these, which is sometimes, but not always, situated at the side of the ventricle, was mistaken by Stilling for another root of the trigeminus, which he calls its "*inconstant* root."

(58) Dean's account of the origin of the facial nerve is by far the fullest and best. He states that at the upper part of the nerve, the whole bundle may be traced inwards to the longitudinal column, where the central portion of the root terminates abruptly, the outer fibres turning off behind and in front of the column, which is thus completely encircled by the roots which afterwards pass onwards to the raphè. But "even," he says, "in the upper portions of the facial course, where the whole bundle seems at first sight traceable to the raphè, the number of bundles actually decussating or passing into the raphè seems so small when compared with the great thickness of the root, that I am inclined to think that many of the fibres do actually turn downwards, passing down in the longitudinal columns on each side of the raphè to the underlying nucleus, justifying in this respect the conclusion of Stilling. I have been confirmed in this supposition, by frequently observing in the columns which Stilling has called the *constant roots of the trifacial*, and Schröder van der Kolk *roots of the auditory*, great numbers of fibres obliquely cut across, which are especially noticeable in connexion with the abrupt transmission of the facial roots just at this point, and I am inclined to consider these columns as, at least, partial channels by means of which the upper portion of the facial roots are conveyed downwards, either to the underlying nucleus or to decussate below in the raphè"‡.

On the Nuclei and Roots of the Abducens Nerve.

(59) The roots of the abducens nerve run nearly directly backward, from the junction of the anterior pyramid with the lower border of the pons Varolii, to the *fasciculus teres* on the floor of the fourth ventricle, at the same level as the roots of the facial nerve. Their course through the pons, however, is slightly undulating upwards and downwards, describing three gentle curves, as shown in the accompanying figure A. As they pass through the pyramid (*a*) they form a single compact bundle, which curves slightly upward. At the back of the pyramid, however, this bundle separates into three or four, which

Fig. A.

* Schröder van der Kolk, Medulla Oblongata, pp. 117, 118. † *Loc. cit.* p. 117 (note).

‡ The Grey Substance of the Medulla Oblongata and Trapezium, by John Dean, M.D., Washington, U.S., 1864, p. 59.

curve first slightly downward, then upward, and again downward as they reach the floor of the ventricle (*b*, *c*). In consequence of this peculiar course, we find that in transverse sections of the pons, at this level, the roots of the abducens nerve are always cut into portions of variable length. Moreover, in transverse sections it is found that they are also somewhat tortuous from *side to side*, and that, unlike the roots of the other cerebral nerves, they curve *outward* instead of inward towards the raphè, as may be seen in figs. 51 and 56, V', Plate XIII., from the adult brain, and fig. 65, Plate XIV., from the brain of a child five years old.

(60) The *lower* bundles of the roots (V', fig. 51, Plate XIII.), as they approach the *fasciculus teres*, give off some lateral branches or tufts of fibres which curve outward to the column lying immediately below, or in front of, the great nucleus (Q''), and containing branched nerve-cells, with numerous longitudinal and transverse fibres, the latter of which might easily be mistaken for these branches. They then taper into a kind of narrow band, composed of the remaining fibres, which curves backward and outward round the inner side of the great nucleus (Q''), and along the outer margin of the longitudinal portion (T'') of the loop of the facial nerve. Amongst this band are numerous cells of considerable size and of different shapes, but for the most part oval, and elongated with their processes in the direction of the fibres. They appear to form part of the great nucleus (Q''). Some of the fibres curve outward into the middle of this nucleus; others curve outward round its posterior border, where they meet fibres coming in an opposite direction from the facial nerve (*q''*), the two sets together forming a kind of loop, interspersed with nerve-cells and some longitudinal bundles. The remaining fibres run directly backwards, and crossing the facial roots between the nucleus (Q'') and the longitudinal column (T'') of the facial nerve, diverge into the superficial grey layer of the *fasciculus teres*, at *g'''*. Some of them run more or less directly backward towards the surface; some turn outward over the convex surface of the facial nerve, amongst nerve-cells of considerable size lying at the bottom of the grey layer; while the rest turn inward around the posterior border of the longitudinal column (T'').

(61) As it ascends beneath the floor of the fourth ventricle the common nucleus (Q'', fig. 56, Plate XIII.) becomes gradually smaller. Its *outer* portion, or that which is in connexion with the roots of the *facial* nerve (*q''*), consists, as it does lower down, of numerous large multipolar cells and many longitudinal bundles; but in its *inner* portion, or that which is more immediately connected with the roots of the *abducens* nerve (V'), the cells are for the most part smaller, less numerous, and elongated in different directions, or crescentic; while the longitudinal bundles (represented by the dark masses), which are often embraced by the cells, and of which many are exceedingly thick, have increased considerably in number and are enclosed in the meshes of a network formed by the rootlets of the abducens nerve, exactly as represented in fig. 56. These rootlets unite rather abruptly, like the roots of a plant, to form four or five bundles (V'), which lean against each other at the point of junction, and then separate or diverge slightly as they traverse the pons Varolii outward. Many of the fibres, however, that con-

tribute to form these bundles proceed, not from the nucleus (Q″), but, as already shown, in the case of the lower roots (fig. 51) from different parts of the superficial grey layer of the *fasciculus teres* (g'' g'''), and in their way outward cross the decussating fibres of the facial nerve, just where these are about to enter the longitudinal column (T″).

(62) It is evident, then, that both the abducens and the facial nerves arise each from two separate nuclei, as well as from the same nuclei. In longitudinal sections of the pons Varolii, both in Man and animals, I have repeatedly endeavoured to ascertain whether any of the fibres of the abducens nerve turn round and run longitudinally upward, but I have never been able to perceive the slightest indication of such a course. Indeed, as already stated, the roots curve slightly downward on reaching the *fasciculus teres*, although I have never succeeded in tracing them any further in that direction*.

(63) Having ascertained the relations and connexions of the facial and abducens nuclei and of the longitudinal column (T″) in transverse sections, and in oblique-longitudinal sections along the plane in which this column is entered by the facial nerve, I wished, further, to ascertain their relations to parts situated both above and below them in the medulla. For this purpose I first made a succession of longitudinal but horizontal sections from side to side along the surface of the fourth ventricle, beginning at the level of the facial nerve and nuclei (along the horizontal dotted line in fig. 51, Plate XIII.), and carrying the section downward through the hypoglossal nucleus to about the horizontal dotted line in fig. 32, Plate XI. Such a section of the left half of the medulla and of a small portion of the right half is represented in fig. 57, Plate XIV. Here we find that a large number of fibres belonging to the lower end of the longitudinal column (T″), instead of curving *forward* to the *antero*-lateral part of the *pons* (s'', figs. 53, 54, & q''', fig. 55, Plate XIII.), spread laterally and curve *horizontally* outward just below the surface of the ventricle. Some of these fibres pass through the facial nucleus (Q″), but the greater number sweep round its lower end across the base of the grey tubercle (e' e') at its junction with the inner auditory nucleus, i (compare fig. 51, Plate XIII.). In this course they are met by another set of horizontal fibres radiating inwards from the *outer* auditory nucleus or column ($d'o$, fig. 57). In fig. 51, Plate XIII. the fibres

* Stilling, who was the first to trace the facial and abducens nerves to the *fasciculus teres*, considers that both these nerves arise from the common nucleus (Q″). He did not, however, trace any of their roots beyond this point. Schröder van der Kolk denies that they arise from a common nucleus, and in forming this opinion he seems to have been influenced, as on many other occasions, by physiological views, rather than by actual observation. His objections on the ground that the two nerves have very different functions, are of no value in the face of positive observation. It is certainly difficult to understand why the two nerves, which are apparently never associated in action (except perhaps in certain cases of facial expression), should arise from the same nuclei; but even if the abducens could not be traced to the nuclei of the facial, it is equally difficult to understand why its fibres are *so closely interlaced* with those of the facial, while it is separated by the whole length of the pons Varolii from the *third nerve* with which it is so closely associated in action. Schröder van der Kolk says that all the roots of the abducens terminate in the floor of the fourth ventricle, and that they seem to bend upward "into a nucleus of ganglionic groups placed externally and superiorly to the facial nerve."

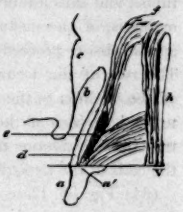
radiating *backward* from *c'o* are well seen, but not so those in question, which run inward from *c'o* along the dotted line. In this figure, however, numerous fibres from *e'i* are seen to sweep through and round the great nucleus (Q''), and in fig. 50 the same are seen at *e* U'''. In fig. 57, Plate XIV. many of the horizontal fibres radiating from the outer auditory nucleus (*c'o* to *e'*) evidently consist of the processes of its large multipolar cells. At the lower end of this nucleus or column (*c' o*) there are numerous longitudinal streaks of grey substance formed from the post-pyramidal and restiform ganglia, as shown in fig. 25, Plate X. The superficial grey layer (*g'''*, fig. 57, Plate XIV.) of the ventricle, at the side of the median furrow (*h''*), as it passes downward becomes reduced to the epithelium of the ventricle on the posterior surface of the raphè; on its outer side some longitudinal fibres descend from the longitudinal portion (T'') of the loop of the facial nerve. This band of fibres inclines somewhat forward as it descends, diminishes in size, and becomes continuous with the surface of the column which runs beneath the front of the hypoglossal nucleus, between the hypoglossal nerve and the raphè (V, fig. 8, Plate VIII. & fig. 32, Plate XI.). At the lower end of fig. 57, Plate XIV., the upper tapering end of the hypoglossal nucleus is represented at J, and beneath its inner side, the longitudinal band of fibres just mentioned is gradually lost to view. On its outer side some plexiform bundles of the white column (J') ascend to the nucleus (Q''). Still further out is the vagal nucleus (H), and beyond this the lower end of the outer auditory column *c'o* The raphè contains numerous cells of considerable size, and a system of antero-posterior fibres. These are crossed nearly at right angles by another set of *horizontal* decussating fibres, which are continued upward beneath the superficial grey layer that lines the median furrow of the fourth ventricle. The decussating fibres cross the longitudinal band on each side of the raphè somewhat obliquely from above downward and from within outward. Some of them are certainly connected with cells *in* the raphè, and through these they probably communicate with antero-posterior *fibres* of the raphè, and thus establish a complicated commissural connexion between distant parts, or levels, of the medulla. Others are continuous with fibres of the lateral bands, and appear also to proceed from the *raphè*, and not from the opposite side of the medulla. The rest of the decussating fibres cross the raphè and the lateral bands from one side of the medulla to the other. The upper set extend obliquely downwards and outwards to the base (*e'e'*) of the grey tubercle (see also *d'*, fig. 51, Plate XIII.). The lower set traverse the anterior portion of the vagal nucleus (H). Many of these proceed from the cells of the hypoglossal nucleus, as shown in fig. 31, Plate X.

(64) Fig. 58, Plate XIV. represents a similar horizontal-longitudinal section a little deeper from the surface of the fourth ventricle. Figs. 59 & 60, Plate XIV. are *transverse* sections of the left lateral half of the medulla at the upper and lower ends of this longitudinal section. In figs. 58 & 59 the upper lines of section correspond. F is the raphè immediately below the median furrow of the ventricle. V is the white column between the raphè and the entrance of the abducens nerve (V') into the nucleus (Q''). *e'* is the base of the grey tubercle. *c' o* the *outer* auditory nucleus. The *lower* line of fig. 58

corresponds to the *upper* line of the transverse section, fig. 60, in which J is the anterior portion of the hypoglossal nucleus. V is the white column between it and the raphè. O″ is the white olivary column lying between it and the vagal nucleus (H). The slender longitudinal column of the vagus is seen at *n*; and *c′ d′* is the lower end of the outer auditory nucleus, formed by the grey substance of the restiform body and posterior pyramid. It will be seen, then, that in fig. 59, the white column (V) between the raphè (F) and the entrance of the abducens nerve into the nucleus (Q″) is continuous, in fig. 60, with the white column (V) between the *hypoglossal* nucleus (J) and the raphè. This column (V) is seen longitudinally in fig. 58, and is only a deeper longitudinal section of the lateral band which descends in fig. 57, from the longitudinal portion of the loop (T″) of the facial nerve. At the lower end of fig. 58, this white column is seen on the *inner* side of the remains of the hypoglossal nucleus (J), instead of immediately beneath it. The olivary bundles (*o″*, fig. 58) run up in a plexiform manner, partly from the remains of the hypoglossal nucleus (J), through the facial nucleus (Q″).

(65) From the longitudinal portion of the loop of the facial nerve, some fibres *ascend* along the side of the median furrow, in company with those which ascend from its *under* surface. They all curve upward and forward beneath the *inner* portion of the superficial grey layer of the ventricle to form part of the longitudinal column immediately beneath the nucleus of the third nerve on the same side. Adjoining them on the outer side, the longitudinal bundles which passed through and beneath the common nucleus (Q″) of the facial and abducens nerves, now ascend beneath the *outer* portion of the superficial grey layer, and contribute to form part of the crescentic column immediately beneath the same side of the third nerve. A longitudinal but horizontal section, from below upward, along the floor of the fourth ventricle, on the left side, and at the inner surface of the superficial grey layer, is represented in fig. B. *a* is the grey tubercle of ROLANDO, seen in the transversely-cut end of the pons Varolii, just above the level of the facial nerve. From its base at *a′* the longitudinal section is carried upward along the fourth ventricle. *b* is the cut surface of the superior peduncle of the cerebellum passing beneath the fillet *c*. The outer border of the grey tubercle running upward and inward, from its base, along the inner side of the superior peduncle is seen at *d*. Along the inner side of this is a layer of black matter or pigmentary cells (*e*), from which a number of fibres ascend and curve inward to the side of the nucleus (*f*) of the third nerve. From the inner side of the grey tubercle (*a′*)—at its junction with the upward continuation of the inner auditory nucleus—and from the pigmentary matter, transverse fibres ascend obliquely to the superficial grey layer (*h*) at the side of the median furrow of the ventricle. On the outer side of this furrow, the band of white fibres (V) ascends from the loop of the facial nerve, and from beneath it, to the side of the nucleus (*f*) of the third nerve.

Fig. B.



(66) Fig. C represents the next deeper, but similar kind of, section. The pigmentary matter at the inner side of the grey tubercle, as well as the oblique fibres, have disappeared; but other fibres issuing from the base of the grey tubercle (a') ascend and curve inward to the nucleus of the third nerve (f). Adjoining these on their inner side, and ascending to the same place, are the white fibres ($i\,i$), which have passed through and beneath the common nucleus of the facial and abducens nerves, as well as those fibres which run along the side of the median furrow. Hence it appears *that both the base of the grey tubercle into which the sensory root of the trigeminus penetrates, and the bundles of fibres which pass through and beneath the common nucleus* (Q''), *are connected with the nucleus of the third nerve.*

Fig. C.

Of the Trapezium and Superior Olivary Bodies.

(67) In the Proceedings of the Royal Society for June 1857, I first showed that the transverse band of fibres known in animals as the trapezium encloses on each side a remarkable grey nucleus which bears a striking resemblance to the olivary body; and I further showed, at the same time, that the trapezium with its nucleus exists also in Man, but that it is covered and concealed by transverse fibres at the lower border of the pons. About a year later the grey nuclei of the trapezium, *in animals*, were described by Schröder van der Kolk, and called by him the superior olivary bodies*. As the structure of these nuclei has since been more fully investigated by Dean†, it will be unnecessary to go over the same ground; and therefore I shall only point out the chief differences in structure and position between the trapezium of Man and that of animals generally.

(68) On turning to fig. 48, Plate XII., which represents a transverse section of the left lateral half of the medulla of the Rabbit on a level with the auditory nerve, a remarkable group or column of large cells (s) will be found imbedded in the anterior part of the antero-lateral column, on the outer side of the anterior pyramid. This group of cells was first pointed out by myself‡, and was subsequently named by Dean the *antero-lateral nucleus*. It begins low down in the medulla, both in Man and animals, and ascends as high as the trapezium §. Here, however, we find it replaced, exactly in the same spot, by another nucleus of a different form and somewhat different structure. This consists of a convoluted grey lamina like that of the olivary body, but the convolutions are few. In the Rabbit (s', fig. 61, Plate XIV.) and in the Cat, in which it is largely developed, it assumes rather a sigmoid form, with some small masses on its inner side. In the Sheep and other Ruminantia there is less trace of convolutions. The cells of the lamina are smaller than those of the *antero-lateral* nucleus, and the course of its

* Schröder van der Kolk, Medulla Oblongata, 1858.

† Medulla Oblongata and Trapezium. By John Dean, M.D.

‡ Philosophical Transactions, 1858. § Ibid.

fibres very much resembles that of the inferior olivary body. Notwithstanding, however, the resemblance of this nucleus, in form and structure, to the olivary body, its *position* is different, being situated *behind the trapezium*; while the lower olivary body in Mammalia is behind the anterior pyramid*. The trapezium consists, for the most part, of transverse bundles, which are partially separated from each other by flat longitudinal bundles of the antero-lateral column. These transverse bundles ($p' p''$, fig. 61, Plate XIV.) proceed, as arciform fibres, from the restiform body on each side, and from the remains of the outer auditory nucleus. They sweep round the front of the grey tubercle (e), then cross the facial nerve (q''), and run inward, in front of the upper olivary body (s''), to the back of the anterior pyramid (y), where they decussate across the raphè with their fellows of the opposite side. Dean states that many separate bundles turn off and curve quite around the nucleus, "either penetrating its interior or completely surrounding it, till the upper side is reached, where the bundle frequently turns still more and enters the central portion of the mass, radiating in the same manner as the bundles which enter the convolutions in the lamina of the human olivaries"†.

(69) The prominence of the trapezium at the surface of the medulla is proportionate to the development of the nucleus within. In the Cat it projects on each side of the anterior pyramid as a large convex and somewhat oval mass‡. In the Dog, the Sheep and other Ruminants, it is less prominent, and appears as a flattened band in the same situation. It is present in all the Quadrumana, but although it varies in size in different species, it is generally small, and, in strong contrast to that of the Cat, is sunk deeply in the fossa at the outer side of the pyramid. Fig. 62, Plate XIV. represents a transverse section of the left lateral half of the medulla of the Orang Outang, on a level with the facial nerves. When compared with that of the Rabbit (fig. 61, Plate XIV.) we find that the anterior pyramid (y) is enormously developed, and that the trapezium (p'') is sunk between it and the facial nerve (q''). Behind the transverse fibres of the trapezium the lamina of the superior olivary body (s'') appears in the form of a simple loop or fold, overlaid, on its outer side, by the group of large multipolar cells constituting the nucleus (U) of the motor root of the trigeminus. On each side of the raphè the decussating fibres of the trapezium are thickly interspersed with small cells in irregular groups, one of which extends forward in a tapering form along the inner side of the pyramid. The surface also of the pyramid is covered with a thick layer of small cells (y'), which extends backward along the inner edge of the raphè, as far as the level of the decussating fibres of the trapezium. Nothing of the kind is found in the Rabbit, or in any of the Mammalia.

(70) Fig. 63, Plate XIV. represents a transverse section of the left side of the human pons Varolii, at the same level, and magnified to the same extent as figs. 61 & 62. After examining the trapezium, first in the Rabbit and then in the Orang Outang, it will not be difficult to recognize the same structure in Man. As usual, it lies partly

* "Medulla Oblongata," Philosophical Transactions, 1858. † Dean, Medulla Oblongata, p. 64.

‡ See my "Medulla Oblongata," Philosophical Transactions, 1858, fig. 4, Plate XII.

behind and partly on the outer side of the anterior pyramid (*y*), where in animals it forms the surface of the medulla (*p''*, figs. 61 & 62); while in Man (fig. 63) it is covered and concealed at this point by the transverse bundles of the lower portion of the *pons* enclosing a quantity of grey substance, which consists of irregular masses of small cells (*y'' y''*). This grey substance extends round the front of the anterior pyramid (*y*) to its inner side, where it forms a deep and broad layer (*y'''*) next the raphè. The transverse bundles of the trapezium are numerous and strongly marked. Along the central line of the *pons* they decussate as usual with their fellows of the opposite side, the most anterior being imbedded in a mass of grey substance (*y'''*), which is denser, and contains larger cells than that which is continuous around the front of the pyramid to *y''*. On the outer side the fibres of the trapezium run into and round the front of the superior olivary body (*s''*), crossing the abducens nerve (V'); and as they reach the inner side of the facial nerve (*q''*), they intersect each other in a complicated manner amongst irregular masses of grey substance consisting of granules and small nuclei. They then cross the facial nerve and sweep round the extremity of the grey tubercle (*e*) to the middle peduncle of the cerebellum. These bundles appear to consist of several sets of commissural fibres: 1, between the superior olivary body, the grey substance at the back of the pyramid, and the cerebellum; 2, between the two opposite olivary bodies.

(71) In Man the superior olivary body is not nearly so distinctly convoluted as in the Cat or the Rabbit, nor is it nearly so large in proportion to the size of the brain. It has the appearance, rather, of two or three ill-defined masses in continuity with each other. The majority of its cells are somewhat smaller than those of the inferior olivary body. There is one remarkable point in which this portion of the medulla, both of Man and the Orang Outang, differs from that of Mammalia. In figs. 62 & 63, the superior olivary is overlaid by the group of large multipolar cells (U) constituting the nucleus of the motor root of the trigeminus, and connected by numerous fibres with the common nucleus (Q'') of the facial and abducens nerves. In Mammalia this nucleus of the trigeminus is not found in the same situation.

(72) Schröder van der Kolk does not appear to have detected the existence of the trapezium in the human pons Varolii. Stilling described and figured this part in the *human pons**, but he was not aware that it is homologous with the trapezium of animals; and as he was not acquainted with the nucleus or superior olivary body, enclosed by the fibres of the trapezium, he called this nucleus, in Man, "quasi appendix nuclei inferioris nervi trigemini, ad formam scafi."

* Stilling, Pons Varolii, tab. ii. *s s*.

CHAPTER III.

(73) Although in the course of these researches I have carefully endeavoured to investigate the facts independently of any particular physiological views with which they might seem to be connected, I shall nevertheless conclude this memoir with a few remarks on some of the physiological and pathological points which they either modify or explain.

(74) Before the publication of my memoir "On the Medulla Oblongata," in 1858, it was believed by anatomists that the decussating fibres of the anterior pyramids are continuous only with the lateral columns of the opposite side of the spinal cord. I have shown, however, that they are connected not only with the *lateral*, but with the *anterior* columns, and with both the anterior and posterior *grey substance*. These facts, which must modify to a certain extent the current opinion concerning the functions of the anterior pyramids, have been confirmed by the investigations of M. VULPIAN, announced in his recent and valuable work on the 'Physiology of the Nervous System*. "Nos recherches anatomiques, d'accord en grande partie avec celles de M. LOCKHART CLARKE, montrent que les pyramides antérieures sont formées par des fibres émanées: (1) des faisceaux antérieurs; (2) des faisceaux latéraux; (3) des faisceaux postérieurs; (4) de la substance grise de la moëlle épinière. A ces fibres, il faut même en ajouter d'autres qui font partie du groupe des fibres arciformes, et qui constituent parfois, soit d'un seul côté, soit, mais plus rarement, des deux côtés du bulbe, une portion assez notable de la pyramide antérieure†."

(75) Besides the antero-lateral white columns, the particular parts with which I have shown the decussating fibres of the anterior pyramids to be connected, are (1) the antero-lateral *grey* substance, (2) the anterior border of the caput cornu, or expanded extremity of the posterior horn, (3) the base of the cervix cornu on each side of the central canal, (4) the continuation of this part of the cervix cornu in the posterior column, forming its grey nucleus, and subsequently contributing to form a large portion of the outer nucleus of the auditory nerve, and (5) the side of the spinal-accessory and hypoglossal nuclei. From some of these facts M. VULPIAN thinks it not improbable that the anterior pyramids are to a certain degree sensitive and excito-motor: "Nous présumons," he remarks, "que les pyramides antérieures pourraient bien être sensitives et excito-motrices, parceque l'anatomie nous démontre qu'elles sont en relation avec les faisceaux postérieurs et avec les diverses parties de la substance grise"‡. Their connexions, however, with the sensory portions of the medulla oblongata and cord do not afford sufficient ground for the conclusion that they are sensitive; although it is presumable that, by virtue of these connexions, they are excito-motor, probably through the grey substance within the pyramids themselves, the pons Varolii, or both.

* Leçons sur la Physiologie Générale et Comparée du Système Nerveux. Par A. VULPIAN, Paris, 1866.
† Ibid. p. 468.
‡ Ibid. p. 484.

(76) Now some of the connexions which I have pointed out between the fibres of the anterior pyramids and certain other parts of the medulla oblongata, are exceedingly interesting in a physiological point of view*. I have shown, both now and on former occasions, that the spinal-accessory nerve may be traced to several different centres of origin. Its upper rootlets arise from a special nucleus or column of cells, which descends behind the canal to about the level of the lower end of the olivary body; while its lower rootlets arise from the *lateral grey substance*, and from the *anterior grey* substance which gives origin to the *lower* roots of the *hypoglossal* nerve and to the anterior roots of the *spinal* nerves in the cervical region. According to BENDZ and CLAUDE BERNARD, the *upper* roots—which I have shown to arise from the special nucleus behind the central canal—go to form the *internal* branch which joins the vagus and is distributed through it to the larynx, pharynx, and palate; while the *lower* rootlets—which we have seen to arise from the antero-lateral grey substance of the cord and of the lower part of the medulla—are collected into the *external* branch, which supplies the trapezius and sternomastoid muscles employed in voluntary and forced efforts of respiration. Now it is particularly interesting to find, as I first showed in 1858, that decussating fibres of the anterior pyramid in their course downward from the cerebrum, turn obliquely backward to the point about which the *special nucleus* and *upper roots cease to arise*, and the *lower roots begin*. In fig. 5, Plate VIII., we see these bundles crossing each other from the opposite anterior pyramids (*y y*), then curving backward round the sides of the hypoglossal nuclei (*t'*), and of the special spinal-accessory nuclei (*t*). Their outer bundles are connected at *n*, fig. 5, with the lateral grey substance where the *lower* roots of the spinal-accessory nerve *r* begin (fig. 28, Plate X.). In fig. 27, Plate X. we have an oblique transverse section of the left side of the medulla in the direction L, *t*, fig. 5, Plate VIII. In this section we see the transversely-cut ends of these decussating bundles (*n''*) of the anterior pyramids, with fibres extending amongst them from the spinal-accessory nucleus (*t t*). On their outer side we see the slender longitudinal column (*n'*) ascending, from the point where the higher roots cease and the lower roots begin, to the side of the vagal nucleus, as already described (§ 15). Moreover, in descending the medulla oblongata and cervical region of the spinal cord, the decussating fibres of the anterior pyramids pass into and through the antero-lateral grey substance where the lowest roots continue to arise (see figs. 4, 3, 2, Plate VIII.). There appears, then, to be scarcely a doubt that *these* particular decussating bundles of the anterior pyramids are the channels through which the will influences the movements of respiration. It is very probable also that they are functionally related, in the same way, to the hypoglossal nuclei (*t'*), or at least, that they are *one* of the channels through which the will acts on the hypoglossal nerves.

(77) The functions of the olivary bodies have been the subject of much discussion amongst physiologists of different ages. WILLIS, I believe, was the first who considered them as the central organs of articulate speech†. BURDACH and RETZIUS regarded them

* The reperusal of the five last paragraphs of section 15 is here recommended.

† Cerebri Anatome.

as subservient to both speech and expression. DUGÈS and SOLLY have adopted the opinion of WILLIS. SERRES concluded, from pathological facts, that the olivary bodies influence the movements of the heart. According to him, isolated and chronic alterations of these bodies cause a jerking and irregular action of the heart*. He also found that stammering was associated with structural alterations of the upper parts of the olivary bodies close to the pons. In my memoir "On the Medulla Oblongata" (1858), I concluded, on anatomical grounds, that these bodies are probably the coordinating centres for the different nuclei of the medulla oblongata; that is, that they are the motor and associating agents by which the different complex movements dependent on the medulla are carried on. SCHRÖDER VAN DER KOLK concluded, both from anatomy and pathology, that the olivary bodies are subservient chiefly to articulate speech and deglutition. He collected from different sources, and recorded, several cases of loss of speech, in which these bodies were more or less altered in structure. Two exceedingly interesting cases of the same kind have come under my own observation.

(78) A lady, ætat 55, fell down suddenly in a state of unconsciousness. When she recovered her senses, she was hemiplegic on the right side, and unable to speak. This was the state in which I found her. She regained the use of her arm and leg, with only some slight remaining weakness, and the power of articulation returned with only some slight impediment. During the next four or five years she had several slight and partial attacks of paralysis, with some additional defect of articulation. She had chalky concretions in the joints of her fingers, and during some of the paralytic attacks, she had what appeared to be gouty inflammation of their joints. On subsiding it left behind chalky deposits. Two years before her death deglutition became much impaired. A year later she had an attack of partial unconsciousness, which rapidly passed away; but from that time the power of articulation was almost lost. She could say "Yes," "No," "Shan't," and one or two other short words, but nothing more. At this time she was almost wholly unable to protrude her tongue. During two months or more she entirely lost the power to swallow, and was nourished by means of injections. She took strychnine and iron; and regained the power of deglutition sufficiently to dispense with injections, although she was a long time in swallowing even a small quantity of liquid food. She swallowed best in a recumbent posture, and it was evident that the pharyngeal muscles were considerably paralysed. A large collection of mucus in the air-passages frequently excited spasmodic coughing; but she had scarcely any voluntary power over the respiratory muscles, and was unable to cough voluntarily. She could scarcely do more than *whisper* the few words she was able to articulate. The power of straining was completely lost, and large quantities of fæcal matter collected in the rectum. Both arms and legs were very weak, especially the latter, so that she could scarcely walk, even with assistance. Sensation was unaffected.

On post-mortem examination the kidneys were found to be very small and very granular on their surface; their cortical portions were much atrophied, with two or three

* Anatomie Comparée du Cerveau, tom. ii. p. 231.

cysts on their surface. The heart was exceedingly flabby, soft, and yellowish, with accumulation of fat outside the left ventricle, which was dilated, and broke down easily under the fingers. There was no disease of mitral valve. The wall of the right ventricle was exceedingly thin—nearly as thin as that of a healthy auricle. The tricuspid valve was healthy.

At the base of the brain the arteries were atheromatous. The membranes at its upper surface were thick and opaque, with some sub-arachnoid fluid and numerous patches of white, flaky, and granular matter. They were adherent to the calvarium which was unusually thick. About four ounces of fluid flowed into the calvarium on removing the brain. The surface of the anterior lobes of the cerebrum was much depressed and atrophied. The grey substance of convolutions was about the usual colour. The under surface of the right crus cerebri was softened. On the outer part of the right corpus striatum were the remains of an old clot, about the size of a horse-bean. The substance beneath it was much softened. In the middle of the right optic thalamus were the remains of another and much larger clot. On the surface of the *left* optic thalamus was a softened and chocolate-coloured depression, and beneath this were the remains of a small clot.

Cerebellum.—In the central white substance, on each side, was a cyst containing turbid fluid. That on the right was about as large as a moderate-sized hazel-nut, and destroyed only part of the convoluted lamina of the corpus dentatum; that on the left was somewhat larger, and destroyed nearly the whole of the lamina. The fluid in the cysts consisted of globular aggregations of oil-globules, some of which were tinted with brown and yellow, and accompanied with crystals and granules of hæmatoidin.

The medulla oblongata was curiously misshapen, and tilted to the right side. This displacement appeared to be due chiefly to atrophy of the right corpus olivare. At the lower two-thirds of the olive the atrophy was considerable, but diminished along the upper third. Fig. 64, Plate XIV. is an exact representation of a transverse section of the medulla through the middle of the olivary bodies. Not only was the right olivary body exceedingly reduced in bulk, but the greater number of its nerve-cells were wasted to granular points or small granular masses tinted brown or yellow by pigment, as the accompanying preparations very distinctly show*. Between the olivary bodies the central part of the medulla was softened, so that its lateral halves readily separated along the median raphè. Neither the hypoglossal nor the vagal nuclei were reduced in bulk to any remarkable degree; but the vagal nucleus, which in Man normally contains a certain number of dark pigmentary cells, contained these in much greater number than usual; and many of the cells of the hypoglossal nucleus had lost some of their sharpness of outline, and contained more than their natural amount of pigment.

* A case of extreme atrophy of the nerve-cells of the spinal cord, in muscular atrophy, has been published by me in the British and Foreign Medico-Chirurgical Review, July 1862, and two others in the Medico-Chirurgical Transactions, 1867 and 1868, with drawings. In No. XIII. of Beale's 'Archives of Medicine,' I recorded another case, in which I described the successive changes by which the cells undergo the process of degeneration.

All the central parts of the pons Varolii were very much softened. The softening involved a large portion of the prolongations of the anterior pyramids, and extended backward to within a sixth of an inch of the fourth ventricle.

(79) The second case was a gentleman, who, having recovered, in a great degree, from an attack of *left* hemiplegia, with some difficulty of articulation, suddenly found that he had altogether lost the power of speech, and then became insensible for a few hours. He had total paralysis of the tongue on both sides. He was unable to cough, and had very great difficulty in swallowing. Sensation was unimpaired. On laryngoscopic examination by Dr. MORELL MACKENZIE, there did not appear to be any paralysis of the vocal cords. The patient was under the care of Dr. HUGHLINGS JACKSON, with whom I saw him, from time to time, to the period of his death. The post-mortem examination was made by Dr. HUGHLINGS JACKSON, who sent me the brain, medulla oblongata, and spinal cord.

I found many of the principal arteries of the brain, particularly the basilar, the posterior, middle, and anterior cerebral, and the right carotid loaded, and in some places nearly choked up, with atheromatous deposit. I shall not give a detailed description of the numerous lesions which, as might be expected from such a condition of the arteries, I found in different parts of the brain, but shall only state that the convolutions of the middle and anterior lobes, especially those along the front of the middle lobes, bordering the fissure of SYLVIUS, and those of the *insula*, were much softer and paler than natural. The anterior perforated space, and the orbital and adjacent convolutions, were particularly soft. Both thalami optici were soft. On the surface of the *left* corpus striatum was a dark brown and softened depression about the size of a pea. Over the posterior half of the *right* corpus striatum and outer and fore part of the thalamus opticus, there was a chocolate-coloured and softened mass about the size of a hazel-nut. Beneath this the substance of both these bodies was very soft, and about a third of an inch from their surface were the remains of an old clot.

Many parts of the cerebellum were exceedingly soft. The corpus dentatum on the *left* side was almost wholly destroyed by a round cyst containing turbid fluid.

The surface of the fourth ventricle was softer than natural, and of a pale-yellow or cream-colour. In the medulla oblongata, on the left side, a small rust-coloured spot was observed at the origin of the vagus nerve; and in the same section there was a dark rust-coloured streak on the inner side of the right anterior pyramid. A third of an inch below the pons Varolii another rust-coloured spot, about the size of an ordinary pin's head, was found in the substance of the posterior convolutions of the olivary body. This was proved by microscopic examination to be the remains of an old clot. It descended the medulla about the third of an inch, gradually diminishing and at length occupying the part immediately behind the convolutions of the olivary lamina, amongst the deep arciform fibres.

(80) Valuable as these cases are in many respects, they by no means prove that the loss of articulate speech was due to the lesions in the olivary bodies; since it is impos-

sible to say how far it might have been dependent on the numerous and extensive alterations which were found in other parts of the brain, particularly about the nuclei of the vagal and hypoglossal nuclei. The same objection may be made to SCHRÖDER VAN DER KOLK'S cases, when they are adduced as proofs that the olivary bodies are the central organs of articulate speech.

(81) Neither are the anatomical points which SCHRÖDER VAN DER KOLK brings forward in support of this opinion exactly as he states them to be. According to his theory, the olivary bodies effect the *bilateral* movement of the tongue by their action on the hypoglossal nuclei, because the actions of these nuclei are *unilateral*; "for the greater part of each nucleus," he says "seems to be isolated in its action, and not to be connected with that of the other side." Now it is not true that the hypoglossal nuclei are scarcely connected with each other other; for, as may be seen, especially in longitudinal and horizontal sections along the fourth ventricle, at different depths of the nuclei, there is a very evident decussation of fine fibres; and at the upper end of the nucleus, the commissural fibres between the anterior groups of cells are particularly striking, as may be seen at J, fig. 57, Plate XIV. VULPIAN speaks almost in derision of SCHRÖDER VAN DER KOLK'S theory of the action of the olivary bundles; but, nevertheless, there are not wanting anatomical facts in favour of the opinion that these bodies are subservient to articulate speech. In this process, the tongue, the lips, the jaws and the expiratory muscles are simultaneously combined in action. Now I have already shown that the connexion of the olivary bodies with the hypoglossal nuclei is very striking. I have also shown that a column of cells connected with longitudinal fibres and continuous with the fasciculus teres, whence the facial nerve takes its origin, descends low down into the medulla and lies at the back of the hypoglossal nucleus (see K', fig. 25, Plate X., and figs. 38, 39 & 40, Plate XI.). Moreover, a band of fibres descending from the loop of the facial nerve (T'', fig. 57, Plate XIV.) passes downwards beneath the hypoglossal nucleus (J). The close commissural connexion between the hypoglossal nucleus (J', fig. 31, Plate X.) and the vagal nucleus (H, H') has been already pointed out. And lastly, we have seen that the lower end of the column of cells constituting the nucleus of the motor root of the trigeminus is imbedded in the columns which ascend from the olivary body. But whether the olivary bodies really act through the channels just mentioned, and thus coordinate the movements of articulation, is not proved by the anatomical facts alone. On the other hand, amongst birds, in which scarcely any traces of olivary bodies are to be found, the Parrot-tribe have the power of distinctly articulating particular words; while in the Porpoise, which emits only a kind of moaning sound, the olivary bodies are enormously developed*. Experiment throws no light whatever on this subject, so

* I obtained, through the kindness of Mr. W. H. FLOWER, of the College of Surgeons, part of the medulla of a Porpoise, and found that the large olivary bodies overlaying the pyramids were everywhere crowded with nerve-cells. The cerebrum of the Porpoise is large and globular, and the convolutions are very numerous and deep. The only remarkable points in the actions of the animal, are its great agility and vivacity of movement, and its extraordinary speed and muscular power.

that our only hope is in some fortunate cases of disease, in which the power of speech is perfect, while the olivary bodies are considerably altered in structure; or in which the olivary bodies, *only*, are damaged, while the power of speech is lost. Either of such cases, determined with accuracy, would set the question at rest. M. VULPIAN has recently recorded a case which seems to be opposed to the opinion that the olivary bodies are concerned in speech, since these bodies were altered in structure while speech remained perfect to the last. The examination, however, does not appear to have been sufficiently precise to justify a positive conclusion. I give his own words: "Les olives du bulbe, surtout la gauche, offraient une sclérose bien manifeste, quoique peu profonde, de leur tiers moyen, et cependant,—ce qui n'est pas sans importance, au point de vue d'une des hypothèses relatives aux fonctions de ces parties,—la parole est restée parfaitement nette jusqu'à la fin de la vie"*.

(82) Deglutition is another process to which the olivary bodies have been regarded as subservient. If they have any share in this process, it is probably only in the first or voluntary stage, in which the food is pressed through the anterior palatine arch by the cooperation of the buccinator, the mylohyoid, the intrinsic muscles of the tongue, and the styloglossi. The second stage, which is purely reflex and involuntary, is probably effected solely through the intimate and important connexions which I have pointed out between the nuclei of the trigeminus, the hypoglossus, the glossopharyngeal, the spinal-accessory, the vagus, and the facial. The third stage is reflex simply through the vagus and spinal-accessory. Cases are on record in which the tongue, while entirely removed from all voluntory influence, co-operates, nevertheless, perfectly with the muscles of the fauces and pharynx in the second reflex stage of deglutition. The demonstration which I have given in this memoir, of the close anatomical connexion, in different ways, between the nuclei of the hypoglossal, vagus, spinal-accessory, facial and trigeminal nerves throws an important light on the manner in which the complex and associate muscular actions concerned in deglutition, vocalization, and articulation may be effected; and explains, in an interesting way, certain forms of partial or complete paralysis to which the muscles employed in these acts are subject. Dr. HUGHLINGS JACKSON was kind enough to show me one of his patients in whom there was complete paralysis and wasting of the right side of the tongue†, with paralysis of the same side of the palate and vocal cords, as shown by the laryngoscope. The man had great difficulty of swallowing, and soreness of throat. He was unable to cough, or rather he could not shut the larynx in coughing. He was weak on both sides, but especially on the right. The right shoulder was much lower than the left, and he was unable to shrug it up as he could the other. He could not *whistle* so well as formerly; but he could *articulate in a whisper very well*. Here we have paralysis of the spinal-accessory nerve on one side, supplying the vocal cords and palate; total paralysis of one hypoglossal nerve, and

* VULPIAN, Leçons sur la Physiologie, p. 495 (note).

† The muscles of the tongue, when paralyzed, suffer atrophy much more rapidly than other muscles under the same conditions.

apparently partial paralysis of the facial supplying the orbicularis oris. I have already shown that some of the roots, both of the spinal-accessory and vagus nerves, may be traced directly into the hypoglossal nucleus, and that, on the other hand, numerous nerve-cells of the hypoglossal nucleus, as may be seen in fig. 31, Plate X., send their processes deeply into the vagal nucleus. Moreover, we have seen in fig. 25, Plate X., just below the olivary body, that a small column of cells (K′) mixed with numerous longitudinal fibres, ascends, and increases as it ascends to the facial nucleus. It is first situated between the spinal-accessory and hypoglossal nuclei, then between the latter nucleus and the vagus (H, fig. 38, Plate XI.), and higher up (fig. 39), where the hypoglossal nucleus ceases, it lies at the inner side of the glossopharyngeal (*t*) and inner auditory nucleus (*i*), with both of which it is connected, as well as with the motor nucleus of the trigeminus (U). Now it is not at all improbable that this column of cells (K′) at fig. 26, Plate X., and a little higher up, is the channel of communication for that portion of the facial nerve which supplies the muscles about the lips and mouth. We know that this portion of the facial is frequently paralyzed alone, and must therefore be, to a certain extent, independent of the rest of the nerve; and we also know how closely and frequently it is associated alone in function with the spinal-accessory and hypoglossal nerve in the operations of articulation and vocalization. By now referring to fig. 25, Plate X. (which of course is considerably magnified*), it will be understood that in this section and in others a little higher up, a small spot of lesion—either a clot or a disintegration in the spinal-accessory nucleus (H)—would more or less paralyze the voice. A similar lesion in the hypoglossal nucleus (J) would paralyze one side of the tongue. One which destroyed K′, would cut off the mouth and lips from co-operation with the voice and tongue; and a lesion that injured, more or less, all three of these nuclei, which are in such close apposition, would paralyze, more or less, at the same time, the voice, tongue, and lips. But of course when I state that the small column (K′) is subservient to the movements of the muscles about the mouth in the co-ordinate movements of articulation, the statement is wholly conjectural, although probably true.

(83) Now there is a form of disease which was first distinguished from ordinary facial paralysis, by Duchenne (de Boulogne), and which has been called by Trousseau "Paralysie Glosso-labio-pharyngée." It consists of paralysis of the orbicular muscle of the lips, of the tongue, of the velum palati, and of some of the muscles of the larynx. The patient *gradually* loses all power of articulation; he is unable to protrude the tongue; deglutition becomes difficult; the velum palati is insensible to the influence of different kinds of stimulants; the posterior nares can no longer be closed by the velum and muscles of the posterior palatine arch, so that the voice becomes nasal; the mucous membrane of the larynx frequently becomes insensible to irritation, and fits of suffocation ensue. It is evident, therefore, that in this curious disease, the hypoglossal, the spinal-accessory, part of the vagus, and part of the facial, are together more or less injured.

* I may here state that this figure is an almost *photographically exact* representation of all the parts of the preparation from which it was drawn.

The disease is not common, and little is known of its morbid anatomy. In one case only, Trousseau examined with the greatest care, the brain, the cord, and the roots of the nerves, but found nothing unusual; nevertheless, something certainly was to be found, and lesions, sufficient to produce the symptoms, might very easily exist in the nuclei at the floor of the fourth ventricle without detection, or in the roots of the nerves arising from them. It is probable, however, that in none of these cases the lesion is limited to the medulla. There is sometimes weakness of one side of the body or of one arm; and sometimes, towards the end, the limbs become flexed on each other, and the patient is drawn up in a heap*.

(84) Whether or not the olivary bodies be subservient to the operations of speech and deglutition, it is quite certain that these are not their only functions. We have seen that in all the Mammalia, except the mute Porpoise, they are very much smaller than in the Monkey, which is wholly unable to articulate, and in which the act of deglutition has nothing peculiar. But even if it should be imagined by some that the "chattering" of the Monkey is a kind of speech, the Orang Outang and Chimpanzee are not more gifted in this respect than the inferior tribes, and yet their olivary bodies are much more highly developed, as I have already shown. Now, except their superior intelligence, the only endowments that distinguish the Ape-tribe from all other Mammalia, are their singular faculty of imitation and gesticulation, and their power of expressing a variety of emotions, of which they are very susceptible; and in these respects the Orang Outang is by far the most highly endowed. Le Comte, in his 'History of China,' says of one of these animals which he saw in the Straits of Molucca, that its actions so strongly resembled those of Man, and its passions were so expressive and lively, that a dumb person could scarcely make himself better understood. It signified its joy and anger by stamping with its foot on the ground; it could dance, and would sometimes cry like a child. These animals are also distinguished for their surprising agility and muscular power. Now since the olivary bodies are larger in the Ape-tribe than in any other animals, and largest of all in the Orang Outang, which is the most highly endowed with the power of expressing its emotions and desires, it appears to be extremely probable that this power is dependent on the co-ordinating functions of the olivary bodies. Nor are these functions limited to the medulla oblongata and the parts supplied by its nerves. They extend to the spinal cord, and to the sympathetic influencing the glandular secretions and the diameter of the capillary vessels, through the vaso-motor nerves.

(85) Anatomists and physiologists are too much in the habit of regarding the olivary bodies as if they were connected only with the medulla oblongata and some other parts

* Since the above was written I have read Trousseau's account of the post-mortem examination of a case in which the roots of the vagus were atrophied; the roots of the right hypoglossal were also atrophied to mere filaments; the roots of the spinal-accessory were on both sides small, and of a greyish colour. In the neurilemma there was a fatty granular substance. The anterior spinal roots were atrophied, especially near the roots of the spinal-accessory. Many other anterior spinal roots were diminished in size. The grey substance of the cord was of deeper colour and harder than natural. I shall be very glad to examine the brain, medulla, and cord of patients who may die of this disease.

of the brain. On this and on other occasions I have pointed out the way in which the different parts of the grey and white substances of the spinal cord are disposed or modified in the medulla oblongata. I have shown (1°) that from the outer (posterior) part of the cervix or neck of the posterior horn are developed the post-pyramidal and restiform nuclei which, higher up, form the nuclei of the auditory nerve, while the caput cornu, or dilated extremity of the posterior horn, is thrown aside to be traversed by the vagus and glossopharyngeal nerves, and ultimately to become the principal nucleus of the large root of the trigeminus; (2°) that from the *base* of the cervix cornu, *behind* the central canal, is developed the *special* nucleus for the spinal-accessory, vagus and glossopharyngeal nerves; (3°) that the *base* of the *anterior* grey substance *in front* and at the side of the central canal is developed into, or at least replaced by, the *special* nucleus giving origin to the *upper* roots of the hypoglossal nerve; (4°) that the *lateral* parts of the grey substance between the extremitities of the anterior and posterior horns, including what I have named the *tractus intermedio-lateralis*, is especially connected with the *lower* roots of the spinal-accessory nerve; and (5°) that, in ascending from the cord to the medulla, the remaining part of the anterior grey substance, which lies against the inner side of the anterior column, or the so-called non-decussating portion of the pyramid, and from which the *lowest* set of hypoglossal roots arise,—gives place to the groups of cells forming the lower end of the olivary body, which, as it swells out, becomes connected with the anterior part of the antero-lateral column (see figs. 6, 7 & 8, Plate VIII.). We have only to make a longitudinal section obliquely inward and backward through this column and the olivary body to see the connexion between them*. We must therefore regard the olivary body as a large motor nucleus, which is directly continuous by its white and grey substances, with the *anterior* white and grey substance of the spinal cord. The only other purely motor centre of the medulla oblongata, is the hypoglossal nucleus, which is developed from the *base* of the anterior grey substance. It is located apart from the olivary body, and is in the closest connexion with the vagus, spinal-accessory and glassopharyngeal nuclei with which it is destined to cooperate in reflex actions. Although distant from the olivary body it is, however, connected with it by a remarkable band of fibres.

(86) It it probable that the olivary bodies are not only the centres through which different movements are coordinated for expressing the passions and emotions, but that they are the motor centres through which different movements are effected by sudden, violent, or peculiar impressions on the special senses; for they are intimately connected with all the sensory ganglia of the medulla,—with the grey tubercle (trigeminus), the vagus nucleus, the post-pyramidal and restiform nuclei (auditory ganglia), the corpora quadrigemina (optic ganglia) through the fillet, and not improbably with the parts about the root of the olfactory bulbs, since I have traced the olivary columns nearly to the anterior perforated space.

(87) I have a few remarks to make on paralysis of the facial and abducens nerves,

* See my "Medulla Oblongata," Plate XV. fig. 24, Philosophical Transactions, 1858.

When paralysis of the facial nerve depends on central lesion it is frequently associated with paralysis of the abducens. Sometimes both nerves are paralyzed on one side, and one of them on the other; and sometimes, but more rarely, both nerves are paralyzed on each side. I have seen two cases of this kind. One of them was in the practice of Dr. Hughlings Jackson. Now, when we consider that both the facial and abducens nerves are connected with the same nucleus on each side of the floor of the fourth ventricle (see fig. 51, Plate XIII. and fig. 65, Plate XIV.), it is evident that a tumour pressing on this part, or any morbid process that injured it, would paralyze at the same time both the facial and abducens nerves; and if the lesion or foreign body extended across the ventricle to the other side, a bilateral paralysis to a greater or less extent would be the result. Dr. Wilks had under his care a little girl four years old, with paralysis of the right facial nerve and of both sixth nerves. The child's arms and legs did not appear to be much affected, but the left arm and leg were a little weaker than the right. On post-mortem examination by Dr. Wilks, a tumour about the size of a large walnut was found to occupy the pons Varolii, and more on the right side than the left. On opening the fourth ventricle, the tumour was seen to project into its cavity, and thus all natural appearance of this part was gone.

(88) I might dwell upon other points in this paper that may serve the purpose of the physiologist and pathologist, and which will probably be noticed by the reader, but I shall now conclude with a few words on the importance of combining together anatomy, experiment, and pathology in physiological investigations. A great physiologist, who conducted a large number of his researches by means of experiment, at length arrived at the conclusion, which he expressed in a paper read before the Royal Society, that anatomy is better adapted for discovery than experiment. "Experiment," he observes, "may take a colour from the preconceived idea, but the accurate investigation of the structure will not deceive us"*. The truth, however, appears to be, that anatomy and pathology, no less than experiment, are liable to deceive, and that neither of them, alone, is sufficient for the purposes of the physiologist. In attempting to determine the function of a particular part, pathology must be employed with great caution, when other parts of the same complex organ are at the same time affected by disease. The many contradictory results obtained by different experimentalists on the same subject are so frequent that this method of investigation cannot be trusted alone. But experiment will be more accurate and precise just in proportion to the accuracy of our knowledge with regard to the structure and relations of the parts on which the experiments are made. Moreover, even if the anatomist cannot immediately draw any physiological conclusions from his observations, those observations may suggest and guide experiment; and with this end in view, the philosophical observer will direct the course of his anatomical investigations. If, therefore, the experimentalist has made no original researches into the structure and relations of parts on which he purposes to operate, he must, if he wishes to avoid error, make himself thoroughly acquainted with the minute details of

* Sir Charles Bell, Nervous System.

structure supplied by the anatomist. From a want of this information serious blunders have been frequently made; for in dividing even a small part, particularly of the medulla oblongata, the operator may injure other parts, with which, as we have abundantly seen, it is so closely connected. LORRY and SERRES divided the medulla oblongata through the olivary bodies, and the latter anatomist concluded that the instant death of the animal was due to injury of those bodies; but if he had known that he had cut through the nucleus of the vagus, which STILLING had the merit of pointing out, his conclusion would of course have been different. On the other hand, the anatomist should make himself acquainted with the labours of the experimentalist, and pursue his researches by the light which they afford. It is in this way that observation, experiment, and induction alternately enlarge or enlighten the field of research and point out each other's path.

APPENDIX (added during the printing of the Paper).

On the Method of making the Preparations.

The method which I introduced some years back (1851) for rendering sections of the brain and spinal cord transparent and preserving them in Canada balsam, has been generally adopted both in Europe and America. The *principle* of the method is this: given a section of opake nervous substance that has been hardened in a fluid not miscible with Canada balsam; to render that section transparent and permeable by Canada balsam *without drying it*. One of the fluids which I employed in the process was oil of turpentine. The whole process may be described as follows.

The spinal cord or medulla oblongata is to be cut into pieces of from half an inch to an inch long and hardened in a solution of chromic acid, composed of one part of the crystallized acid to two or three hundred parts of water*. To this solution I generally add a small quantity of bichromate of potash, which brings out the nuclei of the nerve-cells more distinctly, and causes the grey substance to take the colour of carmine more brightly. At the end of three or four weeks the medulla is in a condition fit for making thin sections by means of a razor or other sharp knife wet with spirit of wine. If the sections are to be coloured with carmine, the ammoniated solution of this substance should be previously filtered, and the sections should be washed free from the spirit used in cutting them, before they are placed in the carmine solution, for spirit readily precipitates the carmine in the form of granules. When sufficiently coloured the sections should be again washed in water, and then soaked in strong spirit for a few minutes to expel the water. Sometimes a few drops of acetic acid may be added with advantage†. On removing them from the spirit, the sections, if thin, are to be floated on the surface of pure oil of turpentine, which will ascend from below and replace the spirit

* For the convolutions of the cerebrum and cerebellum the solution must be much weaker.

† Mr. ADAM ADDISON, late of the Montrose Royal Asylum, sometimes adds a few drops of a saturated spirituous solution of chloride of sodium instead of acetic acid.

that evaporates from their upper surfaces. In a short time, sometimes almost immediately, they will become quite or nearly transparent, when they are to be removed to glass slides on which a little Canada balsam has been dropped. Before they be finally covered with thin glass they should be examined under the microscope, and if all their parts be not perfectly transparent or *distinct* and well defined, a little turpentine and Canada balsam should be alternately dropped on them from time to time until the requisite effect be produced. If the sections be thick, I find it best, instead of floating them on the turpentine, to place them in a shallow vessel the bottom of which is simply wet with turpentine, which then ascends from below while the spirit evaporates from above. Before using the turpentine I also frequently remove a large portion of the spirit by placing the sections for a minute on fine blotting-paper. The process may be further hastened by the following plan: place the section on a glass slide, after the spirit has been partially removed by blotting-paper; drop on it some turpentine, and then tap it gently with the end of the finger, repeating these steps alternately until transparency be produced.

By any one practised in this method, a thin section, after having been coloured, washed, and steeped in the strongest spirit, may be rendered perfectly transparent and fit for mounting in Canada balsam in the course of eight or ten minutes; and a section of moderate thickness in fifteen minutes. But many observers have complained that the use of turpentine renders the process slow, and they have therefore substituted for turpentine a variety of other media, as oil of cloves, creosote, carbolic acid, which so much resembles it, &c. Creosote is that which is now most used on the Continent. In this country another modification of my method has been introduced by Dr. Charlton Bastian. The section having been coloured, washed, and saturated in spirit, is placed on a drop of carbolic acid, instead of turpentine, in the centre of a glass slide. In less than two minutes it is rendered transparent; and when this is accomplished (having got rid of any excess of carbolic acid), he pours over it three or four drops of chloroform, in which the specimen is allowed to remain for two minutes. The superfluous chloroform is then poured off, whilst one or two drops of a solution of Canada balsam in chloroform are dropped over the specimen, and the covering glass is then quickly applied. The whole process extends over ten minutes*. Although these modifications may possess some advantages in point of expedition, the *principle* is the same as that of the original method which I introduced. The preparations so made are certainly not in the slightest degree superior to my own.

Gerlach now hardens the cord in a solution of bichromate of ammonia of the strength of one to two per cent. The sections are then put into a solution of 1 part of chloride

* Journal of Anatomy and Physiology, November 1867. Dr. Bastian here speaks of my original process with turpentine extending over a space of four or five hours! If this were the case it would have been impossible for me to make the thousands of preparations I have made, and to have done the work I have done by means of them. However, for Dr. Bastian's satisfaction, I rendered a section transparent, and fit for mounting in Canada balsam, in his presence, in the course of fifteen minutes.

of gold and potassium to 10,000 parts of water slightly acidulated either with vinegar or hydrochloric acid, for ten or twelve hours, when the white substance becomes of a pale lilac (blass lilla), while the grey substance is scarcely coloured. They are then placed in a mixture of 1 part of hydrochloric acid to 2000 or 3000 parts of water for a few minutes. After this the sections are steeped for about ten minutes in a mixture of 1 part of hydrochloric acid to 1000 parts of 60 per cent. alcohol, and then for some minutes in absolute alcohol; after which they are made transparent by means of creosote and put up in Canada balsam*. This is rather a complicated process.

Explanation of the Plates.

PLATE VIII.

Fig. 1. Transverse section of the lower end of the human medulla oblongata at the points of the anterior pyramids:—*b c*, the posterior column; *d*, the posterior part of the antero-lateral column; *e*, the caput cornu posterioris, or expanded extremity of the posterior horn; *d'*, network of fibres and blood-vessels, mixed with small nerve-cells, in the deep portion of the lateral column, between the anterior and posterior horns; *t*, central grey substance behind the canal; in front of the canal is seen the commencing decussation of the anterior pyramids; *z*, the anterior median fissure.

Fig. 2. A similar section a little higher up. The decussating fibres of the anterior pyramids are much more numerous: V is a portion of the anterior column of the spinal cord forming the non-decussating part of the anterior pyramid of the same side.

Fig. 3. The same, a few sections higher up:—*f*, *d'*, *d'*, bundles of fibres coming from the lateral column and lateral grey substance to form the anterior pyramid of the opposite side; *b*, *b'*, rudiments of the posterior pyramid.

Fig. 4. The same, still higher up:—*f'*, anterior cornu; *o*, conical eminence from the outer part of the cervix cornu forming the rudiment of the grey nucleus of the restiform body; *b*, *b*, posterior pyramids containing grey substance derived from the inner part of the cervix cornu on each side of the posterior median fissure.

Fig. 5. Similar section at the upper part of the decussation of the anterior pyramids:—*b'*, grey nucleus of posterior pyramid very much enlarged; *e*, caput cornu posterioris (grey tubercle of Rolando), also greatly enlarged; *o*, inner grey nucleus of restiform body much increased in size; *o'*, its outer grey nucleus; *k*, posterior median fissure; *t*, rudiment of central spinal-accessory nucleus; *t'*, rudiment of hypoglossal nucleus; *f*, remains of anterior cornu of spinal cord; *n''*, bundles of fibres coming from the restiform nucleus (*o*) and from

* Zur Anatomie des Menschl. Rückenmarks. *Medecin. Centralbl.* Nos. 24, 25.

the postpyramidal nucleus (*b'*) to the opposite anterior pyramid (*y'*), those from the opposite side decussating each other in front of the canal. The anterior pyramids have now ceased to derive their fibres from the lateral columns of the cord.

Fig. 6. Similar section just above the decussation of the pyramids. The restiform nucleus (*o*), the postpyramidal nucleus (*b'*), and the grey tubercle of ROLANDO, *e* (caput cornu posterioris), are still further enlarged. The decussating fibres from *b o*, the postpyramidal and restiform nuclei, instead of entering the anterior pyramids *y y*, now proceed to the lower ends of the olivary bodies (W, W). The spinal-accessory nucleus (*t*) and the hypoglossal nucleus (*t'*) are still further developed. *r* is the spinal-accessory nerve.

Fig. 7. A similar section of the medulla nearer the point of the calamus scriptorius. The inner and outer restiform nuclei (*o* and *c'*) are much enlarged, and the white substance (*c*) of the restiform body is overlain by the posterior pyramid (*b*) and its contained nucleus (*b'*), also increased in size. The hypoglossal nerve (*x*) is seen to enter its nucleus (*t'*), which, like the spinal-accessory nucleus (*t*) is further developed. The olivary bodies (W) increase in size; *s* is the *antero-lateral* nucleus.

Fig. 8 is a similar section just below the calamus scriptorius. The posterior median fissure has just begun to open into the fourth ventricle. The grey nuclei of the posterior pyramids and restiform bodies nearly coalesce, the dotted line, *b''* (on the right side), defining the limits of each. The olivary bodies are much increased in size.

Fig. 9. A similar section through the fourth ventricle, a little above the calamus scriptorius. The grey nuclei of the posterior pyramids and of the restiform bodies on each side have entirely coalesced. The spinal-accessory nucleus has now become the *vagal* nucleus (*g*), und is more distinctly separated from the hypoglossal nucleus (*t'*). At U is seen a small group of cells forming the lowest end of the nucleus of the motor root of the trigeminus.

Fig. 10. Similar section of the medulla on a level with the *upper* roots of the vagus nerve. The hypoglossal nucleus (*t'*) is brought to the surface of the ventricle, while the vagal nucleus (*g*) is partly sunk between it and the rudiment of the inner auditory nucleus (*i*) which is developed out of the posterior part of the vagal nucleus (*g*) and the inner part of the posterior pyramid (*b*, fig. 9). The convolutions of the olivary bodies are more numerous than in fig. 9.

PLATE IX.

Fig. 11. Posterior view of human medulla oblongata:—*a*, its lower end; *b b'*, the posterior pyramid of right side; *c*, right restiform body; *d*, lateral column; *e*, grey tubercle of ROLANDO, or caput cornu posterioris; *g*, pyriform nucleus of vagus

nerve; t', column at side of median furrow constituting the hypoglossal nucleus.

Fig. 13. Side view of the same:—W, olivary body; *y*, anterior pyramid; A, arciform fibres.

Figs. 15 to 21 represent transverse sections of the medulla oblongata of the Monkey; fig. 15, section just below the point of the anterior pyramids. The same letters signify the same parts as in figs. 1 to 10 of the human medulla, Plate VIII.

PLATE X.

Fig. 22. Transverse section of the posterior part of the right lateral half of the medulla oblongata of the Monkey at the level of the highest roots of the vagus nerve.

Fig. 23. Similar section of the whole right lateral half of the medulla:—*i*, the inner auditory nucleus; c' *o*, the outer auditory nucleus, pierced by numerous longitudinal bundles of fibres (represented by the dark masses); t'', nucleus of the lower roots of glossopharyngeal nerve (*g*).

Fig. 24. Longitudinal and horizontal section of the left half of the posterior part of the human medulla oblongata:—$b\,b'$, white and grey substance of the posterior pyramid; *o*, inner grey substance or nucleus of restiform body; c', its outer nucleus; *g*, vagal nucleus; *i*, inner auditory nucleus; c' *o*, outer auditory nucleus, formed by the fusion of the restiform and postpyramidal nuclei.

Fig. 25. Transverse section of the spinal-accessory and hypoglossal nuclei of the left side, at the level of the lower end of the olivary body:—E, bottom of the posterior median fissure; H, group of cells constituting the anterior portion of the spinal-accessory nucleus; *r*, the spinal-accessory nerve; the remainder of the figure behind this group is the posterior division of the spinal-accessory nucleus; B, columns of cells and longitudinal fibres, with transverse fibres decussating across the middle line with those of opposite side; below these are decussating fibres from the anterior division (H) of the nucleus; *h*, central canal lined by columnar epithelium; J, hypoglossal nucleus; J'', fan-shaped set of fibres converging to decussate across the raphé (F); K', small column of cells and longitudinal fibres, ascending and increasing as it ascends, between the hypoglossal and spinal-accessory nuclei to the "fasciculus teres;" *x*, roots of hypoglossal nerve.

Fig. 26. Nerve-cells from the outer restiform nucleus of the Monkey; magnified 220 diameters.

Fig. 27. Oblique-longitudinal section of the medulla oblongata, carried along the line L *t*, fig. 5, and extending as high as fig. 8:—$n\,n'$ is the slender column of the vagus and spinal-accessory nerves; $t\,t$ is vagus and spinal-accessory nucleus; the dark masses at n'' represent the cut-ends of the transverse bundles of fibres

shown at n'', fig. 5, and coming from the postpyramidal and restiform nuclei to decussate into the anterior pyramids.

Fig. 28. Left lateral half of the grey substance of the medulla, a little below fig. 5, and where the *lower* roots (r) of the spinal-accessory nerve begin:—t, lower end of the special nucleus of the *upper* roots of the nerve; t', lower end of the special nucleus of the *upper* roots of the hypoglossal nerve, at the side-front of the canal; x, *lower* hypoglossal roots arising from a group of cells in the remains of the anterior cornu, f'; o, restiform nucleus; b', postpyramidal nucleus. On the left of $t\,t'$ are seen the last traces of the curved bundles of fibres which the anterior pyramids in their course downward send into the posterior grey substance (see n'', fig. 5).

Fig. 29. Transverse section of the grey substance of the lowest end of the medulla oblongata at the point of the anterior pyramids:—e'', posterior roots of first cervical nerves extending forward into the antero-lateral grey substance, to the part where the lowest fibres (f') of the anterior pyramids are connected with it. The same part is also connected with the lower roots of the spinal-accessory nerve (r), which is seen to traverse the lateral grey substance and reach the anterior cornu.

Fig. 30. Longitudinal section of part of the human medulla:—t'', glossopharyngeal nucleus; $n\,n$, slender longitudinal column; g, roots of the glossopharyngeal nerve running partly longitudinally along this column; i, the inner auditory nucleus.

Fig. 31. Longitudinal and horizontal section along the floor of the fourth ventricle, at the dotted line, fig. 32, through the hypoglossal nucleus (J), the vagus nucleus (H, H'), and the slender longitudinal column (n). From the large multipolar cells of the hypoglossal nucleus (J, J'', fig. 31), numerous processes extend outward, between longitudinal bundles of fibres, into the inner and outer portions (H, H') of the vagus nucleus. n is the slender column of longitudinal fibres, continuous with some of the transverse fibres of the vagus nucleus.

PLATE XI.

Fig. 33. Transverse section of the medulla oblongata of the Cod-fish, at the point of the calamus scriptorius:—1, 1, the anterior portion of the medulla; 2, 2, cut-ends of large bundles of fibres at its surface; 3, group of large multipolar cells in its centre; 4, cut-end of column of longitudinal fibres in front of the canal, 6; 5, large group of multipolar cells at the side and front of the canal; 7, fibres radiating from it to the surface (on left side); 8, lower root of vagus nerve arising from it, and joining the other root, 9, which arises from the inner and posterior part of the medulla (10, 10) at the side and behind the canal. The group of cells (5) occupies the position of the hypoglossal nucleus

in mammalia. 11, restiform body; 12, grey substance within it; 13, large band of fibres arising from it, and crossing obliquely to the anterior part of the opposite side of the medulla, decussating with its fellow; 14, trunk of the vagus nerve.

Fig. 34. Group of multipolar cells giving origin to the anterior root (8) of the left vagus nerve, as shown in preceding figure:—6, left wall of the canal lined with epithelium; 13, 13, decussating band of fibres; magnified 100 diameters.

Fig. 35 represents a transverse section of the right anterior pyramid (*y*) and olivary body (W) of the Cat. The olivary body consists of three separate portions.

Fig. 36 represents a transverse section of the right lateral half of the medulla of one of the higher Apes. The structure is peculiar. The highly developed and convoluted olivary body (W) is imbedded in the anterior pyramid, and a separate column (*d*) is on its outer side.

Fig. 37. Transverse section of the right anterior pyramid and olivary body of the Orang Outang. The olivary body largely developed, and its lamina thrown into numerous folds.

Fig. 38. Transverse section of the floor of the fourth ventricle of the left side, from the human medulla, on a level with the upper roots of the vagus nerve:—J, upper part of column of large multipolar cells constituting the hypoglossal nucleus. Some of the cells on its outer side are elongated outward, and send their processes in the direction of the vagus nucleus (*t*, H) and of the nerve (*g*). It is immediately overlain by the column K′, which consists of smaller cells and numerous longitudinal fibres, and which we saw lower down, much reduced in size, in fig. 25. This column is in its turn overlain by the vagus nucleus, between which and the hypoglossal nucleus (J), it is almost entirely imbedded. If we suppose the lower part of the central canal (*h*) in fig. 25 to open into the floor of the fourth ventricle, and the parts on its left to be thrown aside outward, the three nuclei (J, K′, H, fig. 25) will assume the position which they occupy in fig. 38.

Fig. 39. Similar section a little higher up, on a level with the roots of the glossopharyngeal nerve. The hypoglossal nucleus has almost wholly disappeared, a few small cells only (J, J′) remaining with a few nerve-roots. T, dark oval group of cells, increased in size from fig. 38; K′, in the *fasciculus teres*, much increased in size; *i*, inner nucleus of auditory nerve; *t*, glossopharyngeal nucleus with its club-shaped group of cells. It is overlain by the junction of the auditory nucleus (*i*) with the *fasciculus teres*; U, lower end of column of large multipolar cells constituting the nucleus of the motor root of trigeminus nerve. It is connected, by a plexiform system of fibres (U′), with both the *fasciculus teres* and the glossopharyngeal nucleus, especially the latter; *n*, slender longitudinal column of fibres.

Fig. 40. Similar section still higher up. The inner auditory nucleus (*i*) has become

continuous with the fasciculus teres (K′), and increased in size:—Q, Q′, cut ends of oblique *lineæ transversæ*; T, the two oval groups of nuclei much increased in size.

Fig. 41. One lateral half of the human pons Varolii and medulla seen from its under side:—Y, cut middle peduncle; X′, fibres winding round it from fourth ventricle; *c*, restiform body; Z″, band of fibres winding round it from fourth ventricle, to side of pons; X, facial nerve; Z, auditory nerve; Z′, cut end of fifth nerve; *w*, olivary body; Y, anterior pyramid; W W′, pons Varolii.

PLATE XII.

Fig. 42. Transverse section of the posterior portion of the human medulla on left side:—*c*, restiform body; *c′ o*, outer auditory nucleus; *i*, inner auditory nucleus; K′, fasciculus teres; S, a network containing several large multipolar cells, with intervening longitudinal bundles beneath the *fasciculus teres*, and in the place where the hypoglossal nucleus ends; H, nucleus of glossopharyngeal nerve; *n*, slender columnar of longitudinal fibres, now containing a great number of nuclei, and connected with glossopharyngeal nerve (*g*); U, nucleus of motor root of trigeminus; *e e e e*, cut end of descending root of trigeminus at extremity of caput cornu posterioris, or grey tubercle; P, posterior auditory nerve; Q Q′, a large stria medullaris winding over the surface of the ventricle, and passing from behind forward along one side of raphè, R.

Fig. 43. Posterior half of the left side of the human medulla oblongata, just below the pons Varolii:—W, upper end of olivary body; *e*, grey substance of grey tubercle overlain in front by the dark cut ends of the bundles of the descending root of the trigeminus; *i*, inner auditory nucleus; *c′ o*, outer auditory nucleus; P P′, posterior auditory nerve arising from both these nuclei, and from radiating fibres (*p′*) of restiform body. A network of fibres proceeding from the outer auditory nucleus (*c′ o*) crosses the grey tubercle (*e*) to the central part of the medulla and to the raphè; in the higher Apes these fibres are very numerous and distinct (see fig. 36, Plate XI.).

Fig. 44. The same, a few sections higher up:—Z, clusters of small cells on the inner side of the ganglionic enlargement of auditory nerve; Q′, Q, cut ends of striæ medullares.

Fig. 45. Side view of the human medulla oblongata, pons, tubercula quadrigemina, and crus cerebri:—*c*, restiform body entering cerebellum; P″, anterior auditory nerve; one portion of it ascends with restiform body to cerebellum; C′, trigeminus; E, superior peduncle of cerebellum; *f′*, fillet; *q′*, testis.

Fig. 46. Transverse section of left lateral half of the medulla oblongata of a child five years old, at the anterior auditory nerve:—K′, fasciculus teres, connected by numerous plexiform fibres with U, the motor nucleus of trigeminus; *e*, grey

tubercle of ROLANDO, having in front the cut ends of numerous bundles of descending root of trigeminus inclosed in a beautiful network of fibres containing nerve-cells. The tubercle is connected with the motor nucleus (U) of trigeminus by fibres; *i*, upper part of inner auditory nucleus; *c' o*, outer auditory nucleus, extending backward to the cerebellum; P'', the anterior auditory nerve, entering this nucleus at *c' o*, beneath the restiform body. Both the nucleus, and the root of the nerve P'' are intimately connected with the outer side of the grey tubercle and with the network within it; *p'*, radiating fibres of restiform body in the middle peduncle; *p''*, *p'''*, arciform fibres from the same part extending round the olivary body; J', corpus dentatum cerebelli.

Fig. 47. Dissected portion of human fourth ventricle of left side:—*c c'*, the restiform body or inferior peduncle of the cerebellum; *c'''*, middle peduncle of cerebellum; C'', large root of trigeminus; D', its conical nucleus; P'', anterior division of auditory nerve; *i*, concavity left by removal of *inner* auditory nucleus beneath which P'' enters the *outer* auditory nucleus. The anterior and under portion of the trigeminal nucleus D', connected with the point of the fillet (*f*), gives origin to the small or motor root (G') of the trigeminus, which is separated from the large root (C'') by a small bundle of the middle peduncle (A') coming from the pons. The larger root (C'') is separated from the anterior auditory nerve (P'') by a larger bundle of the middle peduncle coming from the pons; E', superior peduncle rolled forward, exposing a thick and broad tract of grey substance (D'') extending obliquely upward beneath it from the conical nucleus (D') of the trigeminus; F, loop of the facial nerve.

Fig. 48. Left lateral half of the medulla with part of the inferior vermiform process of the cerebellum from the Rabbit:—*e*, grey tubercle; *e''*, cut end of descending root of trigeminus in front of it; P'', anterior root of auditory nerve; *e'''*, cut ends of longitudinal fibres at root of grey tubercle, in connexion with fibres of anterior auditory root; *c' o*, outer auditory nucleus giving origin to anterior root of auditory nerve; *i*, upper part of inner auditory nucleus; E' E'', plexus of bundles, with intervening cells running through the lower part of superior peduncle to vermiform process (I' I''); T'', longitudinal portion of loop of facial nerve; *s*, antero-lateral nucleus.

PLATE XIII.

Fig. 49. Transverse section of left lateral half of medulla oblongata of Cat, at level of auditory nerve:—P''', under and whiter portion of nerve entering the outer auditory nucleus (*c' o*); P', bulbous portion crowded with nerve-cells; *e''*, descending root of trigeminus; *s*, antero-lateral nucleus connected with its opposite fellow of decussating fibres across the raphè.

Fig. 50. Transverse section of the fourth ventricle in Man on left side:—*g'' g'''*, super-

ficial grey layer covered with columnar epithelium; h'', median furrow of ventricle; Q'', principal nucleus of facial nerve; i, inner auditory nucleus.

Fig. 51. A similar section a little higher up, including the grey tubercle of ROLANDO:—g'', superficial grey layer of ventricle; T'', cut end of longitudinal portion of loop of facial nerve; Q'', principal nucleus of facial; q'', facial nerve; $e\,e'$, grey tubercle; $c'\,o$, remains of outer auditory nucleus; i, remains of inner auditory nucleus, sending fibres to Q''.

Fig. 52. Transverse section of fourth ventricle of Rabbit on left side.

Fig. 53 shows the course of the root of the facial nerve (q'') from without inward in the Sheep.

Fig. 54 shows the same in Man.

Fig. 55 represents a longitudinal section of loop of the facial nerve in the plane of its course, in Man: magnified 100 diameters.

Fig. 56 shows a transverse section of the fourth ventricle in Man, on the left side along the root of the facial nerve:—g'', superficial grey layer; g''', group of cells on its outer part; above this are the cut-ends of three blood-vessels in their sheaths; T'', cut-end of longitudinal portion of loop of facial nerve; h'', median furrow of ventricle; Q'', principal nucleus of facial nerve; q'', facial nerve; q''', grey streak in its middle, behind which is a blood-vessel and streaks of longitudinal fibres. The under portion of the nerve is seen entering the nucleus (Q''); its remaining fibres decussate with those of the superficial portion (l') of the nerve; the chief portion enter T'' and become longitudinal along the loop; the rest pass in front of the loop to the raphè beneath the median furrow (h''); V', the sixth or abducens nerve, breaking up into a network of bundles enclosing longitudinal fasciculi, and becoming connected with the inner part of Q''.

PLATE XIV.

Fig. 57. Horizontal and longitudinal section along the left side of floor of the fourth ventricle in Man, extending from the upper end of the hypoglossal nucleus (J) to the lower end of the facial nucleus (Q''):—$c'o$ is the outer auditory nucleus; e', the base of the grey tubercle or caput cornu posterioris; Q'', the facial nucleus; T'', lower end of the loop of facial nerve, diverging and curving outward and downward round Q''; g''', the superficial grey layer of ventricle; h'', the median furrow of ventricle overlying the raphè. At the lower end of the figure, H is the vagus nucleus; J, the cells of the upper and tapering end of the hypoglossal nucleus; J', a longitudinal plexus of fibres running from the facial nucleus (Q'') to the hypoglossal nucleus (J).

Fig. 58. A similar horizontal-longitudinal section a little deeper from the surface of the ventricle. Figs. 59 & 60 are transverse sections of the medulla at the

upper and lower ends of the longitudinal section, after it was removed. In figs. 58 & 59, the upper lines of section correspond:—F is the raphè immediately below the median furrow of the ventricle; V, the white column between the raphè and the entrance of the abducens (V') into the nucleus (Q''); e' is the base of the grey tubercle; $c'o$, the outer auditory nucleus. The *lower* line of fig. 58 corresponds to the *upper* line of the transverse section, fig. 60, in which J is the anterior portion of the hypoglossal nucleus; V the white column between it and the raphè; O'', the white olivary column lying between it and the vagus nucleus (H); n, the slender longitudinal column of the vagus; $c'o'$, the lower end of the outer auditory nucleus, formed by the grey substance of the restiform body and posterior pyramid. The white column, V, (fig. 59), between the raphè (F) and the entrance of the abducens nerve into the nucleus (Q'') is continuous in fig. 60 with the white column (V) between the *hypoglossal* nucleus (J) and the raphè.

Fig. 61. Transverse section of part of the left lateral half of the medulla oblongata of the Rabbit, through the trapezium:—e, grey tubercle; e'', descending root of the trigeminus; q'', facial nerve; p', p'', arciform fibres running forward and inward through the trapezium in front of the superior olivary body, s''.

Fig. 62. Transverse section through the whole left lateral half of the medulla at the level of the facial and abducens nerves, from the Orang Outang:—Q'', common nucleus of facial and abducens nerves; T'', descending portion of loop of facial. Transverse fibres of trapezium (p'') more concealed by the enlarged pyramid (y).

Fig. 63. A similar section from Man. Here the transverse fibres of the trapezium (p'') are completely enclosed and concealed by the anterior pyramid and pons Varolii.

Fig. 64, A. Atrophied nerve-cells from the olivary bodies, in a case of paralysis; magnified 350 diameters.

Fig. 64, B. Cells from a healthy olivary body; magnified 350 diameters.

Fig. 65. Transverse section through the fourth ventricle on a level with the facial and abducens nerves, from a child: the letters indicate the same parts as in other figures.

X. *Notices of some Parts of the Surface of the Moon* By John Phillips, *M.A., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Oxford.*

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My first serious attempts to portray the aspect of the moon were made with the noblest instrument of modern times, the great telescope of Lord Rosse, in 1852. The mirror was not in adjustment, so that the axes of the incident and convergent pencils of light were inclined at a very sensible angle. This being met by a large reduction of the working area of the mirror, the performance was found to be excellent. I have never seen some parts of the moon so well as on that occasion. But when I came to represent what was seen, the difficulty of transferring from the blaze of the picture to the dimly lighted paper, on a high exposed station, with little power of arranging the drawing-apparatus, was found to be insuperable, and the effect was altogether disheartening. It was like setting down things *ex memorid*, to give the rude general meaning, not like an accurate and critical copy. I present as a specimen of this memorial a sketch of the great crater of Gassendi, marked No. 1. (See p. 344.)

I next mounted, in my garden at York, a small but fine telescope of Cooke, only 2·4 inches in the aperture; and, aware of the nature of the difficulty which beset me at Birr Castle, I gave it an equatorial mounting, without, however, a clock movement. The whole was adapted to a large solid stone pillar in the open air. It was not possible with $\frac{1}{60}$ of the (reduced) light of the Rosse mirror to *see* so well; but it was easy to represent far better what one saw. A conveniently placed board to hold the drawing-paper, a well-arranged light, no necessity of changing position,—I made in this manner the drawing of Gassendi which is marked No. 2.

My next attempt was made in the same situation with a fine telescope by Cooke, of $6\frac{1}{4}$-inch aperture and 11 feet sidereal focus, mounted equatorially, in the old English mode, and carried by clockwork. With this excellent arrangement I was enabled to use photography very successfully, and to obtain selenographs 2 inches across in 5^s of time. The drawing of Gassendi (Plate XV. fig. 1) was made with this instrument (1853).

From these experiments the conclusion was obvious—that for obtaining good drawings of the moon convenient mounting was actually more important than great optical power; and that for such a purpose it was desirable to increase in every way the comfort of the observer, and furnish him with special arrangements for his own position and the placing of his drawing-board and light.

Having been called to reside in Oxford (1853–54), my plan for continual work on the moon was entirely cut through; it was impossible to mount a large instrument near

my dwelling till (in 1860) the ground was arranged about the museum, so as to give me the requisite space and security close to the house which had been appointed for me by the University. I then arranged with Mr. Cooke for a new telescope of 6 inches aperture, to be protected in a well-planned observatory, the construction of which was aided by the Royal Society. I now propose to give a short notice of some of the results of my work with this instrument, in connexion with remarks on the most advisable course to be followed by other surveyors of the moon.

In making drawings of ring-mountains on different parts of the moon's disk, the artist will be much aided by a projection of the mountain-border on the scale intended, from a few measures, with its proper figure due to the latitude and longitude. Eye-drafts not thus controlled are apt to become absurd, by the heedless substitution of an ideal circle for a real ellipse. Thus I have seen Gassendi forgetfully represented in more than one skilful drawing. Even with the advantage of such a projection (of which I give an example for Gassendi, No. 5) very considerable difficulties occur. One is the variation of outline caused by the displacement of the boundary of light and shade, first when the incidence of light varies through different angles of elevation of the sun, and next when the moon's position causes her to receive the light at the point observed on a different lunar azimuth. Even on so great a ring as Copernicus the variation of the outline as given by different artists is remarkable—hardly any one agreeing with what is really the most accurate drawing of all, that by P. Secchi; and that represents, not a simple ring, but a seven-angled outline. Dates must always be annexed to the figures; and as it is rarely possible to complete a good drawing of a large crater, except in two or three lunations or more, it becomes very essential that a bold free sketch be made of the moon's shadows to control the special work.

Strictly speaking, there should be at least three drawings of a ring-mountain—in morning light, at midday, and in evening. It would be better to have five drawings, one at sunrise and another at sunset being added to the three already named. It will be found most convenient to make the drawings within two hours of the moon's meridian passage.

Shadows thrown from objects on the moon have exactly the same character as those observable on the earth. They are all margined by the penumbra due to the sun's diametral aspect; this is always traceable except very near to the object; but in consequence of the smaller diameter and more rapid curvature of the moon's surface, the penumbral space is narrower. At the boundary of light and shade, on a broad grey level tract, the penumbral space is about nine miles broad, quite undefined, of course, but perfectly sensible in the general effect, and worthy of special attention while endeavouring to trace the minute ridges (of gravel?) or smooth banks (of sand?), which make some of these surfaces resemble the postglacial plains of North Germany, or central Ireland, or the southern parts of the United States, which within a thousand centuries may have been deserted by the sea.

To the same cause is due the curious and transitory extension of half-lights over some

portions of the interior of craters, while other neighbouring portions have the full light. The effect is occasionally to produce half-tints on particular portions of terraces within the crater, as in the case of Theophilus, of which I present two drawings, one (Plate XVI.) showing this peculiarity in the morning light, the other (Plate XV.) not. The central mountains of that great crater are high enough to throw long shadows; and these, as they catch upon other peaks or spread, softening with distance, over the surrounding plains, present far greater variety of shadow-tones than might be expected on a globe deficient, as the moon really appears to be, of both air and water.

If we suppose the moon's mass to have been derived from an outer ring of the earth-nebula, and, following the analogy of the planets in reference to the sun, admit its composition to have been the same as that of the earth, we should have as the originally outstanding lunar atmosphere, only $\frac{0{\cdot}0114}{1{\cdot}0000}$ part of that which surrounds, or rather which at first surrounded, the earth. The surface of the moon being $\frac{0{\cdot}0726}{1{\cdot}0000}$ part of that of the earth, the *barometric pressure* of the atmosphere on the moon would be less than that on the earth, in the proportion of $\frac{0{\cdot}157}{1{\cdot}000}$. The *height* of the lunar atmosphere, taken in terms of the pressure at the surface of the moon, would be the same as that on the earth (5 miles $\pm$). The refractions would be less than on the earth, in the proportion of the function of density, but still very sensible, even in rude observations of occultations of stars.

In like manner the water due to the moon's mass, would have its depth on the moon diminished to $\frac{0{\cdot}157}{1{\cdot}000}$ of that on the earth. If we assume the water on the earth spread over its whole surface to be 1 mile $\pm$, that on the moon would be 829 feet, a quantity not so great as to preclude the possibility of its being accumulated on the surface opposed to us, especially if we remember that the inequalities of internal earth-movement have in fact occasioned the greater part of our oceans to be collected on one half of the globe, while nearly all the land appears on the other half. In reference to this subject the opinion of HANSEN, that the centre of gravity in the spherical moon is removed from the centre of figure $33\frac{1}{2}$ miles in the direction from the earth, may be kept in mind. If it could be admitted, the oceans of water and air might be wholly collected on the remote half of the satellite. Moreover, it must be remembered that in the rocks near the surface of our planet both oxygen in oxides, carbonates, &c., and water in hydrates, are stored up in considerable quantities. We may suppose the process of volcanic incineration and aqueous absorption to have gone to greater extreme in the moon than on the earth, and so to have reduced the original atmosphere and the original oceans to very much smaller amounts. To judge by the absolute freedom from all change but that of degradation, which is observed on the visible disk of the moon, and by the perfect and perpetual clearness of its sharply defined mountain-bordered limb, cutting clear against the unbending rays of the stars, it would seem the most probable opinion that *now* at least there remains no visible trace of atmosphere or ocean.

The different parts of the moon's surface reflect light very unequally; the dark parts have several degrees of darkness, the light parts several degrees of light. On the same

level, as nearly as can be judged, under the same illumination, neighbouring parts are not only unequally reflective, but their light seems to be of different tints. Within the large area of Gassendi, under various angles of illumination, but more conspicuously when the angle of incidence deviates least from verticality, patches of the surface appear distinctly marked out by difference of tint, without shadow. It is well known that in this particular photography has disclosed curious and unexpected differences of the light, which were not apparent, or not so obvious, to the eye. Reflecting telescopes seem to be indicated as most suited for direct observation of differences of the kind of light on the moon.

The surface of the moon is hardly anywhere really smooth, hardly anywhere so smooth as may be supposed to be now the bed of a broad sea on our globe. By watching carefully the curved penumbral boundary of light and shade,—as it passes over ridge and hollow, rift and plain,—broad swells, minute puckerings, and small monticules appear and disappear in almost every part. In several of the maria, minute annular cup-craters about half a mile across are frequent; and on several of the exterior slopes of the crater-rings occur pits, ridges, fissures, and rude craters, something like the sloping surfaces of Etna. Copernicus is a good example of this common occurrence. It appears extremely desirable that the details of this magnificent mountain should be carefully reexamined on the basis of Secchi's fine drawing, for the purpose, amongst others, of determining exactly how many of the bosses and ridges bear cup-hillocks; for many inequalities, which in feeble telescopes have but the indistinct character of being heaped up, appear distinctly crateriform with superior optical power*.

For the purpose of determining the true form of the summits, and the outline of the sides of many mountains, the method of observation of the shadows at different angles of the incident sunlight will be found very useful. Thus, exactly as, in the clear evening hours, one standing on the summit of the Malverns sees the long shadow sweep over the vale of the Severn, and distinguishes the forms of the several beacons and intervening passes, so in the clearer moonlight, the shadows which fall within the craters, and stretch along the plains, often reveal the presence of angular escarpments and deep fissures as well as of peaky summits crowning steep walls of rock.

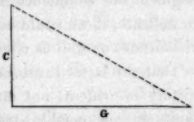
One of the circumstances which is thus often placed in evidence is the exceeding abruptness of many edges, and the uncommon steepness of many of the slopes. In particular, the interior edges of many ring-mountains appear with violently precipitous cliffs and chasms, more or less parallel to the general outline; while, more rarely, deep cross cuts in such situations appear to radiate within the ring, or to traverse its crest. An instance of the latter kind is found in Gassendi.

The steepness of the moon-crater walls and slopes is much greater in general than in any, except very rare examples, known among the volcanic regions of the earth. Popular descriptions are, in this respect, very misleading; and the word-paintings which please so much the gratified ear, lose their reputation for exactness when confronted by

* See "Comparative Remarks on Gassendi and Copernicus," Roy. Soc. Proc. for 1856, p. 74.

the clinometer. When, on the surface of the earth, we observe incoherent materials resting at a slope approaching 45°, we frequently remark some peculiarity of the separable masses which allows of mutual support, or observe that the usual atmospheric agencies, the ordinary changes of temperature and moisture, have had only a very short influence on a newly exposed surface. After a very long exposure, even hard and continuous rocks, such as those of Malvern, sink down to a slope of 18°; and it is chiefly under freshly wasting escarpments of softer strata that we see, as in Mam Tor and the west front of Ingleborough, anything approaching to the steepness of the moon-crater walls. So in the old volcanic regions of Auvergne and Mont Dor, the Eifel and even the Phlegræan fields, which may be the fittest for comparison with the crater-covered tracts of the moon, very steep slopes are almost unknown, except for small spaces of consolidated rocks, such as in the Puy de Dôme, Monte Somma, and the Drachenfels.

Perhaps we may refer the obviously greater steepness of some of the surfaces on the moon to the lesser influence of gravity on the satellite than on the earth. If in a given mass of matter, exposed to various causes by which cohesion is influenced, we represent by C (vertical) the residue of cohesive force, and by G (horizontal) the force of gravity, the slope from C to G will be the final surface, and $\frac{C}{G}$ = tangent of its inclination to the horizon. On the earth, the force of gravity being more than six times as great as on the moon, the tangent of inclination will be relatively less.



Also, if the degrading influences of all kinds on the moon be taken as very much less than on the earth, this again will relatively increase the angle of slope; so that we appear to have no difficulty in understanding why lunar mountains may be so much steeper and less worn than the objects most like them on earth. Perhaps the same general consideration of the greater relative value of heat and other molecular forces, when compared with gravity, on the moon than on the earth, may account for the much greater amplitude, and the generally greater elevation of the moon mountains; for it is only exceptionally that Chimborazo, Teneriffe, and Elburz ascend on the earth to so great a height as the loftiest lunar summits*.

On the very crest of a ring-mountain it is rare to find a cup-crater; quite common to find them in the interior, especially towards the middle, and, in several cases, exactly in the centre. But it happens often that the central mountain-mass of a large crater, such as Gassendi and Theophilus, is of a different structure. In the former a complicated digitated mass of elevated land appeared to me for a long time to be entirely devoid of any small craters; by continued scrutiny at last I see on one of the masses a distinct depression. The area in Gassendi reminded me of the volcanic region of Auvergne, in which,

* On the general relation of the moon's surface to volcanic tracts on the earth, see a graphic passage in Herschel's 'Introduction to Astronomy,' p. 229, Edition 1, 1833.

with many crater-formed mountains, occur also the Puy de Dôme, Puy Sarcoui, Puy Chopin, and others which are heaps of a peculiar trachyte not excavated at the top, while the others are formed of ashes and lava-streams, and are all crateriform. The central masses of Theophilus (Plate XV.) are very lofty and grandly fissured from the middle outwards, with long excurrent buttresses on one side, and many rival peaks separating deep hollows, and catching the light on their small apparently not excavated tops. This is like the upheaved volcanic region of Mont Dor, with its radiating valleys, wide in the central part, and contracted to gorges toward the outside of the district.

The Vesuvian volcanic system, including the Phlegræan fields, exhibits, in all respects but magnitude, remarkable analogy with parts of the moon studded with craters of various sizes, as those adjoining Mount Maurolycus, engraved for comparison by Mr. Scrope in his admirable treatise on Volcanos (p. 232). It is probable that many of the differences which appear on comparing lunar ring-mountains may be understood as the effects of long elapsed time, degrading some craters before others were set up, and turning regular cones and cavities into confused luminous mounds. It would much augment our confidence in the possible history of the moon which these differences seem to indicate, if we could believe it to have ever been under the influence of atmospheric vicissitudes as well as changes of interior pressure.

That the latter cause has been in great activity at some early period of the moon's history is evident, not only by the many sharply cut fissures which range like great faults in our earthly strata for five, ten, twenty, and sixty miles, but is conspicuously proved by the great broken ridges of mountains which, under the names of Alps, Apennines, and Riphæan chains, make themselves known as axes of upward movement, while so many of the craters near them speak of local depression. I have not been able to discover in these great ridges any such marks of successive stratification, or even such concatenation of the crests, as might suggest symmetrical and anticlinal axes. The surface is, indeed, as rough and irregularly broken as that of the Alps and Pyrenees, and marked by as extraordinary transverse rents, of which one, in the Alpine range near Plato, is a well-known example. Must we suppose these mountains to have undergone the same vicissitudes as the mountain-chains of our globe—great vertical displacement, many violent fractures, thousands of ages of rain and rivers, snow and glacial grinding? If so, where are the channels of rivers, the long sweeps of the valleys, the deltas, the sandbanks, the strata caused by such enormous waste? If the broad grey tracts were once seas, as analogy may lead us to expect, and we are looking upon the dried beds, ought we not to expect some further mark of the former residence of water there than the long narrow undulations to which attention has already been called as resembling the escars of Ireland?

Among other curious phenomena probably referable to movement of the solid crust of the moon, are those long, straight, winding, or angularly bent fissures or cracks which the Germans call 'Rillen.' The distinctness of these cracks for such great lengths as some of them reach is consistent with extreme narrowness; very narrow spaces, strongly

contrasted in brightness with the neighbouring surfaces, being easily traceable, if continuous, where much broader tracts of a square or circular figure cannot be discerned. This is familiar to persons who trace by the eye the far extended narrow telegraph wire. But more than this. The rills are, under favourable lights, seen to be really fissures with one dark and one enlightened side—deep dark fissures. Rills of this remarkable character do not appear to be branched; and in this respect the long rill of Hyginus, marked on MÄDLER'S map for ninety-two miles, may be compared with the great North of England dyke, which extends from Teesdale to near the Peak on the coast of Yorkshire, seventy miles, without a branch. Contraction, rather than violent movement of the moon's crust, seems to suit best the facts observed; and the same explanation may be fairly applied to most of the fissures filled with whinstone in the coalfields of Durham and Northumberland. Still the straight rill near Tebit (called the Railroad), nearly on the central meridian of the moon, bears the appearance of dislocation in the shadow thrown by one side.

The rill which descends from Herodotus is of a different but equally remarkable character. Parting from that dark crater—full perhaps of augitic compounds, in strange contrast with its neighbour Aristarchus, which shines as if formed of white trachyte—the rill takes a winding course with some irregularity of width and appearance, till it opens into what resembles an old delta, or dried gulf, margined by cliffs, and undulated by recesses and promontories. The delta-like space is uneven, like the bed of the German Ocean. Still, there are appearances in this seeming valley and outlet to an ancient sea, which can hardly be reconciled to the conjecture. The rill has no branches; in the middle of its course is an oblong crater which makes a part of the seeming channel; and thus this falls into the general rule which makes the rills dependent on the craters, so as to pass commonly, but not always, from one crater to another, and often to traverse them through the ring and athwart the interior space (Plate XVII.)*.

A very different class of phenomena may also be referred to some change of dimensions or some displacement of masses affecting large surfaces of the moon. These are the light-streaks which, from Tycho in particular, radiate, like false meridians, or rather like meridians true to an earlier pole of rotation. Other light-streaks pass off from Copernicus and Kepler, and several other mountains, but none are comparable to those from Tycho. There is this singularity about them; they are most distinctly visible in the full moon, and for some days before and after that phase. When the light falls at a low angle on the part of the moon's disk where one or more of these rays exist, they do not appear; as the sunlight strikes at higher and higher angles they come out bright and clear, again vanishing as the lunar evening comes on.

By the strictest examination these luminous bands are found to have no projection above and no depression below the surface, no shadow on either boundary. They can be traced across what look like seas, and equally traverse the crateriform mountains, continuously; sometimes branching, often varying in breadth. On a first view they seem

* See additional notices in the Supplement.

like a system of radiating fissures, and Mr. NASMYTH very ingeniously imitated their direction at least by a glass globe filled with water, which was made to crack in a nearly regular manner from a centre. But their great and unequal width, their entire want of relief, and the peculiarity of their reflexion of light, seem to point to some other cause. The reflexion seems to be of the kind called "metallic;" perhaps metals or metalloids may there be covered by a translucent crust, which may reflect light of low incidence, but transmit light of higher incidence, to be reflected from the surface below. This is the nearest conjecture which has occurred to me.

The parts of the moon's surface to which I have devoted most time are the ring-mountains of Gassendi and Theophilus. The sketches here given of Cyrillus and Catharina (Plate XVI.), are only first though not careless drawings, in which are omissions, which I hope at some future time to supply. Rather more progress has been made with Posidonius, Aristarchus, and Herodotus. I have also begun to sketch the large and noble group which lies to the south of Ptolemæus, and it is my earnest hope to be able to finish these drawings as well as to complete a good deal of work on the Rills.

In any further attempts of my own to contribute facts toward the survey of the moon, now again taken in hand by the British Association, I shall probably select for careful work some particular features, such as the mountains in the midst of a large crater, the bosses and cup-like hills on the outward slopes of such a crater, the rents in mountain-ridges, and the low winding banks which appear on the broad grey tracts. But for those who desire to perform a work of high value, and lay a sure foundation for accurate surveys of particular mountains, I would earnestly recommend a strict reexamination of every element in the great picture of Copernicus, for which we are indebted to the Roman Astronomer.

The descriptions which follow relate principally to Gassendi and Theophilus.

Gassendi (Plate XV.), whose centre is situate nearly in Lat. S. 16° 56′ and in Long. E. 39° 32′, has the long diameter of its apparently elliptical boundary inclined to the equator about 64° 30′. Measured in this direction, the ring has a diameter of forty-eight miles; the shorter diameter is about thirty-six. Sunrise happens when the moon is 10·5 days old, midday at about eighteen, and sunset at twenty-five days. In consequence of its position on the moon's disk, shadows usually fall from elevated objects a little toward the south.

At the instant of sunrise on the central peaks, the whole of the exterior ring is enlightened, except for a few miles on the south border, where the crests are low and divided (18th May, 1853, 10 P.M.). As the early morning advances, very broad shadows from the western border spread over the interior, and bring out in every part a mingled effect of unequal height and unequal reflective power of the surface. When the moon is eleven or twelve days old, the interior appears everywhere diversified by lights and shades, nowhere smooth, but marked by ridges and hollows in various directions. Within

the great ring in all the S.W., S., and S.E. portions are broken ridges, not terrace edges, separated by considerable depths from the great encircling crest. Toward the northern part there seems a sort of confluence of small ridges from the interior, directed toward the spoon-shaped crater, marked A on MÄDLER's map. The mouldings of the surface of this very deep crater are best seen when the moon is a day older.

Directing our attention to the central mountain-mass, we perceive it to be divided into many digitated parts, each diversified by lights and shades, so as to resemble, more than anything which I remember, dolomitic or trachytic mountains. On their several peaks I have sought carefully for the concavities which might indicate former eruptions, but only on one of them, a little removed from the rest, do I feel sure of this occurrence.

Through these central mountains can be drawn a crescent concave to the S.E., and along it occur four small craters, three of them rising separately in the area, and the one already mentioned near to the eastern slope of the central mountain.

The ring of Gassendi, everywhere rugged and fissured, is broken through in one place completely, in another less distinctly, both in the south part of the contour, so as to open the area to Mare Humorum. In several other places it sinks between points greatly elevated. The highest point is on the east border (9000 feet according to MÄDLER), where a sort of lacuna, rather than crater, occurs in the middle of the crest; nearly opposite to this ridges detach themselves both outwardly and inwardly from the western border.

The north-eastern edge is cut across by three oblique narrow clefts, and the northern border is deflected by the concurrence of the ring of the smaller crater. The country round Gassendi is very ridgy and mountainous in the east, where it is pressed up, as it were, to the great peak and the lacunal crater. This is not the case on the west, which side is much lower. On the south, in Mare Humorum, are many small craters not yet mapped. On the north, in the high grounds leading to the old degraded crater of Letronne, are a few craters apparently of older date than Gassendi.

As the daylight advances, and especially at midday, much of what has been described disappears from sight; but Gassendi, in consequence of its ridges usually having some shadow toward the south, does not lose distinctness in the same proportion as Ptolemæus and the craters near the centre of the moon.

When the sun is on the meridian of Gassendi, the peculiarities of its surface in reflecting light appear conspicuous. Within the south border is a broad dusky space, apparently not much undulated, and resembling the dark parts of the Maria. This space sometimes appears interrupted by a cross band of comparative brightness. A darkly tinted space can also be seen in the N.E. No luminous rays stream from Gassendi. In the afternoon, the appearances described in the morning are discovered, as far as the main features are concerned, with shadows in the opposite direction; but the views have not appeared to me so interesting, nor is the terrestrial hour (early morning) for observation so convenient.

Theophilus, Cyrillus, and Catharina (Plate XVI.), situated south of the equator from

10° to 20°, and in west longitude 21° to 28°, form a remarkable triplet of ring-mountains nearly equal in magnitude, and in contact, yet offering singular differences of structure. They are not of the same date: Theophilus intrudes its continuous outline within the northern part of the older crater of Cyrillus; Cyrillus is linked to Catharina by a concave isthmus; Catharina includes the half-preserved ring of an older and smaller crater. In the centre of Theophilus is an aggregation of peaks; in the same part of Cyrillus is a huge double-horned mountain, and near it a considerable crater; neither of these features occurs in Catharina.

Theophilus, situated in lat. S. 10° to 13° and long. W. 25° to 28°, though nearly circular in figure, shows some diversity in this respect according to the incidence of light, being sometimes almost hexagonal. Copernicus also exhibits this diversity.

The longest diameter of Theophilus is above fifty-five miles in length. Sunrise happens when the moon is 5·2 days old, midday when about 12·7, and sunset at twenty days.

At sunrise the central mountain and all the surrounding ring are brightly illuminated, only narrow channels of darkness running into Cyrillus. But the interior of the crater is then wholly dark, except for a small breadth within and under the eastern crest (1863, April 24). When the sun's rays fall at a low angle on the surface of Theophilus, the shadow thrown from the lofty western crest (15,000 feet high) spreads widely over the interior of the deep crater, and is undulated on the edge so as to mark the effect of higher and lower parts of the crest. Between this shadow and the base of the central mountain the area appears clear like a floor; but beyond this, toward the north, east, and south, all is curiously uneven in heapy little ridges, and long partly fissured surfaces parallel to the ring. I can discern only one distinct crater in this surface, and that is under the north-eastern part of the ring, but there are several smaller pits. Perhaps larger instruments may show that some of the small heaps and ridges have cups on their surface.

To the central mountain (Plate XV.) I have given much and frequent attention, for the purpose of ascertaining the form of its much divided mass, and of discovering whether it contained any cup-formed summits. None were observed among the ten or more bosses which go to make up the rugged mass, elevated about 5000 feet above the central area.

When the sun's rays fall with less obliquity on Theophilus, the part which was wholly in shade under the western crest assumes a quite different aspect—many bold ridges and furrows showing themselves distinctly with light falling at 15° elevation. Some of these appear in the lower part of the figure in Plate XVI., but they really exist all round the inner and western part of the crater.

The phenomena which appear where Theophilus joins Cyrillus, are extremely curious and complicated, not in the least like as if one cone of volcanic eruption had intruded its convex sloping surface within another, but rather as if one great blister had pushed aside another and then burst, leaving a sort of double folding along the line of junction.

Cyrillus varies greatly in appearance, according to the direction of the incident light. The interior is largely rugose, as if ridged and furrowed by lateral pressure. In particular the great double central mountain appears in the two drawings here given (Plate XV. and Plate XVI.) with a difference of outline very remarkable; as well as a difference of shadows, for which, however, the sun's position may perhaps account. The eastern border is remarkably rugged and broken, so as to become indefinite. The crater is unenclosed both to the south and north. Catharina seems to contain traces of more than one old crater enveloped and partly obliterated within it. It is incompletely bordered, or complicated in the boundary on all sides, and open or unenclosed on the south and north. If we regard Catharina and Cyrillus as twin craters of old date, broken into by Theophilus, we may admit the space full of old ridges and hollows to which Cyrillus opens, on the east of Theophilus, as a third and still older, once crateriform, mass which has lost almost every trace of the large ancient ring. In the same way the triplet of larger mountains entitled Arzachel, Alphonso, and Ptolemæus, seem to have been connected, Ptolemæus being the oldest, largest, and least distinct. These mountains, indeed, may be regarded as part of a vast crescent of 60° arc, extending from Barocius and Maurolycus, by Stöffler, Walter, and Regiomontanus, to Purbach, Arzachel, Alphonso, Ptolemæus, and Hipparchus. In each of these cases the oldest mountains, those most degraded and ruined, are near the equator. These are among the greatest of all the lunar mountains.

Another arc, more directly meridional (long. W. 60°), includes the very great rings of Furnerius, Petavius, Vendolinus, and Langrenus, whose diameters are about 5° of lunar latitude. The more degraded and probably the older of these are those nearest the equator.

In various parts of the moon occur twin mountains which do not communicate with one another; as Aristarchus and Herodotus, one light the other dark; Hercules and Atlas; Autolycus and Aristillus; Eudoxus and Aristoteles; Mercator and Campanus. Indications of this kind of symmetry or continuity of action in the old volcanic force, appear worthy of special study, in relation to the physical history of the moon.

Explanation of Plates.

PLATE XV.

Fig. 1. Gassendi, as seen in morning light.
Fig. 2. Theophilus and part of Cyrillus, morning light.
Fig. 3. The central mountain-mass of Theophilus enlarged.

PLATE XVI.

Group of Theophilus, Cyrillus, and Catharina.

PLATE XVII.

The twin craters of Aristarchus and Herodotus, with the rill proceeding from the latter. Several small craters on the side of the rill and about the seeming delta are omitted. The crater edge of Aristarchus appears to me a double crest, with a narrow deep chasm between the ridges. R. The rill from Herodotus. E.W. Borders of the seeming delta. *r*. Smaller rill.

The memoir was accompanied by several other drawings, which it is the intention of the author to present to the Royal Society for reference. The following is a List of these illustrations. Nos. 1, 2, 4, and 5 are referred to in the Memoir, pages 333 and 334.

No. 1. Sketch of Gassendi, taken in 1852, at Birr Castle, with the great telescope of Lord Rosse (morning).

No. 2. Sketch of Gassendi, taken in 1852, at York, with an achromatic by Cooke, of 2·4 inches diameter (morning).

No. 4. Sketch of Gassendi, taken in 1862, at Oxford, with an achromatic by Cooke, of 6 inches diameter (evening).

No. 5. Working Plan of Gassendi, and Scale.

No. 6. Freehand sketches to illustrate the mode of working for general effect. Oxford, 1864.

No. 10. Posidonius, early morning, 1863. Unfinished.

No. 11. Posidonius, nearer to midday, 1863. Unfinished.

SUPPLEMENT.

Received May 23, 1868.

Since the preceding remarks were presented to the Royal Society, I have had the opportunity of reading the valuable memoir of J. F. J. Schmidt, the Director of the Observatory at Athens, entitled "Über Rillen auf dem Monde," 1866, and of comparing the maps of some parts of the moon which accompany his essay with my sketches. I rejoice to see how large an increase to the known number of rills has been made by this experienced observer, aided by the favourable "clearness of the Athenian climate."

In my sketch of Aristarchus (Plate XVII.), the principal purpose was to show the sharp-edged double western crest and the internal broad moulding. In the character of the double crater-wall Schmidt's Map agrees with my drawing, and extends this feature to the eastern side.

There is less agreement in the shape of the interior surface, on which I have bestowed much attention, first with the 3-foot mirror of Lord Rosse (power 200), next with my long achromatic at York, and finally at Oxford with my 6-inch Cooke. The country lying

to the north of Aristarchus has been five times drawn by me, often with difficulty, for the appearances are very variable, with small differences of light. My results agree in general with that of the Athenian astronomer, and with an earlier map published by MÄDLER in 1837. I hope to complete a drawing of this tract.

In Plate XVII. Herodotus and the famous rill from it are merely sketched freely, to show the general course of the rill, and its opening into a seeming delta. The eastern border of the seeming delta (E) is treated as a rill by SCHMIDT and MÄDLER, and so it has always appeared to me in the part marked r, which turns up to the south so as to be rudely parallel to the first or main part of the rill from Herodotus, and seems to begin under a crest of mountain. The smaller notch in this outline (at E) has only been noted once. The western branch (W) is not marked as a rill by SCHMIDT, but may be traced on his map as a kind of coast by the line of the promontories, and so traced agrees with my outline. I propose to complete the drawing of this part of the moon in the next summer.

Plate 2 of SCHMIDT's memoir contains map-sketches of rills lying east of Campanus, which I have also drawn. The appearances in this region differ somewhat from the map of MÄDLER, and the rills in particular are very interesting from their number, general parallelism, partial concurrence, and distinct relation to small and large craters.

In a third plate of the same memoir is a diagram map of Gassendi, and the region to the south, which may be compared with my drawing, Plate XV. fig. 1. The agreement is quite satisfactory in the general sweep and breaks of the border, the digitated central mass, the small craters in the area, the complexity of the eastern crest and slopes.

There is some difference requiring reexamination in regard to the sculpture of the interior, and the linear inequalities, some of which are marked as distinct rills by SCHMIDT. I shall take an early opportunity of making a careful study of the points of difference.

XI. *Results of Observations of Atmospheric Electricity at Kew Observatory, and at King's College, Windsor, Nova Scotia.* *By* JOSEPH D. EVERETT, *D.C.L., F.R.S.E. Communicated by Sir* WILLIAM THOMSON, *F.R.S.*

Received October 14,—Read December 5, 1867.

MY papers of June 18th, 1863 and January 12, 1865, contained a record of observations at Windsor, Nova Scotia, from October 1862 to the end of February 1864. From this latter date they were continued until August 8th of the same year, and I have now to report this concluding series, giving at the same time a summary of results derived from the whole of my observations.

I have also to report, at the request of Sir WILLIAM THOMSON, the results of two years' observations of atmospheric electricity taken at Kew Observatory with his self-recording apparatus, and reduced under his direction and under my own more immediate supervision at the Natural Philosophy Laboratory of the University of Glasgow.

The concluding series of observations at Windsor were taken regularly at the three principal hours previously adopted, that is to say, about 9 A.M., 2 P.M., and 10 P.M., but very few were taken at other hours. The station electrometer alone was used, and the electricity was collected by burning-match. The glass fibre mounted in the electrometer July 31, 1863, remained unchanged to the end of the observations, thus giving a full year's observations with the same fibre.

All the generalizations noticed in my former papers apply also to the concluding series, with the single exception that negative electricity was once observed under a clear sky. This phenomenon was observed at 10 P.M., July 15th, and as its exceptional character struck me at the time, it was carefully verified. The negative potential was equal in absolute amount to about half the average positive potential. The temperature was 59°·1, wet bulb 2°·7 below dry, barometer 30·10, no wind, sky perfectly clear, except a little cirrocumulus near the horizon. There was a faint trace of aurora overhead. The weather had been fine for several days, and continued so for several days after.

The following Table, showing the mean hourly electrical potential in fine weather for each month in the concluding series, has been computed in the same manner as the corresponding Tables in my previous papers. The numbers in this Table are in units of "Station Electrometer with third fibre."

	April.		May.		June.		July.		August.		Total No. of obs.	Gross mean.	Mean of monthly means.
	No. of obs.	Mean.	No. of obs.	Mean.	No. of obs.	Mean.	No. of obs.	Mean.	No. of obs.	Mean.			
5 to 7 A.M.	...	.....	...	.....	...	.....	1	17·2	...	.....	1	17·2	17·2
7 to 8	6	23·9	5	18·9	9	23·4	2	49·4	2	21·5	24	24·6	27·4
8 to 9	12	32·1	18	26·3	14	20·3	14	26·4	3	28·7	61	25·2	26·8
9 to 10	1	28·8	3	25·9	2	23·0	2	31·6	...	.....	8	27·0	27·3
10 to 11	...	.....	...	.....	...	.....	1	33·6	...	.....	1	33·6	33·6
11 to 12													
12 to 1 P.M.													
1 to 2	16	28·5	21	24·9	18	22·3	11	23·3	1	13·4	67	24·6	22·5
2 to 3	4	20·0	5	28·5	5	33·0	10	21·8	3	27·3	27	25·5	26·1
3 to 4	1	32·2	...	.....	3	19·2	1	21·7	...	.....	5	22·3	24·4
4 to 5	...	.....	2	45·8	...	.....	1	13·0	...	.....	3	34·8	29·4
5 to 6													
6 to 7													
7 to 8													
8 to 9													
9 to 10	12	19·2	14	20·3	11	10·9	8	14·4	...	.....	45	16·7	16·2
10 to 11	7	23·5	10	25·3	16	17·5	12	10·0	4	15·3	49	17·9	18·3
11 to 12	1	18·9	2	21·7	1	1·4	1	84·7	...	.....	5	29·7	31·7

In the following Table, which includes the whole of my fine-weather observations from the beginning, all observations, whether actually taken with first, second, or third fibre, are reduced to units of second fibre, that being the unit adopted in my previous papers. Although the numbers for the first seventeen months have already been published in the Proceedings, it seems desirable to reproduce them, in order to show the connexion of the whole series.

	Mean of observations before noon.	Mean of observations from noon to 6 P.M.	Mean of observations after 6 P.M.	Mean of three preceding columns.		Mean of observations before noon.	Mean of observations from noon to 6 P.M.	Mean of observations after 6 P.M.	Mean of three preceding columns.
1862.					1863.				
October ...	3·42	3·68	2·69	3·26	October ...	5·24	4·16	2·74	4·05
November	3·53	2·89	2·58	3·00	November	4·24	4·13	2·82	3·72
December	4·09	5·01	2·77	3·96	December	4·51	5·14	3·39	4·35
1863.					1864.				
January ...	4·11	4·88	3·42	4·14	January ...	3·86	5·74	3·63	4·41
February	6·10	5·77	4·96	5·61	February*	4·78	4·97	3·16	4·30
March.....	6·28	5·10	5·02	5·47	March ...	5·88	6·72	4·05	5·55
April	4·41	4·37	3·26	4·01	April	4·60	4·24	3·24	4·02
May	2·98	3·54	2·85	3·12	May	3·88	4·22	3·49	3·86
June	2·91	3·02	2·52	2·82	June	3·39	3·75	2·24	3·13
July	3·17	3·20	2·50	2·96	July	4·55	3·46	2·39	3·47
August ...	3·98	4·01	3·20	3·73	August ...	4·04	3·72	2·39	3·38
September	3·98	4·41	3·18	3·86					

A portion of the apparatus employed in the observations of atmospheric electricity at Kew Observatory was erected in January 1861, but the observations which have been

* Observations on two days in February 1864 were out of range and have not been included; hence the values here given are too low.

reduced commence with June 1862, and extend to the end of May 1864, thus embracing exactly two years.

The following description of the apparatus is, for the most part, copied verbatim from a lecture delivered by the inventor, Sir WILLIAM THOMSON, at the Royal Institution, May 18th, 1860, as reported in the Proceedings of the Institution*.

I. The water-dropping Collector consists of an insulated copper vessel containing water, which is allowed to flow out in a fine stream through a brass pipe projecting through a hole in a window-frame on the west side of the Observatory into the open air, which frame is, for still greater security, composed of ebonite. The nozzle of the pipe is $11\frac{1}{2}$ feet above the ground, and 3 feet from the wall of the Observatory. The effect of the flow of water is to reduce the copper vessel and its contents to the same electrical potential as that point in the air at which the water-stream breaks into drops.

II. The divided-ring electrometer, of which some of the internal parts are shown in fig. 4, Plate XIX. consists of

(1) A ring of metal (A B) divided into two equal parts (C A D, C B D), of which one is insulated, and the other connected with the metal case of the instrument and so with earth.

(2) A very light needle (E) of sheet aluminium, hung by a fine glass fibre (H) and counterpoised at G so as to make it project only to one side of the axis of suspension.

(3) A Leyden phial, consisting of an open glass jar, coated outside and inside in the usual manner, with the exception that the tinfoil of the inner coating does not extend to the bottom of the jar, which is occupied instead by a small quantity of sulphuric acid.

(4) A stiff straight wire (F F) rigidly attached to the aluminium needle, as nearly as may be in the line of the suspending fibre, bearing a light platinum wire (K) linked to its lower end and hanging down so as to dip into the sulphuric acid.

(5) A case protecting the needle from currents of air, and from irregular electric actions, and maintaining an artificially dried atmosphere round the glass pillar or pillars supporting the insulated half-ring, and the uncoated portion of the glass of the phial.

(6) A light stiff metallic electrode projecting from the insulated half-ring through the middle of a small aperture in the metal case, to the outside.

(7) A wide metal tube of somewhat less diameter than the Leyden jar, attached to a metal ring borne by its inside coating, and standing up vertically to a few inches above the level of the mouth of the jar.

(8) A stiff wire projecting horizontally from this metal tube above the edge of the Leyden jar, and out through a wide hole in the case of the instrument to a convenient position for applying electricity to charge the jar with.

* This lecture, as reported in the Proceedings of the Royal Institution, was sent in the following year, with the photographic curves for four successive days, and an accompanying description, to the Philosophical Magazine, but was not inserted; and down to the present time no full description of the apparatus has been published, the most successful attempt that we have seen being the description of the Electrometers at Kew in the 'Engineer' for August 9th of the present year.

(9) A very light glass mirror, about three-quarters of an inch in diameter, attached by its back to the wire (4), and therefore rigidly connected with the aluminium needle.

(10) A circular aperture in the case, shut by a convex lens, and a long horizontal slit, shut by plate glass, with its centre immediately above or below that of the lens, one of them above, and the other equally below the level of the centre of the mirror.

(11) A large aperture in the wide metal tube (7), on a level with the mirror (9), to allow light from a lamp outside the case, entering through the lens, to fall upon the mirror, and be reflected out through the plate-glass window; and three or four fine metal wires stretched across this aperture to screen the mirror from irregular electric influences, without sensibly diminishing the amount of light falling on and reflected off it.

The divided ring (1) is cut out of thick strong sheet metal (generally brass). Its outer diameter is about 4 inches, its inner diameter $2\frac{1}{2}$; and it is divided into two equal parts by cutting it along a diameter with a saw. The two halves are fixed horizontally; one of them on a firm metal support, and the other on glass, so as to retain as nearly as may be their original relative position, with just the saw cut, from $\frac{1}{10}$ to $\frac{1}{20}$ of an inch broad, vacant between them. They are placed with their common centre as nearly as may be in the axis of the case (5), which is cylindrical, and placed vertically. The Leyden jar (3) and the tube (7), carried by its inside coating, have their common axis fixed to coincide as nearly as may be with that of the case and divided ring. The glass fibre hangs down from above in the direction of this axis, and supports the needle about an inch above the level of the divided ring. The stiff wire (4) attached to the needle hangs down as nearly as may be along the axis of the tube (7).

Before using the instrument, the Leyden phial (3) is charged by means of its projecting electrode (8). When an electrical machine is not available, this is very easily done by the aid of a stick of vulcanite, rubbed by a piece of chamois leather. The potential of the charge thus communicated to the phial, is to be kept as nearly constant as is required for the accuracy of the investigation for which the instrument is used. Two or three rubs of the stick of vulcanite once a day, or twice a day, are sufficient when the phial is of good glass, well kept dry. The most convenient test for the charge of the phial is a proper electrometer or electroscope, of any convenient kind, kept constantly in communication with the charging electrode (8). The gauge-electrometer described below was used for that purpose at Kew. Failing any such electrometer or electroscope, a zinc-copper-water battery of ten, twenty, or more small cells, may be very conveniently used to test directly the sensibility of the reflecting electrometer, which is to be brought to its proper degree by charging its Leyden phial as much as is required.

In the use of the divided-ring electrometer, the two bodies of which the difference of potentials is to be tested, are connected one of them with the metal case of the instrument, and the other with the insulated half-ring. In the Kew observations of atmospheric electricity these two bodies were the earth and the water-dropping collector. The needle being, let us suppose, negatively electrified, will move towards or from the

insulated half-ring, according as the potential of the conductor connected with this half-ring differs positively or negatively from that of the other conductor (earth) connected with the case. The mirror turns accordingly in one direction or the other through a small angle from its zero position, and produces a corresponding motion in the image of the lamp on the screen on which it is thrown.

In the Kew apparatus, this image was thrown upon photographic paper, which was drawn upwards with a uniform motion by clockwork, and a continuous trace of the variations of electrical potential was thus produced. A zero line was at the same time drawn by the image of the same flame reflected from a fixed mirror.

The curves of atmospheric electricity thus obtained are about 18 inches long; and each sheet contains two. Each curve embraces a period of about twenty-four hours, the paper having been regularly shifted or changed at about half-past 10 A.M.

Generally speaking, the curves are distinctly traceable through the whole twenty-four hours. The interruptions which do occur are owing, in some cases, to the spot of light having moved too fast to leave a trace, in others to its having passed off the paper. As regards this latter source of failure, it may be remarked that it is not detrimental to the investigation of the law of diurnal variation. It merely does for us what General SABINE found it necessary to do in combining magnetic observations, that is to say, it rejects observations at times of unusual disturbance.

Specimens of the curves, of the actual size, are given in Plate XIX.

The ordinates (positive or negative) of the curves are to a close degree of approximation proportional to the potential (positive or negative) of the air at the place of observation, provided that the charge of the Leyden phial (3) be preserved constant; and if this charge be allowed to vary, the ordinates will vary in simple proportion.

The charge was tested daily by the gauge-electrometer, which we shall now describe, and which is identical with the station electrometer used in my own Windsor observations. Its external appearance is shown in fig. 1, Plate XVIII., and some internal parts in figs. 2, 3, Plate XVIII. The same letters denote the same parts in all three figures. It consists of

(1) A thin flint-glass bell (fig. 1, Plate XVIII.) coated outside and inside like a Leyden phial, with the exception of the bottom inside, which contains a little sulphuric acid (H). The dotted line A A indicates the boundary of the tinfoil.

(2) A cylindrical metal case (shaded in fig. 1, Plate XVIII.), inclosing the glass jar, cemented to it round its mouth outside, extending upwards about $1\frac{1}{2}$ inch above the mouth, and downwards to a metal base supporting the whole instrument, and protecting the glass against the danger of breakage.

(3) A cover of plate glass (C) with a metal rim, closing the top of the cylindrical case of the instrument.

(4) A torsion-head (B, fig. 1, Plate XVIII) after the manner of COULOMB'S balance, supported in the centre of the glass cover, and bearing a glass fibre (E, figs. 1, 2, Plate XVIII.) which hangs down through an aperture in its centre.

(5) A light aluminium needle (L L, figs. 1, 2, Plate XVIII.) attached across the lower end of the fibre (which is somewhat above the centre of the glass bell), and a stiff platinum wire (F, figs. 1, 2, Plate XVIII.) attached to it at right angles and hanging down to near the bottom of the jar.

(6) A very light platinum wire (G, fig. 1, Plate XVIII.), long enough to hang within one-eighth of an inch or so of the bottom of the jar and to dip into the sulphuric acid (H).

(7) A metal ring attached to the inner coating of the jar, bearing two plates (M, M, figs. 1, 2, 3, Plate XVIII.) in proper positions for reflecting the two ends of the aluminium needle when similarly electrified, and proper stops (as O, fig. 3, Plate XVIII.) to limit the angular motion of the needle to within about 45° from these plates.

(8) A cage (P P, figs. 1, 2, Plate XVIII.) of fine brass wire stretched on brass framework, supported from the main case above by two glass pillars (Q Q) and partially inclosing the two ends of the needle and the repelling plates, from all of which it is separated by clear spaces of nowhere less than one-fourth of an inch of air.

(9) A charging electrode (J, fig. 1, Plate XVIII.) attached to the ring (7) and projecting over the mouth of the jar to the outside of the metal case (2), through a wide aperture, which is commonly kept closed by a metal cap (K), leaving at least one-quarter of an inch of air round the projecting end of the electrode.

(10) An electrode (S T, figs. 1, 2, Plate XVIII.) attached to the cage (P P) and projecting over the mouth of the jar to the outside of the metal case (2) through the centre of an aperture. In order to dry any air which may enter through this aperture, a hollow cylinder of pumice soaked in sulphuric acid is inserted in the leaden receptacle (U, fig. 1, Plate XVIII.), through the centre of which the wire (T) passes, and the leaden cover (V V) is pierced with a hole large enough for the wire to pass through without contact. This cover has a depression (W) to receive the droppings of acidulated water from the pumice.

This instrument is adapted to measure differences of potential between two conducting systems, namely, as one, the aluminium needle (5), the repelling plates (7), and the inner coating of the jar, and, as the other, the insulated cage (8). This latter is commonly connected, by means of its projecting electrode (10), with the conductor to be tested. The two conducting systems, if connected through their projecting electrodes by a metallic wire, may be electrified to any degree, without causing the slightest sensible motion in the needle. If, on the other hand, the two electrodes of these two systems are connected with two conductors, electrified to different potentials, the needle moves away from the repelling plates; and if, by turning the torsion-head, it is brought back to one accurately marked position, the number of degrees of torsion required is proportional to the square of the difference of potentials thus tested.

In the ordinary use of the instrument, the inner coating of the Leyden jar is charged negatively, by an external application of electricity through its projecting electrode (9). The degree of the charge thus communicated is determined by putting the cage in connexion with the earth through its electrode (10), and bringing the needle by torsion to

its marked position. The square root of the number of degrees of torsion required to effect this measures the potential of the Leyden charge. This result is called the reduced earth-reading. When the atmosphere inside the jar is kept sufficiently dry, this charge is retained from day to day with little loss, not more, often, than 1 per cent. in twenty-four hours.

In using the instrument the charging electrode (9) of the jar is left untouched, with the aperture through which it projects closed over it by the metal cap referred to above. The electrode (10) of the cage, when an observation is to be made, is connected with the conductor to be tested, and the needle is brought by torsion to its marked position. The square root of the number of degrees of torsion now required measures the difference of potentials between the conductor tested and the interior coating of the Leyden jar. The excess, positive or negative, of this result above the reduced air-reading, measures the excess of the potential, positive or negative, of the conductor tested above that of the earth; or simply the potential of the conductor tested, if we regard that of the earth as zero.

The mode of employing this instrument at Kew was to keep its Leyden phial (1) always connected with the Leyden phial of the self-recording electrometer by means of a wire protected by an air-tight tube, and to keep the cage-electrode (10) always connected with the earth.

Readings of the gauge-electrometer were taken daily at about $10^h\ 30^m$ A.M. and entered in a book. Whenever the charge of the Leyden jars as thus tested was found to have fallen too low, a fresh charge was given. The earth-readings of the gauge-electrometer were thus always kept between 245° as a lower, and 255° as an upper limit, or, to speak more strictly, in the few instances in which these limits were exceeded, the observations were rejected in the reductions. These readings, however, require to be corrected by subtracting the index-error, which was carefully ascertained from time to time, and never varied much from 230°. The corrected readings were therefore contained between the limits 15° and 25°; and as the square roots of these numbers are as 1 : 1·3, the weakest and strongest charges must have differed by about 30 per cent. This difference, however, only affects the comparison of one day with another. The loss of charge in twenty-four hours was from 1 to 3 per cent., and it is this loss only which affects the diurnal curve. As the fresh charge was always given at about $10^h\ 30^m$ A.M., the disturbing effect is to be looked for in a sudden rise of the curve about this time, a consideration to which we shall hereafter recur.

Since the erection of the Kew instruments the divided-ring electrometer has undergone considerable modification at the hands of its inventor. The flat ring (fig. 4) divided into two segments is changed for a hollow box (figs. 5, 6) divided into four segments, of which one opposite pair are connected with earth, and the other pair with the conductor to be tested. This box encloses the needle, which is represented by the dotted outline in fig. 5, and projects symmetrically on both sides of the suspending fibre, thus obviating the necessity for a counterpoise.

The use of a gauge-electrometer has been superseded by the introduction of a micrometer-screw attached to one of the four segments, and regulating its distance from the others. By diminishing or increasing this distance the sensibility of the instrument can be increased or diminished at pleasure; and by attending to this adjustment as often as may be necessary (say, once a day), the sensibility can be kept practically constant in spite of variations in the charge of the Leyden jar*.

The retention of charge by the jar has been greatly improved by closing up the space round the open electrode (as T, fig. 1) with vulcanite.

The portable electrometer employed in some of my Windsor observations has been superseded by a smaller and at the same time more sensitive instrument, in which the distance between two parallel plates, one of which is connected with a charged Leyden jar and the other with the conductor to be tested, is varied at pleasure by a micrometer-screw so as to obtain a constant force of attraction between them. Equal differences of potential, whether in the conductor tested or in the charge of the jar, correspond to equal movements of the micrometer-screw, and the potential of the conductor can thus, by comparison with an earth-reading, be found by mere addition and subtraction.

The curves which have been measured and reduced comprise the two years commencing with June 1862 and ending with May 1864. The method of procedure was as follows:—

1. Ordinates were erected at every hour and half hour, careful attention being paid to the times of commencing and ending, which were in every case indicated by entries made on the photographic sheets by the Kew observers. In placing the ordinates, it was assumed that each sheet had moved with uniform velocity through its whole length, but it was not assumed that different sheets had moved with the same velocity; in fact the difference in the lengths of the curves, taken in connexion with the times of beginning and ending, showed that the velocities of different sheets must have differed by (in extreme instances) about 5 per cent. As it was impossible therefore to use one time-scale for all the curves, about twenty different time-scales were prepared differing by small gradations one from another, and for the measurement of each curve that scale was employed which suited it best.

2. The ordinates erected at the half-hours (*e. g.* half-past one, half-past two, &c.) were joined by straight lines drawn by hand in such a manner as to give and take equal areas as nearly as the draughtsman could judge.

3. The lengths intercepted by these joining lines on the hourly ordinates were measured with a scale divided to millimetres, and were adopted as the mean heights of the curve for each hour.

Whenever the curve for part of an hour was not traceable, a blank was left for that hour, and whenever the curve was partly above and partly below the zero-line (showing that the electricity was partly positive and partly negative), the algebraic mean was taken.

The measurements made in the manner above described were entered in a book in

* Still more recently a "replenisher" acting by induction and convection has been added, by means of which the jar can easily be kept at a nearly constant charge.

order of date, and from these entries the results shown in the subjoined Tables were computed.

Table I. shows the mean electrical potential for each day, omitting those days on which the number of blanks exceeded two.

Table II. shows the mean electrical potential for each of the twenty-four hours, month by month, and the last line and column contain the means of the other lines and columns respectively.

From the numbers in the body of this Table the curves (Plate XX.) have been drawn, the first twelve lines of numbers being represented by the continuous curves, and the next twelve lines by the dotted curves. These curves very clearly show a double maximum and minimum, the principal maximum occurring about 8 P.M. in autumn and winter, 9 P.M. in spring, and 10 P.M. in summer, and the secondary maximum about 8 A.M. in spring and summer and 9 A.M. in autumn. The principal minimum occurs at 4 A.M. in spring and summer and 5 A.M. in autumn and winter. The curvature is greater in the neighbourhood of the maxima than in the neighbourhood of the minima. The mean diurnal curve at Kew (Plate XXI.) has been drawn by projecting the numbers in the last line of the Table, the vertical scale adopted being twice as large as for the twenty-four curves belonging to single months. Above this is placed the diurnal barometric curve for Halle, drawn from data contained in KAEMTZ's 'Meteorology,' and below is placed the diurnal electrical curve for Windsor, N.S., obtained by taking the gross means of my own observations at each hour, observations from 2 to 6 A.M. being entirely wanting*. The electrical curves for the two places are remarkably dissimilar, both, however, having a maximum between 8 and 9 A.M.† The principal maximum at Kew occurs between 8 and 9 P.M., and the principal minimum between 4 and 5 A.M. The barometric curve for Halle bears a strong resemblance to the Kew electrical curve, but is upwards of an hour later in phase. The slight rise in the Kew curve at 11 A.M. is attributable to the fact that the Leyden jar was recharged at 10.30 A.M. By projecting the numbers in the last column of the Table, we obtain annual curves of electricity for two years. I have projected these so as to form one continuous curve, and along

* These means, together with the number of observations from which they are deduced, are as follows, in units of "station electrometer with second fibre."

Hour	17 to 19	19 to 20	20 to 21	21 to 22	22 to 23	23 to 0	0 to 1
Number of observations	7	114	227	103	43	30	32
Mean	3·17	3·93	4·51	4·23	3·32	3·29	3·90
Hour	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8
Number of observations	155	223	35	53	45	38	26
Mean	4·44	4·53	3·67	3·84	3·76	3·73	3·44
Hour	8 to 9	9 to 10	10 to 11	11 to 12	12 to 14		
Number of observations	34	272	68	32	8		
Mean	3·18	3·28	2·71	2·97	2·81		

† At Windsor, in every month without exception, electricity was weaker at 10 P.M. than either at 9 A.M. or 2 P.M., but the reverse of this rule would appear to hold at Kew.

with this I have projected the corresponding curve for Windsor, N.S. The Windsor observations commence four months later than those of Kew and terminate three months later, the time from October 1862 to May 1864 being common to both. In order to ensure a fair comparison, as I have no means of comparing the units in which the observations at the two places are stated, I have calculated the ratios of the several monthly means to the annual mean, and have projected these ratios.

Inspection of the curves for the two places shows that they agree pretty well from January to October, but take reverse directions from October to January, the Windsor curve having a decided minimum in November, which is about the time of the principal maximum at Kew. The annual range (as a fraction of the mean annual potential) appears to be greater for Kew than for Windsor. The following are the ratios thus plotted:—

Ratio of mean monthly to mean annual potential.

At Kew.				At Windsor, N.S.			
1862.		1863.		1862.		1863.	
June	·770	June	·681	October	·832	October	1·033
July	·773	July	·643	November	·766	November	·949
August	·836	August	·685	December	1·010	December	1·110
September	·845	September	·854	1863.		1864.	
October	·981	October	1·000	January	1·057	January	1·125
November	1·600	November	1·390	February	1·432	February	?
December	1·188	December	1·460	March	1·396	March	1·416
1863.		1864.		April	1·023	April	1·026
January	1·033	January	1·226	May	·796	May	·985
February	1·333	February	1·263	June	·720	June	·799
March	1·160	March	1·375	July	·755	July	·885
April	·920	April	·831	August	·952	August	·862
May	·672	May	·549	September	·985		

The final step in the reductions has been to express the variations of electrical potential approximately by harmonic series. Both the diurnal and the annual variations have been thus treated by calculating the values of the coefficients A_0, A_1, E_1, A_2, E_2 in the formula

$$A_0+A_1 \sin\left(\frac{t}{T}\,360°+E_1\right)+A_2 \sin\left(\frac{2t}{T}\,360°+E_2\right),$$

T denoting twenty-four hours in the case of diurnal, and a year in the case of annual variations; and t denoting the time reckoned from noon in the former case and from the middle of January in the latter.

The first step in this calculation consists in finding the values of P_1, Q_1, P_2, Q_2 which are connected with the above-mentioned coefficients by the relations

$$P_1=A_1 \sin E_1,\quad Q_1=A_1 \cos E_1,\quad P_2=A_2 \sin E_2,\quad Q_2=A_2 \cos E_2.$$

Commencing with the diurnal variations, we have the following values of the latter coefficients for the twenty-four months of observation.

	First year.				Second year.			
	P_1.	Q_1.	P_2.	Q_2.	P_1.	Q_1.	P_2.	Q_2.
June	−·139	+ ·185	+·002	−·241	−·218	+ ·114	−·099	−·328
July	−·050	+ ·053	+·164	−·456	−·230	− ·066	+·014	−·364
August	−·083	− ·011	+·029	−·507	−·029	+ ·203	−·009	−·298
September	−·284	+ ·409	−·002	−·490	−·255	+ ·378	−·115	−·467
October	−·053	+ ·835	−·216	−·315	+·284	+ ·586	−·642	−·310
November	−·264	+ ·384	−·036	−·214	+·109	+ ·509	−·166	−·371
December	+·028	+ ·767	−·058	−·563	+·297	+ ·957	−·017	−·449
January	+·006	+1·060	−·039	−·455	+·287	+1·271	−·160	−·297
February	−·051	+ ·743	−·033	−·582	−·089	+ ·630	−·236	−·561
March	−·119	+ ·008	−·144	−·516	−·211	+ ·441	−·127	−·565
April	−·402	+ ·056	−·141	−·535	−·304	+ ·311	−·141	−·440
May	−·279	+ ·006	−·025	−·322	−·381	+ ·044	+·046	−·195

From these we derive the following values of amplitude (A_1 and A_2) and epoch (E_1 and E_2), regarding the former as essentially positive.

	First year.				Second year.			
	A_1.	E_1.	A_2.	E_2.	A_1.	E_1.	A_2.	E_2.
June	·232	− 36° 53′	·241	+179° 24′	·246	− 62° 28′	·343	+196° 45′
July	·073	− 43 26	·484	160 15	·240	−105 53	·364	177 46
August	·084	− 97 46	·508	176 45	·205	− 8 5	·299	181 45
September	·498	− 34 47	·490	180 17	·456	− 34 2	·481	193 50
October	·837	− 3 33	·382	214 21	·651	+ 25 53	·313	187 46
November	·466	− 34 26	·218	189 39	·321	+ 12 3	·407	204 9
December	·768	+ 2 4	·566	185 55	1·003	+ 17 15	·450	182 8
January	1·060	+ 0 8	·457	184 39	1·303	+ 12 43	·337	208 19
February	·743	− 1 38	·583	183 14	·636	− 8 1	·609	202 51
March	·119	− 87 53	·536	195 37	·489	− 25 36	·580	192 41
April	·406	− 82 6	·553	194 45	·435	− 44 19	·462	197 46
May	·279	− 88 48	·323	184 30	·384	− 83 29	·201	166 36
Year	·400	− 20 36	·435	185 28	·452	− 7 50	·395	192 50

The value of A_0, or the mean electrical potential, is 2·14 for the first and 2·12 for the second year.

It will be observed that the values of A_1 and E_1 are subject to much greater irregularities than those of A_2 and E_2.

The values of P_1, Q_1, P_2, Q_2 for the two years combined can be correctly found by taking the means of their values for the two years, and the values of amplitude and epoch (which can *not* be correctly found by taking the means of their values for the two years) may be hence derived. The following Table has been thus computed.

Two years combined.

	A_1.	E_1.	Hour of maximum from E_1.	A_2.	E_2.	Hours of maxima from E_2.	
			h m			h m	h m
June	·232	−5° 4′	9 20	·292	+190° 3′	8 40	20 40
July	·140	−92 27	12 10	·395	166 40	9 27	21 27
August	·111	−30 15	8 1	·398	177 59	9 4	21 4
September	·477	−34 30	8 18	·483	187 1	8 46	20 46
October	·720	+ 9 16	5 23	·339	202 24	8 15	20 15
November	·454	− 9 46	6 39	·310	199 6	8 22	20 22
December	·877	+10 39	5 17	·507	184 14	8 52	20 52
January	1·174	+ 7 9	5 31	·389	194 45	8 30	20 30
February	·689	− 5 25	6 22	·587	193 18	8 33	20 33
March.....	·279	−36 15	8 25	·558	194 7	8 32	20 32
April	·398	−62 28	10 10	·508	196 9	8 28	20 28
May	·331	−85 40	11 43	·259	177 34	9 5	21 5
Year	·424	−13 50	6 55	·413	189 0	8 42	20 42

The term involving A_1 and E_1 takes one maximum in the twenty-four hours, whose times are given in the third column of the above Table. The term involving A_2 and E_2 takes two maxima which are always twelve hours apart. Their times are given in the last two columns. In deducing the hours of maxima from the values of E_1 and E_2, it is to be borne in mind that the phases are earlier in proportion as the epochs are greater, 15° in the value of E_1 and 30° in the value of E_2 corresponding respectively to differences of an hour. It will be observed that the earliest and latest hours of maxima differ by about seven hours in the case of E_1, and by only one hour twelve minutes in the case of E_2.

The values in the last line of the Table are derivable either from the last line of Table II. or from the means of the values of P_1, Q_1, P_2, Q_2. The amplitudes of the diurnal and semidiurnal terms are, it will be seen, nearly equal. Practically, the hours of electrical maxima month by month agree, within an hour or two, with those of the semidiurnal term, and the hourly values for the average of the year, given in the last line of Table II., show a still closer agreement. The diurnal term, without having much effect on the times of maxima, causes one maximum to be much greater than the other.

It may be interesting to inquire into the connexion, if any, between electrical and barometrical maxima, an inquiry which is suggested by the fact that the latter, like the former, occur twice in the twenty-four hours. In default of the necessary barometric data for Kew, I have compared the numbers in the last line of Table II. with the following numbers which represent the mean heights of the barometer at Halle for all hours on the average of the whole year, and are taken from Kaemtz's 'Meteorology,' page 248.

Barometric Heights at Halle (lat. 54° 29′), in millimetres,

750 plus the following numbers.

Noon.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
3·29	3·11	2·99	2·89	2·84	2·86	2·91	3·02	3·14	3·24	3·31	3·29
12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
3·23	3·14	3·05	2·99	2·99	3·34	3·12	3·24	3·37	3·44	3·46	3·40

The value of E_2 derived from these numbers gives maxima at $10^h\ 28^m$ and $22^h\ 28^m$, or an hour and forty-six minutes later than E_2 for Kew electricity. About the same amount of retardation can be roughly inferred either from inspection of the numbers themselves, or from comparison of the curve representing them with the Kew electrical curve (see Plate XXI.).

Thus far we have been speaking of diurnal variations. For annual variations, taking the numbers in the last column of Table II. as our data, we have the following results:—

For the first year,

$A_0=2{\cdot}14$, $A_1={\cdot}643$, $E_1=107°\ 56'$, $A_2={\cdot}080$, $E_2=257°\ 31'$;

for the second year,

$A_0=2{\cdot}12$, $A_1={\cdot}888$, $E_1=102°\ 38'$, $A_2={\cdot}030$, $E_2=358°\ 26'$;

and for the two years combined,

$A_0=2{\cdot}13$, $A_1={\cdot}765$, $E_1=104°\ 51'$, $A_2={\cdot}040$, $E_2=279°\ 19'$.

The corresponding values for Windsor, N.S., derived from the numbers given in the earlier part of this paper under the heading "mean of three preceding columns," are as follows, the two years being combined, except for the months of February, August, and September, which are taken from the first year's observations alone, from defect of complete observations in the second year.

$A_0=3{\cdot}92$, $A_1={\cdot}725$, $E_1=63°\ 43'$, $A_2=1{\cdot}014$, $E_2=354°\ 29'$.

In order to render these results for the two places more comparable, seeing that we have no direct means of comparing the units employed, we shall, as we have already done in the comparison of monthly means, take A_0 (or mean annual potential) as our unit at both places. We thus obtain the following values for the two years combined:—

Kew . . . $A_1={\cdot}359$, $E_1=104°\ 51'$, $A_2={\cdot}019$, $E_2=279°\ 19'$,

Windsor . . $A_1={\cdot}185$, $E_1=63°\ 43'$, $A_2={\cdot}259$, $E_2=354°\ 29'$.

This comparison brings out the astonishing fact that while at Kew the half-yearly term is almost inappreciable, at Windsor its amplitude is actually greater than that of the annual term. As regards phase, confining our attention to the annual term, Kew is earlier than Windsor by about forty-two days. The semiannual term at Kew is too small and too fluctuating to admit of reliable comparison.

It must be borne in mind that the Kew reductions include wet as well as fine weather, the only exclusions being those produced automatically by the spot of light passing out of range or moving too rapidly to leave a trace; whereas, in the Windsor reductions, all observations taken during rain, snow, hail, sleet, fog, thunder, or lightning were excluded. It is assumed, however, that this difference can go but a little way towards explaining the great differences which we have detected in the electrical variations at the two places.

I am happy to be able to state that preparations are now in progress for resuming the photographic registration of atmospheric electricity at Kew with new apparatus containing all Sir William Thomson's most recent improvements.

TABLE I.

	1862.							1863.											1864.				
Day.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May.
1	.....	·94	1·72	.....	.....	1·42	.....	2·86	1·80	2·19	2·27	2·67	1·84	.....	1·51	1·88	1·26	.....	1·96	.....	1·63	3·23	2·74	
2	.....	·60	1·01	.....	2·42	2·39	3·58	.....	3·25	3·86	2·30	.....	1·18	·74	1·74	.....	·82	·80	1·90	.....	.....	.....	2·02	
3	.....	1·69	.....	2·62	1·88	2·18	.....	2·25	1·70	2·64	2·72	1·50	·96	1·27	1·43	1·80	2·30	1·03	2·10	.....	.....	3·70		
4	.....	1·49	.....	1·89	1·84	2·94	1·05	.....	2·96	2·32	1·51	1·64	.....	.....	.....	1·86	1·26	2·10	1·87	.....	2·24			
5	.....	1·34	.....	1·16	.....	1·80	·83	.....	.....	1·84	1·40	1·38	·88	.....	.....	.....	1·56	3·04	2·58	.....	4·58	1·90	2·17	
6	.....	.....	1·36	1·02	.....	3·20	1·93	.....	3·02	.....	2·10	.....	.....	1·75	1·11	1·20	1·08	2·08	1·45	.....	.....	.....	3·22	
7	.....	2·11	.....	1·97	3·50	3·58	3·22	.....	2·77	2·65	1·72	1·97	.....	1·42	2·30	1·20	2·44	1·39	1·96	.....	.....	.....	2·29	
8	.....	1·00	1·84	1·41	3·91	2·74	1·62	2·70	2·85	.....	1·98	1·51	.....	1·62	.....	.....	.....	.....	2·59	.....	.....	.....	2·84	
9	1·08	2·12	1·22	1·66	.....	.....	3·81	.....	2·11	3·68	2·33	1·08	·82	.....	1·91	3·03	.....	.....	4·54	.....	.....	2·84	1·96	
10	1·50	1·15	1·71	.....	.....	3·91	3·12	.....	3·05	1·93	1·33	.....	.....	1·14	1·03	1·98	.....	5·29	1·69	3·80	.....	2·57	1·64	
11	·36	2·47	.....	1·90	.....	.....	3·34	.....	2·64	3·10	.....	·59	·69	.....	1·10	·98	.....	4·64	2·86	.....	2·53	2·57	2·06	
12	·64	1·57	1·32	1·48	2·03	.....	4·38	2·81	4·34	3·46	3·11	.....	1·13	·95	.....	·58	.....	3·16	2·75	.....	2·02	1·48	2·97	
13	.....	.....	.....	2·53	2·01	.....	4·61	4·18	.....	.....	2·91	·76	1·35	·97	.....	1·50	.....	4·55	3·67	2·23	2·86	1·79	3·98	
14	.....	1·55	.....	.....	1·67	2·90	3·56	3·18	5·18	1·99	1·80	1·46	1·49	.....	.....	1·93	2·39	2·42	3·24	2·63	2·18	1·85		
15	.....	.....	.....	.....	1·38	3·69	2·63	2·02	.....	2·46	1·86	1·62	1·31	·80	.....	1·86	2·82	2·09	2·80	2·63	2·76	3·25		
16	.....	1·64	.....	.....	1·71	5·17	4·09	1·56	.....	2·32	1·53	2·10	2·21	·68	.....	1·88	3·13	2·48	3·63	.....	4·48	4·70		
17	2·50	1·78	.....	1·95	2·76	3·42	1·15	1·22	.....	1·90	3·02	.....	2·42	1·24	.....	1·68	1·20	2·93	4·66	.....	4·66	5·09	1·66	
18	2·49	.....	.....	1·91	.....	.....	·94	1·38	.....	2·20	1·90	.....	2·42	·74	1·30	.....	1·51	3·52	4·20	.....	.....	5·23	2·26	
19	1·54	1·25	1·64	1·55	.....	.....	.....	1·32	.....	1·20	1·41	.....	1·72	.....	1·00	1·19	2·41	3·63	3·28	·90	.....	3·78	2·11	1·10
20	2·05	.....	.....	1·58	.....	2·82	2·08	1·32	3·26	2·35	1·13	2·36	1·17	.....	1·29	2·08	2·49	1·98	2·88	1·84	.....	3·38	1·19	1·38
21	2·19	1·12	1·78	2·80	.....	.....	2·72	1·90	2·43	.....	1·90	1·60	1·35	1·28	1·71	2·06	2·21	3·88	4·53	·94	.....	1·58	1·91	1·38
22	2·57	.....	2·43	.....	.....	.....	3·72	2·35	1·43	2·60	1·85	·98	1·43	1·22	2·74	.....	.....	2·87	4·09	3·10	.....	3·30	1·77	2·11
23	.....	.....	2·31	2·03	2·69	4·06	2·32	2·06	1·51	2·48	1·86	·76	1·08	1·26	1·78	.....	3·57	2·92	4·88	3·70	.....			
24	.....	2·03	2·89	2·01	1·80	2·33	2·23	1·98	3·53	2·02	1·54	1·44	1·37	.....	.....	1·89	.....	3·81	3·25	4·28	1·60	.....	·15	·73
25	.....	1·88	.....	1·58	2·56	3·40	2·27	1·23	2·07	.....	1·82	1·20	.....	1·04	·95	1·74	.....	2·48	1·44	3·07	1·80	.....	1·21	1·37
26	.....	2·15	2·52	.....	1·82	5·14	1·91	3·77	4·12	.....	·95	1·13	.....	.....	2·03	.....	2·39	3·59	4·37	2·52	2·22	.....	·81	·89
27	.....	1·74	1·90	.....	.....	3·22	·85	2·34	3·24	1·42	1·29	1·51	1·29	1·24	2·07	2·23	.....	5·02	2·75	.....	3·84	.....	·83	·78
28	.....	1·11	1·79	.....	1·68	.....	·98	1·72	1·80	1·60	.....	1·70	1·88	.....	2·16	3·69	.....	.....	2·64	.....	.....	.....	·96	1·26
29	.....	1·74	1·68	.....	.....	.....	.....	1·90	.....	1·80	.....	1·33	1·71	2·25	1·39	.....	.....	.....	.....	4·34	.....	3·41	·52	
30	·96	1·06	1·92	.....	1·17	6·22	.....	2·41	.....	3·50	.....	1·45	1·27	2·71	1·32	.....	1·82	.....	.....	.....	.....	1·66		·96
31	.....	1·18	.....	.....	.....	.....	·60	2·04	.....	3·54	.....	1·42	.....	2·01	1·30	.....	.....	.....	.....	2·23	.....	2·62	.....	

TABLE II.

	23h.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	Mean.
1862.																									
June	1·41	1·43	1·44	1·49	1·63	1·72	1·40	1·79	1·84	1·99	2·22	2·32	1·91	1·64	1·64	1·46	1·42	1·27	1·31	1·35	1·59	1·96	1·73	1·42	1·64
July	1·69	1·60	1·54	1·41	1·14	1·05	1·11	1·36	1·58	1·92	2·18	2·17	2·07	1·65	1·37	1·26	1·10	1·08	1·04	1·21	1·70	2·04	1·94	1·67	1·54
August	1·66	1·50	1·46	1·35	1·30	1·42	1·59	1·57	1·89	2·31	2·39	2·22	2·14	1·89	1·69	1·60	1·24	1·33	1·44	1·58	1·88	2·44	2·57	2·28	1·78
September	1·60	1·40	1·30	1·49	1·34	1·48	1·69	2·31	2·68	2·90	2·84	2·38	2·31	2·06	1·66	1·49	1·38	1·39	1·28	1·10	1·18	1·79	2·08	1·97	1·80
October	1·68	1·81	2·03	2·16	2·17	2·55	2·76	3·18	3·47	3·41	2·97	2·70	2·07	1·81	1·70	1·43	1·26	1·40	1·39	1·21	1·41	1·78	1·92	1·90	2·09
November	3·00	3·08	3·03	3·20	3·28	3·33	3·64	3·94	4·09	4·13	4·10	3·83	3·71	3·61	3·64	3·17	3·21	3·19	2·94	2·98	2·95	3·01	3·38	3·46	3·41
December	2·55	2·74	2·64	2·29	2·30	2·44	3·04	3·49	3·80	3·88	3·78	3·12	2·64	2·34	1·97	1·64	1·62	1·59	1·66	1·71	1·75	2·23	2·73	2·71	2·53
1863.																									
January	2·16	2·30	2·29	2·40	2·42	2·61	2·86	3·25	3·62	3·73	3·50	3·10	2·52	1·97	1·56	1·29	1·13	1·09	1·03	1·11	1·25	1·66	1·93	1·99	2·20
February	3·09	2·82	2·67	2·65	2·61	2·78	3·12	3·77	4·33	4·01	3·86	3·64	3·25	2·82	2·30	2·09	1·92	1·88	1·90	1·92	2·26	2·78	2·78	2·90	2·84
March	2·17	2·12	1·90	1·94	1·98	2·17	2·14	2·65	2·93	3·08	3·02	2·89	2·65	2·45	2·21	2·26	2·21	2·03	2·12	2·28	2·90	3·26	3·15	2·72	2·47
April	1·36	1·43	1·23	1·44	1·34	1·28	1·71	1·98	2·47	2·76	2·79	2·88	2·59	2·22	1·88	1·68	1·71	1·79	1·65	1·90	2·61	2·49	2·17	1·70	1·96
May	1·07	1·05	1·16	1·10	1·04	1·07	1·13	1·26	1·51	1·81	2·08	2·16	1·97	1·59	1·48	1·32	1·14	1·18	1·33	1·62	1·79	1·74	1·38	1·23	1·43
June	1·11	1·10	1·20	1·12	1·13	1·24	1·30	1·46	1·85	2·01	2·02	2·04	1·87	1·63	1·33	1·10	1·08	1·21	1·30	1·49	1·74	1·83	1·39	1·20	1·45
July	1·08	1·06	·92	·87	·95	1·06	1·07	1·18	1·38	1·63	1·85	1·91	1·88	1·67	1·40	1·34	1·24	1·13	1·05	1·29	1·71	1·98	1·74	1·38	1·37
August	1·50	1·54	1·42	1·27	1·27	1·41	1·56	1·56	1·78	1·90	1·95	1·92	1·69	1·50	1·28	1·13	1·00	·98	1·02	1·27	1·71	1·68	1·48	1·29	1·46
September	1·41	1·51	1·41	1·44	1·45	1·44	1·95	2·36	2·76	2·71	2·66	2·62	2·27	1·87	1·63	1·49	1·24	1·40	1·26	1·52	1·68	1·91	1·90	1·76	1·82
October	1·85	2·23	2·56	2·60	2·41	2·36	2·72	2·71	2·86	2·92	2·79	2·38	1·90	1·62	1·60	1·55	1·42	1·36	1·13	1·22	1·74	2·24	2·77	2·43	2·13
November	2·80	2·88	2·72	2·77	2·98	3·35	3·56	3·88	3·69	3·54	3·55	3·20	3·03	2·83	2·49	2·30	2·22	2·14	2·26	2·37	2·74	3·24	3·24	3·18	2·96
December	3·14	3·55	3·32	3·18	3·48	3·51	3·95	4·16	4·36	4·17	3·89	3·53	3·15	2·64	2·28	2·11	1·95	1·85	1·75	1·85	2·20	2·82	3·31	3·41	3·11
1864.																									
January	2·32	2·92	2·99	2·98	3·20	3·54	3·94	4·31	4·29	3·92	3·52	2·97	2·31	2·28	1·85	1·52	1·42	1·31	1·06	1·18	1·75	1·96	2·33	2·72	2·61
February	2·40	2·48	2·31	2·50	2·47	2·64	3·19	3·46	3·85	4·00	3·91	3·55	2·86	2·37	1·97	1·94	1·94	1·97	2·01	2·15	2·37	2·90	2·75	2·62	2·69
March	2·51	2·53	2·60	2·57	2·57	2·87	3·01	3·26	3·76	4·07	4·14	3·90	3·25	2·91	2·64	2·34	2·32	2·20	2·33	2·54	2·85	3·20	3·15	2·84	2·93
April	1·33	1·22	1·22	1·35	1·64	1·68	1·62	2·06	2·56	2·60	2·70	2·58	2·21	1·96	1·72	1·46	1·28	1·24	1·28	1·52	1·97	2·15	1·84	1·34	1·77
May	·92	·82	·72	·77	·81	·91	·94	1·07	1·38	1·60	1·72	1·68	1·76	1·56	1·39	1·42	1·36	1·13	·91	1·01	1·24	1·20	1·10	·93	1·17
Sums	45·81	47·12	46·08	46·34	46·91	49·91	55·00	62·02	68·73	71·00	70·43	65·69	58·01	50·89	44·68	40·39	37·91	37·05	36·45	39·38	46·97	54·29	54·76	51·05	51·16
Means	1·91	1·96	1·92	1·93	1·95	2·08	2·29	2·58	2·86	2·96	2·93	2·74	2·42	2·12	1·86	1·68	1·58	1·54	1·52	1·64	1·96	2·26	2·28	2·13	2·13

XII. *Account of Experiments on Torsion and Flexure for the Determination of Rigidities.*—Third Paper.

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These experiments are a continuation of those described in my papers read February 26th, 1866 and February 7th, 1867. They were made during the Winter Session of 1866–67, in the same place, and with the same apparatus as the later experiments on the steel rod (see p. 150 of last paper), the observer being Mr. Zaccheus Walker, student of Engineering, who has been already mentioned as joint observer in the experiments on the steel rod. I personally inspected the apparatus from time to time, and assisted in the taking of those measurements which only required to be made once for all in the case of each rod. The weights, in air and water, of the portions of rods operated on, were observed by Mr. John Tatlock, Laboratory Assistant to Sir William Thomson, who also performed this portion of the labour in the previous experiments.

The rods operated on in the experiments now to be described were three in number, and were of malleable iron, cast iron, and copper respectively, those previously operated on being of flint glass (two specimens), brass, and steel.

Experiments on Malleable Iron Rod.

The mean values of T (proportional to torsion) and F (proportional to flexure) were as follows, the pointer-readings indicating the amounts by which the rod had been rotated about its axis from a certain initial position.

Pointer-reading.	T.	F.	Pointer-reading.	T.	F.
30°	210·3	165·3	120°	210·7	165·0
60°	212·0	164·8	150°	210·1	165·0
90°	210·3	165·3	180°	210·8	165·2

Combining, as theory requires, those positions which are mutually at right angles, we have—

From 30° and 120° . . .	T=210·5	F=165·2
„ 60° „ 150° . . .	„ 211·0	„ 164·9
„ 90° „ 180° . . .	„ 210·6	„ 165·2

The cross-piece on whose arms the weights are hung, ought to have been turned through a right angle in passing from the position 90° to the position 120°, in order to

destroy the effect of inequality in the length of the arms. This operation was, however, omitted through oversight, and it is therefore necessary, in comparing the torsional and flexural rigidities, to correct for this difference.

The arm of couple, that is the distance SS' or TT' in fig. 1 of last paper, was 55·71 centims. for torsion and 55·83 for flexure.

Also the units in which the numbers T and F are expressed, being tenths of scale-divisions perpendicular and parallel to the rod respectively, are slightly different, being $\frac{23 \cdot 88}{1710}$ and $\frac{23 \cdot 97}{1710}$ centims. for torsion and flexure respectively.

The corrections for these two inequalities are in opposite directions. Their resultant is a correction of $\cdot 0016 \frac{T}{F}$ to be subtracted.

The values of $\frac{T}{F}$ thus corrected are—

From 30°	and 120°	1·272,
„ 60°	„ 150°	1·278,
„ 90°	„ 180°	1·273,

which when diminished by unity are determinations of σ or Poisson's ratio, subject to two other corrections, which we now proceed to calculate.

For the mechanical correction the data are—

Torsion in portion of rod between mirrors	·00661,
Flexure in portion of rod between mirrors	·00519,

these numbers (which are in circular measure) being obtained by dividing T and F respectively reduced to centimetres by 445·6, which is twice the height of scale above mirrors. They must now be multiplied by $\frac{452}{944}$, which is the ratio of all that portion of the rod subjected to torsion and flexure to the portion between mirrors, and also by the constant number ·729 (see last paper, p. 143). The products thus obtained are ·0074 and ·0058; hence the mechanical corrections of T and F are

$$+\cdot 0074\ T \text{ and } +\cdot 0058\ F.$$

The optical correction (which I am told was too briefly described in my last paper) is dependent on the fact that the ray from scale to mirror is not precisely vertical, and therefore not truly perpendicular to the scale, which is horizontal. It is determined by measuring—

(1) Height of scale above mirrors.

(2) Distance of vertical through centre of scale from line joining centres of mirrors.

(3) Distances, resolved parallel to rod, of vertical through centre of scale from centre of each mirror.

These three distances suffice to determine the corrections both for torsion and flexure, on the hypothesis that the centre of the scale comes into the field of view of both telescopes in every observation, an hypothesis which, though not strictly fulfilled, gives a fair approximation to the actual obliquities.

Let β (as in last paper) denote the quotient of one of the distances (2) or (3) according as we are dealing with torsion or flexure, by the distance (1). Then since β is the tangent of a small angle (which call B), we have

$$\delta \tan B = (1+\tan^2 B)\delta B = (1+\beta^2)\delta B,$$

and therefore the angle turned by mirror (δB) is found by dividing the observed deflection ($\delta \tan B$) by $1+\beta^2$, or, what amounts to the same thing, multiplying it by $1-\beta^2$.

Let B_1, β_1 and B_2, β_2 be the values of B, β for the nearer and further mirrors respectively; then $\delta \tan B_2 - \delta \tan B_1 = (1+\beta_2^2)\delta B_2 - (1+\beta_1^2)\delta B_1$, which may be shown to be equal to

$$(\delta B_2 - \delta B_1)\left(1+\frac{m_2\beta_2^2 - m_1\beta_1^2}{m_2 - m_1}\right),$$

m_2 and m_1 being any two numbers which have the ratio $\delta \tan B_2 : \delta \tan B_1$. The observed values of T and F are proportional to $\delta \tan B_2 - \delta \tan B_1$, and must therefore be multiplied by the correcting factor $1-\frac{m_2\beta_2^2 - m_1\beta_1^2}{m_2 - m_1}$, which, when β_1 and β_2 are equal, becomes simply $1-\beta^2$.

In the present series of experiments β_1 and β_2 were sensibly equal, their common value being about ·069 for torsion and ·064 for flexure. By squaring these numbers we obtain the corrections

$$-\cdot0047\ \mathrm{T} \text{ and } -\cdot0041\ \mathrm{F},$$

which are applicable not only to the experiments on the malleable iron rod, but also to those on the cast-iron and copper rods.

Adding together the mechanical and optical corrections, we obtain the total corrections

$$+\cdot0027\ \mathrm{T} \text{ and } +\cdot0017\ \mathrm{F}.$$

The resulting correction for $\frac{\mathrm{T}}{\mathrm{F}}$ is $+\cdot001\frac{\mathrm{T}}{\mathrm{F}}$, which applied to the values 1·272, 1·278, 1·273 gives, after subtracting unity, ·273, ·279, ·274 as the corrected determinations of the value of σ, and we adopt the mean of these, which is ·275.

We proceed to calculate the torsional and flexural rigidities t and f.

The mean values of T and F, uncorrected, are respectively 210·7 and 165·1, which, when corrected as above, become 211·3 and 165·4.

Now we have

$$t = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{1710}{23\cdot88} \div \mathrm{T},$$
$$f = \text{twice distance} \times \text{force} \times \text{arm} \times \text{length} \times \tfrac{1710}{25\cdot97} \div \mathrm{F},$$

where, in centimetres and grammes weight, we have twice distance=445·6, force=100, length=29·45, arm=55·71 for torsion and 55·83 for flexure.

Hence

$$\log t = 9\cdot71893 - \log \mathrm{T} - 7\cdot39403,$$
$$\log f = 9\cdot71823 - \log \mathrm{F} - 7\cdot49969.$$

The rod having been cut at the places where the mirrors were attached, the central

piece was weighed in air and water, the weights found being 100·485 and 87·400 grammes. The temperature of the water was 10·4 Cent., at which temperature the expansion of water is 1·0003. From these data the volume of the rod in centimetres is found to be

$$13{\cdot}085 \times 1{\cdot}0003 = 13{\cdot}089,$$

and the specific gravity $100{\cdot}485 \div 13{\cdot}089 = 7{\cdot}6771$. Dividing the volume by the length, which was 29·27, we find the sectional area $\pi r^2 = {\cdot}44566$, $r = {\cdot}37664$. Hence we have

$$\text{M (Young's modulus)} = \frac{4f}{\pi r^4} = 1{,}999{,}400{,}000,$$

$$n \text{ (Rigidity)} = \frac{2l}{\pi r^4} = 783{,}790{,}000,$$

$$k \text{ (Incompressibility)} = \frac{\text{M}n}{3(3n - \text{M})} = 1{,}484{,}100{,}000,$$

$$\sigma \text{ (Poisson's ratio)} = \frac{\text{M}}{2n} - 1 = {\cdot}275 \text{ as above.}$$

Experiments on Cast-Iron Rod.

These experiments gave the following values for T and F:—

Pointer-reading.	T.	F.	Pointer-reading.	T.	F.
0°	311·2	245·8	90°	311·8	245·2
30°	310·7	245·3	120°	310·8	245·0
60°	311·8	244·7	150°	311·7	245·3

Combining positions at right angles, we have—

From 0° and 90° . .	T = 311·5	F = 245·5
„ 30° „ 120° . .	„ 310·8	„ 245·2
„ 60° „ 150° . .	„ 311·7	„ 245·0

Here no correction is required for difference of arms, as the cross-piece was turned through a right angle in passing from the third to the fourth position.

The values of $\frac{\text{T}}{\text{F}}$, corrected for difference of scale-divisions by subtracting one part in 266, are 1·264, 1·263, and 1·267, which are first approximations to $1 + \sigma$.

To find the mechanical correction, we have length between mirrors 28·69, length subjected to torsion and flexure 43·5, height of scale above mirrors 222·8, as before. From these data, together with the above values of T and F, we have—

Torsion in portion of rod between mirrors	·0098
Flexure „ „ „ „	·0077

which numbers being multiplied by $\frac{43{\cdot}5}{28{\cdot}69} \times {\cdot}729$, give ·0108 and ·0085. Hence the mechanical corrections are

$$+{\cdot}0108\,\text{T} \text{ and } +{\cdot}0085\,\text{F};$$

and the optical corrections being the same as before, the total corrections are

$$+{\cdot}0061\,\text{T} \text{ and } +{\cdot}0044\,\text{F}.$$

The resulting correction for $\frac{T}{F}$ is $+\cdot0017\ \frac{T}{F}$, the application of which gives as the corrected determinations of σ, ·266, ·265, and ·269, and we adopt the mean ·267.

The mean uncorrected values of T and F are 311·3 and 245·2, which, when corrected as above, become 313·2 and 246·3.

For the torsional and flexural rigidities, we have

$$t=445\cdot6\times100\times55\cdot77\times28\cdot69\times\tfrac{1710}{93\cdot85}\div T,$$

$$f=445\cdot6\times100\times55\cdot77\times28\cdot69\times\tfrac{1710}{93\cdot97}\div F.$$

Hence

$$\log t=9\cdot70805-\log T=7\cdot21223,$$

$$\log f=9\cdot70641-\log F=7\cdot31495.$$

The rod having been cut at the places where the mirrors were attached, the middle piece was found to weigh 90·110 in air, and 77·658 in water at temperature 9°·9 C. The expansion of water at this temperature being 1·00026, we have for the volume of the piece

$$12\cdot452\times1\cdot00026=12\cdot455,$$

and for the specific gravity,

$$90\cdot110\div12\cdot455=7\cdot2347.$$

The volume divided by the length, which was 28·66, gives $\pi r^2=\cdot43458$, $r=\cdot37193$.

Hence we have

$$M=1{,}374{,}100{,}000,$$
$$n=\ 542{,}340{,}000,$$
$$k=\ 982{,}180{,}000,$$
$$\sigma=\ \cdot267.$$

Experiments on Copper Rod.

These experiments gave the following values for T and F:—

Pointer-reading.	T.	F.	Pointer-reading.	T.	F.
0°	365·0	265·0	90°	365·8	264·8
30°	365·2	265·0	120°	364·7	265·0
60°	365·8	264·7	150°	365·7	265·2

Combining positions at right angles, we have—

From 0° and 90° . .	T=365·4	F=264·9
„ 30° „ 120° . .	„ 365·0	„ 265·0
„ 60° „ 150° . .	„ 365·7	„ 264·9

The values of $\frac{T}{F}$ corrected for difference of scale-divisions, are 1·374, 1·372, and 1·375.

To find the mechanical correction, we have length between mirrors 28·72, length subject to torsion and flexure 45·5, height of scale 222·8.

Hence we find

Torsion in portion of rod between mirrors	.	.	.	·0114,
Flexure „ „ „ „	.	.	.	·0083,

which numbers being multiplied by $\frac{445}{365} \times ·729$ give ·0132 and ·0096. Hence the mechanical corrections are

$$+·0132\,\mathrm{T} \text{ and } +·0096\,\mathrm{F},$$

which being added to the optical corrections already specified, give the total corections

$$+·0085\,\mathrm{T} \text{ and } +·0055\,\mathrm{F},$$

and a resultant correction $+·003\,\frac{\mathrm{T}}{\mathrm{F}}$. The application of this correction gives as the corrected determinations of σ, ·378, ·376, and ·379, and we adopt the mean ·378.

The corrected values of T and F are 368·5 and 266·4.

For the torsional and flexural rigidities, we have

$$t = 445·6 \times 100 \times 55·77 \times 28·72 \times \tfrac{1710}{33·88} \div \mathrm{T},$$

$$f = 445·6 \times 100 \times 55·77 \times 28·72 \times \tfrac{1710}{33·97} \div \mathrm{F}.$$

Hence

$$\log t = 9·70850 - \log \mathrm{T} = 7·14206,$$

$$\log f = 9·70686 - \log \mathrm{F} = 7·28133.$$

The rod having been cut at the places where the mirrors were attached, the middle piece was found to weigh 110·645 in air, and 98·136 in water at temperature 10° C. The expansion of water at this temperature being 1·00026, we have for the volume of the piece

$$12·502 \times 1·00026 = 12·5122;$$

and for its specific gravity,

$$110·645 \div 12·5122 = 8·84293.$$

The volume divided by the length, which was 28·61, gives $\pi r^2 = ·43733$, $r = ·37310$.

Hence

$$\mathrm{M} = 1{,}255{,}800{,}000,$$

$$n = 455{,}640{,}000,$$

$$k = 1{,}716{,}400{,}000,$$

$$\sigma = ·378,$$

the units of length and force being the centimetre and gramme weight.

The values of n obtained by Sir W. Thomson for brass and copper from observations on the torsional vibrations of wires* were, in millions of grammes weight per square centimetre,

Brass, three specimens	410·3	354·8	350·1
Copper, two specimens	448·7	448·4.	

Other specimens of copper in abnormal states gave results ranging from 393·4 to 472·9.

* Proceedings, Royal Society, May 18, 1865.

The following Table contains the collected results of all my experiments, the figures I., II., III. indicating the paper in which the experiments are described and the results deduced. The values of M, n, and k are given in millions of grammes weight per square centimetre.

	M (Young's modulus).	n (Rigidity).	k (Resistance to compression).	σ (Poisson's ratio).	Specific gravity.
Glass, flint (I.)	614·3	244·2	423·0	·258	2·942
„ „ (II.)	585·1	239·0	353·3	·229	2·935
Brass, drawn (II.) ..	1094·8	372·9	5701 (?)	·469	8·471
Steel (II.)	2179·3	834·1	1875·6	·310	7·849
Iron, wrought (III.)	1999·4	783·8	1484·1	·275	7·677
„ cast (III.)....	1374·1	542·3	982·2	·267	7·235
Copper (III.)	1255·8	455·6	1716·4	·378	8·843

The values of M and n are derived directly from the observed amounts of flexure and torsion respectively, and their probable errors (estimated as percentages) may be considered equal.

Again, the comparison between flexure and torsion is so direct (with the exception of the first set of results for glass), that the probable error of $\frac{M}{2n}$ may be considered about equal to that of M or n.

The values of σ and k are deduced on the hypothesis that the substances operated on are isotropic; and on this hypothesis, if e denote the probable error of $\frac{M}{2n}$, the probable error of σ is $\frac{1+\sigma}{\sigma}e$, and that of k is $\frac{1+\sigma}{\frac{1}{2}-\sigma}e$ nearly.

For all the substances in the Table except brass, the value of the coefficient $\frac{1+\sigma}{\frac{1}{2}-\sigma}$ is from 5 to 11, but for brass it is 47; hence the enormous value of k found for brass cannot be depended on.

The tendency of anything like fibrous structure in the rods operated on, the fibres being supposed to run in the direction of the length, is to make σ and k come out too large. This follows from the theoretical considerations adduced on page 152 of my last paper, and I have verified it by experiments on a rod of wood which gave values of T four or five times greater than those of F, whereas the ratio $\frac{T}{F}$ for isotropic substances must always lie between 1 and 1·5.

As the intention was expressed in my last paper of introducing a modification of the apparatus with the view of diminishing the mechanical correction, I should state by way of explanation that the experiments here described were nearly completed before that intention was expressed.

XIII. *Contributions to Terrestrial Magnetism.—No.* XI.

By Lieut.-General EDWARD SABINE, *R.A., President of the Royal Society.*

Received and Read June 18, 1868.

THE object of the present (*i. e.* the Eleventh) Number of the Contributions to Terrestrial Magnetism is the completion of the great national undertaking, the Magnetic Survey of the South Polar Regions of the Globe, corresponding to the Epoch 1840–1845.

The Survey originated in a Report presented to the British Association for the Advancement of Science at the Liverpool Meeting in 1837, entitled "On the Variations of the Intensity of the Earth's Magnetic Force observed at different points of the Earth's Surface:" copies of this Report were widely circulated amongst the Members of the Association previously to the Meeting at Newcastle in the following year, 1838; and having received a favourable notice in the opening address of the then Secretaries of the Association, Dr. GEORGE PEACOCK and Sir RODERICK MURCHISON, resolutions were passed by the General Committee, which are printed in pages xxi and xxii of the "Annual Report of the Proceedings at Newcastle in August 1838." These resolutions having been formally communicated to the Royal Society, a joint committee of the two scientific institutions was appointed to bring the subject of the equipment of a naval expedition for magnetic observations in the Arctic Seas under the consideration of Her Majesty's Government. A single sentence from the address of this Committee may be cited as evidencing the spirit in which the joint application of the Royal Society and of the British Association was made to Her Majesty's Government.

"The Committee consider the subject of Terrestrial Magnetism to have now attained a degree of theoretical as well as of practical importance, and to afford a scope for the application of exact inquiry, which it has never before enjoyed, and which are such as fully to justify its recommendation to the revival of that national support to which we are indebted for the first Chart of the Declinations, constructed by our illustrious countryman HALLEY in 1701, on the basis of observations collected in a voyage expressly equipped for that purpose by the British Government"*.

The Report of the Committee thus prepared and presented to Her Majesty's Government (of which Lord MELBOURNE was at the time the Prime Minister) dwelt in some detail on the principal objects to be accomplished by the expedition recommended. The following passage may be especially cited as evidencing the early and just appreciation of the leading desiderata in magnetical science, and as furnishing to the readers of this paper the opportunity of judging of the degree in which the anticipations therein expressed have been realized.

* Proceedings of the Royal Society, December 22, 1838.

"The intensity lines in the southern regions rest on observations far too few to justify any sure reliance on their courses over a large part of their extent, and over the rest are altogether conjectural. Nevertheless there is good reason to believe in the existence and accessibility of two points of maximum intensity in the southern as in the northern hemisphere, the attainment of which would be highly interesting and important. A correct knowledge of the courses of these lines, especially when they approach their respective poles, is to be regarded as a first, and indeed indispensable preliminary step to the construction of a rigorous and complete theory of terrestrial magnetism."

Two ships, the 'Erebus' and 'Terror,' commanded respectively by Sir James Clark Ross and Captain Francis Rawdon Crozier, who, besides their other qualifications, were already favourably known for their magnetic observations in the course of several north polar voyages, were selected for this service. The magnetical instruments to be employed were prepared under my own direction, at an establishment then existing at Woolwich, but since transferred in part to the Physical Observatory at Kew. It was a most fortunate incident that instruments specially designed for the observation of the Dip and Intensity *at sea* had been devised a few years previously by Mr. Robert Were Fox, F.R.S., as with the instruments previously employed for these purposes the results would scarcely have been such as to have justified the undertaking.

The 'Erebus' and 'Terror' sailed from the Thames in September 1839, arrived at the Cape of Good Hope in March 1840, and, after a sojourn of some days in Kerguelen Island, anchored at Hobarton in Tasmania, which was selected as one of the base-stations of the Survey, and where a Magnetic Observatory was established. Quitting Hobarton in November 1840, the two ships made good a nearly south course, surmounting the difficulties occasioned by the ice, and carefully observing the Magnetic Declination, Dip, and Force in every twenty-four hours with very few exceptions, until, in the latitude of 70°, the discovery was made of the great Southern Continent of South Victoria; and its coast was followed and examined until further progress towards the south was arrested, in a latitude little short of 80°, by a vast glacier extending in an east and west direction for about 30° of longitude. In returning to Hobarton on the approach of the southern winter, a route was chosen which led to the vicinity which had been named to Sir James Ross as the probable locality of a higher intensity than would be found in the region of the higher dips, and as the approximate locality of the second southern magnetic pole. The expedition arrived at Hobarton in April 1841, having completed the first year of the Survey, and having sustained no injury either to the ships or to the instruments.

Quitting Hobarton in the following month (May 1841), it was Sir James Ross's purpose to penetrate a second time to the southward in the large interval of longitude comprised between the great glacier which had arrested their southern progress in the preceding year and the land named South Shetland, to the south of Cape Horn. This endeavour was frustrated by the ice, which admitted a progress through it between the

latitudes of 60° and 70° only by a continual inclination towards the west, which at length conducted them in 70° of latitude to the vicinity of their previous explorations and of the great glacier which had barred their southerly progress in the preceding year. After revisiting some of the geographical positions in which the magnetic observations of the preceding year had been made, and thus verifying their earlier results, the ships returned to the more navigable parallels of 59° and 60°, and, traversing the South Pacific in about the latitude of 60°, rounded Cape Horn and arrived at Port Louis in the Falkland Islands in August 1842. Here the ships were refitted, and, sailing thence in September 1842, passed some days at St. Martin's Cove near Cape Horn, where very careful examinations and comparisons of the magnetic instruments were made. Returning to Port Louis, and sailing thence on the 17th of December 1842, the ships resumed their eastern progress, continuing the habit of daily observation of the three magnetic elements, and availing themselves of every occasion which presented itself of attaining a higher south latitude. One notable opportunity occurred towards the end of February and beginning of March, when the 71st parallel was reached, but with great difficulty and considerable peril.

On the 4th of April 1843 the ships arrived at Simon's Bay in the vicinity of the Cape of Good Hope, thus completing the third year of the Survey, as well as the circumnavigation of the globe.

It was the practice of Sir James Ross and Captain Crozier on all convenient occasions during the Survey to transmit the successive records of their observations to the Admiralty, by whom on their arrival they were sent to me for examination and reduction, pursuant to an arrangement between the Departments of the Admiralty and of the Ordnance, in which provision had been made for those purposes in an office under my superintendence at Woolwich. These original records have been carefully preserved, and will be deposited for all future reference (if required) in the Archives of the Royal Society. No time was lost, on the reception of these documents, in making such a provisional calculation of the results as circumstances would then permit, and in communicating both the observations themselves and the results to the Royal Society. The instruments employed being for the most part of novel design, and the conduct and execution of a magnetic survey *at sea* being scarcely less so (especially of a survey which should include the *three* magnetic elements), the provisional character of these communications was from the first distinctly recognized, and a final publication, such as the present purports to be, was throughout contemplated. With this understanding the observations of the first year of the Antarctic Survey, with their results provisionally computed, were communicated to the Royal Society in 1843, and printed in the Philosophical Transactions for that year, forming the V.th Number of these Contributions; those of the second year in 1844, forming the VI.th Number; and those of the third year in 1866, forming the X.th Number of the Contributions to Terrestrial Magnetism.

The success with which such Surveys could be prosecuted at sea having been shown by the first two of the publications thus referred to, the Royal Society was emboldened to

propose to Her Majesty's Government, in 1844, the employment of a third ship, supplied with instruments of a similar construction, in a portion of the high southern latitudes comprised between the meridians of 0° and 125° E., which was not comprehended in the voyages of the 'Erebus' and 'Terror.' The Admiralty having acceded to this proposition, the 'Pagoda,' a barque of 300 tons, was hired for the purpose by the Admiral Commanding at the Cape of Good Hope, and, having been duly strengthened for ice-navigation, was fitted for a voyage of some months' duration. Two officers were employed conjointly in the direction of this portion of the Survey, Lieut. (since Rear-Admiral) J. E. L. Moore and Lieut. (since Colonel) Henry Clerk of the Royal Artillery. Lieut. Moore had been one of the officers of the 'Terror' in Sir James Ross's expedition, and was consequently accustomed to the navigation of the high latitudes, as well as practised in magnetic observations, having taken a very prominent share in those of the 'Terror.' Lieut. Clerk had been attached by Lord Vivian, Master-General of the Ordnance, to the Magnetic Observatory at the Cape of Good Hope, with the express view of being employed in a Magnetic Survey, either of the Colony itself, or of such part of the globe as might be conveniently accessible from it, and, in the passage from England to the Cape, had had an opportunity of practising with the instruments employed in a magnetic survey conducted on the ocean. The 'Pagoda' completed her survey between the 60th and 70th parallels, and between the 4th and 96th meridians of East longitude, returning to the Cape by South Australia and Mauritius. The results were communicated to the Royal Society in the same provisional form with those of the 'Erebus' and 'Terror' in the VIII.th Number of these Contributions, in which Number were also included the determinations of the Inclination and Intensity made at sea by Lieut. Alexander Smith, R.N., and by Lieut. Joseph Dayman, R.N. (officers employed at the Magnetic Observatory at Hobarton), on passages from Hobarton to the Cape of Good Hope, in which they were provided with instruments similar to those employed by Sir James Ross.

One of the chief difficulties anticipated in these undertakings arose from the circumstance that magnetic determinations made at sea are necessarily subject to the disturbing influence of the iron which cannot be wholly dispensed with in the equipment of a ship fitted for general navigation, and which, even when reduced to its smallest practicable amount, and kept as far as may be possible at a distance from the magnetic instruments, still exercises a disturbing influence which in the high latitudes becomes excessive, and requires to be met by appropriate corrections. The disturbing influence of the ship's iron on the pointing of the compasses had attracted the notice of some of the most careful marine surveyors towards the close of the last century, and had been met by empirical rules of inconsiderable extent and partial application—when in 1818 the 'Isabella' and 'Alexander,' ships of a class corresponding in many respects to the 'Erebus' and 'Terror,' were employed in the first expedition to the Arctic Polar Regions, and were led by the objects of the expedition into localities where the navigation had to be conducted in magnetic Inclinations exceeding 85°, when the superior influence of

the ship's iron so far prevailed over the directive influence of the earth (upon the horizontal needles), that it was obvious that we had nearly attained the limit within which the compass, as it had been previously employed, could be available in navigation. In a paper presented to the Royal Society on the return of the expedition to England, and printed in the Philosophical Transactions for 1819, Art. XVI., I have related these particulars, together with the partial remedies which suggested themselves at the time and on the spot, and which were so far practically successful that, both in that voyage and in the next (viz. in 1819 and 1820, when Barrow's Straits were passed and still higher magnetic inclinations were encountered), we were still enabled to use the ship's compasses in navigation, and in some degree also as aids in fixing geographical positions by compass bearings, until the inclination became so great that the directive force of the earth on the horizontal needle ceased to be appreciable, and the compass pointed uniformly to the general resultant of the ship's attraction, whatever might be the direction of the ship's head at the time.

In 1824 M. Poisson communicated two Memoirs to the French Institute, in the first of which he propounded a mathematical theory of transient induced magnetism founded on the physical theory of Coulomb, that by induction each particle of soft iron becomes a magnet, having an intensity proportional to that of all the forces which act on it, including the force of the magnetism developed by induction in all the other particles of the mass. In a subsequent Memoir, published in 1839, "Sur les Déviations de la Boussole produites par le fer des vaisseaux," M. Poisson adapted his formulæ to observations made on shipboard in the particular case of the soft iron being symmetrically distributed on either side of the principal section of the ship. The memoir was accompanied by a practical application and verification of the theory, showing the accordance of the calculated results with the facts recorded in the Arctic voyages of 1818, 1819, and 1820, spoken of above. A careful examination of the disturbances of the needle in the 'Erebus' and 'Terror' in their passage from England to the Cape of Good Hope in 1839, showed that in both ships they were occasioned chiefly, if not wholly, by the magnetism induced in the iron of the fittings and equipment by the vertical part of the earth's force, and which was distributed symmetrically on either side of the fore-and-aft vertical section passing through the compass. In the case of these ships, therefore, M. Poisson's method of analysis was strictly applicable; and its practical application was greatly facilitated by a memorandum drawn up by Mr. Archibald Smith, F.R.S., with which he obligingly supplied me, and which was printed in the provisional discussion of the observations of Sir James Ross's expedition in the first year of the Survey, in the V.th Number of these Contributions (Philosophical Transactions, 1843, Art. VIII.),—and by a supplementary memorandum, printed in the VIII.th Number of the Contributions (Philosophical Transactions, 1846, Art. XVIII.). Mr. Smith has now furnished me with a third memorandum, which I subjoin.

Formulæ for the correction of observations of Dip and Total Intensity made in Wood-built Ships.

In the Memorandum printed in the Contributions to Terrestrial Magnetism, No. V., Philosophical Transactions, 1843, p. 147, the following expressions will be found which are immediately derived from the formulæ given by POISSON in the Memoirs of the Institute, vol. v. p. 533.

$$\frac{\varphi'}{A'\varphi}\cos\theta'\cos\zeta' = \cos\theta\cos\zeta + a\sin\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{\varphi'}{A'\varphi}\cos\theta'\sin\zeta' = b\cos\theta\sin\zeta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{\varphi'}{A'\varphi}\sin\theta' = c\cos\theta\cos\zeta + d\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In these expressions

φ is the total Magnetic Intensity expressed in any unit.

θ is the Dip.

ζ is the magnetic azimuth of the ship's head.

φ', θ', ζ' are the same quantities affected by the induced magnetism of the soft iron in the ship.

A', a, b, c, d are coefficients depending on the amount and distribution of the soft iron.

These expressions are based on the assumptions that all the iron of the ship is (magnetically) *soft*, and that it is symmetrically arranged on each side of the fore-and-aft section. These assumptions are nearly true in ships such as the 'Erebus' and 'Terror.'

In the same memorandum, p. 148, expressions are given for θ' and φ' in terms of θ, φ, ζ, ζ', from which the coefficients may be determined if we have a sufficient number of corresponding observations; and from them Tables of double entry may then be constructed, giving the corrections to be applied to the observed values θ' and φ' in any required dip. These formulæ are *exact*, and may be used whatever be the amount of disturbing force, but are not very conveniently adapted for calculation.

When, as in the 'Erebus' and 'Terror,' the disturbing force is small, we may treat the errors of Dip and Force as small quantities of which the squares and products may be neglected, and we may then obtain the errors of dip and force by simple expressions of the form

$$\partial\theta = \mathrm{L} + \mathrm{M}\cos\zeta + \mathrm{N}\cos 2\zeta,$$

$$\frac{\partial\varphi}{\varphi} = \mathrm{P} + \mathrm{Q}\cos\zeta + \mathrm{R}\sin 2\zeta.$$

If the observations of $\partial\theta$ and $\frac{\partial\varphi}{\varphi}$ are made on any number of equidistant magnetic points exceeding two, the coefficients L, M, N, P, Q, R may be obtained with great facility

by applying the method of least squares, as in the Memorandum in Contribution No. VIII., Philosophical Transactions, 1846, p. 346, and as in the 'Admiralty Manual for the Deviation of the Compass.'

In this way, from observations made on sixteen points in the 'Erebus' at Hobarton in 1840–1841, we obtain

$$\delta\theta = -2' - 66' \cos\zeta + 6' \cos 2\zeta,$$

$$\frac{\delta\varphi}{\varphi} = +\cdot0053 - \cdot0146 \cos\zeta + \cdot0009 \cos 2\zeta.$$

These formulæ give the error, and therefore, changing the sign, the correction of the Dip and Total Intensity on any magnetic azimuth at the place of observation.

If we wish to know how the coefficients L, M, N, P, Q, R are affected by a change of dip, we may proceed as follows.

From equations (1), (2), (3) we obtain

$$\frac{\varphi' \cos\theta' \cos\zeta' - \varphi \cos\theta \cos\zeta}{\varphi \cos\theta} = A'a \tan\theta + (A'-1)\cos\zeta, \quad \ldots\ldots (4)$$

$$\frac{\varphi' \cos\theta' \sin\zeta' - \varphi \cos\theta \sin\zeta}{\varphi \cos\theta} = (A'b-1)\sin\zeta, \quad \ldots\ldots (5)$$

$$\frac{\varphi' \sin\theta' - \varphi \sin\theta}{\varphi \sin\theta} = A'd - 1 + A'c \cot\theta \cos\zeta. \quad \ldots\ldots (6)$$

Let x, y, z be the components of φ in the three rectangular directions, to head, to starboard, and to nadir, so that $x = \varphi\cos\theta\cos\zeta$, $y = \varphi\cos\theta\sin\zeta$, $z = \varphi\sin\theta$, and let x', y', z' be the same quantities affected by the magnetism of the ship, and let h be the horizontal force $= \sqrt{x^2+y^2} = \varphi\cos\theta$, and let $x'-x = \delta x$, $y'-y = \delta y$, $z'-z = \delta z$, then equations (4), (5), (6) become

$$\frac{\delta x}{h} = A'a\tan\theta + (A'-1)\cos\zeta,$$

$$\frac{\delta y}{h} = (A'b-1)\sin\zeta,$$

$$\frac{\delta z}{z} = A'd - 1 + A'c\cot\theta\cos\zeta;$$

and as $\tan\theta = \frac{z}{h}$,

$$\delta\theta = \frac{h\delta z - z\delta h}{\varphi^2}$$

$$= \frac{hz}{\varphi^2}\left\{\frac{\delta z}{z} - \frac{\delta h}{h}\right\},$$

$$= \sin\theta\cos\theta\left\{\frac{\delta z}{z} - \frac{\delta x}{h}\cos\zeta - \frac{\delta y}{h}\sin\zeta\right\}$$

$$= L + M\cos\zeta + N\cos 2\zeta,$$

where

$$L = +\tfrac{1}{2}A'\left(d - \frac{1+b}{2}\right)\sin 2\theta,$$

$$M=+\tfrac{1}{2}A'\{c-a+(c+a)\cos 2\vartheta\},$$

$$N=-\tfrac{1}{2}A'\frac{1-b}{2}\sin 2\vartheta.$$

Also

$$\varphi^2=x^2+y^2+z^2,$$

$$\varphi\delta\varphi=x\delta x+y\delta y+z\delta z,$$

$$\frac{\delta\varphi}{\varphi}=\cos^2\vartheta\left\{\frac{\delta x}{h}\cos\zeta+\frac{\delta y}{h}\sin\zeta\right\}+\sin^2\vartheta\frac{\delta z}{z}$$

$$=P+Q\cos\zeta+R\cos 2\zeta,$$

where

$$P=+\tfrac{1}{2}A'\left(d+\frac{1+b}{2}\right)-1-\tfrac{1}{2}A'\left(d-\frac{1+b}{2}\right)\cos 2\vartheta,$$

$$Q=+\tfrac{1}{2}A'(c+a)\sin 2\vartheta,$$

$$R=+\tfrac{1}{2}A'\frac{1-b}{2}(1+\cos 2\vartheta).$$

If we suppose ϑ_1, φ_1, L_1, M_1, P_1, Q_1, R_1 to be values of ϑ, φ, L, &c. at a base-station, then at any other station at which the dip is ϑ, we have

$$L=\frac{L_1}{\sin 2\vartheta_1}\sin 2\vartheta,$$

$$M=M_1-Q_1\cot 2\vartheta_1+\frac{Q_1}{\sin 2\vartheta_1}\cos 2\vartheta,$$

$$N=\frac{N_1}{\sin 2\vartheta_1}\sin 2\vartheta,$$

$$P=P_1+L_1\cot\vartheta_1-\frac{L_1}{\sin 2\vartheta_1}\cos 2\vartheta,$$

$$Q=\frac{Q_1}{\sin 2\vartheta_1},$$

$$R=\frac{R_1}{1+\cos 2\vartheta_1}(1+\cos 2\vartheta).$$

It will be observed that P, Q, R are abstract numbers, while L, M, N are angles the numerical values of which depend on the assumed unit of angle. The values just given may be used without modification if the angular unit be the angle subtended by the arc=radius, or 57°·3. If the unit of angle be, say 1′, then in the expression for M we must divide Q_1 by sin 1′, and in the expression for P we must multiply L_1 by sin 1′.

As a check on the values of N and R, we may observe that we ought to have

$$-\frac{N_1}{\sin 2\vartheta_1}=+\frac{R_1}{1+\cos 2\vartheta_1}=+\tfrac{1}{2}A'\tfrac{1}{2}(1-b)$$

$$=\tfrac{1}{2}\lambda\mathfrak{D}$$

in the notation of the Admiralty Manual;

And that if we have the value of $\frac{1}{2}A'\frac{1}{2}(1-b)$, or $\frac{1}{2}\lambda\mathfrak{D}$, determined independently from

the observation of deviation and horizontal intensity, it will in general be better to use it than the value of $-\frac{N_1}{\sin 2\theta_1}$ and $\frac{R_1}{1+\sin 2\theta_1}$ obtained from the observations of dip and total intensity.

The observations for the 'Erebus' at Hobarton treated in this way give

$$\delta\theta = 3' \sin 2\theta + \{-3' + 81' \cos 2\theta\} \cos\zeta - 14' \sin 2\theta \cos 2\zeta,$$

$$\frac{\delta\varphi}{\varphi} = \cdot 0045 - \cdot 001 \cos 2\theta + \cdot 0234 \sin 2\theta \cos\zeta + \cdot 004(1+\cos 2\theta)\cos 2\zeta.$$

This is the formula from which the Tables used for correcting the dips and intensities observed in the 'Erebus' have been computed.

P.S. It may be convenient for persons who make use of the Admiralty Manual to have the expressions for L, M, N, P, Q, R, A′, a, b, c, d in the notation of that work.

$$L = +\tfrac{1}{2}(k+1-\lambda)\sin 2\theta,$$

$$M = +\tfrac{1}{2}(g-c)+\tfrac{1}{2}(g+c)\cos 2\theta,$$

$$N = -\tfrac{1}{2}\lambda\mathfrak{D}\sin 2\theta,$$

$$P = \tfrac{1}{2}(k-1+\lambda)-\tfrac{1}{2}(k+1-\lambda)\cos 2\theta,$$

$$Q = +\tfrac{1}{2}(g+c)\sin 2\theta,$$

$$R = +\tfrac{1}{2}\lambda\mathfrak{D}(1+\cos 2\theta).$$

This Memorandum.		Admiralty Manual.
A′	=	$1+a=\lambda(1+\mathfrak{D})$,
a	=	$\frac{c}{1+a}=\frac{\mathfrak{B}}{1+\mathfrak{D}}\cot\theta$,
b	=	$\frac{1+c}{1+a}=\frac{1-\mathfrak{D}}{1+\mathfrak{D}}$,
c	=	$\frac{g}{1+a}$,
d	=	$\frac{1+k}{1+a}$.

Before the 'Erebus' and 'Terror' sailed from Hobarton in November 1840 to commence the first year of the Survey, the deviations of the Declination, as shown by the Standard Compass in each ship, were very carefully observed; and the observations were repeated on the return to Hobarton at the conclusion of the first year's Survey, in June 1841. The results are printed in Contributions No. V., those of the 'Erebus' in page 154, and those of the 'Terror' in page 159. Tables for the correction of the Deviation in the different values of the Inclination, in the first and second years of the Survey, were computed from these observations, employing Mr. ARCHIBALD SMITH'S

Table in Contribution No. VIII. pp. 352, 353; and the same was done for the third year of the Survey from a mean of similar observations at Port Louis in the Falkland Islands in August 1842, and at the Cape of Good Hope in April 1843, as stated in Contribution No. X. pp. 455–457. The corrections for the Deviation in the observations of the Inclination and of the Intensity were computed in all cases by the method described in Mr. SMITH's Memorandum in the present Number, page 379,—employing for the first and second year's Surveys the mean of the observations of the 'Erebus' at Hobarton in October 1840, before her departure for the south, and at the same place, in June 1841, on her return*, and for the third year from the 'Erebus' Table in Contribution X. p. 459, and the 'Terror' Table in Contribution X. p. 461.

The Deviations of the Declination became very large when the Inclination approached 90°; in 88° of Inclination, for example, the influence of the ship's iron occasioned a deviation amounting to the excessive value of 50° and upwards at the extreme points; so that an observation with the ship's head *East* would in consequence differ 100° or even more from an observation taken at the same place with the ship's head *West*. It was doubtless the experience of such extreme differences which led to what might almost appear an excessive multiplication of observations, as well as to the constant endeavour to diversify the circumstances of the direction of the ship's head, both which precautions have contributed in no small degree to reduce within such comparatively small limits the differences which are seen in the Tables between the *mean* values of the Declination on the same or on successive days. Even the remaining minor differences which appear might probably have been still further reduced by another precaution, the desirability of which may not have been so clearly apprehended at the time, namely, the recording the direction of the ship's head at the *precise instant* when the pointing of the compass is read. Unless the two observed facts strictly synchronize, the very slight and almost momentary change in the direction of the ship's head will occasion an apparent discordance in successive results when the ship's head is near the extreme points of East or West, though it may have little ultimate influence on the position or the direction of the isogonic lines in the final result of a great body of observations. This remark applies in a far minor degree to the differences of successive or of nearly adjacent results of the Inclination or Intensity, where no such excessive increase in the amount of the deviation-corrections takes place when approaching the point of 90° of Inclination. A critical eye may well regard the general

* Mr. Fox's apparatus, designed for the use of the 'Terror' in the Antarctic Survey, not having been quite ready when the 'Erebus' and 'Terror' sailed from England in 1839, a smaller apparatus was supplied for immediate use; and the apparatus actually employed (in the 'Terror') in the Survey did not reach that ship until she was on the point of sailing from Hobarton in 1840. It was found, however, on careful subsequent examination that in the two ships the corrections for the deviations of the Inclination and of the Force were so nearly the same in the course of the first two years of the Survey, that the Table computed from the observations of the 'Erebus' referred to in the text might safely be employed for the correction of the observations of these elements in the 'Terror.'

accordance of the results of the Declination on successive days, or on the return in successive years to the same localities, as satisfactory to a degree which might scarcely have been anticipated, supposing the existence of the above-noticed fact to have been known and duly borne in mind, viz. that the observed Declinations were subject to variations extending to 100 degrees and upwards, according to the direction of the ship's head at the moment when the compass was observed*.

Tables in which all the required corrections have been applied to the individual observations have been prepared, and will accompany this paper with a view to their being deposited in the Archives of the Royal Society, in case a reference to them should be hereafter desired: and from these, *General Tables* have been prepared showing the mean Geographical Positions and the mean Magnetical Values after the application of the Corrections (including the latest of these), together with the number of observations from which each result has been obtained. These *General Tables* are subjoined at the close of the paper.

The intensities of the magnetic Force are expressed in Absolute Value, in British units, depending upon the determinations of these values at the Magnetic Observatories of Hobarton and the Cape of Good Hope, regarded as the base-stations of the Survey (Contribution X., Philosophical Transactions, 1866, Art. XX. pp. 463, 464).

The mean results have been inserted in the Maps of Declination, Inclination, and Intensity of the Force, respectively, and constitute the authorities upon which the endeavour has been made to trace the general course of the isogonic, isoclinal, and isodynamic lines, in conformity with the observations of the Survey. The maps with the lines thus traced upon them have supplied the groundwork for the next step, viz. the assignment, as far as could safely be done, of the approximate values of each of the three elements at the intersections of every fifth degree of latitude between $-40°$ and $-90°$, and of every 10th degree of longitude from 0° to 360°, thus supplying the numerical coefficients which may be required, in whole or in part, in a revision of Gauss's 'Allgemeine Theorie des Erdmagnetismus.'

We learn from the original publication of that important work, in the 'Resultate aus den Beobachtungen des Magnetischen Vereins,' im Jahre 1838, and the 'Atlas des Erdmagnetismus,' 1840, that the numerical data employed in the calculations were for "twelve meridional points on each of seven parallels of latitude"—the greater part of the parallels being taken north of the equator, and none south of $-20°$. To those who duly considered the possible influence of this incompleteness of the numerical coefficients, it will not have been matter of surprise that where these were wanting the result of the calculations should have been found to differ widely from the facts made known by

* No record appears to have been made of the amount of the ship's "heeling" at the times of observation; in reference to this Mr. Archibald Smith remarks in a note to myself, that in the 'Erebus,' on North and South courses, when the effect produced is a maximum, it would only be $30'' \tan \theta$ for every degree of heel; which at a dip of 88° (nearly the highest observed) would give a correction of only about 15′ for each degree of heel, and may therefore be neglected.

the Survey. The application of the theory professed, indeed, to be simply a first attempt from which we were entitled to expect little more than a rough approximation. Viewed in this light (and it must be remembered that this was all that the author himself claimed for it), its publication could only be regarded as strengthening the grounds on which the desire was felt for more complete and trustworthy data than were possessed by M. Gauss in 1839. The failure of the theory, as provisionally applied, to reproduce in the Southern Hemisphere the peculiar and characteristic features of the Terrestrial Magnetism which were so well established by observation and represented by the theory in the Northern Hemisphere, and the strong objection to receiving on less than conclusive evidence so improbable an anomaly as this diversity in the two hemispheres, tended, without doubt, greatly to stimulate the endeavours of those who desired to render the facts of observation in the Southern Hemisphere more commensurate with the theory in which they were to be employed. Accordingly in the VI.th Number of the Contributions (Philosophical Transactions, 1844, Art. VII.) I exhibited in Plate XIII. the lines of equal intensity deduced from the observations in the first two years of Sir James Ross's Survey, in comparison with Gauss's theoretical lines taken from Plates XVIII. and XIX. of the 'Atlas des Erdmagnetismus,' showing the difference between the two. The immediate object of this comparison was to strengthen the application which was then being made for the additional employment of the 'Pagoda;' whilst the two following sentences which I venture to extract from the paper accompanying the Plate, are sufficient to manifest that a full respect for the theory itself and for its illustrious author was not wanting.

"The very imperfect resemblance between the two systems of lines in the southern hemisphere is of course no impeachment of the sufficiency of the theory, *with corrected numerical elements*, to represent the natural phenomena in parts of the globe which observation may not have reached. The degree of approximation to which it will do this must depend upon the extent and correctness of the observation-basis from whence the numerical elements are derived, and upon the order of the magnitudes comprehended in the calculation.

"The evidence which the Plate affords that the calculation in the elaborate work referred to differs so widely from the facts in the southern latitudes, shows how much observations were wanting in those latitudes for the purpose of perfecting the theory, and is an ample justification (if indeed any justification were necessary) of the exertions which the last few years have witnessed to obtain them."

I am greatly indebted to the Hydrographer, Captain Richards, R.N., F.R.S., for his kind permission to have the maps which accompany this Number of the Contributions prepared at the Hydrographic Office, and am particularly obliged to the Assistant Hydrographer, Captain Frederick John Evans, R.N., F.R.S., for the very valuable superintendence which he has kindly given to their preparation and execution.

It may be desirable that I should add a few words in explanation of the sense in which, in the present and in earlier papers, I have employed certain of the technical ex-

pressions appertaining to this branch of the physical sciences. Thus:—I have used the terms magnetic *pole* and *poles* in their original and natural sense, as designating those points on the surface of the globe where the intensity of the magnetic force is a maximum in comparison with adjacent localities—in other words, the centres of the lemniscates into which, at our present epoch in the magnetic cycle, the isodynamic lines of highest intensity resolve themselves.

In one of the earliest systematic treatises on the phenomena of terrestrial magnetism as derived from and resting on observation, (that of HALLEY in the Philosophical Transactions for 1683,) it is unhesitatingly affirmed, on the evidence of well-assured facts, that "the globe of the earth may be regarded as one great magnet having four magnetical *poles*, or *points of attraction*, two of them near each pole of the equator; and that in those parts of the globe which lie near any of those magnetical poles, the needle is chiefly governed thereby, the nearest pole being always predominant over the more remote." It is in this, its original signification, bearing date so early as 1683, that I have employed the term "magnetic pole." The magnetic surveys which have been so carefully and systematically made in the course of the present century over almost all the accessible parts of the globe, may now be considered to have fully confirmed the truth of HALLEY'S remarkable anticipation, and may be claimed in full justification by those who, following in the steps of our sagacious and illustrious countryman, employ the terms which he was amongst the first to use (and the first to assign to them their true significancy), in the same sense in which he himself employed them. The Halleian hypothesis, and the incontrovertible facts on which it rested, were for a time obscured by the prevalence of a uniaxial hypothesis, in which the points of maximum force were assumed to be identical with the points where the inclination is 90°; around which, as their centre, the isodynamic and isoclinal lines were supposed to be arranged symmetrically in parallel lines; hence the term magnetic pole came, in the uniaxal hypothesis, to be attached to the points where the inclination is 90°, that element being the easier of determination, and consequently the one most popularly regarded. But the acceptance of the uniaxial hypothesis has gradually yielded to the progressive advance of observational knowledge; and one of its ablest expositors (BIOT), as far back as the commencement of the present century, was constrained to admit that, even in the case of apparently the most simple of the three elements (viz. the Inclination), a single magnetic axis would afford no sufficient explanation of the phenomena, unless it were supplemented by the supposition of subordinate centres; whilst as regarded the known facts of the Declination and Intensity, they must be held to be wholly inexplicable on the supposition of a single magnetic axis (Report of the British Association, 1837, p. 64).

Preserving, therefore, the sense in which HALLEY employed the term magnetic pole, and in accordance with the evidence, now fully established by observation, of the existence of four such points on the surface of the globe, the *magnetic equator* is most properly defined as the line connecting those points in each geographical meridian where the intensity of the earth's magnetism is less than in any other point situated in the

same geographical meridian. This line, at the present magnetic epoch, has a double curvature. I was myself the first, I believe, to trace (Philosophical Magazine for February 1829, Art. XV.) the position on the globe of the magnetic equator as thus defined, and to exhibit it in comparison with the line of no dip, from which it differs very considerably in geographical position*.

In concluding this paper, I should be unjust to the memory of Sir James Ross and to my own high regard to his memory, if I failed to record my conviction that, by the remarkable character of his geographical discoveries, by the perseverance and indomitable resolution which he displayed on so many occasions, and by that which we of the Royal Society are peculiarly able to appreciate and peculiarly bound to honour and applaud, *i. e.* the large extent and high character of his contributions to the advancement of the sciences connected with physical geography in the polar regions of *both* hemispheres, he has established a claim to be regarded as the first scientific navigator of his country and of his age.

* In an earlier paper (Philosophical Transactions, 1864, Art. VI.) I availed myself of what appeared to me a suitable occasion to express the conjectural belief, which I have long entertained, that of the two magnetic systems which are distinctly recognizable in the phenomena of the magnetism of the globe, one has a terrestrial and the other a cosmical source. It is, I believe, the latter of these two systems which, by its progressive translation, gives rise to the phenomena of secular change, and to those magnetical cycles which owe their origin to the operation of the secular change. I have naturally seen with great pleasure that this conjectural anticipation is received, and is viewed in the same light, by the author of the able Essay which obtained the Adams Prize at Cambridge in 1865, and which has been since published under the title of "Terrestrial and Cosmical Magnetism, by Edward Walker, M.A., late Fellow of Trinity College, Cambridge, &c." I may add my entire concurrence with the following statement of that gentleman in § 185, p. 293, of the work referred to.

"Our final conclusion, therefore, upon the whole subject seems to be, that the magnetic influence at any point of the globe is the result of two distinct magnetic systems, the principal of which is the magnetism proper of the globe, having its [northern] point of greatest attraction in the north of the American Continent, whilst the weaker system is that which results from the magnetism induced in the earth by cosmical action, and of which the [northern] point of greatest attraction is at present in the north of the Asiatic Continent. Thus the direction of the magnet at any point results from the superposition of the two systems,—'the nearest pole being always predominant over the more remote.'"

TABLES.

I. Numerical Coefficients, for a revision of GAUSS's Allgemeine Theorie, pp. 386 & 387.

II. General Tables of the Values entered in the Maps (Plates XXII. to XXIV.), pages 388–416.

NOTE.—In the vicinity of the Great Glacier, where the Southerly progress of the Ships was arrested, and many days were passed in the endeavour to discover some channel by which a further advance might be made, the rapid convergence both of the geographical meridians and of the lines which represent every 10th degree of Declination has made it desirable to depart, in the Map of the Declination, from the practice which has been generally followed elsewhere in the Map, of inserting the mean of each day's observations in the geographical position to which it corresponds. In the "General Tables" the daily means in this part of the voyage are inserted as usual; but in the Map the number of entries in latitudes exceeding 75° S. has been reduced by combining the observations in larger groups. The manner in which this has been done is shown in a small subsidiary Table in page 391, where the small letters *a*, *b*, *c*, &c., referring to the results characterized by the same letters in pp. 389 and 391 of the General Tables, supply the requisite connexion between the Tables and the Map.

Approximate Numerical Coefficients of the Magnetism of the Earth, Epoch 1840–1845, prepared for a revision of GAUSS's 'Allgemeine Theorie des Erdmagnetismus.'

Lat.	Longitudes....	0°.	10°.	20°.	30°.	40°.	50°.	60°.	70°.	80°.	90°.	100°.	110°.	120°.	130°.	140°.	150°.	160°.	170°.
−40°	Declination...	21·2 w.	25·6 w.	30·0 w.	31·1 w.	30·0 w.	27·0 w.	24·4 w.	22·2 w.	20·9 w.	20·0 w.	18·8 w.	11·9 w.	3·1 w.	0·4 w.	5·9 e.	10·5 e.	13·3 e.	14·9 e.
	Inclination ...	47·5	52·8	56·9	60·2	62·7	64·8	66·1	66·8	67·4	68·0	68·7	69·5	70·0	69·9	69·7	68·4	66·2	64·1
	Intensity	7·0	7·4	7·9	8·5	9·0	9·6	10·2	10·8	11·5	12·1	12·5	13·0	13·5	13·75	13·6	13·3	12·95	12·6
−45°	Declination...	20·0 w.	25·4 w.	30·0 w.	33·5 w.	33·3 w.	31·1 w.	29·0 w.	27·0 w.	26·0 w.	24·7 w.	21·0 w.	15·0 w.	8·2 w.	0·9 w.	6·0 e.	11·4 e.	14·5 e.	16·4 e.
	Inclination ...	51·0	55·7	58·6	61·7	64·2	66·2	67·7	68·9	69·8	71·2	72·2	73·1	73·9	73·9	73·1	71·7	70·0	68·0
	Intensity	7·55	7·9	8·35	8·85	9·4	10·0	10·6	11·2	11·8	12·4	12·9	13·4	14·1	14·6	14·2	13·8	13·5	13·1
−50°	Declination...	18·6 w.	23·8 w.	29·6 w.	33·5 w.	34·6 w.	34·3 w.	33·4 w.	32·5 w.	32·0 w.	31·0 w.	26·5 w.	19·5 w.	10·7 w.	1·8 w.	6·4 e.	12·4 e.	16·1 e.	18·0 e.
	Inclination ...	54·2	57·5	60·5	63·0	65·5	67·4	69·0	70·6	72·0	73·6	75·1	76·5	77·2	77·2	76·4	75·0	73·5	71·9
	Intensity	8·1	8·4	8·8	9·2	9·8	10·35	10·95	11·5	12·1	12·7	13·2	13·75	14·5	15·4	14·5	14·25	13·95	13·6
−55°	Declination...	17·4 w.	22·9 w.	29·0 w.	33·5 w.	36·3 w.	37·3 w.	37·7 w.	38·7 w.	39·0 w.	38·5 w.	33·3 w.	24·0 w.	14·0 w.	3·5 w.	6·6 e.	13·8 e.	18·0 e.	19·7 e.
	Inclination ...	57·0	59·6	62·0	64·4	66·6	68·7	70·3	72·0	73·7	75·7	77·7	79·4	80·3	80·3	79·7	78·4	77·0	75·5
	Intensity	8·7	8·9	9·3	9·8	10·25	10·75	11·25	11·8	12·4	13·0	13·5	13·95	14·5	15·0	14·8	14·6	14·3	14·1
−60°	Declination...	15·5 w.	21·5 w.	28·0 w.	33·5 w.	37·2 w.	39·4 w.	43·0 w.	45·2 w.	47·0 w.	46·4 w.	41·0 w.	30·5 w.	19·5 w.	7·4 w.	6·1 e.	15·8 e.	21·0 e.	22·6 e.
	Inclination ...	59·8	61·7	63·9	65·9	68·0	69·8	71·7	73·4	75·0	77·4	79·6	81·8	83·3	83·5	83·0	81·7	80·3	78·8
	Intensity	9·4	9·6	9·95	10·3	10·7	11·1	11·6	12·1	12·6	13·2	13·7	14·2	14·7	15·3	15·2	14·95	14·7	14·25
−65°	Declination...	14·0 w.	20·0 w.	27·0 w.	33·0 w.	37·3 w.	41·4 w.	46·2 w.	51·8 w.	55·6 w.	55·0 w.	49·0 w.	41·0 w.	29·0 w.	15·0 w.	4·2 e.	17·5 e.	25·0 e.	27·4 e.
	Inclination ...	62·5	64·2	65·9	67·8	69·7	71·5	73·1	74·6	76·5	78·6	80·9	83·4	85·4	86·1	86·0	85·0	83·5	82·2
	Intensity	10·25	10·35	10·55	10·85	11·15	11·5	11·9	12·4	12·9	13·35	13·85	14·35	14·9	15·25	15·35	15·2	15·05	14·8
−70°	Declination...	13·0 w.	19·5 w.	26·3 w.	31·9 w.	37·0 w.	41·5 w.	47·5 w.	55·0 w.	62·0 w.	64·2 w.	60·5 w.	55·0 w.	47·5 w.	35·0 w.	9·5 w.	22·5 e.	35·5 e.	38·8 e.
	Inclination ...	65·7	66·9	68·5	70·0	71·6	73·2	74·6	76·1	77·8	79·5	81·6	84·0	86·0	87·7	88·3	87·6	86·5	85·4
	Intensity	11·0	11·05	11·15	11·4	11·65	11·9	12·3	12·7	13·1	13·65	14·0	14·5	15·0	15·2	15·4	15·4	15·25	15·05
−75°	Declination...	12·4 w.	18·8 w.	25·0 w.	30·8 w.	36·0 w.	41·3 w.	47·5 w.	55·0 w.	64·5 w.	72·8 w.	77·0 w.	78·0 w.	78·0 w.	80·0 w.	105·0 w.	150·0 e.	91·0 e.	80·0 e.
	Inclination ...	69·3	70·0	71·0	72·1	73·5	74·7	76·1	77·4	78·8	80·2	82·0	83·9	85·6	87·0	88·5	89·0	88·5	87·5
	Intensity	11·8	11·8	11·85	12·0	12·15	12·4	12·7	13·0	13·5	13·75	14·15	14·55	14·95	15·2	15·25	15·35	15·25	15·15
−80°	Declination...	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	Inclination ...	72·6	73·2	73·8	74·6	75·5	76·6	77·4	78·4	79·5	80·6	81·9	83·1	84·3	85·2	86·0	86·4	86·5	86·3
	Intensity	12·5	12·5	12·5	12·6	12·8	12·9	13·1	13·4	13·65	13·95	14·2	14·5	14·8	15·0	15·1	15·15	15·15	15·1
−85°	Declination...	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
	Inclination ...	76·4	76·7	76·9	77·0	77·7	78·2	78·8	79·5	80·0	80·7	81·2	81·7	82·5	82·7	83·0	83·3	83·3	83·3
	Intensity	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Note.—All the Inclinations are South.

Approximate Numerical Coefficients of the Magnetism of the Earth, Epoch 1840–1845, prepared for a revision of GAUSS's 'Allgemeine Theorie des Erdmagnetismus.'

180°.	190°.	200°.	210°.	220°.	230°.	240°.	250°.	260°.	270°.	280°.	290°.	300°.	310°.	320°.	330°.	340°.	350°.	360°.	Longitudes.	Lat.
14·5 E.	13·2 E.	11·5 E.	10·0 E.	9·3 E.	9·3 E.	10·6 E.	12·7 E.	15·1 E.	18·3 E.	19·2 E.	17·6 E.	14·5 E.	9·4 E.	3·1 E.	3·1 W.	9·4 W.	15·5 W.	21·2 W.	Declination	
62·6	61·3	60·0	58·5	57·8	57·4	56·6	55·0	53·3	51·0	48·8	45·1	41·7	39·5	38·0	38·0	39·7	42·9	47·5	Inclination	−40
12·2	12·0	11·8	11·6	11·55	11·45	11·25	11·0	10·7	10·4	9·8	9·2	8·6	8·1	7·7	7·2	6·9	6·8	7·0	Intensity	
15·8 E.	14·4 E.	12·8 E.	11·7 E.	11·0 E.	11·0 E.	12·3 E.	14·3 E.	17·4 E.	20·5 E.	21·2 E.	19·4 E.	16·0 E.	10·7 E.	4·7 E.	1·7 W.	7·7 W.	14·0 W.	20·0 W.	Declination	
66·6	65·6	64·7	63·5	62·5	62·0	61·2	60·2	58·4	56·2	53·7	50·5	47·0	44·2	42·6	42·6	44·1	46·9	51·0	Inclination	−45
12·8	12·5	12·3	12·2	12·15	12·1	11·9	11·75	11·5	11·1	10·5	9·9	9·3	8·75	8·3	7·9	7·6	7·45	7·55	Intensity	
17·5 E.	15·9 E.	14·2 E.	13·1 E.	12·7 E.	12·8 E.	13·8 E.	16·3 E.	19·6 E.	22·6 E.	23·6 E.	21·6 E.	17·6 E.	12·4 E.	6·3 E.	0·2 W.	6·2 W.	12·5 W.	18·0 W.	Declination	
70·4	69·5	68·6	67·6	66·7	66·0	65·1	64·3	62·7	60·8	58·0	55·0	51·8	49·0	47·4	47·2	48·4	50·8	54·2	Inclination	−50
13·3	13·05	12·85	12·8	12·75	12·7	12·6	12·4	12·15	11·8	11·25	10·55	9·9	9·4	8·95	8·6	8·25	8·1	8·1	Intensity	
19·1 E.	17·6 E.	16·0 E.	14·7 E.	14·3 E.	14·5 E.	16·1 E.	19·3 E.	22·5 E.	24·8 E.	25·8 E.	24·0 E.	19·2 E.	14·3 E.	8·2 E.	1·8 E.	4·3 W.	11·0 W.	17·4 W.	Declination	
74·2	73·0	72·3	71·3	70·4	69·7	69·0	68·1	66·7	64·5	62·0	59·2	56·3	53·7	52·0	51·9	52·6	54·4	57·0	Inclination	−55
13·85	13·6	13·5	13·4	13·4	13·35	13·25	13·1	12·8	12·5	11·9	11·2	10·65	10·1	9·7	9·3	8·95	8·7	8·75	Intensity	
21·8 E.	19·6 E.	18·6 E.	17·8 E.	17·5 E.	18·4 E.	20·6 E.	23·6 E.	25·9 E.	27·4 E.	27·9 E.	25·5 E.	21·1 E.	16·0 E.	9·9 E.	3·8 E.	2·5 W.	9·5 W.	15·5 W.	Declination	
77·6	76·6	75·6	74·7	73·8	72·8	72·0	71·2	69·7	67·5	65·4	62·9	60·5	58·3	56·9	56·4	56·7	57·8	59·6	Inclination	−60
14·2	14·1	14·05	14·0	14·0	14·0	13·9	13·65	13·4	13·1	12·5	11·9	11·3	10·8	10·4	10·0	9·7	9·5	9·15	Intensity	
27·0 E.	25·5 E.	24·0 E.	23·2 E.	23·0 E.	24·2 E.	26·0 E.	27·5 E.	28·6 E.	29·7 E.	29·7 E.	26·7 E.	22·5 E.	17·2 E.	11·0 E.	4·6 E.	1·4 W.	8·0 W.	14·0 W.	Declination	
81·0	80·0	79·0	78·0	76·9	75·7	74·7	73·6	71·9	70·0	68·0	66·0	64·2	62·5	61·2	60·8	60·9	61·4	62·5	Inclination	−65
14·6	14·45	14·35	14·3	14·25	14·2	14·15	14·05	13·85	13·45	13·0	12·4	11·9	11·45	11·05	10·7	10·45	10·25	10·25	Intensity	
38·0 E.	36·6 E.	34·8 E.	33·1 E.	32·0 E.	32·0 E.	32·0 E.	32·7 E.	34·2 E.	34·7 E.	32·0 E.	28·0 E.	23·4 E.	17·8 E.	11·6 E.	5·0 E.	0·8 W.	7·0 W.	13·0 W.	Declination	
84·2	83·2	82·2	81·0	79·6	78·3	77·0	75·5	73·9	72·1	70·5	68·8	67·3	66·1	65·2	64·7	64·6	65·0	65·7	Inclination	−70
14·9	14·75	14·6	14·55	14·45	14·4	14·35	14·2	14·05	13·7	13·3	12·8	12·4	12·0	11·7	11·4	11·2	11·1	11·0	Intensity	
73·0 E.	66·0 E.	60·0 E.	56·6 E.	53·0 E.	49·5 E.	47·5 E.	45·5 E.	43·4 E.	40·0 E.	34·8 E.	28·5 E.	23·6 E.	17·8 E.	12·0 E.	5·0 E.	0·8 W.	6·7 W.	12·4 W.	Declination	
86·5	85·6	84·6	83·2	81·7	80·2	78·8	77·2	75·7	74·3	72·8	71·5	70·4	69·5	68·9	68·6	68·5	68·6	69·0	Inclination	−75
15·0	14·9	14·8	14·7	14·65	14·55	14·4	14·25	14·1	13·9	13·5	13·2	12·85	12·55	12·3	12·1	11·9	11·8	11·8	Intensity	
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	Declination	
86·0	85·3	84·6	83·6	82·3	81·2	80·0	78·7	77·5	76·4	75·5	74·5	73·8	73·1	72·7	72·4	72·3	72·5	72·6	Inclination	−80
15·05	14·95	14·8	14·7	14·6	14·5 E.	14·35	14·25	14·1	13·95	13·7	13·5	13·2	13·1	12·9	12·8	12·6	12·55	12·5	Intensity	
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	Declination	
83·2	83·0	82·6	82·2	81·6	81·0	80·5	79·8	79·3	78·6	78·0	77·5	77·0	76·6	76·4	76·3	76·1	76·1	76·4	Inclination	−85
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	Intensity	

The South Pole of the Earth { Declination 180·0 / Inclination 80·0 / Intensity 14·0 }

General Table of the Declinations observed on board Her Majesty's Ship 'Erebus,' by Captain Sir James Clark Ross, between the Cape of Good Hope and Van Diemen Island in 1840, April to August.

Date.	Position.		No. of observations.	Declination.	
	Lat. S.	Long. E.		In degrees and minutes.	In degrees and decimals.
1840.	° ′	° ′		° ′	°
April 8.	36 01	18 38	6	+30 08	+30·1
10.	36 16	20 41	2	+30 15	+30·3
11.	36 24	21 14	14	+30 40	+30·7
12.	37 18	21 25	8	+30 51	+30·9
13.	38 15	21 19	4	+32 10	+32·2
14.	39 55	20 35	1	+31 09	+31·1
15.	41 07	22 11	2	+29 51	+29·9
16.	41 28	23 14	6	+30 25	+30·4
17.	41 50	26 24	2	+31 49	+31·8
18.	43 09	28 46	5	+33 15	+33·3
25.	46 31	48 03	1	+32 41	+32·7
28.	46 34	52 43	2	+33 28	+33·5
April 30 and May 1.	46 32	52 01	2	+33 16	+33·3
May 2.	47 03	56 22	3	+31 32	+31·5
4.	47 45	62 27	2	+29 00	+29·0
July 27.	47 18	93 36	5	+25 38	+25·6
31.	47 34	105 47	1	+18 59	+19·0
Aug. 2.	47 45	113 49	2	+11 29	+11·5
4.	47 40	121 50	3	+ 9 34	+ 9·6
7.	46 18	132 00	13	− 0 50	− 0·8
10.	44 23	141 11	7	− 7 52	− 7·9

General Table of the Declinations observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between November 1840 and April 1841.

Position.		Ship.	No. of observations.	Declination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
° ′	° ′			° ′	°
45 36	152 53	Erebus.	8	−13 05	−13·1
46 08	154 15	Erebus.	8	−13 26	−13·4
46 30	154 55	Erebus.	7	−14 04	−14·1
49 47	161 00	Erebus.	8	−16 49	−16·8
50 32	166 12	Erebus.	8	−17 44	−17·7*
50 54	166 35	Erebus.	7	−15 34	−15·6
52 33	169 50	Terror.	10	−16 03	−16·1
52 33	169 09	Erebus.	2	−17 40	−17·7†
54 14	169 06	Erebus.	5	−18 17	−18·3
57 33	170 30	Terror.	4	−21 58	−22·0
57 25	170 20	Erebus.	14	−21 19	−21·3
58 59	170 59	Terror.	7	−21 45	−21·7
59 39	169 59	Erebus.	8	−21 11	−21·2
59 32	170 05	Terror.	13	−20 45	−20·7
59 33	169 55	Terror.	10	−20 06	−20·1
60 13	170 25	Erebus.	10	−22 32	−22·5
60 47	170 51	Erebus.	11	−22 10	−22·2
60 26	170 23	Terror.	10	−24 06	−24·1
64 16	172 31	Erebus.	9	−25 26	−25·4
64 26	173 00	Terror.	9	−25 44	−25·7
64 40	172 44	Erebus.	8	−25 42	−25·7
65 26	172 20	Erebus.	8	−24 53	−24·9
65 57	171 45	Erebus.	7	−26 58	−27·0
65 42	172 13	Terror.	7	−25 31	−25·5
66 23	169 51	Erebus.	17	−28 07	−28·1
65 32	172 40	Terror.	7	−28 15	−28·3
65 27	172 23	Erebus.	11	−27 22	−27·4
65 32	173 44	Terror.	8	−29 35	−29·6
65 30	173 43	Erebus.	10	−28 11	−28·2
65 35	173 33	Terror.	9	−29 01	−29·0
67 16	174 41	Erebus.	16	−31 22	−31·4
68 00	175 05	Erebus.	13	−33 42	−33·7
68 24	175 45	Terror.	10	−33 36	−33·6
68 31	176 05	Erebus.	12	−33 37	−33·6
68 55	176 22	Erebus.	5	−34 52	−34·9
70 33	172 57	Erebus.	6	−38 22	−38·4
70 53	173 00	Terror.	5	−42 49	−42·8
71 00	172 25	Erebus.	9	−43 43	−43·7
71 21	171 14	Terror.	13	−47 01	−47·0
71 22	170 55	Erebus.	10	−43 47	−43·8
71 51	171 19	Terror.	12	−45 59	−46·0
71 51	171 53	Erebus.	9	−45 06	−45·1
71 55	172 00	Erebus.	9	−49 47	−49·8
72 16	174 09	Erebus.	6	−52 43	−52·7
72 40	175 17	Terror.	11	−51 03	−51·1
73 01	175 55	Erebus.	12	−52 21	−52·3
72 36	173 40	Erebus.	5	−48 08	−48·1
72 33	172 51	Terror.	8	−48 41	−48·7
72 36	173 39	Erebus.	8	−51 49	−51·8
72 34	172 44	Terror.	8	−53 16	−53·3

* On shore at Auckland Island.

† Campbell Island.

General Table of the Declinations (continued).

Position.		Ship.	No. of observations.	Declination.		Position.		Ship.	No. of observations.	Declination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
73° 37′	171° 13′	Erebus.	12	− 65° 05′	− 65°·1	70° 11′	167° 27′	Erebus.	17	−39° 12′	−39°·2
73 47	171 23	Terror.	10	− 62 45	− 62·7	70 15	166 38	Terror.	10	−37 50	−37·8
74 01	171 42	Erebus.	10	− 65 14	− 65·2	69 19	167 46	Terror.	4	−34 09	−34·1
73 53	171 37	Terror.	8	− 63 37	− 63·6	69 33	167 31	Erebus.	11	−38 34	−38·6
74 01	172 35	Terror.	5	− 66 57	− 66·9	67 27	167 43	Terror.	3	−42 08	−42·1
74 39	169 00	Erebus.	7	− 71 04	− 71·1	68 59	167 46	Erebus.	12	−36 00	−36·0
74 26	168 37	Terror.	6	− 68 27	− 68·5	67 56	167 31	Erebus.	8	−31 55	−31·9
74 46	168 42	Erebus.	10	− 68 54	− 68·9	68 12	167 40	Terror.	4	−30 32	−30·5
75 04	168 40	Terror.	3	− 84 03	− 84·1	65 43	165 10	Erebus.	13	−28 44	−28·7
75 39	168 33	Erebus.	9	− 85 32	− 85·5	65 32	167 30	Terror.	4	−24 53	−24·9
75 40	168 28	Terror.	6	−100 47	−100·8	65 50	164 39	Terror.	9	−28 04	−28·1
75 58	168 50	Terror.	2	−107 40	−107·7	64 59	162 42	Erebus.	9	−26 09	−26·1
a 76 47	169 26	Terror.	2	−112 22	−112·4	65 34	162 08	Terror.	6	−24 15	−24·3
77 21	172 04	Terror.	4	−115 44	−115·7	64 33	163 23	Terror.	10	−26 22	−26·4
77 41	175 57	Erebus.	7	−104 43	−104·7	64 10	163 14	Erebus	11	−25 50	−25·8
77 47	176 20	Terror.	5	−102 44	−102·7	64 08	163 04	Terror.	7	−24 57	−24·9
77 49	178 31	Erebus.	5	−103 28	−103·5	64 05	161 13	Erebus.	9	−24 37	−24·6
77 22	188 43	Erebus.	12	− 82 37	− 82·6	64 05	161 47	Terror.	7	−23 53	−23·9
77 04	188 28	Terror.	5	− 83 12	− 83·2	63 10	156 25	Erebus.	9	−23 48	−23·8
77 09	188 22	Erebus.	9	− 91 28	− 91·5	62 51	157 02	Terror.	7	−25 24	−25·4
77 43	187 05	Terror.	8	− 93 23	− 93·4	63 51	151 49	Erebus.	12	−18 44	−18·7
b 77 35	186 40	Erebus.	9	− 93 46	− 93·8	63 50	151 35	Terror.	10	−17 59	−18·0
77 45	189 00	Terror.	6	− 94 32	− 94·5	64 18	149 03	Erebus.	5	−17 31	−17·5
77 46	186 54	Erebus.	8	− 93 15	− 93·3	64 17	148 53	Terror.	7	−14 08	−14·1
77 51	186 38	Erebus.	10	− 96 40	− 96·7	65 03	142 47	Erebus.	2	−13 48	−13·8
77 44	186 06	Terror.	5	− 97 00	− 97·0	64 20	140 40	Erebus.	6	− 6 25	− 6·4
77 18	192 38	Erebus.	13	− 82 06	− 82·1	64 05	140 00	Terror.	2	− 3 32	− 3·5
77 06	192 22	Terror.	13	− 81 46	− 81·8	63 13	139 50	Erebus.	12	− 5 04	− 5·1
77 11	192 59	Erebus.	6	− 82 21	− 82·3	63 08	139 35	Terror.	6	− 3 42	− 3·7
c 77 16	191 45	Erebus.	6	− 83 57	− 83·9	62 37	138 04	Erebus.	6	− 3 47	− 3·8
77 08	189 02	Erebus.	6	− 82 02	− 82·0	62 15	136 22	Erebus.	7	− 0 23	− 0·4
76 57	188 30	Terror.	9	− 82 30	− 82·5	62 13	136 12	Terror.	5	+ 0 49	+ 0·8
76 57	186 39	Erebus.	9	− 83 15	− 83·3	62 06	136 07	Erebus.	6	+ 0 54	+ 0·9
77 03	186 40	Terror.	10	− 88 26	− 88·4	60 22	131 28	Erebus.	25	+ 7 45	+ 7·7
77 12	187 01	Erebus.	10	− 88 20	− 88·3	60 22	131 34	Terror.	21	− 1 01	− 1·0
77 25	186 27	Erebus.	12	− 91 32	− 91·5	58 54	129 38	Erebus.	11	+ 8 14	+ 8·2
77 36	186 42	Terror.	9	− 92 04	− 92·1	59 03	129 42	Terror.	7	+ 9 08	+ 9·1
77 49	187 32	Erebus.	12	− 97 11	− 97·2	57 21	127 46	Erebus.	10	+ 8 35	+ 8·6
d 77 52	189 27	Erebus.	13	− 93 38	− 93·6	57 22	127 40	Terror.	8	− 2 01	− 2·0
77 51	189 42	Terror.	7	− 95 08	− 95·1	56 14	130 44	Erebus.	8	+ 5 17	+ 5·3
77 44	187 59	Erebus.	8	− 93 31	− 93·5	56 39	129 45	Terror.	7	+ 6 13	+ 6·2
76 39	188 27	Erebus.	5	− 77 44	− 77·7	55 11	131 21	Erebus.	14	+ 0 35	+ 0·6
76 23	177 25	Erebus.	2	− 84 55	− 84·9	55 05	132 07	Terror.	18	+ 0 59	+ 1·0
76 25	177 34	Terror.	2	− 88 44	− 88·7	55 04	132 34	Erebus.	9	+ 0 34	+ 0·6
76 07	168 45	Erebus.	4	− 99 28	− 99·5	54 05	134 44	Terror.	11	− 3 01	− 3·0
76 31	166 31	Terror.	9	−122 41	−122·7	54 04	134 38	Erebus.	13	− 2 04	− 2·1
76 22	165 44	Erebus.	6	−119 17	−119·3	52 52	135 26	Erebus.	7	− 1 33	− 1·5
e 76 36	166 18	Erebus.	14	−113 24	−113·4	52 56	135 05	Terror.	8	− 3 34	− 3·6
76 33	164 45	Erebus.	8	−115 06	−115·1	51 10	136 54	Erebus.	5	− 5 05	− 5·1
76 03	166 26	Terror.	6	− 96 30	− 96·5	51 15	136 49	Terror.	3	− 5 29	− 5·5
76 11	166 02	Erebus.	4	− 99 05	− 99·1	44 52	143 31	Erebus.	3	− 6 02	− 6·0
75 52	167 07	Erebus.	8	− 95 32	− 95·5	44 38	143 57	Terror.	11	− 8 11	− 8·2
74 46	167 53	Erebus.	2	− 79 14	− 79·2	44 02	145 51	Erebus.	6	−10 32	−10·5
74 55	168 14	Terror.	5	− 82 05	− 82·1						

The small letters *a*, *b*, *c*, *d*, and *e* refer to the Subsidiary Table in page 391.

General Table of the Declinations observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between May 1841 and August 1842.

Position.		Ship.	No. of observations.	Declination.		Position.		Ship.	No. of observations.	Declination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
42° 52′	147° 24′	Erebus.	2	−10° 24′*	−10°4	56° 15′	211° 47′	Erebus.	9	−15° 40′	−15°7
43 30	147 20	Terror.	4	−11 59	−12·0	56 20	211 40	Terror.	14	−14 51	−14·9
42 40	148 45	Erebus.	2	− 9 54	− 9·9	56 23	211 58	Erebus.	9	−13 45	−13·7
42 17	149 31	Terror.	5	−12 25	−12·4	56 54	212 23	Erebus.	8	−13 08	−13·1
40 40	149 23	Erebus.	2	−10 17	−10·3	57 03	212 16	Terror.	10	−14 42	−14·7
40 51	149 21	Terror.	5	−11 39	−11·7	57 15	212 45	Erebus.	13	−13 41	−14·7
37 48	150 21	Erebus.	10	−11 27	−11·5	58 25	213 13	Erebus.	12	−14 11	−14·2
37 54	150 20	Terror.	8	−11 13	−11·2	59 25	213 20	Terror.	10	−16 52	−16·9
37 18	151 34	Erebus.	12	− 9 52	− 9·9	62 49	212 00	Erebus.	12	−20 15	−20·3
37 10	151 32	Terror.	10	−11 44	−11·7	62 56	212 07	Terror.	14	−19 55	−19·9
33 51	151 17	Erebus.	1	− 9 52†	− 9·9	63 20	210 22	Erebus.	6	−20 47	−20·8
33 54	153 50	Erebus.	2	− 9 56	− 9·9	63 23	210 05	Erebus.	5	−19 69	−20·0§
33 56	151 00	Terror.	4	−11 29	−11·5	63 23	210 03	Erebus.	19	−20 38	−20·6
33 33	162 01	Erebus.	8	−11 52	−11·9	63 21	209 48	Terror.	17	−20 42	−20·7
33 35	162 47	Terror.	10	−14 24	−14·4	64 29	206 55	Erebus.	11	−21 49	−21·8
33 41	166 26	Erebus.	8	−13 22	−13·4	64 54	206 04	Erebus.	8	−22 39	−22·7
33 48	166 29	Terror.	16	−13 33	−13·5	64 48	206 10	Terror.	9	−22 15	−22·3
33 32	167 35	Erebus.	7	−13 16	−13·3	65 13	205 54	Erebus.	8	−21 48	−21·8
33 22	168 04	Terror.	12	−14 58	−15·0	65 29	205 55	Erebus.	8	−22 35	−22·6
33 42	169 44	Erebus.	11	−12 45	−12·7	65 32	204 57	Terror.	7	−23 51	−23·9
34 15	172 33	Terror.	9	−13 52	−13·9	66 04	203 51	Erebus.	4	−24 06	−24·1
34 31	173 28	Erebus.	11	−13 48	−13·8	66 22	203 40	Erebus.	11	−25 34	−25·6
34 32	173 47	Terror.	5	−13 39	−13·7	66 04	203 16	Erebus.	8	−27 08	−27·1
35 16	174 00	Erebus.	1	−13 36‡	−13·6	66 10	203 37	Terror.	7	−26 40	−26·7
36 40	177 58	Erebus.	11	−14 07	−14·1	66 16	204 39	Erebus.	6	−26 37	−26·6
38 03	179 32	Terror.	10	−14 46	−14·8	66 08	204 20	Erebus.	26	−25 43	−25·7
38 12	179 51	Erebus.	13	−14 23	−14·4	66 02	204 00	Terror.	18	−26 39	−26·7
39 10	182 43	Erebus.	13	−14 31	−14·5	65 58	204 13	Erebus.	24	−25 24	−25·4
39 29	182 42	Terror.	11	−15 44	−15·7	65 58	203 54	Terror.	11	−25 04	−25·1
40 53	183 17	Erebus.	11	−12 36	−12·6	65 58	203 49	Terror.	17	−25 18	−25·3
41 33	183 30	Terror.	15	−14 37	−14·6	65 58	203 28	Erebus.	17	−24 49	−24·8
42 02	183 31	Erebus.	11	−14 09	−14·1	66 00	202 45	Terror.	9	−26 11	−26·2
43 37	183 05	Terror.	4	−14 46	−14·8	65 47	202 13	Erebus.	6	−25 15§	−25·3
46 09	183 43	Erebus.	13	−16 13	−16·2	67 38	204 20	Erebus.	9	−27 47	−27·8
45 29	183 10	Terror.	2	−17 17	−17·3	67 39	204 08	Terror.	11	−28 18	−28·3
47 32	184 52	Erebus.	11	−15 21	−15·3	67 20	202 03	Erebus.	8	−27 13	−27·2
47 27	184 48	Terror.	9	−14 49	−14·8	67 15	202 16	Terror.	9	−27 51	−27·9
48 52	186 47	Erebus.	15	−16 02	−16·0	67 19	201 56	Erebus.	8	−28 53	−28·9
49 23	188 32	Terror.	7	−16 41	−16·7	67 20	201 40	Terror.	11	−28 34	−28·6
49 29	189 00	Erebus.	8	−17 27	−17·5	68 28	200 16	Erebus.	14	−30 09	−30·1
50 02	191 28	Erebus.	12	−18 10	−18·2	68 31	199 59	Terror.	10	−33 08	−33·1
49 57	191 10	Terror.	7	−16 24	−16·4	68 46	199 47	Erebus.	13	−31 40	−31·7
50 51	192 32	Erebus.	6	−17 54	−17·9	68 36	199 43	Terror.	10	−30 59	−31·0
50 53	192 30	Terror.	8	−16 20	−16·3	70 13	186 20	Erebus.	9	−35 23	−35·4
51 39	194 50	Erebus	18	−15 33	−15·5	70 30	185 25	Terror.	23	−38 13	−38·2
51 55	195 28	Terror.	9	−15 25	−15·4	70 33	185 21	Erebus.	11	−38 40	−38·7
52 43	202 14	Erebus.	10	−13 40	−13·7	70 22	184 32	Erebus.	10	−37 34	−37·6
53 07	205 21	Erebus.	12	−12 44	−12·7	70 13	183 52	Erebus.	11	−35 52	−35·9
53 08	205 11	Terror.	10	−14 20	−14·3	70 14	184 00	Terror.	17	−36 54	−36·9
54 54	209 24	Erebus.	8	−14 09	−14·1	71 04	180 46	Terror.	5	−40 46	−40·8

* Declinometers at the Observatory.

† Magnetometers on shore.

‡ Bay of Islands.

§ Observed on ice.

General Table of Declinations (continued).

Position.		Ship.	No. of observations.	Declination.		Position.		Ship.	No. of observations.	Declination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
72° 10′	180° 58′	Erebus.	1	−45° 43′	−45°·7	60° 20′	205° 12′	Terror.	4	−18° 02′	−18°·0
73 14	181 08	Terror.	2	−50 11	−50·2	60 16	212 59	Erebus.	4	−17 03	−17·1
75 06	173 14	Erebus.	3	−77 41	−77·7	60 05	213 52	Terror.	6	−17 05	−17·1
75 40	174 56	Terror.	5	−79 09	−79·1	58 59	220 29	Erebus.	6	−15 16	−15·3
76 48	182 33	Erebus.	6	−86 51	−86·9	58 50	222 00	Terror.	4	−15 34	−15·6
76 54	182 17	Terror.	4	−79 15	−79·3	59 04	229 00	Erebus.	4	−17 23	−17·4
76 39	194 04	Terror.	10	−79 22	−79·4	58 58	227 00	Terror.	1	−16 31	−16·5
76 40	194 37	Erebus.	10	−79 34	−79·6	60 14	236 32	Erebus.	3	−20 34	−20·6
77 44	198 07	Erebus.	10	−87 15	−87·3	60 14	237 30	Terror.	4	−20 44	−20·7
78 03	197 31	Erebus.	10	−88 40	−88·7	59 17	245 40	Erebus.	1	−20 11	−20·2
77 54	198 08	Terror.	9	−87 04	−87·1	58 40	251 52	Erebus.	3	−21 30	−21·5
74 45	193 57	Erebus.	6	−60 03	−60·1	58 28	251 40	Terror.	4	−22 28	−22·5
74 44	193 52	Terror.	4	−59 14	−59·2	58 41	254 59	Erebus.	4	−23 14	−23·2
71 56	186 36	Erebus.	2	−44 47	−44·8	58 36	255 20	Terror.	7	−24 27	−24·5
71 08	184 54	Erebus.	5	−38 20	−38·3	58 46	258 07	Erebus.	3	−25 11	−25·2
70 58	184 03	Terror.	3	−37 29	−37·5	58 48	259 04	Terror.	4	−27 04	−27·1
69 50	180 16	Erebus.	12	−31 04	−31·1	59 01	268 34	Erebus.	9	−26 19	−26·3
70 10	180 20	Terror.	3	−30 53	−30·9	59 03	270 00	Terror.	11	−26 11	−26·2
68 07	183 42	Erebus.	12	−27 41	−27·7	59 02	272 04	Erebus.	1	−26 57	−26·9
68 01	183 35	Terror.	4	−28 45	−28·7	58 51	276 04	Erebus.	5	−26 24	−26·4
67 25	186 42	Erebus.	4	−27 08	−27·1	58 55	276 26	Terror.	7	−28 46	−28·8
67 30	185 00	Terror.	7	−29 24	−29·4	58 20	280 27	Erebus.	5	−25 17	−25·3
67 19	187 25	Terror.	2	−23 47	−23·8	58 21	279 48	Terror.	10	−27 30	−27·5
65 07	192 24	Erebus.	7	−24 04	−24·1	58 30	282 05	Erebus.	8	−26 17	−26·3
65 06	191 55	Terror.	4	−25 54	−25·9	58 30	282 00	Terror.	10	−27 22	−27·4
63 33	194 53	Erebus.	1	−21 06	−21·1	58 30	282 30	Terror.	5	−27 18	−27·3
62 20	196 15	Erebus.	5	−19 54	−19·9	58 32	283 40	Erebus.	7	−26 46	−26·8
62 26	195 40	Terror.	4	−19 55	−19·9	58 14	286 11	Terror.	4	−25 52	−25·9
60 57	199 31	Erebus.	8	−18 43	−18·7	56 46	294 30	Terror.	2	−20 50	−20·8
61 00	199 00	Terror.	4	−19 34	−19·6	52 16	301 06	Erebus.	6	−16 41	−16·7
60 29	203 40	Erebus.	4	−17 13	−17·2	52 14	301 09	Terror.	4	−18 12	−18·2

The small letter, *f*, refers to the Subsidiary Table which follows.

Subsidiary Table; comprising "Groups" in Latitudes exceeding 75° S.

Group.	Lat. S.	Long. E.	Ship.	No. of observations.	Declination.	Group.	Lat. S.	Long. E.	Ship.	No. of observations.	Declination.
a	76° 50′	173° 22′	Erebus.	21	96°·2	*d*	77° 51′	188° 59′	Erebus.	25	95°·3
a	76 30	171 03	Terror.	22	103·3	*d*	77 39	187 00	Terror.	9	93·7
b	77 16	188 34	Erebus.	21	86·4	*d*	77 11	186 44	Erebus.	15	87·1
b	77 36	187 40	Terror.	24	92·3	*e*	76 32	165 44	Erebus.	28	115·2
b	77 44	186 43	Erebus.	27	94·7	*e*	76 20	166 29	Terror.	15	112·2
c	77 16	192 45	Erebus.	19	82·1	*e*	76 00	167 15	Erebus.	16	97·6
c	77 02	190 47	Terror.	22	82·1	*f*	76 13	178 12	Terror.	9	79·2
c	77 06	188 47	Erebus.	21	83·1	*f*	76 28	187 26	Erebus.	19	81·6
d	77 19	186 41	Terror.	19	90·1	*f*	77 15	196 00	Terror.	19	83·0
d	77 19	186 44	Erebus.	22	90·0	*f*	77 53	197 49	Erebus.	20	88·0

General Table of the Declinations observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between September 1842 and April 1843.

Position.		Ship.	No. of observations.	Declination.		Position.		Ship.	No. of observations.	Declination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
° ′	° ′			° ′	°	° ′	° ′			° ′	°
51 32	301 53	Erebus.	1	−17 36	−17·6*	64 09	303 03	Erebus.	4	−21 32	−21·5
54 10	305 35	Terror.	2	−16 02	−16·0	64 38	303 30	Terror.	5	−22 50	−22·8
53 55	304 19	Erebus.	3	−18 18	−18·3	64 42	303 20	Terror.	2	−24 08	−24·1
53 54	304 25	Terror.	3	−19 55	−19·9	64 30	303 00	Erebus.	3	−23 00	−23·0
55 07	300 19	Terror.	2	−21 39	−21·7	64 44	303 12	Erebus.	8	−22 03	−22·1
55 41	296 41	Terror.	3	−23 54	−23·9	64 44	303 10	Erebus.	3	−21 13	−21·2‡
56 07	292 53	Terror.	4	−22 21	−22·3	64 41	302 52	Terror.	3	−23 52	−23·9§
55 39	295 23	Erebus.	4	−24 03	−24·1	64 40	303 06	Erebus.	5	−23 32	−23·5
55 42	295 20	Terror.	6	−24 50	−24·8	64 40	302 07	Terror.	2	−24 37	−24·6
55 33	299 12	Erebus.	8	−21 29	−21·5	64 38	302 40	Terror.	2	−24 07	−24·1†
55 29	299 03	Terror.	11	−24 06	−24·1	64 39	302 36	Erebus.	5	−23 04	−23·1
54 32	299 53	Terror.	5	−22 04	−22·1	64 41	302 36	Terror.	6	−24 30	−24·5
54 53	299 59	Erebus.	5	−21 10	−21·2	64 39	302 36	Erebus.	1	−22 14	−22·2†
52 43	301 12	Terror.	12	−21 15	−21·3	64 36	302 38	Erebus.	7	−23 11	−23·2
53 04	300 51	Erebus.	1	−19 48	−19·8	64 31	302 36	Erebus.	4	−23 37	−23·6
52 04	302 47	Erebus.	5	−18 03	−18·1	64 32	302 55	Terror.	4	−23 31	−23·5
52 46	303 12	Terror.	4	−19 41	−19·7	64 25	304 46	Erebus.	8	−21 17	−21 3
52 50	303 12	Erebus.	6	−18 29	−18·5	64 03	305 10	Terror.	8	−22 23	−22·4
53 50	303 49	Terror.	2	−20 22	−20·4	64 20	306 00	Terror.	5	−21 30	−21·5
53 56	303 52	Erebus.	6	−17 35	−17·6	64 19	304 20	Erebus	8	−21 49	−21·8
55 45	305 17	Terror.	6	−20 12	−20·2	64 17	304 42	Terror.	4	−21 49	−21·8
55 46	305 17	Erebus.	7	−18 47	−18·8	64 14	303 50	Erebus.	11	−20 56	−20·9
56 36	306 38	Erebus.	5	−17 47	−17·8	64 09	304 10	Terror.	5	−22 30	−22·5
56 54	306 41	Terror.	4	−21 09	−21·1	64 04	304 18	Erebus.	11	−20 49	−20·8
56 28	306 45	Erebus.	2	−17 59	−18·0	64 05	303 55	Terror.	6	−22 27	−22·5
58 29	308 13	Erebus.	7	−18 03	−18·1	64 08	304 12	Erebus.	4	−21 07	−21·1
58 25	308 00	Terror.	7	−20 06	−20·1	64 04	303 58	Erebus.	6	−21 37	−21·6
59 28	308 20	Terror.	3	−21 29	−21·5	64 00	304 22	Terror.	5	−23 02	−23·0
59 34	308 28	Erebus.	5	−17 56	−17·9	64 08	303 47	Erebus.	12	−21 53	−21·9
62 00	307 52	Terror.	2	−20 22	−20·4	64 09	303 57	Terror.	7	−22 13	−22·2
62 18	308 03	Erebus.	5	−18 24	−18·4	63 53	304 51	Erebus.	8	−20 36	−20·6
62 31	308 05	Terror.	2	−23 50	−23·8	64 12	305 20	Terror.	10	−21 56	−21·9
62 20	308 12	Terror.	5	−22 43	−22·7	64 16	305 23	Erebus.	4	−20 37	−20·6
62 54	305 41	Erebus.	8	−20 16	−20·3	64 10	309 30	Terror.	10	−19 58	−20·0
62 39	306 12	Terror.	22	−21 41	−21·7	64 44	315 41	Erebus.	4	−14 00	−14·0
63 47	304 52	Erebus.	5	−22 19	−22·3	64 37	315 51	Erebus.	7	−13 51	−13·9
63 40	304 45	Terror.	1	−21 15	−21·3	64 31	316 22	Erebus.	3	−13 49	−13·8
63 51	304 24	Terror.	5	−23 04	−23·1	64 41	316 00	Terror.	9	−16 25	−16·4
64 14	304 21	Erebus.	5	−20 56	−20·9	64 50	316 41	Terror.	10	−15 32	−15·5
64 17	304 17	Terror.	7	−21 34	−21·6	64 40	316 50	Erebus.	12	−13 46	−13·8
64 26	303 52	Erebus.	5	−22 51	−22·9	65 04	318 29	Terror.	8	−14 27	−14·5
64 26	304 20	Erebus.	2	−20 50	−20·8†	65 06	318 20	Erebus.	6	−13 20	−13·3
64 35	303 47	Erebus.	6	−23 51	−23·9	65 13	319 20	Erebus.	3	−12 47	−12·8
64 33	304 05	Terror.	13	−22 23	−22·4	63 58	321 43	Terror.	7	−10 25	−10·4
64 31	304 28	Erebus.	7	−21 50	−21·8	63 56	321 58	Erebus.	10	− 9 20	− 9·3
64 11	303 50	Terror.	9	−22 11	−22·2	62 38	328 00	Terror.	8	− 7 37	− 7·6
64 20	304 34	Erebus.	7	−20 08	−20·1	62 50	328 20	Erebus.	3	− 7 54	− 7·9
64 04	304 41	Erebus.	4	−20 44	−20·7	62 37	328 30	Erebus.	7	− 2 50	− 2·8
64 20	303 12	Terror.	7	−24 02	−24·0	62 16	330 30	Terror.	3	− 6 30	− 6·5
64 20	304 28	Erebus.	3	−19 19	−19·3	62 20	330 30	Erebus.	8	− 4 41	− 4·7

* Magnetometers on shore at Port Louis.

† On ice.

‡ On ice: mean of 3 of Kater's compasses.

§ On ice: mean of 3 compasses.

General Table of the Declinations (continued).

Position.		Ship.	No. of observations.	Declination.		Position.		Ship.	No. of observations.	Declination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
62° 06′	333° 43′	Erebus.	8	− 3° 41′	− 3·7	61° 16′	349° 00′	Terror.	4	+ 7° 16′	+ 7·3
62 09	332 58	Terror.	15	− 2 38	− 2·6	57 46	351 52	Erebus.	5	+11 17	+11·3
61 55	333 48	Erebus.	16	− 3 38	− 3·6	57 30	351 40	Terror.	9	+10 36	+10·6
61 36	336 20	Erebus.	10	− 0 44	− 0·7	57 22	352 12	Erebus.	7	+13 10	+13·2
61 32	335 33	Terror.	6	− 0 44	− 0·7	57 16	352 54	Terror.	12	+11 24	+11·4
62 24	343 58	Erebus.	8	+ 4 40	+ 4·7	57 10	352 53	Erebus.	8	+12 28	+12·5
62 18	343 44	Terror.	12	+ 3 39	+ 3·7	57 04	352 52	Erebus.	4	+14 57	+14·9
62 52	344 33	Erebus.	7	+ 4 40	+ 4·7	56 42	353 40	Terror.	2	+11 22	+11·4
64 04	345 07	Terror.	4	+ 4 17	+ 4·3	56 34	353 50	Erebus.	3	+13 46	+13·8
64 04	345 16	Erebus.	4	+ 5 07	+ 5·1	55 56	355 38	Erebus.	4	+13 19	+13·3
64 24	349 04	Terror.	4	+ 4 46	+ 4·8	55 58	355 30	Terror.	3	+12 59	+13·0
65 08	349 50	Erebus.	7	+ 7 02	+ 7·0	54 28	357 45	Erebus.	2	+16 48	+16·8
65 01	349 04	Terror.	8	+ 5 58	+ 6·0	54 30	357 50	Terror.	1	+13 52	+13·9
66 00	351 00	Terror.	6	+ 8 12	+ 8·2	54 06	359 38	Erebus.	6	+17 11	+17·2
66 01	353 00	Erebus.	5	+ 9 06	+ 9·1	54 18	359 38	Terror.	8	+14 49	+14·8
67 12	350 36	Terror.	6	+ 7 08	+ 7·1	50 00	9 35	Erebus.	4	+24 12	+24·2
67 04	350 39	Erebus.	7	+ 8 56	+ 8·9	48 12	10 29	Terror.	6	+24 15	+24·3
68 00	348 21	Erebus.	7	+ 6 34	+ 6·6	47 50	10 54	Erebus.	5	+25 27	+25·5
68 08	348 10	Terror.	13	+ 4 15	+ 4·3	43 36	13 47	Terror.	7	+26 59	+27·0
68 18	347 20	Erebus.	7	+ 5 37	+ 5·6	43 49	13 38	Erebus.	3	+29 28	+29·5
68 32	347 09	Erebus.	2	+ 4 43	+ 4·7	43 15	14 40	Erebus.	11	+28 25	+28·4
68 30	346 50	Terror.	6	+ 4 15	+ 4·3	41 51	15 03	Terror.	2	+27 29	+27·5
69 36	345 18	Erebus.	4	+ 3 05	+ 3·1	41 43	15 10	Erebus.	7	+29 01	+29·0
69 42	345 20	Terror.	7	+ 1 53	+ 1·9	39 52	16 04	Terror.	5	+26 53	+26·9
70 55	343 30	Erebus.	6	+ 2 24	+ 2·4	39 43	15 45	Erebus.	5	+28 29	+28·5
70 43	343 12	Terror.	11	+ 2 23	+ 2·4	38 26	16 39	Terror	4	+27 44	+27·7
70 24	341 56	Terror.	3	− 2 06	− 2·1	37 50	16 35	Erebus.	2	+29 28	+29·5
70 33	343 18	Erebus.	2	+ 3 23	+ 3·4	36 15	16 31	Terror.	3	+28 48	+28·8
56 10	346 40	Terror.	3	+ 5 35	+ 5·6	35 50	16 29	Erebus.	11	+30 02	+30·0
64 29	346 02	Erebus.	4	+ 4 25	+ 4·4	35 26	16 14	Terror.	3	+28 04	+28·1
64 06	346 15	Terror.	7	+ 5 29	+ 5·5	35 01	17 08	Erebus.	5	+30 32	+30·5
61 16	348 56	Erebus.	4	+ 8 59	+ 9·0						

General Table of the Declinations observed on board Her Majesty's hired Bark 'Pagoda.'

Lat. S.	Long. E.	No. of observations.	Declination.		Lat. S.	Long. E.	No. of observations.	Declination.	
			In degrees and minutes.	In degrees and decimals.				In degrees and minutes.	In degrees and decimals.
° ′	° ′		° ′	°	° ′	° ′		° ′	°
34 42	17 36	4	+29 51	+29·9	38 40	116 17	4	+ 6 56	+ 6·9
35 26	15 08	4	+28 39	−28·7	36 51	116 36	3	+ 4 31	+ 4·5
35 17	14 00	3	+27 15	+27·3	36 10	116 43	8	+ 4 52	+ 4·9
35 10	13 25	3	+25 40	+25·7	35 12	117 41	5	+ 6 20	+ 6·3
38 43	14 25	1	+25 09	+25·1	35 28	117 04	1	+ 6 55	+ 6·9
39 18	14 28	11	+28 20	+28·3	35 02	117 56	3	+ 5 33*	+ 5·5
40 15	14 35	11	+27 40	+27·7	35 10	118 06	1	+ 5 57	+ 5·9
44 45	13 19	4	+26 34	+26·6	35 42	115 40	1	+ 5 59	+ 6·0
46 24	13 34	3	+25 54	+25·9	35 33	114 42	3	+ 5 41	+ 5·7
48 27	10 51	4	+24 50	−24·8	34 18	113 12	4	+ 6 37	+ 6·6
50 45	10 18	8	+23 55	+23·9	32 42	111 43	5	+ 6 36	+ 6·6
51 47	9 34	17	+23 37	+23·6	30 25	109 07	4	+ 7 20	+ 7·3
52 56	7 53	4	+23 46	+23·8	29 20	106 55	2	+ 6 30	+ 6 5
53 52	6 12	12	+21 34	+21·6	27 41	106 35	2	+ 6 33	+ 6·5
55 29	5 54	3	+21 23	+21·4	26 10	105 16	3	+ 5 30	+ 5·5
59 02	4 09	4	+17 30	+17·5	24 07	102 28	2	+ 5 32	+ 5·5
61 12	9 30	10	+20 29	+20·5	23 58	99 21	4	+ 5 14	+ 5·2
62 03	12 45	4	+22 07	+22·1	24 01	97 34	4	+ 7 08	+ 7·1
61 54	16 40	8	+23 11	+23·2	23 56	95 46	6	+ 6 10	+ 6·2
61 49	19 13	27	+26 16	+26·3	24 17	94 06	4	+ 5 31	+ 5·5
62 05	20 58	9	+28 05	+28·1	24 05	92 11	1	+ 6 34	+ 6·6
63 18	21 10	6	+28 56	+28·9	22 46	90 40	9	+ 5 56	+ 5·9
64 25	25 05	9	+30 24	+30·4	21 53	89 42	2	+ 4 23	+ 4·4
65 43	28 33	11	+31 37	+31·6	20 46	88 06	5	+ 4 45	+ 4·7
66 26	37 25	3	+35 39	+35·7	20 37	85 32	3	+ 5 20	+ 5·3
66 55	38 32	5	+37 43	+37·7	20 25	82 32	5	+ 5 08	+ 5·1
67 34	39 41	7	+38 13	+38·2	20 36	79 20	7	+ 4 43	+ 4·7
66 41	39 22	3	+37 18	+37·3	20 44	78 34	21	+ 5 29	+ 5·5
67 06	40 03	5	+36 59	+37·0	20 39	77 45	3	+ 5 22	+ 5·4
67 01	40 30	2	+37 12	+37·2	20 28	76 23	3	+ 6 01	+ 6·0
64 52	38 37	5	+36 38	+36·6	20 45	73 20	1	+ 5 54	+ 5·9
64 52	40 12	6	+36 54	+36·9	20 27	70 49	2	+ 6 35	+ 6·6
64 22	40 49	2	+36 32	+36·5	20 34	69 37	2	+ 6 07	+ 6·1
63 57	41 37	3	+37 34	+37·6	20 58	68 12	5	+ 8 00	+ 8·0
63 22	43 31	6	+39 39	+39·7	21 11	67 54	3	+ 6 34	+ 6·6
63 37	47 01	7	+40 03	+40·1	21 12	67 29	2	+ 7 37	+ 7·6
63 43	49 29	2	+39 21	+39·3	21 01	66 24	3	+ 7 46	+ 7·8
61 33	53 40	6	+40 30	+40·5	20 39	63 01	2	+ 8 27	+ 8·5
61 18	57 41	6	+41 57	+41·9	20 30	59 42	3	+ 9 44	+ 9·7
61 04	63 45	4	+45 17	+45·3	19 54	57 55	1	+ 9 27	+ 9·5
61 43	69 36	1	+45 51	+45·9	20 09	57 31	2	+ 9 44†	+ 9·7
62 10	72 25	7	+46 01	+46·0	20 50	55 32	2	+11 15	+11·3
62 46	76 30	11	+50 35	+50·6	21 54	53 00	4	+13 44	+13·7
63 05	80 20	1	+52 17	+52·3	23 44	51 48	3	+14 22	+14·4
61 41	85 20	7	+47 19	+47·3	25 47	49 40	2	+15 09	+15·1
60 50	87 41	3	+47 47	+47·8	26 30	49 20	3	+16 23	+16·4
61 23	91 26	10	+49 28	+49·5	27 12	46 02	5	+20 25	+20·4
61 16	91 43	7	+49 02	+49·0	28 19	43 07	3	+21 19	+21·3
60 46	92 20	6	+48 01	+48·0	28 49	42 07	4	+21 57	+21·9
60 03	95 15	4	+44 53	+44·9	28 36	40 14	5	+22 34	+22·6
59 22	100 31	7	+41 02	+41·0	28 57	37 49	7	+23 37	+23·6
58 31	98 59	1	+39 50	+39·8	30 18	35 55	2	+26 29	+26·5
58 30	98 32	3	+40 37	+40·6	30 30	33 42	6	+27 28	+27·5
56 53	101 15	2	+37 37	+37·6	30 35	33 13	1	+25 09	+25·1
55 42	103 15	3	+32 54	+32·9	31 09	31 31	9	+26 46	+26·8
54 45	106 17	6	+29 24	+29·4	32 54	29 49	4	+28 44	+28·7
54 05	108 15	2	+26 34	+26·6	34 44	26 50	9	+28 41	+28·7
53 12	110 24	6	+21 52	+21·9	35 39	23 35	5	+29 26	+29·4
49 05	112 47	10	+17 09	+17·1	35 40	21 37	2	+29 16	+29·3
46 32	115 54	1	+12 02	+12·0	35 06	20 46	1	+27 56	+27·9
44 59	116 53	2	+ 9 43	+ 9·7	35 08	20 24	2	+28 47	+28·8
43 41	116 57	6	+ 7 04	+ 7·1	34 55	19 33	3	+28 57	+28·9
41 02	116 42	3	+ 4 10	+ 4·2	34 12	18 27	16	+29 15	+29·3

* On shore at King George's Sound.

† On shore at Port Louis, Mauritius.

General Table of the Inclinations observed in Her Majesty's Ship 'Erebus' with Needle F 1, from the Cape of Good Hope to Kerguelen Island, and from Kerguelen Island to Hobarton, between April 1840 and August of the same year.

Position.		No. of observations.	Inclination.		Position.		No. of observations.	Inclination.	
Lat. S.	Long. E.		In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.		In degrees and minutes.	In degrees and decimals.
35° 14′	18° 27′	2	−54° 32′	−54°·5	47° 40′	62° 25′	2	−69° 37′	−69°·6
35 48	18 47	4	−54 18	−54·3	48 32	69 24	6	−70 59	−71·0
36 00	19 00	2	−55 24	−55·4	48 41	68 54	9	−70 00	−70·0*
36 07	20 55	2	−55 50	−55·8	48 29	76 55	2	−71 08	−71·1
36 29	21 16	4	−55 38	−55·6	48 17	80 15	2	−72 09	−72·1
37 19	21 37	5	−56 03	−56·1	47 55	83 00	2	−72 49	−72·8
38 11	22 00	2	−55 35	−55·6	47 46	86 18	2	−73 48	−73·8
40 05	20 38	2	−56 33	−56·5	47 12	89 45	2	−73 51	−73·9
40 29	22 22	2	−57 28	−57·5	47 03	93 00	2	−74 53	−74·9
41 24	25 00	4	−58 21	−58·3	47 39	102 42	2	−74 46	−74·8
41 47	26 38	2	−58 43	−58·7	47 35	106 26	2	−74 48	−74·8
43 04	28 40	5	−59 34	−59·6	47 45	110 39	2	−75 25	−75·4
44 19	31 06	2	−60 52	−60·9	47 34	114 15	2	−75 33	−75·5
45 40	34 08	2	−62 23	−62·4	47 41	121 30	2	−76 22	−76·4
46 59	37 14	2	−64 00	−64·0	47 34	124 43	2	−76 58	−77·0
47 00	38 14	2	−64 30	−64·5	46 44	128 26	2	−75 58	−76·0
46 46	42 23	3	−65 47	−65·8	46 13	132 00	2	−75 35	−75·6
47 01	46 10	2	−66 36	−66·6	45 59	135 38	2	−74 05	−74·1
46 41	50 52	2	−67 41	−67·7	45 17	139 19	2	−73 38	−73·6
46 17	52 04	2	−67 10	−67·2	44 24	141 39	2	−72 55	−72·9
46 25	52 01	4	−67 30	−67·5	44 16	142 38	2	−73 19	−73·3
46 57	55 34	2	−68 12	−68·2	42 52	147 24	12	−70 38	−70·6†
47 19	59 10	4	−68 42	−68·7					

* On shore at Kerguelen Island. † On shore at Van Diemen Island.

General Table of the Observations of the Inclination between the Cape of Good Hope and Van Diemen Island, by Lieut. Alexander Smith, R.N., 1844.

Lat. S.	Long. E.	Inclination, in degrees and minutes.	In degrees, and decimals of a degree.	Lat. S.	Long. E.	Inclination, in degrees and minutes.	In degrees, and decimals of a degree.
38° 00′	4° 20′	−52° 00′	−52°·0	38° 46′	77° 50′	−67° 14′	−67°·2
38 28	7 45	−53 03	−53·1	39 04	79 45	−67 17	−67·3
39 06	12 00	−55 42	−55·7	39 58	84 00	−67 43	−67·7
39 42	15 44	−57 06	−57·1	40 06	87 00	−68 13	−68·2
39 33	23 05	−59 32	−59·5	40 06	90 52	−68 41	−68·7
39 33	26 52	−61 47	−61·8	40 02	95 10	−69 08	−69·1
39 50	28 36	−62 08	−62·1	39 52	99 22	−69 00	−69·0
40 01	32 22	−62 56	−62·9	39 54	102 00	−69 00	−69·0
40 32	36 40	−64 09	−64·1	40 08	105 55	−69 22	−69·4
41 06	41 40	−64 42	−64·7	40 31	109 06	−70 02	−70·0
41 00	46 13	−65 19	−65·3	41 16	113 25	−70 30	−70·5
40 43	49 12	−66 08	−66·1	41 54	117 40	−71 20	−71·3
40 56	53 30	−66 25	−66·4	41 58	119 00	−71 33	−71·5
39 34	60 55	−67 27	−67·5	42 17	122 30	−71 45	−71·7
39 00	65 44	−67 19	−67·3	42 35	125 40	−72 08	−72·1
38 31	68 45	−66 45	−66·7	43 00	129 36	−72 08	−72·1
38 22	70 10	−67 06	−67·1	43 16	133 44	−72 13	−72·2
38 08	73 35	−66 45	−66·7	43 28	137 10	−71 55	−71·9
38 10	75 22	−66 42	−66·7	44 06	141 37	−72 14	−72·2
38 25	76 44	−66 54	−66·9				

General Table of the Observations of the Inclination between Van Diemen Island and the Cape of Good Hope, by Lieut. JOSEPH DAYMAN, R.N., 1844 and 1845.

Lat. S.	Long. E.	Inclination, in degrees and minutes.	In degrees and decimals of a degree.	Lat. S.	Long. E.	Inclination, in degrees and minutes.	In degrees and decimals of a degree.
° ′	° ′	° ′	°	° ′	° ′	° ′	°
Hobarton Observatory.		−70 40	−70·7	23 11	95 40	−53 44	−53·7
44 48	144 51	−71 36	−71·6	22 54	93 48	−53 26	−53·4
44 30	143 56	−70 50	−70·8	22 19	91 16	−53 11	−53·2
44 34	142 51	−71 31	−71·5	22 17	89 57	−53 11	−53·2
44 34	139 37	−73 11	−73·2	22 11	86 30	−53 20	−53·3
43 21	138 37	−70 33	−70·5	22 08	84 17	−53 52	−53·9
42 24	137 18	−70 42	−70·7	22 34	80 10	−54 23	−54·4
41 46	133 26	−70 40	−70·7	22 35	78 08	−54 26	−54·4
41 24	129 30	−70 22	−70·4	22 38	76 10	−54 45	−54·7
42 08	128 45	−71 11	−71·2	22 28	74 18	−54 41	−54·7
40 05	128 23	−70 08	−70·1	22 33	72 00	−55 04	−55·1
39 25	124 04	−69 40	−69·7	22 41	69 54	−55 54	−55·9
38 00	123 38	−68 21	−68·3	23 52	64 59	−56 52	−56·9
38 21	122 46	−67 27	−67·5	24 23	62 54	−57 57	−57·9
37 52	122 29	−67 39	−67·7	24 50	61 11	−58 15	−58·3
37 14	121 58	−67 07	−67·1	24 43	59 46	−57 59	−58·0
37 13	120 40	−68 06	−68·1	24 36	58 37	−58 17	−58·3
36 28	118 57	−67 08	−67·1	24 45	57 03	−59 13	−59·2
35 22	117 46	−65 53	−65·9	25 13	51 29	−58 34	−58·6
36 42	118 35	−67 03	−67·1	25 42	49 06	−58 43	−58·7
36 58	117 38	−67 19	−67·3	26 54	45 47	−58 41	−58·7
36 06	116 42	−66 27	−66·5	28 15	42 18	−59 37	−59·6
36 24	115 33	−66 44	−66·7	29 21	39 06	−60 01	−60·0
37 00	115 10	−66 35	−66·6	30 10	36 17	−59 08	−59·1
35 46	114 14	−65 48	−65·8	31 19	32 21	−58 32	−58·5
34 58	112 59	−65 16	−65·3	32 17	29 34	−58 16	−58·3
33 47	111 04	−63 23	−63·4	34 02	26 53	−57 23	−57·4
32 37	108 24	−62 18	−62·3	34 35	25 31	−56 45	−56·7
29 40	105 28	−59 37	−59·6	34 36	25 23	−56 35	−56·6
28 04	105 06	−58 33	−58·5	34 40	24 16	−56 32	−56·5
26 44	104 26	−56 54	−56·9	34 04	22 45	−56 43	−56·7
25 52	102 58	−55 50	−55·8	34 29	21 44	−55 36	−55·6
24 50	101 31	−54 44	−54·7	34 48	19 33	−54 50	−54·8
24 00	99 33	−54 20	−54·3				

General Table of the Magnetic Inclinations observed in Her Majesty's Ships 'Erebus' and 'Terror,' between November 13, 1840 and April 6, 1841.

Ship.	Lat. S.	Long. E.	No. of observations.	Inclination.	In degrees, and decimals of a degree.	Ship.	Lat. S.	Long. E.	No. of observations.	Inclination.	In degrees, and decimals of a degree.
Erebus.	44° 16′	149° 29′	3	−70° 56′	−70°·9	Erebus.	70° 23′	174° 50′	4	−85° 02′	−85°·0
Erebus.	45 13	151 57	3	−72 03	−72·1	Terror.	71 15	171 38	1	−85 52	−85·9
Erebus.	45 33	152 45	3	−71 53	−71·9	Erebus.	71 20	171 00	8	−85 52	−85·9
Erebus.	46 18	154 30	3	−72 19	−72·3	Erebus.	71 47	170 52	3	−86 43	−86·7
Erebus.	47 46	157 40	3	−73 29	−73·5	Terror.	71 54	171 00	1	−85 26	−85·4
Erebus.	49 20	160 13	3	−74 32	−74·5	Erebus.	72 07	172 19	4	−86 43	−86·7
Terror.	50 28	164 09	3	−74 40	−74·7	Terror.	72 12	171 47	1	−86 12	−86·2
Erebus.	50 33	166 19	3	−73 24*	−73·4	Erebus.	71 53	172 00	6	−86 26	−86·4
Erebus.	52 34	169 10	8	−74 20†	−74·3	Terror.	71 54	172 56	1	−86 19	−86·3
Erebus.	54 06	169 09	4	−74 57	−74·9	Terror.	71 52	171 20	1	−86 42	−86·7
Terror.	55 40	169 38	1	−76 23	−76·4	Erebus.	72 10	172 54	7	−86 42	−86·7
Erebus.	55 50	170 06	3	−76 23	−76·4	Terror.	72 12	171 49	1	−86 47	−86·8
Erebus.	57 35	170 32	6	−77 55	−77·9	Terror.	72 08	173 39	1	−86 15	−86·3
Terror.	58 54	170 50	1	−78 25	−78·4	Erebus.	72 57	176 06	5	−86 40	−86·7
Erebus.	58 57	170 57	4	−78 09	−78·1	Terror.	72 54	176 27	1	−86 21	−86·3
Terror.	59 38	169 51	1	−78 56	−78·9	Erebus.	72 33	173 36	8	−87 00	−87·0
Erebus.	59 44	169 40	6	−78 47	−78·8	Terror.	72 32	173 10	1	−87 18	−87·3
Erebus.	60 14	170 15	3	−78 59	−79·0	Erebus.	73 58	171 11	15	−87 30	−87·5
Erebus.	60 38	170 38	8	−79 09	−79·1	Terror.	73 45	171 41	2	−87 22	−87·4
Erebus.	61 34	170 40	3	−79 36	−79·6	Erebus.	73 56	171 35	8	−87 14	−87·2
Erebus.	62 07	172 54	4	−79 49	−79·8	Terror.	74 15	171 20	2	−87 27	−87·5
Terror.	62 35	174 54	1	−80 02	−80·0	Terror.	74 05	174 48	1	−86 11	−86·2
Erebus.	62 40	173 40	1	−80 17	−80·3	Erebus.	74 34	173 23	7	−87 36	−87·6
Erebus.	62 46	174 34	6	−80 15	−80·3	Terror.	74 29	173 30	1	−86 58	−87·0
Terror.	62 57	174 42	1	−80 26	−80·4	Erebus.	74 43	169 09	9	−88 13	−88·2
Erebus.	64 04	172 41	9	−81 10	−81·2	Terror.	75 03	169 01	2	−88 02	−88·0
Terror.	64 05	172 52	1	−81 36	−81·6	Erebus.	74 55	169 01	3	−88 40	−88·7
Terror.	64 18	172 29	1	−81 35	−81·6	Terror.	74 56	168 55	2	−87 58	−88·0
Erebus.	64 28	173 22	1	−81 20	−81·3	Erebus.	75 22	168 48	4	−88 40	−88·7
Erebus.	64 30	172 56	6	−81 19	−81·3	Terror.	75 36	168 27	3	−88 09	−88·1
Terror.	65 44	171 59	1	−82 26	−82·4	Erebus.	76 06	168 11	4	−88 50	−88·8
Erebus.	66 07	171 22	8	−82 27	−82·5	Terror.	77 09	170 57	2	−87 52	−87·9
Terror.	66 23	170 23	2	−83 30	−83·5	Erebus.	77 07	169 56	2	−88 29	−88·5
Erebus.	66 31	169 29	8	−82 41	−82·7	Terror.	77 54	176 28	2	−87 13	−87·2
Erebus.	66 01	170 28	8	−82 30	−82·5	Erebus.	77 44	178 18	6	−86 37	−86·6
Terror.	66 18	170 11	1	−82 50	−82·8	Terror.	77 45	178 21	2	−86 47	−86·8
Terror.	65 40	171 17	1	−81 34	−81·6	Terror.	76 56	188 35	3	−87 02	−87·0
Terror.	65 19	172 24	1	−82 00	−82·0	Erebus.	77 08	189 01	14	−86 28	−86·5
Erebus.	65 22	172 40	4	−82 06	−82·1	Terror.	77 33	186 10	2	−86 53	−86·9
Terror.	65 35	173 18	1	−83 32	−83·5	Erebus.	77 50	186 47	5	−86 28	−86·5
Erebus.	65 55	174 31	3	−82 33	−82·5	Erebus.	77 07	190 14	13	−86 22	−86·4
Terror.	67 22	174 42	1	−83 31	−83·5	Terror.	77 10	190 18	2	−86 25	−86·4
Erebus.	67 27	174 51	4	−83 24	−83·4	Erebus.	77 17	192 52	9	−86 15	−86·3
Terror.	68 08	175 18	1	−83 28	−83·5	Terror.	76 56	186 05	2	−86 25	−86·4
Erebus.	68 17	175 00	5	−83 29	−83·5	Erebus.	77 10	188 43	6	−86 12	−86·2
Terror.	68 29	175 30	2	−84 09	−84·1	Terror.	77 37	186 43	3	−86 25	−86·4
Erebus.	68 32	175 49	6	−83 56	−83·9	Erebus.	77 00	186 37	5	−86 32	−86·5
Terror.	68 25	176 25	1	−83 56	−83·9	Terror.	77 54	190 34	1	−86 27	−86·5
Erebus.	68 28	176 31	10	−83 39	−83·7	Erebus.	77 55	189 05	2	−86 06	−86·1
Erebus.	69 02	176 20	5	−83 48	−83·8	Terror.	77 42	188 10	1	−86 24	−86·4
Terror.	69 23	176 17	1	−84 38	−84·6	Erebus.	77 43	187 10	5	−86 16	−86·3
Terror.	70 10	174 33	1	−85 07	−85·1	Erebus.	77 35	186 52	5	−86 28	−86·5

* Auckland Island: observed on shore with needles whose poles were inverted.

† Campbell Island: observed on shore.

General Table of the Magnetic Inclinations, 'Erebus' and 'Terror,' 1840 and 1841 (continued).

Ship.	Lat. S.	Long. E.	No. of observations.	Inclination.	In degrees, and decimals of a degree.	Ship.	Lat. S.	Long. E.	No. of observations.	Inclination.	In degrees, and decimals of a degree.
	° ′	° ′		° ′			° ′	° ′		° ′	
Terror.	76 27	183 03	1	−86 52	−86·9	Terror.	64 21	148 43	1	−84 22	−84·4
Erebus.	76 22	188 05	4	−86 22	−86·4	Erebus.	64 33	148 03	4	−85 15	−85·3
Terror.	76 49	184 06	2	−87 14	−87·2	Terror.	65 19	144 14	7	−85 41	−85·7
Erebus.	76 50	183 26	3	−86 50	−86·8	Erebus.	65 12	143 45	12	−85 40	−85·7
Terror.	76 24	177 36	1	−87 12	−87·2	Erebus.	64 20	140 40	5	−84 58	−85·0
Erebus.	76 16	175 11	7	−87 25	−87·4	Terror.	64 32	140 46	1	−85 34	−85·6
Terror.	76 31	170 51	2	−87 47	−87·8	Erebus.	63 09	139 28	7	−85 06	−85·1
Erebus.	76 06	168 07	7	−88 25	−88·4	Terror.	62 54	139 10	1	−84 59	−85·0
Terror.	76 30	166 00	6	−88 56	−88·9	Erebus.	62 30	137 21	6	−84 40	−84·7
Erebus.	76 28	165 02	9	−88 39	−88·7	Terror.	62 11	135 52	1	−84 28	−84·5
Terror.	76 35	165 09	4	−88 33	−88·5	Erebus.	61 18	134 02	5	−84 14	−84·2
Erebus.	75 30	167 47	6	−88 26	−88·4	Terror.	61 40	133 26	1	−83 59	−84·0
Terror.	76 03	166 05	1	−88 25	−88·4	Erebus.	60 20	131 23	5	−83 43	−83·7
Terror.	75 03	168 27	2	−88 20	−88·3	Terror.	60 25	131 26	1	−83 22	−83·4
Erebus.	73 09	171 26	1	−87 48	−87·8	Erebus.	59 25	130 14	5	−83 04	−83·1
Terror.	71 10	172 08	1	−86 42	−86·7	Terror.	59 22	130 12	1	−83 10	−83·2
Erebus.	71 10	170 25	4	−86 27	−86·4	Terror.	58 02	128 40	1	−82 22	−82·4
Terror.	70 20	167 50	2	−86 18	−86·3	Erebus.	58 06	128 43	1	−82 18	−82·3
Erebus.	70 42	167 26	7	−86 12	−86·2	Terror.	57 25	127 35	1	−82 09	−82·1
Terror.	70 09	167 50	2	−86 10	−86·2	Erebus.	57 22	127 37	2	−82 02	−82·0
Erebus.	70 13	167 27	9	−86 24	−86·4	Terror.	56 20	129 50	1	−80 35	−80·6
Terror.	69 30	167 16	3	−85 43	−85·7	Erebus.	55 16	131 30	6	−80 32	−80·5
Erebus.	69 37	167 51	11	−85 57	−85·9	Erebus.	54 55	132 50	6	−80 17	−80·3
Terror.	68 55	168 07	2	−85 16	−85·3	Terror.	55 11	132 10	2	−79 27	−79·5
Erebus.	68 27	167 45	15	−85 28	−85·4	Erebus.	54 01	134 59	8	−79 52	−79·9
Terror.	66 01	165 51	3	−83 21	−83·3	Terror.	53 49	134 54	2	−79 56	−79·9
Erebus.	65 48	167 14	4	−83 22	−83·4	Erebus.	53 13	135 18	5	−79 15	−79·3
Terror.	65 32	162 07	3	−83 48	−83·8	Terror.	53 10	134 37	1	−79 47	−79·8
Erebus.	65 51	163 31	6	−84 15	−84·3	Erebus.	51 16	136 50	4	−78 08	−78·1
Terror.	64 37	162 46	6	−82 55	−82·9	Terror.	51 10	136 56	4	−77 44	−77·7
Erebus.	64 30	163 02	8	−83 06	−83·1	Erebus.	48 40	138 33	4	−77 04	−77·1
Terror.	64 17	163 22	3	−83 19	−83·3	Erebus.	49 14	139 05	1	−77 31	−77·5
Erebus.	63 52	160 55	6	−83 41	−83·7	Terror.	46 55	139 55	3	−75 57	−75·9
Terror.	63 43	160 15	2	−82 32	−82·5	Erebus.	46 29	140 40	4	−75 34	−75·6
Erebus.	62 41	156 59	5	−82 54	−82·9	Terror.	44 59	143 12	1	−73 30	−73·5
Terror.	62 42	157 05	1	−82 15	−82·3	Erebus.	46 22	141 06	4	−75 24	−75·4
Erebus.	64 10	154 23	15	−84 15	−84·3	Erebus.	44 57	144 19	8	−74 11	−74·2
Terror.	63 54	155 19	1	−82 12	−82·2	Terror.	44 11	145 51	1	−72 36	−72·6
Erebus.	64 06	150 32	6	−84 47	−84·8	Erebus.	43 49	146 00	5	−72 30	−72·5
Terror.	64 25	152 55	2	−83 25	−83·4	Terror.	42 52	147 24	3	−70 40	−70·7
Terror.	63 50	156 21	2	−84 11	−84·2						

General Table of the Inclinations observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between July 1841 and July 1842.

Position.		Ship.	No. of observations.	Inclination.		Position.		Ship.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
° ′	° ′			° ′	°	° ′	° ′			° ′	°
43 00	148 25	Erebus.	5	−70 44	−70·7	39 20	182 59	Terror.	14	−61 02	−61·0
43 03	148 20	Terror.	3	−71 25	−71·4	40 47	183 03	Erebus.	5	−62 29	−62·5
42 13	149 25	Erebus.	5	−69 57	−69·9	40 42	183 05	Terror.	13	−61 49	−61·8
42 24	149 30	Terror.	3	−70 24	−70·4	41 49	183 41	Erebus.	5	−63 30	−63·5
40 55	149 12	Erebus.	4	−69 01	−69·0	42 07	183 43	Terror.	12	−63 13	−63·2
40 51	149 28	Terror.	3	−69 25	−69·4	43 32	183 03	Erebus.	5	−64 48	−64·8
37 50	150 22	Erebus.	4	−67 00	−67·0	43 52	183 05	Terror.	13	−65 16	−65·3
38 17	150 22	Terror.	3	−67 19	−67·3	45 40	183 20	Erebus.	5	−66 47	−66·8
37 21	151 33	Erebus.	5	−66 24	−66·4	45 39	183 18	Terror.	12	−66 45	−66·7
37 28	151 30	Terror.	3	−66 36	−66·6	47 19	184 40	Erebus.	5	−68 06	−68·1
36 01	151 48	Erebus.	4	−65 29	−65·5	47 26	184 42	Terror.	12	−67 38	−67·6
36 21	151 39	Terror.	2	−66 21	−66·3	48 43	186 30	Erebus.	6	−69 16	−69·3
33 52	151 21	Erebus.	5	−63 47	−63·8	48 44	186 29	Terror.	13	−68 48	−68·8
34 06	151 19	Terror.	3	−63 24	−63·4	49 20	187 41	Erebus.	5	−69 55	−69·9
33 51	151 17	Erebus*.	8	−62 52	−62·9	49 24	187 23	Terror.	13	−69 14	−69·2
33 51	151 17	Terror.	6	−62 57	−62·9	49 27	189 13	Erebus.	4	−69 46	−69·8
33 52	154 07	Erebus.	5	−63 05	−63·1	49 30	189 19	Terror.	12	−69 08	−69·1
33 57	153 35	Terror.	6	−62 43	−62·7	50 00	191 00	Erebus.	4	−69 46	−69·8
33 51	157 18	Erebus.	5	−62 26	−62·4	49 58	191 11	Terror.	12	−68 55	−68·9
33 56	156 38	Terror.	3	−62 03	−62·1	50 48	192 20	Erebus.	6	−69 57	−69·9
33 27	160 43	Erebus.	5	−61 50	−61·8	50 38	192 05	Terror.	12	−69 30	−69·5
33 31	160 20	Terror.	3	−61 18	−61·3	51 34	194 29	Erebus.	5	−70 20	−70·3
33 38	163 42	Erebus.	5	−61 05	−61·1	51 48	194 26	Terror.	13	−70 05	−70·1
33 42	164 05	Terror.	3	−61 01	−61·0	52 21	197 53	Erebus.	6	−70 35	−70·6
33 38	166 28	Erebus.	5	−60 38	−60·6	52 17	198 55	Terror.	7	−70 24	−70·4
33 44	166 37	Terror.	8	−60 08	−60·1	53 01	202 11	Erebus.	5	−71 04	−71·1
33 32	167 40	Erebus.	5	−60 00	−60·0	53 01	202 16	Terror.	3	−70 24	−70·4
33 32	167 39	Terror.	7	−60 15	−60·3	52 48	203 50	Erebus.	5	−70 47	−70·8
32 58	169 20	Erebus.	5	−59 28	−59·5	52 51	203 56	Terror.	7	−70 18	−70·3
33 00	169 20	Terror.	8	−59 06	−59·1	53 01	205 08	Erebus.	6	−70 21	−70·3
32 12	170 27	Erebus.	4	−58 36	−58·6	53 13	205 43	Terror.	13	−70 03	−70·1
32 12	170 27	Terror.	3	−57 04	−57·1	55 08	210 01	Erebus.	15	−71 14	−71·2
33 27	171 21	Erebus.	1	−57 45	−57·7	54 42	209 04	Terror.	19	−70 19	−70·3
32 11	171 20	Terror.	6	−57 33	−57·5	56 07	211 45	Erebus.	10	−72 04	−72·1
33 55	171 54	Erebus.	5	−58 55	−58·9	56 22	211 46	Terror.	18	−71 46	−71·8
33 56	172 02	Terror.	10	−58 20	−58·3	56 22	212 25	Erebus.	11	−72 29	−72·5
34 10	172 54	Erebus.	2	−58 26	−58·4	57 04	212 11	Terror.	15	−72 15	−72·3
34 15	172 50	Terror.	9	−59 03	−59·1	58 41	213 17	Erebus.	11	−73 54	−73·9
34 29	173 36	Erebus.	6	−58 38	−58·6	58 19	213 06	Terror.	18	−73 28	−73·5
34 24	173 43	Terror.	6	−59 01	−59·0	61 20	213 57	Erebus.	11	−75 37	−75·6
35 16	174 00	Terror.	12	−59 26†	−59·4	61 12	213 52	Terror.	12	−75 33	−75·5
35 16	174 00	Erebus.	10	−59 36†	−59·6	62 40	212 53	Erebus.	7	−76 41	−76·7
35 15	174 39	Erebus.	6	−59 21	−59·3	62 42	212 42	Terror.	7	−76 39	−76·7
36 27	177 34	Erebus.	5	−59 59	−60·0	63 23	210 02	Erebus.	12	−77 33	−77·5
36 20	177 27	Terror.	6	−59 32	−59·5	63 13	210 25	Terror.	11	−77 42	−77·7
38 35	180 31	Erebus.	7	−60 41	−60·7	63 47	208 26	Erebus.	5	−78 02	−78·0
38 13	179 46	Terror.	12	−60 31	−60·5	63 42	208 24	Terror.	18	−78 00	−78·0
39 01	182 12	Erebus.	5	−61 28	−61·5	64 47	206 40	Erebus.	13	−78 39	−78·7
38 55	182 17	Terror.	15	−61 13	−61·2	64 31	206 27	Terror.	12	−78 34	−78·6
39 18	182 58	Erebus.	5	−61 58	−62·0	65 30	205 41	Erebus.	6	−79 12	−79·2

* Observed on shore, Garden Island, Sydney.

† Observed on shore, Bay of Islands.

General Table of the Inclinations (continued).

Position.		Ship.	No. of observations.	Inclination.		Position.		Ship.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
65° 26′	205° 04′	Terror.	13	−79° 20′	−79°·3	67° 32′	203° 59′	Erebus.	4	−80° 29′	−80°·5
65 59	204 16	Erebus.	9	−79 53	−79·9	67 24	204 05	Terror.	5	−80 49	−80·8
65 47	204 19	Terror.	23	−79 36	−79·6	67 18	203 39	Erebus.	5	−81 05	−81·1
65 57	203 52	Erebus.	13	−79 24	−79·4	67 21	202 14	Erebus.	6	−80 18	−80·3
65 50	204 08	Terror.	10	−79 40	−79·7	67 12	202 24	Terror.	8	−80 47	−80·8
66 00	203 46	Erebus.	6	−79 56	−79·9	67 14	201 34	Terror.	13	−80 50	−80·8
66 01	204 00	Terror.	4	−79 47	−79·8	67 57	200 00	Erebus.	7	−80 56	−80·9
66 11	203 36	Erebus.	6	−80 04	−80·1	67 56	199 48	Terror.	6	−81 18	−81·3
65 57	204 27	Terror.	4	−80 02	−80·0	68 21	200 03	Erebus.	5	−81 14	−81·2
66 16	203 31	Erebus.	2	−79 52	−79·9	68 21	200 06	Terror.	6	−81 27	−81·5
66 08	203 50	Terror.	9	−79 53	−79·9	68 43	199 43	Erebus.	6	−81 15	−81·3
66 20	203 22	Erebus.	2	−79 52	−79·9	68 46	199 39	Terror.	8	−81 34	−81·6
66 11	202 54	Terror.	3	−80 15	−80·3	68 59	196 00	Erebus.	6	−82 05	−82·1
66 24	203 51	Erebus.	3	−80 07	−80·1	68 52	198 24	Terror.	7	−82 48	−82·8
66 20	203 20	Terror.	4	−80 13	−80·2	69 48	192 25	Erebus.	5	−82 41	−82·7
66 31	203 07	Erebus.	4	−79 51	−79·9	69 55	192 17	Terror.	8	−83 06	−83·1
66 25	203 12	Terror.	6	−80 22	−80·4	70 16	190 06	Erebus.	11	−83 10	−83·2
66 32	203 33	Erebus.	2	−80 04	−80·1	70 05	191 03	Terror.	8	−83 28	−83·5
66 30	203 08	Terror.	6	−80 34	−80·6	70 18	186 01	Erebus.	5	−83 25	−83·4
66 32	203 32	Erebus.	5	−80 00	−80·0	70 12	186 21	Terror.	14	−83 33	−83·5
66 36	203 29	Terror.	6	−80 50	−80·8	70 39	185 31	Erebus.	6	−83 47	−83·8
66 35	203 29	Erebus.	5	−79 58	−80·0	70 32	185 38	Terror.	8	−83 51	−83·9
66 33	203 21	Terror.	4	−80 27	−80·5	70 11	183 50	Erebus.	5	−83 52	−83·9
66 34	203 51	Erebus.	2	−79 56	−79·9	69 56	184 43	Terror.	7	−84 30	−84·5
66 14	203 17	Terror.	1	−80 22	−80·4	70 08	181 42	Erebus.	4	−84 05	−84·1
66 06	204 24	Erebus.	5	−79 48	−79·8	69 53	182 49	Terror.	6	−84 33	−84·5
66 11	203 48	Terror.	3	−79 54	−79·8	71 00	180 44	Erebus.	5	−84 30	−84·5
66 13	204 22	Erebus.	7	−79 44	−79·7	71 03	180 56	Terror.	7	−84 33	−84·5
66 20	203 39	Terror.	8	−80 02	−80·0	72 46	181 46	Erebus.	6	−85 14	−85·2
66 13	204 33	Erebus.	14	−79 38	−79·6	72 07	181 50	Terror.	8	−85 12	−85·2
66 05	204 02	Terror.	13	−79 57	−79·9	73 23	183 04	Erebus.	1	−85 57	−85·9
66 04	204 19	Erebus.	3	−79 48	−79·8	72 59	181 30	Terror.	7	−85 34	−85·6
66 01	204 04	Terror.	11	−80 05	−80·1	74 24	177 09	Erebus.	1	−86 35	−86·6
65 59	204 12	Erebus.	6	−79 51	−79·9	74 20	177 55	Terror.	1	−86 47	−86·8
65 57	203 46	Terror.	13	−80 05	−80·1	75 03	173 22	Erebus.	11	−87 12	−87·2
65 56	203 38	Erebus.	5	−79 46	−79·8	75 00	173 38	Terror.	11	−87 27	−87·5
65 56	203 31	Terror.	6	−79 51	−79·9	75 57	175 10	Erebus.	6	−86 52	−86·9
65 45	203 23	Terror.	3	−80 08	−80·1	76 05	174 59	Terror.	7	−87 39	−87·7
66 12	203 04	Erebus.	11	−79 38	−79·6	76 58	181 03	Erebus.	5	−86 54	−86·9
66 09	202 11	Terror.	12	−79 54	−79·9	77 03	181 35	Terror.	7	−87 32	−87·5
66 14	203 09	Erebus.	4	−79 39	−79·7	76 44	185 12	Erebus.	6	−85 52	−85·9
66 08	201 46	Terror.	10	−79 46	−79·8	76 48	185 00	Terror.	7	−86 33	−86·5
66 02	202 30	Erebus.	1	−79 35	−79·6	76 14	192 35	Erebus.	1	−85 16	−85·3
65 58	202 21	Terror.	4	−80 01	−80·0	76 19	190 44	Terror.	7	−86 18	−86·3
65 49	202 02	Erebus.*	5	−79 39	−79·7	75 53	194 52	Erebus.	1	−85 40	−85·7
65 47	202 08	Terror.	7	−79 53	−79·9	75 45	195 02	Terror.	1	−86 01	−86·0
65 47	201 56	Terror.	1	−79 44	−79·7	76 42	194 39	Erebus.	11	−85 42	−85·7
66 18	201 22	Erebus.	2	−79 57	−79·9	76 46	189 22	Terror.	13	−85 32	−85·5
66 11	200 45	Terror.	3	−80 20	−80·3	77 58	197 34	Erebus.	4	−85 01	−85·0
67 37	200 12	Terror.	6	−80 31	−80·5	77 47	197 25	Terror.	8	−85 09	−85·1
66 49	202 40	Erebus.	4	−80 12	−80·2	76 55	198 40	Erebus.	1	−85 28	−85·5
66 43	202 50	Terror.	3	−80 40	−80·7	77 12	199 23	Terror.	7	−85 51	−85·9
67 12	203 12	Terror.	9	−80 23	−80·4	74 50	193 45	Erebus.	6	−85 06	−85·1
67 38	204 01	Erebus.	5	−80 22	−80·4	75 20	194 36	Terror.	8	−86 17	−86·3
67 47	204 17	Terror.	26	−80 55	−80·9	72 46	189 59	Erebus.	5	−84 45	−84·7

* Observed on ice.

General Table of the Inclinations (continued).

Position.		Ship.	No. of observations.	Inclination.		Position.		Ship.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
° ′	° ′			° ′	°	° ′	° ′			° ′	°
73 10	189 21	Terror.	6	−85 30	−85·5	60 01	241 31	Erebus.	5	−72 45	−72·7
72 01	187 35	Erebus.	5	−84 33	−84·5	59 39	243 13	Terror.	2	−71 48	−71·8
72 00	187 37	Terror.	8	−85 19	−85·3	59 17	245 40	Erebus.	5	−71 35	−71·6
71 08	184 59	Erebus.	6	−84 22	−84·4	59 07	248 39	Erebus.	11	−71 23	−71·4
71 06	184 11	Terror.	9	−85 04	−85·1	59 04	247 30	Terror.	7	−71 37	−71·6
69 48	179 58	Erebus.	10	−83 42	−83·7	58 29	252 18	Erebus.	5	−71 01	−71·0
69 54	179 55	Terror.	7	−84 58	−85·0	58 26	251 42	Terror.	6	−71 13	−71·2
68 04	183 25	Erebus.	10	−82 19	−82·3	58 35	255 10	Erebus.	5	−70 23	−70·4
68 09	183 10	Terror.	6	−82 50	−82·8	58 33	254 45	Terror.	6	−70 40	−70·7
67 30	185 21	Erebus.	6	−82 01	−82·0	58 47	258 12	Erebus.	5	−69 59	−70·0
67 34	185 20	Terror.	9	−82 14	−82·2	58 42	257 46	Terror.	9	−70 07	−70·1
67 22	188 10	Erebus.	5	−81 08	−81·1	58 54	263 35	Terror.	1	−69 14	−69·2
67 40	187 40	Terror.	4	−81 34	−81·6	58 59	265 55	Erebus.	6	−67 49	−67·8
67 09	188 02	Terror.	6	−81 21	−81·3	58 59	267 50	Terror.	6	−68 08	−68·1
65 15	192 04	Erebus.	5	−79 24	−79·4	59 02	271 58	Erebus.	5	−67 09	−67·1
65 07	191 55	Terror.	9	−80 06	−80·1	59 01	272 06	Terror.	7	−67 04	−67·1
63 30	194 22	Erebus.	6	−78 18	−78·3	58 52	276 51	Erebus.	6	−65 34	−65·6
63 30	194 15	Terror.	6	−78 58	−79·0	58 54	276 18	Terror.	6	−66 24	−66·4
62 16	196 10	Erebus.	5	−77 26	−77·4	58 23	280 03	Erebus.	5	−64 54	−64·9
62 17	195 55	Terror.	6	−77 56	−77·9	58 25	279 44	Terror.	7	−64 58	−65·0
60 58	199 32	Erebus.	10	−76 12	−76·2	58 29	282 10	Erebus.	5	−63 50	−63·8
61 06	198 08	Terror.	8	−77 00	−77·0	58 31	281 37	Terror.	8	−64 04	−64·1
60 18	204 03	Erebus.	6	−75 31	−75·5	58 31	285 54	Erebus.	5	−63 07	−63·1
60 57	199 03	Terror.	6	−75 30	−75·5	58 36	285 33	Terror.	6	−63 08	−63·1
60 18	208 29	Erebus.	1	−74 14	−74·2	57 22	289 50	Erebus.	5	−61 19	−61·3
60 15	208 06	Terror.	6	−74 41	−74·7	57 21	289 36	Terror.	6	−61 42	−61·7
60 12	212 03	Erebus.	6	−74 14	−74·2	57 11	292 15	Erebus.	6	−59 07	−59·1
60 17	211 53	Terror.	7	−74 35	−74·6	57 26	291 36	Terror.	7	−60 14	−60·2
60 00	216 12	Erebus.	1	−73 10	−73·2	56 40	294 46	Erebus.	5	−59 13	−59·2
59 53	216 28	Terror.	6	−74 04	−74·1	56 37	294 34	Terror.	6	−59 15	−59·3
59 20	219 14	Erebus.	10	−73 47	−73·8	54 50	298 08	Erebus.	4	−56 33	−56·5
59 22	218 14	Terror.	8	−74 57	−74·9	54 48	297 21	Terror.	6	−56 49	−56·8
58 52	222 44	Erebus.	7	−73 51	−73·9	52 54	300 27	Erebus.	5	−54 09	−54·1
58 49	221 33	Terror.	7	−73 17	−73·3	52 33	300 20	Terror.	13	−53 30	−53·5
59 22	230 47	Erebus.	16	−73 06	−73·1	52 11	301 46	Erebus.	3	−52 54	−52·9
59 01	227 41	Terror.	7	−73 50	−73·8	51 42	301 36	Terror.	6	−52 13	−52·2
59 32	231 46	Terror.	1	−73 16	−73·3	51 32	301 53	Erebus.	8	−52 39*	−52·7
60 19	237 03	Erebus.	19	−73 01	−73·0	51 32	301 53	Terror.	21	−52 15*	−52·3
60 15	236 41	Terror.	18	−73 26	−73·4						

* Observed on shore, Port Louis, Falkland Islands.

General Table of the Inclinations observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between September 1842 and April 1843.

Position.		Ship.	No. of observations.	Inclination.		Position.		Ship.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
51° 32′	301° 53′	Erebus.	4	−52° 34′	−52°·6	62° 25′	307° 58′	Terror.	15	−60° 52′	−60°·9
52 32	301 53	Terror.	4	−52 02	−52·0	62 20	308 12	Erebus.	11	−60 44	−60·7
53 33	302 05	Erebus.	5	−53 18	−53·3	62 18	308 17	Terror.	13	−61 17	−61·3
52 48	303 10	Terror.	7	−53 46	−53·8	62 30	306 30	Erebus.	5	−61 07	−61·1
54 03	305 26	Erebus.	5	−54 27	−54·5	62 36	306 09	Terror.	20	−61 52	−61·9
54 02	305 30	Terror.	1	−53 36	−53·6	62 36	306 20	Erebus.	6	−61 45	−61·7
53 47	304 48	Erebus.	5	−54 08	−54·1	63 24	305 40	Terror.	13	−62 30	−62·5
53 42	305 04	Terror.	1	−53 34	−53·6	63 36	305 00	Erebus.	5	−62 18	−62·3
54 42	304 39	Erebus.	10	−54 22	−54·4	63 42	304 40	Erebus.	13	−62 09	−62·1
54 40	304 35	Terror.	1	−53 29	−53·5	63 57	304 32	Terror.	12	−62 42	−62·7
54 42	305 30	Terror.	8	−54 39	−54·7	64 23	304 00	Erebus.	6	−63 04	−63·1
55 08	300 44	Erebus.	1	−56 24	−56·4	64 15	304 25	Terror.	7	−63 31	−63·5
55 10	301 00	Terror.	1	−56 05	−56·1	64 26	303 52	Erebus.	5	−63 16†	−63·3
55 40	296 52	Erebus.	8	−57 21	−57·3	64 27	303 54	Terror.	11	−63 21§	−63·3
55 30	297 00	Terror.	7	−57 29	−57·5	64 12	304 11	Erebus.	1	−62 37	−62·6
55 51	292 28	Erebus.	7	−58 12*	−58·2	64 26	303 54	Terror.	10	−63 07	−63·1
55 51	292 28	Terror.	14	−58 03†	−58·1	64 32	304 20	Erebus.	5	−62 50	−62·8
56 25	293 07	Erebus.	5	−58 29	−58·5	64 21	304 04	Terror.	19	−63 09	−63·1
56 02	292 57	Terror.	6	−59 20	−59·3	64 12	304 04	Erebus.	3	−62 58	−63·0
55 38	296 00	Erebus.	5	−56 53	−56·9	64 13	304 06	Terror.	18	−63 10	−63·2
55 45	295 47	Terror.	8	−57 31	−57·5	64 12	303 04	Terror.	8	−63 27	−63·5
55 58	299 12	Erebus.	1	−56 54	−56·9	64 34	302 50	Erebus.	6	−63 29	−63·5
55 56	299 17	Terror.	1	−56 46	−56·8	64 36	302 42	Terror.	15	−63 44	−63·7
55 32	299 08	Erebus.	2	−56 14	−56·2	64 35	302 13	Erebus.	4	−63 27	−63·5
55 28	298 35	Terror.	3	−56 57	−56·9	64 40	303 08	Terror.	29	−63 48	−63·8
54 24	300 08	Erebus.	9	−55 07	−55·1	64 44	303 07	Erebus.	10	−63 21‡	−63·3
54 59	300 12	Terror.	9	−56 31	−56·5	64 43	303 10	Erebus.	2	−63 37	−63·6
52 52	301 05	Erebus.	9	−53 34	−53·6	64 40	303 08	Terror.	8	−63 49	−63·8
52 31	301 16	Terror.	9	−53 40	−53·7	64 42	302 54	Erebus.	1	−63 19	−63·3
51 32	301 53	Terror.	11	−52 13†	−52·2	64 35	303 06	Terror.	1	−64 04	−64·1
51 32	301 53	Terror.	11	−52 18†	−52·3	64 39	302 04	Erebus.	2	−63 12	−63·2
51 32	301 53	Erebus.	10	−52 49†	−52·8	64 36	302 07	Terror.	14	−64 04	−64·1
51 32	301 53	Erebus.	10	−52 30	−52·5	64 35	302 37	Erebus.	2	−63 17	−63·3
52 50	303 07	Erebus.	5	−53 20	−53·3	64 38	302 42	Terror.	7	−63 53	−63·9
52 49	303 19	Terror.	13	−52 57	−52·9	64 31	302 34	Erebus.	4	−63 39	−63·7
54 23	303 59	Erebus.	5	−54 29	−54·5	64 28	303 03	Erebus.	8	−63 25	−63·4
53 38	303 43	Terror.	7	−53 11	−53·2	64 26	303 05	Terror.	8	−63 07	−63·1
55 51	305 18	Erebus.	5	−55 25	−55·4	64 25	304 27	Terror.	15	−62 45	−62·7
55 48	305 26	Terror.	21	−55 03	−55·1	63 58	304 46	Erebus.	1	−62 57	−62·9
56 34	306 39	Erebus.	5	−56 02	−56·0	64 22	305 01	Erebus.	5	−63 09	−63·1
56 57	306 41	Terror.	12	−56 13	−56·2	64 22	305 44	Terror.	2	−63 36	−63·6
58 16	308 00	Erebus.	5	−57 08	−57·1	64 18	304 18	Erebus.	5	−63 19	−63·3
58 07	307 55	Terror.	21	−57 19	−57·3	64 17	304 20	Terror.	20	−63 22	−63·4
59 28	308 20	Erebus.	8	−58 33	−58·5	64 19	304 04	Erebus.	2	−63 00	−63·0
59 50	307 57	Terror.	8	−58 33	−58·5	64 12	303 50	Erebus.	5	−62 34	−62·6
61 23	307 41	Erebus.	3	−59 35	−59·6	64 12	304 07	Terror.	11	−63 05	−63·1
61 21	307 41	Terror.	7	−59 55	−59·9	64 24	304 49	Erebus.	3	−62 41	−62·7
62 14	307 55	Erebus.	1	−60 55	−60·9	64 20	304 08	Terror.	6	−63 23	−63·4
62 12	307 47	Terror.	7	−60 50	−60·8	64 16	304 40	Terror.	13	−63 03	−63·1
62 31	308 05	Erebus.	4	−60 34	−60·6	64 15	304 00	Erebus.	2	−62 54	−62·9

* Observed on shore, St. Martin's Cove, with needles whose poles were reversed.

† Observed on shore, St. Martin's Cove. ‡ Observed on ice.

§ Dip on ice with needle whose poles were reversed.

General Table of the Inclinations (continued).

Position.		Ship.	No. of observations.	Inclination.		Position.		Ship.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
° ′	° ′			° ′	°	° ′	° ′			° ′	°
64 04	305 19	Erebus.	2	−62 53	−62·9	63 58	345 10	Erebus.	5	−60 30	−60·5
64 01	304 20	Terror.	8	−63 09	−63·1	64 14	345 30	Terror.	7	−60 57	−60·9
64 08	304 09	Erebus.	3	−62 39	−62·7	64 38	348 00	Erebus.	5	−61 16	−61·3
64 08	304 08	Erebus.	2	−62 42	−62·7	64 33	347 52	Terror.	8	−61 39	−61·7
64 10	304 04	Terror.	11	−63 12	−63·2	65 12	350 05	Erebus.	5	−61 43	−61·7
64 05	304 00	Erebus.	5	−62 47	−62·8	65 06	349 45	Terror.	10	−61 49	−61·8
64 09	303 57	Erebus.	1	−63 01	−63·0	66 08	352 43	Erebus.	5	−63 08	−63·1
64 07	303 58	Terror.	6	−63 11	−63·2	66 01	353 06	Terror.	10	−63 12	−63·2
64 00	304 42	Erebus.	3	−62 29	−62·5	67 06	351 04	Erebus.	5	−63 12	−63·2
64 00	304 46	Terror.	11	−62 55	−62·9	67 00	351 07	Terror.	9	−63 30	−63·5
63 53	304 51	Erebus.	3	−62 25	−62·4	68 14	347 08	Erebus.	9	−64 17	−64·3
64 16	304 38	Erebus.	2	−62 38	−62·6	68 20	347 41	Terror.	10	−64 07	−64·1
64 19	304 43	Terror.	9	−63 14	−63·2	68 32	347 09	Erebus.	5	−64 14	−64·2
64 18	305 11	Terror.	7	−63 10	−63·2	69 26	345 31	Erebus.	5	−64 37	−64·6
64 17	305 20	Erebus.	3	−62 21	−62·3	69 24	345 30	Terror.	10	−65 20	−65·3
64 16	304 47	Erebus.	4	−62 42	−62·7	71 10	344 13	Erebus.	5	−66 13	−66·2
64 11	305 15	Terror.	9	−62 58	−63·0	71 09	344 10	Terror.	8	−66 54	−66·9
63 34	307 00	Erebus.	5	−61 38	−61·6	70 55	343 14	Erebus.	1	−65 38	−65·6
63 34	306 59	Terror.	2	−62 05	−62·1	71 08	344 18	Terror.	1	−66 52	−66·9
63 35	307 33	Erebus.	5	−61 38	−61·6	70 33	343 23	Terror.	1	−64 54	−64·9
63 36	307 35	Terror.	8	−62 18	−62·3	70 28	342 39	Erebus.	1	−65 39	−65·7
63 54	308 00	Erebus.	2	−61 47	−61·8	70 28	342 26	Terror.	4	−66 12	−66·2
64 00	308 01	Terror.	15	−61 56	−61·9	69 56	344 03	Erebus.	1	−64 07	−64·1
63 49	309 00	Erebus.	5	−62 06	−62·1	69 36	344 15	Terror.	1	−64 43	−64·7
64 03	309 16	Terror.	16	−62 15	−62·3	68 06	344 40	Erebus.	4	−63 04	−63·1
64 19	309 36	Erebus.	10	−62 34	−62·6	68 07	346 13	Terror.	7	−63 23	−63·4
64 36	311 53	Erebus.	5	−61 47	−61·8	65 56	346 24	Erebus.	5	−61 48	−61·8
64 43	312 06	Terror.	8	−62 09	−62·1	65 57	346 40	Terror.	8	−62 10	−62·2
64 37	314 21	Erebus.	6	−62 03	−62·1	64 31	346 01	Erebus.	4	−59 53	−59·9
64 38	314 01	Terror.	9	−61 50	−61·8	63 58	346 25	Terror.	4	−60 33	−60·5
64 39	316 04	Erebus.	5	−61 34	−61·6	61 34	348 37	Erebus.	5	−58 52	−58·9
64 49	315 07	Terror.	10	−61 47	−61·8	61 35	349 00	Terror.	7	−58 59	−59·0
64 38	316 57	Erebus.	5	−61 49	−61·8	59 34	350 34	Erebus.	5	−57 31	−57·5
64 47	316 57	Terror.	9	−62 23	−62·4	59 21	350 36	Terror.	7	−57 40	−57·7
65 06	318 46	Erebus.	5	−61 47	−61·8	57 27	352 08	Erebus.	8	−56 43	−56·7
64 58	318 26	Terror.	16	−61 54	−61·9	57 31	352 04	Terror.	12	−56 33	−56·5
64 40	320 12	Erebus.	5	−61 12	−61·2	57 09	352 45	Erebus.	5	−56 27	−56·5
64 37	320 28	Terror.	8	−61 10	−61·2	57 09	352 44	Terror.	7	−56 13	−56·2
63 54	321 36	Erebus.	5	−60 34	−60·6	56 38	353 57	Erebus.	5	−56 09	−56·1
64 02	321 55	Terror.	10	−60 30	−60·5	56 44	353 45	Terror.	4	−56 02	−56·0
63 36	324 36	Erebus.	14	−59 50	−59·8	55 38	355 32	Erebus.	5	−56 55	−56·9
63 59	324 18	Terror.	7	−60 15	−60·3	55 56	355 39	Terror.	3	−55 10	−55·2
62 39	328 16	Erebus.	8	−59 11	−59·2	54 31	357 35	Erebus.	1	−55 41	−55·7
62 37	328 17	Terror.	9	−59 03	−59·1	54 32	357 26	Terror.	4	−54 56	−54·9
62 20	330 00	Erebus.	4	−58 45	−58·7	54 07	359 56	Erebus.	7	−55 33	−55·5
62 13	330 38	Terror.	12	−58 37	−58·6	54 06	359 50	Terror.	10	−55 33	−55·5
61 59	333 43	Erebus.	5	−58 17	−58·3	50 37	9 03	Erebus.	4	−56 14	−56·2
62 05	333 38	Terror.	10	−58 02	−58·0	50 35	9 01	Terror.	7	−56 19	−56·3
61 37	336 05	Erebus.	5	−58 17	−58·3	47 38	10 51	Erebus.	5	−55 46	−55·8
61 32	336 10	Terror.	7	−58 06	−58·1	47 36	10 41	Terror.	5	−55 55	−55·9
61 30	338 00	Erebus.	5	−57 55	−57·9	45 32	11 54	Erebus.	5	−55 24	−55·4
61 28	337 42	Terror.	15	−58 05	−58·1	45 38	11 52	Terror.	7	−55 28	−55·5
61 46	341 02	Erebus.	5	−58 17	−58·3	43 57	13 16	Erebus.	8	−54 35	−54·6
61 13	340 00	Terror.	2	−58 04	−58·1	43 55	13 16	Terror.	7	−55 19	−55·3
62 36	344 08	Erebus.	5	−58 57	−58·9	43 10	14 44	Erebus.	8	−54 45	−54·7
62 41	343 18	Terror.	13	−59 02	−59·0	43 13	14 36	Terror.	15	−55 24	−55·4

General Table of the Inclinations (continued).

Position.		Ship.	No. of observations.	Inclination.		Position.		Ship.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
41° 40′	15° 09′	Erebus.	5	−54° 48′	−54°8	36° 02′	16° 32′	Terror.	8	−53° 41′	−53°7
41 58	15 11	Terror.	7	−55 01	−55·0	35 26	16 22	Erebus.	5	−53 42	−53·7
40 15	15 47	Erebus.	5	−54 54	−54·9	35 21	16 22	Terror.	14	−53 10	−53·2
40 07	16 08	Terror.	8	−54 43	−54·7	35 04	17 08	Erebus.	2	−53 19	−53·3
37 40	16 40	Erebus.	5	−54 40	−54·7	35 03	17 06	Terror.	15	−53 18	−53·3
38 00	16 45	Terror.	7	−54 20	−54·3	34 11	18 26	Erebus.	11	−53 40*	−53·7
35 59	16 34	Erebus.	5	−54 15	−54·3	34 11	18 26	Terror.	12	−53 35*	−53·6

* Observed on shore, Simon's Bay.

General Table of the Inclinations observed on board Her Majesty's hired Bark 'Pagoda' in 1844 and 1845.

Position.		Observer.	No. of observations.	Inclination.		Position.		Observer.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
34° 44′	17° 50′	Clerk.	3	−52° 56′	−52°·9	67° 00′	40° 07′	Clerk.	4	−69° 39′	−69°·7
34 46	17 46	Moore.	7	−53 34	−53·6	66 55	40 16	Moore.	7	−69 30	−69·5
35 29	15 09	Moore.	6	−51 27	−51·5	66 24	40 01	Moore.	4	−69 15	−69·3
35 17	14 00	Moore.	7	−51 16	−51·3	64 52	38 37	Clerk.	3	−68 40	−68·7
35 12	13 28	Clerk.	3	−51 35	−51·6	64 52	38 37	Moore.	6	−68 53	−68·9
35 24	13 23	Moore.	6	−51 19	−51·3	64 52	40 12	Clerk.	4	−68 44	−68·7
37 25	13 24	Clerk.	4	−51 44	−51·7	64 43	40 12	Moore.	7	−68 18	−68·3
38 42	14 27	Moore.	7	−53 31	−53·5	64 22	40 49	Clerk.	3	−68 40	−68·7
38 37	14 27	Clerk.	4	−52 39	−52·7	64 05	41 09	Moore.	2	−70 02	−70·0
39 10	14 38	Moore.	7	−54 12	−54·2	63 49	42 00	Clerk.	4	−69 36	−69·6
39 10	14 41	Clerk.	4	−54 14	−54·2	63 19	45 52	Moore.	4	−70 14	−70·2
40 41	14 16	Moore.	4	−54 59	−55·0	63 22	45 35	Clerk.	3	−70 03	−70·1
40 21	14 29	Clerk.	4	−55 10	−55·2	63 22	45 58	Moore.	4	−70 15	−70·3
42 50	13 00	Clerk.	4	−55 34	−55·6	63 36	46 41	Clerk.	4	−70 02	−70·0
44 50	13 19	Clerk.	4	−56 14	−56·2	63 36	46 48	Moore.	11	−70 13	−70·2
50 21	10 31	Moore.	6	−55 34	−55·6	62 36	51 40	Clerk.	5	−69 13	−69·2
47 40	12 25	Clerk.	2	−56 29	−56·5	61 34	53 49	Moore.	7	−70 49	−70·8
48 35	10 51	Clerk.	4	−56 44	−56·7	61 25	53 38	Clerk.	4	−70 46	−70·8
50 48	10 18	Moore.	7	−57 19	−57·3	61 17	57 28	Clerk.	4	−72 01	−72·0
50 30	10 25	Clerk.	4	−57 02	−57·0	61 19	57 33	Moore.	4	−71 26	−71·4
51 44	9 36	Moore.	2	−57 28	−57·5	61 22	57 41	Moore.	6	−71 27	−71·5
51 48	9 33	Clerk.	4	−57 13	−57·2	61 00	64 03	Clerk.	4	−73 27	−73·5
51 56	9 30	Moore.	3	−57 42	−57·7	61 10	64 20	Moore.	3	−72 18	−72·3
52 53	7 53	Clerk.	4	−57 03	−57·1	61 36	70 46	Clerk.	4	−74 02	−74·0
53 21	7 32	Moore.	7	−57 24	−57·4	61 49	71 30	Moore.	10	−73 38	−73·6
53 52	6 07	Clerk.	4	−57 01	−57·0	62 10	72 25	Clerk.	4	−74 35	−74·6
54 02	6 02	Moore.	7	−56 58	−57·0	62 10	72 25	Moore.	6	−74 33	−74·5
55 08	5 50	Clerk.	4	−57 26	−57·4	62 40	76 09	Clerk.	4	−74 50	−74·8
55 18	5 55	Moore.	3	−58 12	−58·2	62 48	76 15	Moore.	11	−75 05	−75·1
60 43	4 00	Clerk.	6	−59 58	−60·0	64 20	79 38	Clerk.	4	−76 34	−76·6
61 14	9 07	Moore.	7	−61 43	−61·7	64 20	79 38	Moore.	5	−75 57	−75·9
61 05	9 03	Clerk.	4	−61 01	−61·0	61 38	84 40	Clerk.	4	−76 27	−76·5
62 06	12 52	Moore.	4	−63 17	−63·3	61 42	85 07	Moore.	5	−76 58	−77·0
61 54	16 23	Clerk.	4	−63 00	−63·0	60 42	88 12	Clerk.	4	−75 43	−75·7
61 56	16 36	Moore.	7	−63 55	−63·9	60 48	88 33	Moore.	7	−77 04	−77·1
61 50	19 13	Clerk.	4	−63 55	−63·9	61 23	91 15	Clerk.	8	−77 35	−77·6
61 50	19 14	Moore.	5	−64 44	−64·7	61 23	91 15	Moore.	7	−77 41	−77·7
62 00	20 25	Clerk.	4	−64 55	−64·9	61 07	92 10	Moore.	5	−77 57	−77·9
63 00	20 40	Moore.	6	−64 25	−64·4	61 14	92 03	Clerk.	4	−78 26	−78·4
63 19	21 48	Moore.	7	−65 35	−65·6	60 30	92 34	Moore.	5	−77 33	−77·5
64 20	24 05	Clerk.	4	−66 37	−66·6	60 35	92 25	Clerk.	4	−77 30	−77·5
64 25	24 18	Moore.	5	−66 41	−66·7	60 03	96 03	Moore.	4	−77 38	−77·6
65 34	28 30	Clerk.	4	−66 59	−67·0	60 03	95 36	Clerk.	3	−77 35	−77·6
65 39	28 48	Moore.	6	−67 56	−67·9	59 45	99 50	Moore.	7	−79 23	−79·4
66 27	30 45	Moore.	7	−68 31	−68·5	59 55	99 35	Clerk.	8	−79 36	−79·6
66 30	36 46	Clerk.	4	−68 16	−68·3	57 46	99 17	Moore.	4	−77 43	−77·7
66 36	36 50	Moore.	7	−69 22	−69·4	57 35	99 28	Clerk.	2	−78 36	−78·6
66 43	38 49	Clerk.	4	−69 22	−69·4	56 56	101 36	Moore.	5	−78 11	−78·2
67 10	38 51	Moore.	7	−71 07	−71·1	56 53	101 24	Clerk.	4	−78 40	−78·7
67 35	39 31	Clerk.	3	−69 49	−69·8	55 40	103 18	Moore.	5	−78 09	−78·1
67 39	40 28	Moore.	2	−69 27	−69·5	55 52	103 06	Clerk.	4	−78 56	−78·9
66 45	39 23	Clerk.	4	−70 12	−70·2	54 38	106 15	Moore.	5	−79 13	−79·2
67 18	40 22	Moore.	7	−70 20	−70·3	54 48	106 04	Clerk.	4	−78 09	−78·1

General Table of the Inclinations observed in the 'Pagoda' (continued).

Position.		Observer.	No. of observations.	Inclination.		Position.		Observer.	No. of observations.	Inclination.	
Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.	Lat. S.	Long. E.			In degrees and minutes.	In degrees and decimals.
° ′	° ′			° ′	°	° ′	° ′			° ′	°
54 10	108 15	Moore.	5	−79 19	−79·3	20 38	85 26	Clerk.	4	−51 14	−51·2
54 17	108 05	Clerk.	6	−78 49	−78·8	20 25	82 00	Moore.	7	−51 05	−51·1
53 00	110 35	Moore.	7	−77 39	−77·7	20 26	82 22	Clerk.	8	−51 39	−51·7
53 00	110 08	Clerk.	4	−77 28	−77·5	20 36	79 10	Moore.	5	−51 46	−51·8
51 00	111 29	Moore.	1	−77 36	−77·6	20 36	79 22	Clerk.	8	−51 50	−51·8
51 20	111 23	Clerk.	3	−76 41	−76·7	20 44	78 31	Moore.	18	−52 00	−52·0
48 57	112 56	Moore.	6	−76 04	−76·1	20 44	78 31	Clerk.	19	−52 03	−52·1
49 01	111 47	Clerk.	4	−76 30	−76·5	20 39	77 43	Moore.	12	−51 58	−52·0
47 21	115 15	Moore.	7	−75 32	−75·5	20 39	77 43	Clerk.	12	−51 59	−52·0
47 21	115 15	Clerk.	4	−75 31	−75·5	20 29	76 22	Clerk.	8	−52 20	−52·3
45 08	116 50	Moore.	4	−73 27	−73·5	20 26	69 19	Moore.	5	−52 19	−52·3
43 20	116 52	Clerk.	4	−72 45	−72·7	20 28	70 46	Clerk.	4	−52 51	−52·9
43 22	116 49	Moore.	6	−72 10	−72·2	20 34	69 24	Moore.	12	−53 01	−53·0
41 18	116 09	Clerk.	4	−70 11	−70·2	21 06	68 12	Clerk.	4	−53 10	−53·2
41 00	116 42	Moore.	7	−71 14	−71·2	21 08	68 04	Moore.	1	−53 01	−53·0
38 52	116 15	Clerk.	4	−68 49	−68·8	21 11	67 54	Clerk.	8	−53 02	−53·0
38 40	116 15	Moore.	5	−68 04	−68·1	21 11	67 54	Moore.	5	−53 46	−53·8
37 03	116 57	Clerk.	4	−66 46	−66·8	21 12	67 29	Clerk.	8	−53 39	−53·7
37 00	116 57	Moore.	6	−66 21	−66·3	21 12	67 29	Moore.	5	−53 59	−54·0
36 12	116 50	Clerk.	4	−65 28	−65·5	21 02	66 02	Clerk.	4	−54 03	−54·1
36 11	116 48	Moore.	4	−66 00	−66·0	21 01	66 50	Moore.	7	−53 49	−53·8
35 18	117 07	Clerk.	4	−65 48	−65·8	20 40	62 58	Moore.	6	−53 53	−53·9
35 07	117 28	Moore.	4	−65 24	−65·4	20 31	59 42	Clerk.	4	−53 59	−54·0
35 02	117 56	Clerk.	8	−65 11*	−65·2	20 09	57 31	Moore.	10	−53 52†	−53·9
35 02	117 56	Moore.	12	−64 54*	−64·9	21 44	53 34	Clerk.	4	−54 38	−54·6
35 36	114 44	Clerk.	4	−65 28	−65·5	21 50	53 25	Moore.	4	−54 51	−54·9
35 30	114 35	Moore.	5	−64 54	−64·9	26 25	49 12	Clerk.	4	−58 36	−58·6
34 16	113 01	Clerk.	4	−64 44	−64·7	26 26	48 20	Moore.	4	−58 46	−58·8
32 24	111 26	Moore.	7	−62 22	−62·4	27 12	46 09	Clerk.	4	−58 44	−58·7
32 32	111 36	Clerk.	4	−62 14	−62·2	27 14	45 50	Moore.	4	−58 32	−58·5
29 16	106 49	Moore.	5	−59 30	−59·5	28 02	42 40	Moore.	1	−58 10	−58·2
29 20	106 55	Clerk.	4	−59 19	−59·3	28 24	43 00	Clerk.	4	−58 52	−58·9
27 35	106 32	Moore.	7	−57 26	−57·4	28 44	42 01	Clerk.	4	−59 01	−59·0
27 47	106 36	Clerk.	4	−57 17	−57·3	28 35	40 24	Clerk.	4	−58 54	−58·9
25 46	104 55	Moore.	4	−55 05	−55·1	28 57	37 52	Clerk.	4	−59 08	−59·1
26 00	105 11	Clerk.	4	−55 09	−55·1	28 57	37 45	Moore.	6	−59 14	−59·2
23 58	99 06	Moore.	4	−53 46	−53·8	30 27	33 41	Moore.	2	−56 37	−56·6
24 00	99 23	Clerk.	4	−54 28	−54·5	30 33	33 19	Clerk.	4	−57 19	−57·3
24 01	97 25	Moore.	4	−54 18	−54·3	31 06	31 26	Moore.	5	−57 24	−57·4
24 01	97 30	Clerk.	4	−54 03	−54·1	31 06	31 34	Clerk.	4	−57 28	−57·5
23 50	95 56	Moore.	5	−54 26	−54·4	33 01	29 36	Clerk.	4	−57 34	−57·6
24 00	96 06	Clerk.	4	−54 16	−54·3	34 31	27 04	Clerk.	4	−57 06	−57·1
24 17	93 50	Moore.	5	−54 07	−54·1	35 46	23 35	Clerk.	4	−56 08	−56·1
24 02	92 07	Moore.	3	−52 44	−52·7	35 40	21 40	Moore.	4	−55 31	−55·5
22 47	91 00	Clerk.	4	−52 49	−52·8	35 36	21 40	Clerk.	4	−55 18	−55·3
21 44	89 38	Moore.	4	−51 45	−51·7	35 07	20 46	Clerk.	4	−55 08	−55·1
21 50	89 44	Clerk.	4	−52 17	−52·3	34 12	18 26	Clerk.	2	−53 37‡	−53·6
20 38	87 50	Moore.	5	−51 33	−51·5	34 12	18 26	Moore.	4	−53 37‡	−53·6
20 46	87 59	Clerk.	8	−50 57	−50·9	33 56	18 29	Clerk.	8	−53 31§	−53·5
20 37	85 02	Moore.	5	−51 21	−51·3	33 56	18 29	Moore.	18	−53 29§	−53·5

* On shore, King George's Sound.

‡ Observed on shore, Simon's Bay.

† Observed on shore, Port Louis, Mauritius.

§ Magnetic Observatory, Cape of Good Hope.

The — sign prefixed to the values of the Magnetic Force indicates that they belong to that portion of the values of the Force which is on the South side of the Magnetic Equator.

General Table of the Observations of the Intensity made by Her Majesty's Ships 'Erebus' and 'Terror,' on the passage from the Cape of Good Hope to Kerguelen Island in 1840, previous to the commencement of the regular Survey.

Lat. S.	Long. E.	Ship.	Intensity.	Lat. S.	Long. E.	Ship.	Intensity.
° ′	° ′			° ′	° ′		
34 11	18 26	Erebus.	7·60*	46 41	29 06	Terror.	8·73
34 11	18 26	Terror.	7·60*	45 44	34 16	Erebus.	9·08
35 14	18 27	Erebus.	7·65	47 00	37 14	Erebus.	9·18
35 48	18 47	Erebus.	7·70	46 45	40 05	Terror.	9·27
36 04	19 19	Erebus.	7·62	47 00	38 48	Erebus.	9·04
36 16	20 04	Terror.	7·76	47 00	43 48	Terror.	9·51
36 11	20 42	Erebus.	7·76	46 46	42 41	Erebus.	9·55
36 52	18 25	Terror.	7·67	47 50	45 20	Terror.	9·82
36 35	21 20	Erebus.	7·89	47 01	46 10	Erebus.	9·82
37 44	16 36	Terror.	7·65	46 41	50 52	Erebus.	9·91
38 47	17 00	Terror.	7·81	46 28	52 43	Erebus.	9·95
38 58	17 26	Terror.	7·94	46 29	52 26	Erebus.	10·17
40 45	19 20	Terror.	7·94	46 18	52 04	Erebus.	10·30
38 13	21 30	Erebus.	8·08	46 25	52 01	Erebus.	10·26
40 05	20 38	Erebus.	7·94	46 57	55 39	Erebus.	10·28
42 40	22 02	Terror.	8·25	47 19	59 10	Erebus.	10·66
40 29	22 22	Erebus.	8·38	47 41	62 59	Erebus.	11·28
42 56	23 12	Terror.	8·31	48 36	69 21	Erebus.	11·43
41 24	25 00	Erebus.	8·24	48 36	69 07	Erebus.	11·55
44 28	24 55	Terror.	8·57	48 30	69 52	Erebus.	11·60
41 58	26 38	Erebus.	8·45	48 39	68 57	Erebus.	11·48
46 00	26 12	Terror.	8·76	48 41	68 54	Erebus.	11·32†
43 07	28 43	Erebus.	8·56	48 41	68 54	Erebus.	11·31†
44 19	31 06	Erebus.	8·77	48 41	68 54	Terror.	11·34†

* Simon's Bay.

† On shore at Kerguelen Island.

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General Table of the Intensities of the Magnetic Force observed on board Her Majesty's Ship 'Erebus,' between September 1840 and April 1841.

Lat. S.	Long. E.	No. of observations.	Intensity.	Lat. S.	Long. E.	No. of observations.	Intensity.	Lat. S.	Long. E.	No. of observations.	Intensity.
42° 52′	147° 24′	6	13·54*	72° 03′	172° 00′	4	15·07	68° 28′	168° 10′	2	15·17
44 10	149 29	2	13·69	72 09	173 35	1	15·03	67 52	167 28	2	15·13
45 13	151 57	2	13·62	72 57	176 06	2	15·14	65 31	167 42	1	15·13
45 33	152 45	2	13·51	72 33	173 37	4	15·15	65 33	162 14	1	14·84
46 18	154 30	2	13·53	73 47	171 50	2	15·24	64 41	162 34	2	15·05
47 46	157 40	2	13·52	74 08	170 54	4	15·23	64 13	163 18	2	14·95
49 20	160 13	2	13·72	73 56	172 20	2	15·11	63 57	161 11	2	14·95
50 28	164 09	2	13·76	74 36	173 01	3	15·16	62 41	156 59	2	15·01
50 33	166 19	20	13·79†	74 44	169 43	2	15·16	63 50	156 06	2	14·98
52 42	169 10	10	14·08‡	75 22	168 48	2	15·25	64 13	154 03	2	14·95
54 06	169 09	2	13·85	76 26	168 46	4	15·12	64 20	153 02	2	15·11
55 50	170 06	1	14·22	77 47	175 43	2	14·99	63 54	151 56	1	15·02
57 35	170 32	2	14·26	77 04	188 24	4	14·97	64 26	148 20	2	15·34
58 57	170 57	2	14·36	77 09	188 15	2	15·12	65 10	143 21	2	15·20
59 41	169 38	2	14·42	77 17	185 26	2	15·11	64 20	140 40	2	15·44
60 34	170 33	5	14·50	77 00	192 18	4	15·13	63 09	139 28	7	15·15
61 49	171 44	2	14·72	77 12	192 25	4	15·07	62 13	136 20	2	15·05
62 40	174 40	2	14·78	77 09	188 50	2	15·09	61 20	134 05	2	15·25
64 03	172 41	4	14·71	76 58	180 40	2	15·13	60 19	131 20	2	15·36
64 31	172 55	2	14·81	77 47	187 18	4	15·11	58 00	128 40	2	15·25
66 08	171 22	4	14·83	77 39	187 06	5	15·03	55 00	131 43	2	15·00
66 31	169 24	3	14·95	76 55	188 40	1	15·18	54 58	132 50	4	14·95
66 23	170 12	2	14·94	76 50	183 26	1	15·11	54 00	132 02	2	15·19
65 30	170 42	4	15·11	76 16	175 50	2	15·00	53 13	135 18	5	15·01
66 55	174 31	2	14·93	76 03	169 30	1	15·15	51 16	136 50	2	15·12
68 17	175 00	1	14·91	76 20	165 32	2	15·13	48 24	138 32	1	14·88
68 32	175 49	4	14·98	75 03	168 44	2	15·15	46 29	140 40	2	14·40
68 28	176 31	4	15·07	71 25	174 25	4	15·06	46 22	141 06	2	14·45
68 28	176 32	4	15·06§	70 41	167 26	2	15·22	45 02	143 10	4	14·21
69 35	175 47	4	15·05	70 14	167 25	3	15·03	43 41	146 03	8	13·98
71 20	171 00	4	15·10	69 32	167 51	2	15·05				
71 57	171 36	4	15·20	69 08	167 44	1	15·03				

* On shore at the Observatory, Hobarton.

† On shore at Auckland Island.

‡ On shore at Campbell Island.

§ Observed on ice.

General Table of the Intensity of the Magnetic Force observed in Her Majesty's Ship 'Terror,' between January 7 and April 16, 1841.

Position.		No. of observations.	Intensity.	Position.		No. of observations.	Intensity.	Position.		No. of observations.	Intensity.
Lat. S.	Long. E.			Lat. S.	Long. E.			Lat. S.	Long. E.		
72° 54′	176° 27′	1	15·03	77° 40′	186° 40′	2	15·02	64° 06′	152° 30′	2	15·01
73 07	172 17	2	15·05	77 48	189 23	2	15·01	64 53	142 29	2	15·02
74 15	171 20	2	15·00	76 40	186 25	2	15·02	61 55	134 36	2	15·02
74 17	174 09	2	14·80	76 27	174 14	2	15·19	59 54	130 50	2	15·01
75 03	169 01	2	14·71	76 19	165 36	2	15·23	57 43	128 07	2	14·90
74 56	168 55	2	14·93	75 03	168 27	2	15·06	56 20	129 50	1	14·99
75 33	168 27	2	15·05	71 10	172 08	1	15·03	55 11	132 10	2	15·01
76 13	168 56	2	15·05	70 21	167 43	1	15·02	53 50	134 53	2	15·01
77 43	174 29	2	15·00	70 09	167 50	2	15·02	53 10	134 37	1	14·98
77 45	179 30	2	14·95	69 28	167 35	3	15·02	51 10	136 56	3	14·66
77 21	186 58	3	15·00	65 00	164 50	4	15·19	49 14	139 05	1	14·66
77 06	192 25	2	15·00	64 18	163 37	2	15·38	44 59	143 12	2	14·03
77 01	187 07	2	15·04	63 25	159 12	2	15·00	44 52	147 24	2	13·54*

* Hobarton Observatory.

General Table of the Intensities of the Magnetic Force observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between July 1841 and August 1842.

Position.		Ship.	No. of observations.	Corrected Intensity.	Position.		Ship.	No. of observations.	Corrected Intensity.
Lat. S.	Long. E.				Lat. S.	Long. E.			
42° 52′	147° 24′	Erebus.	12	−13·54*	45° 39′	183° 18′	Terror.	8	−12·77
42 52	147 24	Terror.	20	−13·54*	46 29	184 00	Erebus.	4	−12·76
43 00	148 28	Erebus.	2	−13·62	47 26	184 42	Terror.	8	−12·91
43 03	148 20	Terror.	2	−13·67	48 18	185 54	Terror.	10	−13·06
42 13	149 25	Erebus.	2	−13·43	48 43	186 30	Erebus.	3	−12·96
42 24	149 30	Terror.	2	−13·49	49 05	186 54	Terror.	10	−13·07
40 54	149 13	Erebus.	2	−13·38	49 20	187 41	Erebus.	4	−12·97
40 51	149 28	Terror.	2	−13·4)	49 24	187 23	Terror.	11	−13·10
38 17	150 22	Terror.	2	−13·20	49 27	189 13	Erebus.	5	−12·89
37 31	151 09	Erebus.	3	−12·99	49 23	188 54	Terror.	10	−13·12
37 28	151 30	Terror.	2	−13·00	49 38	189 44	Terror.	4	−13·06
36 21	151 39	Terror.	1	−13·12	50 00	191 00	Erebus.	5	−13·03
34 35	151 30	Erebus.	3	−12·72	49 50	190 46	Terror.	10	−13·15
34 06	151 19	Terror.	2	−12·72	50 08	191 39	Terror.	6	−13·09
33 51	151 17	Erebus.	12	−12·60†	50 48	192 20	Erebus.	2	−13·10
33 51	151 17	Terror.	12	−12·65†	50 32	191 52	Terror.	4	−13·18
32 52	154 07	Erebus.	2	−12·52	50 45	192 19	Terror.	10	−13·03
33 57	153 35	Terror.	4	−12·59	51 34	194 29	Erebus.	5	−13·24
33 51	157 18	Erebus.	2	−12·32	51 37	194 00	Terror.	10	−13·24
33 43	158 29	Terror.	4	−12·38	52 00	195 00	Terror.	4	−13·25
33 27	160 43	Erebus.	2	−12·24	52 21	197 53	Erebus.	2	−13·31
33 42	163 50	Terror.	4	−12·25	52 14	197 49	Terror.	4	−13·32
33 38	163 42	Erebus.	2	−12·13	53 01	202 16	Terror.	1	−13·22
33 44	166 37	Terror.	5	−12·02	52 53	203 10	Erebus.	5	−13·37
33 41	166 23	Erebus.	2	−12·02	52 51	203 56	Terror.	10	−13·53
33 34	167 37	Terror.	5	−11·82	52 53	205 07	Terror.	10	−13·30
33 22	167 40	Erebus.	2	−11·95	53 01	205 08	Erebus.	5	−13·42
32 58	169 20	Terror.	4	−11·87	53 31	206 14	Terror.	10	−13·53
32 12	170 53	Terror.	6	−11·72	54 37	208 57	Terror.	9	−13·39
32 35	169 53	Erebus.	4	−11·70	55 08	210 01	Erebus.	6	−13·52
33 57	172 04	Terror.	6	−12·01	55 04	209 58	Terror.	4	−13·35
34 12	172 45	Erebus.	4	−11·76	56 20	211 52	Erebus.	6	−13·53
34 15	172 50	Terror.	5	−11·80	56 14	211 43	Terror.	8	−13·54
34 24	173 43	Terror.	4	−11·98	56 55	212 06	Erebus.	4	−13·64
35 16	174 00	Erebus.	24	−11·81‡	56 30	211 50	Terror.	10	−13·58
35 16	174 00	Terror.	24	−11·98‡	57 21	212 46	Erebus.	2	−13·74
35 15	174 39	Erebus.	2	−11·90	57 05	212 12	Terror.	8	−13·60
36 20	177 27	Terror.	4	−11·88	57 44	212 59	Terror.	8	−13·75
36 27	177 34	Erebus.	2	−11·89	58 45	213 19	Erebus.	7	−13·85
38 00	179 34	Terror.	4	−11·95	58 36	213 10	Terror.	14	−13·92
38 17	179 31	Erebus.	2	−11·90	60 48	213 51	Terror.	10	−14·07
38 27	179 59	Terror.	4	−12·08	61 20	213 57	Erebus.	4	−14·10
39 10	182 35	Erebus.	4	−11·93	61 37	213 54	Terror.	4	−14·00
38 54	182 20	Terror.	10	−12·05	62 34	212 34	Terror.	10	−14·15
40 47	183 03	Erebus.	2	−12·25	62 40	212 53	Erebus.	2	−14·24
39 22	182 57	Terror.	8	−12·04	63 32	209 16	Terror.	12	−14·14
40 42	183 05	Terror.	8	−12·29	63 23	210 02	Erebus.	2	−14·36
41 49	183 41	Erebus.	2	−12·32	63 23	210 02	Erebus.	2	−14·30§
41 34	183 40	Terror.	10	−12·26	63 47	208 26	Erebus.	6	−14·20
42 40	183 46	Terror.	4	−12·38	64 38	206 53	Erebus.	2	−14·26
43 32	183 03	Erebus.	2	−12·55	64 38	206 24	Terror.	12	−14·31
43 54	183 06	Terror.	8	−12·56	64 53	206 30	Erebus.	6	−14·38

* Hobarton.

‡ Bay of Islands.

† Garden Island, Sydney.

§ Observed on ice.

General Table of the Intensities of the Magnetic Force (continued).

Position.		Ship.	No. of observations.	Corrected Intensity.	Position.		Ship.	No. of observations.	Corrected Intensity.
Lat. S.	Long. E.				Lat. S.	Long. E.			
° ′	° ′				° ′	° ′			
65 27	205 04	Terror.	8	−14·25	76 42	194 48	Erebus.	2	−14·83
65 36	205 32	Erebus.	2	−14·47	76 20	191 26	Terror.	4	−14·97
65 47	204 19	Terror.	4	−14·46	76 24	184 54	Terror.	4	−14·83
65 59	204 08	Erebus.	7	−14·49	77 05	194 38	Erebus.	6	−14·78
65 54	204 08	Terror.	4	−14·43	77 13	193 52	Terror.	10	−14·88
66 10	203 42	Erebus.	6	−14·47	77 47	197 25	Terror.	4	−14·80
66 08	203 39	Terror.	5	−14·42	77 14	199 29	Terror.	10	−14·74
66 33	203 28	Erebus.	4	−14·57	74 50	193 45	Erebus.	2	−14·69
66 36	203 29	Terror.	10	−14·50	75 20	194 36	Terror.	4	−14·81
66 09	204 27	Erebus.	11	−14·49	72 46	189 59	Erebus.	2	−14·64
66 20	203 39	Terror.	4	−14·40	73 10	189 21	Terror.	4	−14·78
65 58	204 07	Erebus.	8	−14·53	72 01	187 35	Erebus.	2	−14·64
66 05	204 02	Terror.	10	−14·43	72 03	187 40	Terror.	10	−14·80
66 12	203 05	Erebus.	4	−14·42	71 43	187 15	Terror.	6	−14·86
66 01	204 04	Terror.	4	−14·33	71 08	184 59	Erebus	4	−14·77
65 57	203 56	Terror.	14	−14·41	71 20	184 30	Terror.	4	−14·68
65 56	203 31	Terror.	4	−14·31	69 48	179 57	Erebus.	4	−14·68
65 49	202 02	Erebus.	7	−14·45*	69 54	179 55	Terror.	4	−14·78
65 47	202 08	Terror.	10	−14·49*	68 04	183 25	Erebus.	4	−14·59
66 49	202 40	Erebus.	2	−14·52	68 09	183 10	Terror.	4	−14·68
66 07	201 58	Terror.	8	−14·43	67 32	185 09	Erebus.	2	−14·65
67 12	203 12	Terror.	5	−14·59	67 35	185 18	Terror.	10	−14·66
67 32	204 00	Erebus.	3	−14·52	67 24	187 51	Terror.	8	−14·67
67 46	204 17	Terror.	16	−14·57	66 20	189 59	Erebus.	2	−14·47
67 18	203 39	Erebus.	3	−14·51	65 05	191 58	Terror.	10	−14·48
67 24	204 05	Terror.	4	−14·35	63 30	194 52	Erebus.	4	−14·42
67 21	202 15	Erebus.	6	−14·46	63 30	194 15	Terror.	4	−14·39
67 12	202 24	Terror.	6	−14·38	62 17	195 55	Terror.	10	−14·17
68 14	200 09	Erebus.	4	−14·64	62 16	196 10	Erebus.	2	−14·17
67 15	201 34	Terror.	8	−14·31	61 06	198 08	Terror.	4	−14·25
68 42	199 44	Erebus.	5	−14·66	61 07	199 05	Erebus.	7	−14·17
68 08	199 57	Terror.	8	−14·45	60 19	203 42	Terror.	4	−14·20
68 45	199 40	Terror.	13	−14·50	60 18	204 11	Erebus.	2	−13·80
68 52	198 24	Terror.	10	−14·53	60 15	209 55	Terror.	8	−14·09
69 24	194 08	Erebus.	4	−14·66	60 13	211 34	Erebus.	2	−13·91
70 00	191 44	Terror.	9	−14·53	59 53	216 28	Terror.	4	−14·13
70 05	191 10	Erebus.	4	−14·73	59 20	219 14	Erebus.	4	−13·82
70 14	186 16	Terror.	15	−14·62	59 22	218 14	Terror.	4	−14·06
70 18	186 01	Erebus.	6	−14·67	58 04	222 04	Erebus.	3	−13·83
70 32	185 38	Terror.	6	−14·69	58 49	221 23	Terror.	4	−14·15
70 39	185 31	Erebus.	2	−14·72	59 04	228 57	Erebus.	2	−13·85
69 54	183 52	Terror.	8	−14·68	59 01	227 43	Terror.	4	−14·02
70 07	181 45	Erebus.	3	−14·65	59 42	233 20	Erebus.	4	−13·91
71 03	180 56	Terror	4	−14·66	60 05	235 56	Terror.	4	−13·91
71 00	180 44	Erebus.	2	−14·74	60 19	237 07	Erebus.	9	−14·01
72 07	181 50	Terror.	4	−14·79	60 17	236 38	Terror.	4	−13·99
72 46	181 46	Erebus.	4	−14·62	60 24	237 29	Terror.	4	−14·11
72 55	181 33	Terror.	4	−14·83	60 01	241 38	Erebus.	4	−13·71
75 05	173 17	Erebus.	9	−14·87	59 05	247 27	Terror.	4	−13·87
74 51	174 02	Terror.	4	−14·90	59 07	248 48	Erebus.	4	−13·62
75 09	173 16	Terror.	9	−14·86	58 26	251 42	Terror.	4	−13·94
76 29	178 09	Erebus.	4	−14·87	58 29	252 22	Erebus.	2	−13·42
76 06	174 57	Terror.	4	−14·80	58 33	254 45	Terror.	10	−13·49
77 02	181 37	Terror.	4	−14·82	58 46	258 13	Terror.	8	−13·54
76 42	184 09	Erebus.	2	−14·85	58 35	255 10	Erebus.	2	−13·42
76 48	184 46	Terror.	4	−14·86	58 44	257 49	Erebus.	2	−13·56

* Observed on ice.

General Table of the Intensities of the Magnetic Force (continued).

Position.		Ship.	No. of observations.	Corrected Intensity.	Position.		Ship.	No. of observations.	Corrected Intensity.
Lat. S.	Long. E.				Lat. S.	Long. E.			
° ′	° ′				° ′	° ′			
58 59	267 50	Terror.	4	−13·35	57 22	289 50	Terror.	2	−11·62
59 02	268 30	Erebus.	2	−13·33	57 26	291 32	Erebus.	4	−11·60
59 01	272 06	Terror.	4	−13·10	57 10	292 11	Terror.	2	−11·20
59 02	272 02	Erebus.	2	−12·88	56 37	294 34	Erebus.	4	−11·51
58 24	276 18	Terror.	4	−12·92	56 40	294 46	Terror.	2	−11·14
58 50	277 12	Erebus.	2	−12·68	54 48	297 21	Erebus.	4	−10·86
58 25	279 44	Terror.	8	−12·44	54 50	298 10	Terror.	1	−10·58
58 23	280 03	Erebus.	2	−12·37	52 40	299 52	Erebus.	4	−10·04
58 31	281 33	Terror.	4	−12·66	52 40	299 52	Terror.	5	−10·17
58 29	282 04	Erebus.	2	−12·36	52 28	301 42	Terror.	4	− 9·99
58 36	285 33	Terror.	4	−12·35	52 54	300 57	Terror.	5	−10·11
58 29	286 04	Erebus.	1	−11·94	51 42	301 36	Erebus.	4	−10·00
57 21	289 36	Terror.	4	−12·13	51 32	301 53	Terror.	12	− 9·91*

* Observed on shore, Falkland Islands.

General Table of the Intensities of the Magnetic Force observed on board Her Majesty's Ships 'Erebus' and 'Terror,' between September 1842 and April 1843.

Lat. S.	Long. E.	Ship.	No. of observations.	Corrected Intensity.	Lat. S.	Long. E.	Ship.	No. of observations.	Corrected Intensity.
° ′	° ′				° ′	° ′			
53 03	302 05	Erebus.	2	− 9·89	64 13	304 06	Terror.	8	−11·87
52 48	303 10	Terror.	2	−10·18	64 12	303 04	Terror.	2	−11·95
54 03	305 26	Erebus.	2	− 9·89	64 34	302 50	Erebus.	2	−11·69
53 47	304 48	Erebus.	2	− 9·89	64 28	303 20	Terror.	8	−11·86
54 43	304 30	Erebus.	1	− 9·99	64 44	303 07	Erebus.	7	−11·76§
54 42	304 46	Erebus.	2	−10·03	64 41	302 52	Terror.	8	−11·86
54 42	305 30	Terror.	2	−10·32	64 48	303 09	Terror.	8	−11·85‡
55 40	296 52	Erebus.	5	−10·71	64 38	302 40	Terror.	2	−11·82
55 30	297 00	Terror.	2	−10·99	64 40	302 07	Terror.	2	−11·89
55 51	292 28	Erebus.	8	−11·13*	64 31	302 34	Erebus.	2	−11·65
55 51	292 28	Terror.	8	−11·19*	64 48	303 09	Erebus.	2	−11·95
56 02	292 57	Terror.	2	−11·48	64 48	303 09	Erebus.	2	−11·71§
55 39	296 00	Erebus.	2	−10·82	64 30	303 04	Terror.	2	−12·06
55 52	295 41	Terror.	2	−10·99	64 04	305 00	Terror.	8	−11·75
54 24	300 08	Erebus.	5	−10·29	64 22	305 01	Erebus.	2	−11·54
55 05	299 49	Terror.	2	−11·01	64 18	304 18	Erebus.	2	−11·53
52 52	301 05	Erebus.	5	−10·05	64 16	304 42	Terror.	1	−11·81
52 26	301 16	Terror.	2	−10·05	64 20	304 40	Terror.	2	−11·80
51 32	301 53	Erebus.	18	− 9·95†	64 12	303 50	Erebus.	2	−11·91
51 32	301 53	Terror.	14	− 9·92†	64 12	304 07	Terror.	6	−11·84
52 50	303 07	Erebus.	2	− 9·98	64 05	304 08	Terror.	6	−11·82
52 46	303 16	Terror.	7	− 9·92	64 05	304 00	Erebus.	2	−11·56
54 23	303 59	Erebus.	2	−10·22	63 56	305 22	Terror.	2	−11·68
53 38	303 43	Terror.	2	−10·00	64 24	305 30	Terror.	6	−11·66
55 51	305 18	Erebus.	2	−10·22	64 16	304 47	Terror.	4	−11·63
55 51	305 26	Terror.	9	−10·34	63 30	306 59	Erebus.	2	−11·36
56 34	306 39	Erebus.	2	−10·28	63 46	307 23	Erebus.	2	−11·37
56 55	306 40	Terror.	6	−10·42	63 47	308 00	Terror.	2	−11·51
58 16	308 00	Erebus.	2	−10·51	63 49	308 53	Erebus.	2	−11·28
58 17	307 54	Terror.	9	−10·68	63 42	308 45	Terror.	2	−11·58
59 28	308 00	Erebus.	4	−10·81	64 19	309 36	Erebus.	6	−11·35
59 57	307 53	Terror.	2	−10·86	64 19	309 40	Terror.	8	−11·49
61 23	307 41	Erebus.	1	−10·86	64 36	311 53	Erebus.	2	−11·28
61 20	307 42	Terror.	2	−11·10	64 43	312 06	Terror.	2	−11·48
62 12	307 47	Terror.	2	−11·10	64 37	314 21	Erebus.	2	−11·26
63 31	308 05	Erebus.	2	−11·06	64 38	314 01	Terror.	2	−11·53
62 25	307 58	Terror.	7	−11·32	64 39	316 04	Erebus.	2	−11·13
62 22	308 00	Erebus.	7	−11·14	64 49	315 07	Terror.	2	−11·37
62 18	308 17	Terror.	7	−11·32	64 56	317 01	Erebus.	2	−11·07
62 39	305 48	Terror.	8	−11·45	64 47	316 57	Terror.	2	−11·13
62 30	306 30	Erebus.	2	−11·22	65 06	318 46	Erebus.	2	−11·02
62 36	306 20	Erebus.	2	−11·32	64 58	318 26	Terror.	8	−11·34
63 35	305 47	Terror.	2	−11·73	64 40	320 12	Erebus.	2	−10·95
63 36	305 00	Erebus.	2	−11·34	64 37	320 28	Terror.	2	−11·22
63 39	304 40	Erebus.	7	−11·54	63 54	321 36	Erebus.	2	−10·71
63 57	304 32	Terror.	7	−11·67	64 02	321 55	Terror.	2	−11·38
64 23	304 00	Erebus.	4	−11·76	63 36	324 36	Erebus.	7	−10·62
64 26	303 52	Erebus.	2	−11·69‡	63 59	324 18	Terror.	2	−10·88
64 27	303 54	Terror.	7	−11·79‡	62 39	328 16	Erebus.	5	−10·26
64 30	304 10	Terror.	2	−11·86	62 37	328 17	Terror.	2	−10·19
64 32	304 20	Erebus.	2	−11·59	62 20	330 00	Erebus.	2	−10·16
64 38	304 20	Terror.	8	−11·81	62 13	330 28	Terror.	6	−10·35
64 18	308 39	Erebus.	2	−11·50	61 59	333 43	Erebus.	2	− 9·94

* On shore at St. Martin's Cove.

† On shore at Port Louis.

‡ On ice.

§ On an ice-floe.

General Table of the Intensities of the Magnetic Force (continued).

Lat. S.	Long. E.	Ship.	No. of observations.	Corrected Intensity.	Lat. S.	Long. E.	Ship.	No. of observations.	Corrected Intensity.
° ′	° ′				° ′	° ′			
62 05	333 38	Terror.	2	−10·33	59 21	350 36	Terror.	2	−9·69
61 37	336 05	Erebus.	2	− 9·93	57 27	352 08	Erebus.	5	−9·13
61 32	336 10	Terror.	2	−10·33	57 30	352 05	Terror.	6	−9·14
61 30	338 00	Erebus.	2	− 9·78	57 09	352 45	Erebus.	2	−8·73
61 28	337 42	Terror.	8	−10·05	57 09	352 44	Terror.	2	−9·42
61 46	341 02	Erebus.	2	− 9·77	56 38	353 57	Erebus.	2	−8·59
62 36	344 08	Erebus.	2	− 9·98	56 44	353 45	Terror.	2	−9·17
62 41	343 18	Terror.	7	−10·06	55 38	355 32	Erebus.	2	−8·57
63 58	345 10	Erebus.	2	−10·19	54 32	357 26	Terror.	2	−8·73
64 14	345 30	Terror.	2	−10·49	54 07	359 56	Erebus.	4	−8·24
64 38	348 00	Erebus.	2	−10·27	54 05	359 33	Terror.	3	−8·76
64 33	347 52	Terror.	2	−10·60	50 37	9 03	Erebus.	2	−8·22
65 12	350 05	Erebus.	2	−10·30	50 41	8 56	Terror.	3	−8·52
65 00	349 30	Terror.	2	−10·54	47 38	10 51	Erebus.	2	−8·08
66 08	352 43	Erebus.	2	−10·44	47 36	10 41	Terror.	2	−8·40
66 09	353 00	Terror.	2	−10·90	45 32	11 54	Erebus.	2	−7·89
67 06	351 04	Erebus.	2	−10·68	45 38	11 52	Terror.	2	−8·19
66 54	351 15	Terror.	2	−11·01	43 57	13 16	Erebus.	5	−7·90
68 14	347 40	Erebus.	5	−10·91	43 55	13 16	Terror.	2	−7·96
68 08	348 10	Terror.	2	−11·34	43 10	14 44	Erebus.	5	−7·75
68 32	347 09	Erebus.	2	−10·94	43 13	14 36	Terror.	8	−7·99
69 26	345 31	Erebus.	2	−11·17	41 48	15 09	Erebus.	2	−7·78
69 24	345 30	Terror.	2	−11·51	41 58	15 11	Terror.	2	−7·96
71 10	344 13	Erebus.	2	−11·47	40 15	15 47	Erebus.	2	−7·63
71 09	344 10	Terror.	2	−11·80	40 12	16 06	Terror.	2	−7·83
70 28	342 26	Terror.	2	−11·74	37 40	16 40	Erebus.	2	−7·60
68 06	344 40	Erebus.	1	−10·91	38 00	16 45	Terror.	2	−7·77
68 07	346 23	Terror.	2	−11·27	35 59	16 34	Erebus.	2	−7·52
65 56	346 24	Erebus.	2	−10·59	36 04	16 32	Terror.	2	−7·63
65 57	346 40	Terror.	2	−10·83	35 26	16 22	Erebus.	2	−7·50
64 31	346 01	Erebus.	1	−10·20	35 21	16 22	Terror.	6	−7·56
63 58	346 25	Terror.	2	−10·39	35 03	17 06	Terror.	4	−7·60
61 34	348 39	Erebus.	2	− 9·65	35 00	17 00	Erebus.	2	−7·50
61 35	349 00	Terror.	2	− 9·98	34 11	18 26	Erebus.	7	−7·59*
59 34	350 34	Erebus.	2	− 9·23	34 11	18 26	Terror.	7	−7·56*

* On shore at Simon's Bay, Cape of Good Hope.

General Table of the Intensities of the Magnetic Force observed in Her Majesty's Ship 'Pagoda' in 1845.

Lat. S.	Long. E.	Observer.	No. of observations.	Intensity of the Force, in British units.	Lat. S.	Long. E.	Observer.	No. of observations.	Intensity of the Force, in British units.
° ′	° ′				° ′	° ′			
33 56	18 29	Clerk.	12	− 7·57	66 55	40 16	Moore.	5	−11·29
34 12	18 26	Clerk.	6	− 7·62	66 24	40 01	Moore.	2	−11·32
34 44	17 50	Clerk.	3	− 7·43	66 43	40 12	Moore.	5	−11·22
34 46	17 46	Moore.	5	− 7·46	64 52	38 37	Moore.	5	−10·98
35 12	13 28	Clerk.	6	− 7·20	64 52	39 24	Clerk.	5	−11·10
35 09	15 09	Moore.	6	− 7·33	64 22	40 49	Clerk.	2	−11·06
35 17	14 00	Moore.	5	− 6·99	64 05	41 09	Moore.	1	−11·00
35 24	13 23	Moore.	7	− 7·07	63 49	42 00	Clerk.	3	−10·72
37 25	13 24	Clerk.	3	− 7·31	63 21	45 55	Moore.	6	−11·07
38 37	14 27	Clerk.	3	− 7·63	63 22	45 35	Clerk.	2	−10·88
38 42	14 27	Moore.	5	− 7·41	63 36	46 41	Clerk.	3	−11·04
39 10	14 38	Moore.	5	− 7·30	63 36	46 48	Moore.	8	−11·13
39 10	14 21	Clerk.	6	− 7·49	62 36	51 40	Clerk.	3	−11·10
40 21	14 29	Clerk.	3	− 7·53	61 34	53 49	Moore.	5	−11·35
40 41	14 16	Moore.	3	− 7·45	61 25	53 38	Clerk.	3	−11·58
42 50	13 00	Clerk.	3	− 7·55	61 26	57 37	Moore.	9	−11·41
44 50	13 19	Clerk.	3	− 7·63	61 17	57 28	Clerk.	3	−11·63
47 40	12 25	Clerk.	1	− 7·96	61 00	64 03	Clerk.	3	−11·76
50 21	10 31	Moore.	5	− 7·96	61 10	64 20	Moore.	2	−11·82
48 35	10 51	Clerk.	3	− 8·03	61 36	70 46	Clerk.	3	−12·15
50 30	10 25	Clerk.	3	− 8·29	61 49	71 31	Moore.	8	−12·16
50 48	10 18	Moore.	5	− 8·28	62 10	72 25	Clerk.	5	−12·55
51 48	9 33	Clerk.	3	− 8·48	62 10	72 25	Moore.	6	−12·44
51 50	9 33	Moore.	3	− 8·40	62 40	76 09	Clerk.	5	−12·54
52 53	7 53	Clerk.	3	− 8·50	62 48	76 15	Moore.	9	−12·52
53 21	7 52	Moore.	5	− 8·59	64 20	79 38	Clerk.	3	−12·92
53 52	6 07	Clerk.	6	− 8·66	64 20	79 38	Moore.	3	−12·71
54 02	6 02	Moore.	6	− 8·64	61 38	84 40	Clerk.	3	−12·79
55 08	5 50	Clerk.	3	− 8·79	61 42	85 07	Moore.	3	−13·10
55 18	5 55	Moore.	1	− 8·66	60 42	88 12	Clerk.	3	−13·10
60 43	4 00	Clerk.	3	− 9·39	60 48	88 33	Moore.	5	−13·23
61 05	9 03	Clerk.	6	− 9·73	61 23	91 15	Clerk.	6	−13·31
61 14	9 07	Moore.	5	− 9·76	61 23	91 15	Moore.	5	−13·25
62 06	12 52	Moore.	2	−10·22	61 14	92 03	Clerk.	3	−13·35
61 54	16 23	Clerk.	3	−10·08	61 07	92 10	Moore.	3	−13·31
61 56	16 36	Moore.	5	−10·01	60 35	92 25	Clerk.	3	−13·22
61 50	19 13	Clerk.	6	−10·10	60 30	92 34	Moore.	3	−13·25
62 00	20 25	Clerk.	3	−10·25	60 03	95 36	Clerk.	2	−13·62
63 00	20 40	Moore.	6	−10·20	60 03	96 03	Moore.	2	−13·41
63 19	21 48	Moore.	6	−10·32	59 55	99 35	Clerk.	5	−13·42
64 20	24 05	Clerk.	6	−10·61	59 45	99 50	Moore.	5	−13·91
64 25	24 18	Moore.	3	−10·59	57 46	99 17	Moore.	2	−13·73
65 34	28 30	Clerk.	3	−10·85	56 53	101 24	Clerk.	3	−13·53
65 39	28 48	Moore.	6	−10·85	56 56	101 36	Moore.	3	−13·65
66 27	30 45	Moore.	5	−10·97	55 52	103 06	Clerk.	3	−13·75
66 30	36 46	Clerk.	3	−11·22	55 40	103 18	Moore.	3	−13·75
66 36	36 50	Moore.	5	−11·13	54 48	106 04	Clerk.	3	−13·64
66 43	38 39	Clerk.	3	−11·29	54 38	106 15	Moore.	3	−13·76
67 11	38 51	Moore.	5	−11·23	54 17	108 05	Clerk.	4	−13·75
67 35	39 31	Clerk.	3	−11·50	54 10	108 15	Moore.	3	−13·79
67 39	40 28	Moore.	1	−11·33	53 00	110 08	Clerk.	3	−13·74
66 45	39 23	Clerk.	3	−11·32	53 00	110 30	Moore	5	−13·82
67 00	40 07	Clerk.	6	−11·35	51 20	111 23	Clerk.	2	−13·53
67 18	40 22	Moore.	5	−11·33	48 57	112 56	Moore.	7	−13·79

General Table of Intensities observed in the 'Pagoda' (continued).

Lat. S.	Long. E.	Observer.	No. of observations.	Intensity of the Force, in British units.	Lat. S.	Long. E.	Observer.	No. of observations.	Intensity of the Force, in British units.
° ′	° ′				° ′	° ′			
49 01	111 47	Clerk.	5	−13·62	20 36	79 10	Moore.	4	−9·19
47 21	115 15	Moore.	5	−13·95	20 36	79 22	Clerk.	6	−9·44
47 21	115 15	Clerk.	3	−13·82	20 44	78 31	Moore.	12	−9·35
45 08	116 50	Moore.	2	−13·78	20 44	78 31	Clerk.	9	−9·38
43 20	116 52	Clerk.	5	−13·33	20 39	77 43	Moore.	6	−9·34
43 22	116 49	Moore.	6	−13·66	20 34	77 02	Clerk.	12	−9·31
41 18	116 09	Clerk.	3	−13·22	20 26	70 36	Moore.	3	−9·13
41 00	116 42	Moore.	5	−13·31	20 28	70 46	Clerk.	3	−9·08
38 52	116 15	Clerk.	3	−13·16	20 34	69 24	Moore.	8	−9·16
38 40	116 15	Moore.	3	−13·04	21 06	68 12	Clerk.	3	−9·02
37 03	116 57	Clerk.	6	−12·84	21 11	67 54	Clerk.	9	−9·11
37 00	116 57	Moore.	7	−12·70	21 11	67 42	Moore.	10	−9·10
36 12	116 50	Clerk.	3	−12·67	21 12	67 29	Clerk.	6	−9·10
36 11	116 48	Moore.	3	−12·65	21 02	66 02	Clerk.	6	−8·94
35 18	117 07	Clerk.	3	−12·89	21 01	66 10	Moore.	5	−8·92
35 07	117 38	Moore.	3	−12·83	20 40	62 58	Moore.	4	−8·88
35 02	117 56	Clerk.	11	−12·78*	20 31	59 42	Clerk.	3	−8·87
35 02	117 56	Moore.	10	−12·78*	20 09	57 31	Moore.	7	−8·75‡
35 30	114 35	Moore.	3	−12·66	21 44	53 34	Clerk.	3	−8·93
35 36	114 44	Clerk.	3	−12·78	21 50	53 25	Moore.	3	−8·79
34 16	113 01	Clerk.	3	−12·43	26 25	49 12	Clerk.	3	−8·79
32 32	111 36	Clerk.	3	−12·22	26 26	48 20	Moore.	3	−8·82
32 24	111 26	Moore.	5	−11·91	27 12	46 09	Clerk.	3	−8·55
29 20	106 55	Clerk.	3	−11·76	27 14	45 50	Moore.	3	−8·78
29 16	106 49	Moore.	3	−11·35	28 24	43 00	Clerk.	3	−8·52
27 47	106 36	Clerk.	3	−11·29	28 44	42 01	Clerk.	3	−8·46
26 41	105 43	Moore.	7	−11·13	28 35	40 24	Clerk.	3	−8·54
26 00	105 11	Clerk.	3	−11·13	28 57	37 55	Moore.	5	−8·41
24 00	98 15	Moore.	6	−10·46	28 57	37 52	Clerk.	6	−8·29
24 00	97 40	Clerk.	12	−10·37	30 33	33 19	Clerk.	3	−8·22
24 03	94 52	Moore.	9	−10·37	30 27	33 41	Moore.	1	−8·37
24 02	92 07	Moore.	2	−10·35	31 06	31 34	Clerk.	3	−8·04
22 47	91 00	Clerk.	3	−10·03	31 06	31 26	Moore.	3	−8·05
21 44	89 38	Moore.	3	− 9·95	33 01	29 36	Clerk.	3	−7·92
21 50	89 44	Clerk.	3	−10·04	34 31	27 04	Clerk.	5	−8·02
20 46	87 59	Clerk.	6	− 9·80	35 46	23 35	Clerk.	3	−7·82
20 37	86 26	Moore.	6	− 9·70	35 40	21 40	Moore.	2	−7·82
20 38	85 26	Clerk.	3	− 9·58	35 36	21 40	Clerk.	3	−7·76
20 25	82 10	Moore.	5	− 9·45	35 07	20 46	Clerk.	3	−7·67
20 26	82 22	Clerk.	6	− 9·52	33 56	18 29	Clerk.	12	−7·58†

* King George's Sound, West Australia. † Magnetic Observatory, Cape of Good Hope.

‡ On shore, Port Louis, Mauritius.

Intensities of the Force observed by Lieut. Alexander John Smith, R.N., between Hobarton and the Cape of Good Hope, in July to September 1844.

Lat. S.	Long. E.	No. of observations.	Intensity, in British units.	Lat. S.	Long. E.	No. of observations.	Intensity, in British units.
42° 52′	147° 24′	2	−13·54	40° 06′	87° 00′	2	−12·01
43 28	137 10	2	−14·04	38 10	75 22	2	−10·79
43 14	133 22	2	−14·06	38 08	73 35	2	−10·94
42 35	125 40	2	−14·25	39 00	65 44	2	−10·40
40 31	109 06	2	−13·17	41 00	46 13	2	− 9·69
39 54	102 00	2	−12·97	39 50	28 36	2	− 8·40

Intensities of the Force observed by Lieut. Joseph Dayman, R.N., between Hobarton and the Cape of Good Hope, between December 1844 and March 1845.

Lat. S.	Long. E.	No. of observations.	Intensity, in British units.	Lat. S.	Long. E.	No. of observations.	Intensity, in British units.
44° 39′	144° 23′	4	−13·70	22° 34′	80° 10′	2	−9·49
44 34	141 46	3	−13·75	22 35	78 08	2	−9·31
42 24	137 18	2	−13·75	22 38	76 10	2	−9·30
41 46	133 26	2	−13·89	22 28	74 18	2	−9·15
40 05	128 23	2	−13·96	22 33	72 20	2	−9·11
38 42	123 51	4	−13·51	22 41	69 54	2	−9·14
37 47	123 52	4	−13·34	23 52	64 59	2	−9·00
35 55	118 22	4	−12·93	24 36	62 02	4	−8·93
36 06	116 42	2	−12·90	24 43	59 46	2	−8·91
34 58	112 59	2	−12·68	24 36	58 37	2	−8·86
33 47	111 04	2	−12·10	25 13	51 29	2	−8·57
33 37	108 24	2	−12·06	25 42	49 06	2	−8·38
29 40	105 28	2	−11·56	28 15	42 18	2	−8·17
28 04	105 06	2	−11·23	29 21	39 06	2	−8·13
26 44	104 26	2	−11·03	30 10	36 17	2	−8·09
25 52	102 58	2	−10·69	31 19	32 21	2	−7·87
24 50	101 31	2	−10·64	32 17	29 34	2	−7·72
24 00	99 33	2	−10·58	34 02	26 53	2	−7·61
23 02	94 44	4	−10·05	34 36	25 23	2	−7·68
22 18	90 36	4	− 9·65	34 40	24 16	2	−7·55
22 11	86 30	2	− 9·58	34 48	19 33	2	−7·40
22 08	84 17	2	− 9·41				

ERRATUM.

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for	62° 37′	328° 30′	Erebus.	7	−2° 50′	−2°·8
read	62 37	328 30	Erebus.	7	−5 46	−5·8

XIV. *On the Resistance of the Air to the Motion of Elongated Projectiles having variously formed Heads.* *By* F. Bashforth, *B.D., Professor of Applied Mathematics to the Advanced Class of Artillery Officers, Woolwich, and late Fellow of St. John's College, Cambridge. Communicated by Professor* Stokes, *Sec. R.S.*

Received January 30,—Read February 20, 1868.

The famous theory of the parabolic motion of projectiles was at an early period found to give results not in accordance with practice. Manifestly, then, the air must offer a very sensible resistance to a body which is moving through it with a high velocity. This resistance will depend upon the *form* of the moving body, and upon the *velocity* with which it is moving. Hence, before the path of a projectile can be calculated, it will be necessary to determine experimentally the resistance opposed by the air to the motion of the projectile, corresponding to various velocities. According to Newton's law, the resistance of the air varies as the square of the velocity. But the velocities were low in the experiments made under his direction. In 1719 John Bernoulli gave equations for finding by the method of Quadratures the path &c. of a projectile, when the resistance of the air was supposed to vary according to any power of the velocity. But in spite of grave doubts respecting the accuracy of Newton's law, it has been adopted by most of the eminent mathematicians who have written on the subject, such as Euler (1753), Lambert (1765), Borda (1769), Bezout (1789), Tempelhof (1788–9), d'Ehrenmalm (1788), Lombard (1796), and Poisson.

The first good experiments made with a view to determine the resistance of the air to the motion of projectiles were those of Robins in 1742. The projectiles used were leaden bullets of small size. When we consider the great density of the material used, its liability to change its form in the barrel of the gun, and the smallness of the *solid* projectiles, it is truly wonderful that Robins was able to accomplish so much with his ballistic pendulum. Afterwards Hutton carried on Robins' system of experimenting both with the whirling machine and ballistic pendulum, introducing additional precautions, and using iron projectiles of greater size. In recent times MM. Didion, Morin, and Piobert have carried on experiments in France with heavier spherical projectiles, by the help of an improved ballistic pendulum; but they have done little more than confirm the results of Robins and Hutton, and extend them to spherical projectiles of larger diameter.

Robins came to the conclusions:—"*First*, That, till the velocity of the projectile surpasses that of 1100 feet in a second, the resistance may be esteemed to be in the duplicate proportion of the velocity; and its mean quantity may be taken to be nearly the

same with that I have assigned in the former paper. *Second*, That, if the velocity be greater than that of 1100 or 1200 feet in a second, then the absolute quantity of that resistance in these greater velocities will be near three times as great as it should be by a comparison with the smaller velocities"*. HUTTON remarks in a note on these conclusions:—"These suppositions are not nearly correct. In fact, by more accurate experiments with cannon-balls, it appears that the law of the resistance begins to increase above the ratio of the square of the velocity, from the very slowest motions, and thence goes on increasing gradually more and more above what is assigned by that ratio, till we arrive at the velocity of 1600 or 1700 feet per second, where it is at the greatest, amounting in that maximum state to only $2\frac{1}{10}$ times the quantity resulting from the ratio of the square of the velocity. And at the velocity of 1100 feet, instead of answering to that law, it amounts to 1·86 times the same." EULER, in the remarks which accompany his translation of ROBINS' 'Gunnery,' states that, the greater the velocity of the shot, so much the more does theory deviate from the truth†. HUTTON's formula of resistance consisted of two terms, one varying as the velocity, and the other as the square of the velocity.

In the year 1836 M. PIOBERT reexamined HUTTON's experiments, and found that the resistance of the air for various velocities was sufficiently well represented by a formula of two terms, one of which varied as the square, and the other as the cube of the velocity. In 1839 and 1840 numerous experiments were made at Metz, under the direction of a commission, by means of an improved ballistic pendulum. The projectiles used were spherical solid shot of 24, 12, and 8, or 26·47 lbs., 13·38 lbs., and 8·86 lbs. in weight, and 5·85 inches, 4·66 inches, and 4·06 inches in diameter, and a shell 50·71 lbs. in weight and 8·67 inches in diameter. The distances from the gun at which the pendulum was placed were 49 feet, 82 feet, 164 feet, 246 feet, and 328 feet. The resistance of the air to these projectiles was found to be represented by the formula

$$\pi R^2 v^2 \times 0{\cdot}027(1+0{\cdot}0023v)\ddagger;$$

and the new calculation of HUTTON's experiments gave

$$\pi R^2 v^2 \times 0{\cdot}02786(1+0{\cdot}0023v)\ddagger.$$

When spherical balls and smooth-bored guns were used, it was only possible to strike the receiver properly when at a moderate distance from the gun; and thus the variation of velocity to be measured was confined within very narrow limits. There was also the disadvantage that, as the velocity of the ball had to be reduced to that of the receiver in order to determine the striking velocity of the ball, only one velocity could be measured for each round fired. It would therefore be quite impossible to employ ROBINS' ballistic pendulum to find the velocities of the heavy elongated projectiles in use at the present day.

* ROBINS' Tracts on Gunnery, by HUTTON, 1805, p. 181.

† Neue Grundsätze der Artillerie, 1745, p. 508.

‡ DIDION, Traité de Balistique, 1860, pp. 61 & 64.

Various attempts have been made to measure the velocities of cannon-balls by the aid of electricity. The machines with revolving cylinders were in general failures, because their inventors made their success depend upon the known uniform angular velocity of the cylinder. These failures opened the way for the introduction of Major NAVEZ' electro-ballistic pendulum, and others of the same class, which worked with two screens, and therefore furnished no means for testing the probable accuracy of the velocity determined. The apparent convenience and portability of these instruments led to their general use both in Europe and America. Major NAVEZ' instrument, in its original complicated form, is now out of fashion; whilst Colonel BENTON'S two-pendulum instrument and Colonel LEURS' modification of it are in common use, as they are simpler than Major NAVEZ' instrument, and give results quite as much to be relied upon. Even if the electro-ballistic pendulum were perfect in itself as a measurer of time, considerable errors might be expected to arise from the imperfections of the indications of two screens placed at moderate distances apart. A reference to a paper by Major NAVEZ, "Considérations sur les expériences de balistique en ce qui concerne la mesure du temps"*, will show how little had been accomplished when that was written (1865). And Colonel BENET has well remarked:—"Electro-ballistic machines heretofore used have been powerless to solve one of the most important problems in ballistics—the law of the movement of a projectile through the air,—and this because of the limited number of points of the trajectory that could be determined"†. From preliminary experiments already made, I feel certain that a simpler, cheaper, and better instrument might be substituted with advantage for electro-ballistic pendulums where such instruments can be used. The time occupied by a body in falling from rest through a given space, or the time occupied by the sound of the explosion in travelling over a given space, might be made the foundation of the measurement of a velocity; or the velocity of the shot might be directly compared with the velocity of the sound of the explosion.

In the spring of 1864, when I was appointed Professor of Applied Mathematics to the Advanced Class of Artillery Officers at Woolwich, and Referee of the Ordnance Select Committee, I strongly recommended the construction of a chronograph capable of recording the time occupied by a projectile in passing over *nine* or more *successive equal spaces*. The principle of the chronograph used at the Greenwich Observatory was plainly the one to be adopted. The chief difficulties to be overcome were found (1) in the arrangement of a proper system of screens, so that the ball in passing might merely cause a *momentary interruption* (not a rupture) of the galvanic current, and that the resistance of the circuit might be kept *perfectly constant* during the experiment, (2) in the arrangement of a system of marking, which should give definite records on the surface of the cylinder when moving with a velocity of about 10 inches per second, and (3) in the compensation for the want of uniformity in the angular velocity of the cylinder.

* Revue de Technologie Militaire, t. iv.

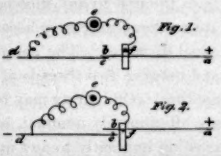
† Electro-ballistic Machines, 1866, p. 39.

The chronograph was completed in the summer of 1865, and in November and December of the same year it was tried with ten screens, placed at intervals of 120 feet. Satisfactory records were obtained for eleven out of eighteen rounds of elongated shot fired from a 12-pounder B.L. gun, which very plainly indicated that the resistance of the air varied as the cube of the velocity. A full account of the chronograph and of the method of using it, accompanied by a detailed statement of the eleven successful rounds, was printed in the Proceedings of the Royal Artillery Institution, Woolwich, for August 1866, and was also published in a separate form. At present it will be sufficient to state that the axis of the cylinder is vertical, and in a line with the axis of the horizontal fly-wheel, to which it is attached. The fly-wheel is spun by hand. When the gun is ready to be fired, the markers are brought into contact with the paper; and after the clock has recorded three or four seconds the gun is fired. The clock is allowed to record three or four seconds more, and then the markers are raised and the experiment is completed. The friction on the axis of the fly-wheel, the resistance of the air to the motion of the wheel, and the friction of the markers on the paper tend to reduce the angular velocity of the cylinder. But as the pendulum of a half-seconds' clock raises a lever once each double swing, it interrupts the clock galvanic current once a second, and thus the clock-records show what spaces have passed under the markers each second. The changing angular velocity of the cylinder thus becomes accurately known. For if we commence measuring from some arbitrary point taken two or three seconds before the screen-records, and measure along the spiral traced by the clock marker, noting each record of the clock, and continuing our measurements two or three seconds beyond the screen-records, and if we difference these quantities, we shall find whether the angular velocity has been sufficiently regular in its change during the experiment. If so, we can by interpolation find what would have been the records if one had been made every *tenth* of a second. It has always been found to be sufficient to suppose the angular velocity constant during each tenth of a second, and to calculate smaller intervals of time by proportional parts.

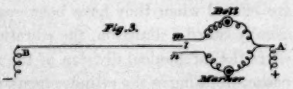
As the clock goes on all day breaking the current once a second, every record of the clock is made under precisely the same circumstances. If there be a loss of time between the breaking of the current and the making of the corresponding record, the loss of time may always be expected to remain the same for any single experiment, and therefore there can be no error; for in experiments on gunnery the exact length of a second, only, is required, and not the exact time of the day. The galvanic current which works the screen-marker is kept constantly circulating through all the screens, excepting during the momentary interruption caused by the breaking or repairing of a screen, or some accidental rupture of the conducting wire. Although there is no necessity for this arrangement, it is found to be practically convenient. The ordinary screens used for other instruments are formed of fine copper wire stretched across a frame repeatedly, and through which the galvanic current circulates. When, then, a shot passes through the screen the current is permanently broken, unless some of the broken wires happen

to rest in contact. But for my chronograph it was necessary to make provision for the breaking and immediate restoration of the current, and that without changing the resistance of the circuit. The galvanic current passes along the top of each of the ten screens simultaneously. Equal weights are attached by long pieces of sewing-cotton to certain wire springs which project through holes in sheet copper. When the shot cuts one or more threads, the corresponding springs are released and fly from the bottom to the top of their holes. So long as any single spring is not in contact with the side of the hole through which it projects, the current is interrupted. This kind of screen secures a perfectly constant resistance to the screen galvanic current. But it has been objected that if the galvanic current circulated about the screen electro-magnet for several minutes, which elapse between successive rounds, the loss of time between the breaking of the galvanic current and its corresponding registration would *not be equal* for all the ten screens. This error would be the more pernicious because it might be expected to *follow some law*, and therefore could not be eliminated. The following arrangements have been made partly to meet this difficulty, and partly for the sake of the convenience of keeping up a constant communication between the instrument and the range.

A self-acting contact-breaker and ringer (figs. 1 & 2) is placed by the side of the gun. Ordinarily the lever $a\,b$ is down, as in fig. 1, in which case the galvanic current takes the direct course, $a\,b\,c\,d$. When the lever $a\,b$ is raised, the current is permanently interrupted, but the insertion of a metallic pin f (fig. 2) opens a passage through the contact-breaker, e, when all the bells in the circuit are continuously rung. After the screens have been mended the lever $a\,b$ is raised to try whether the current is good. If so, it is lowered and the gun is loaded. When the range is clear and all is ready, the lever $a\,b$ is raised and the pin f is inserted. The fly-wheel of the chronograph is now put in motion. The rapid interruptions of the current by the contact-breaker are recorded on the cylinder, till a pull at the lanyard, to fire the gun, simultaneously withdraws also the pin f, and so shuts out the contact-breaker. The screen-records often follow so closely that it is difficult, before measurement, to say which is the first screen-record.



Another improvement, represented in fig. 3, has recently been introduced. The two ends of the circuit are at A and B, near the chronograph. When the markers are being raised from the paper by a lever, the spring l is simultaneously brought into contact with m, which turns the screen-current from the marker to the bell. Thus every interruption of the screen-current caused by the repair of the screens is signaled by the bell. When the bell rings continuously, it is known that the lever $a\,b$ (fig. 2) is raised ready for firing. The fly-wheel is caused to spin, the markers are brought down upon the paper, and *simultaneously* the spring l is brought into contact with n, when the bell is silenced and the



marker registers the breaks of the contact-breaker till the pin f is withdrawn and the gun is fired. The clock is allowed to make a few beats; and then the markers are raised from the paper, and contact is reestablished between l and m. Thus the galvanic current only circulates for eight or ten seconds about the screen electro-magnet for each experiment, and the current is always being rapidly interrupted quite up to the firing of the gun; so that there is no opportunity for the development of a varying strength of remaining magnetism. The whole arrangement is found to work so satisfactorily that on one occasion nine rounds (23 to 31) were fired in forty-five minutes.

After all possible precautions have been taken, it is found that there are small corrections required in order to make the successive records of both clock and screen consistently regular. The unit of the scale used in measuring is about half an inch, and the scale is read off to two places of decimals, or to the $\frac{1}{100}$ of an inch. The corrections are carried to three places of decimals of the scale. The final calculations are carried to four places of decimals of a second. This is done to secure accuracy to the nearest $\frac{1}{10000}$ of a second of time, giving an opening for an error of $\pm\frac{1}{30000}$ of a second of time, or 6 or 8 inches of space, in finding the time occupied by the shot in passing from the first to any succeeding screen. These corrections of the readings of the scale are rendered necessary because the screens cannot be practically maintained at perfectly equal distances. The point of the shot may strike fairly upon a thread at one screen, and between two threads at the next screen. One spring may act more promptly than another. One string may bend more than another before breaking. These corrections are often merely nominal, but there are some sufficiently large to warn us to beware of trusting implicitly to any measurement of a velocity by two screens only.

Shortly after the publication of the description of my chronograph, my attention was directed to a chronograph with a cylinder, the invention of Captain Schultz*, of the French Artillery, which had been tried in France and America. The instrument is adapted for making any number of records; and, like my own, its success does not depend upon the uniformity of rotation of the cylinder. My instrument makes the clock- and screen-records side by side, on glazed paper which covers the cylinder, so that the original records of the experiments can be preserved for future reference. Captain Schultz makes his records on the slightly smoked metal surface of his cylinder, which are effaced when they have been read off. Captain Schultz uses a large tuning-fork, usually called a diapason, the vibrations of which are sustained by electro-magnetism, to effect the mechanical division of the second into 250 or more equal intervals. The diapason, vibrating as the cylinder turns, traces a sinuous spiral line. The pendulum in swinging interrupts a galvanic circuit once a second, and causes a spark from a Ruhmkorff's coil to strike the cylinder and make a record. Thus it is found how many vibrations the diapason makes per second. The clock is then taken out of the circuit, and the current is made to pass through the Ruhmkorff coil and the first screen. When the first screen is broken the coil gives a spark, and the galvanic current is made to pass through the

* Colonel Benet's 'Electro-ballistic Machines,' 1866, p. 32.

second screen*. When the second screen is broken, a recording spark is given by the coil, and the current is passed on to the third screen, and so on to the end. All the time, the diapason is tracing its sinuous spiral. Unless a careful system of compensation be provided, this method of working the screens would cause great variation in the resistance of the circuit. In the arrangement of my screens, I was careful to maintain a *constant resistance* to the current, which end is secured by making the current pass through all the screens simultaneously, and by providing for it to be interrupted, but not broken. I am not aware that Captain SCHULTZ' instrument has been tried with more than two screens; but Colonel BENET has given a Table showing the number of vibrations per second made by the diapason as determined at the Frankfort Arsenal. In this Table there are striking variations in the numbers of vibrations made in successive seconds, as in the second trial, extending to twenty-five seconds, we find 249·1, 252·0, 249·5, 248·5, 246·0, 249·0, &c. The result of the trials is stated as if the constancy of the *mean* number of vibrations per second was all that was required. This is not the case. A succession of *equal* intervals of time must be marked out by the clock, or diapason, so that, when the gun is fired, the time of passing the screens may be noted by the side of a *correct* scale of time. Now it would make an important difference in the resulting velocity, if the gun were fired when the diapason was making 252·0, 248·5, or 246·0 vibrations per second. If the vibrations of the diapason be maintained by the vibrations of a second fork which alternately makes and breaks contact, as described by HELMHOLTZ†, it is hardly to be expected that the number of vibrations per second can be maintained with sufficient constancy. There is another question. The point from which the spark is discharged cannot be allowed to touch the smoked surface of the cylinder; and it may be asked whether the spark is not liable to deviate in its passage.

After my chronograph had passed its first trial (in December 1865), it appeared to be desirable to institute experiments with a view to find the resistance of the air to various forms of heads of elongated shot, but such as were likely to be of practical utility.

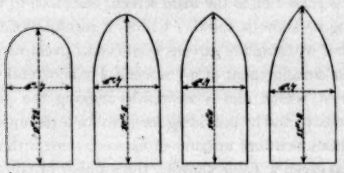
The proposed experiments received the sanction of the Right Honourable the Secretary of State for War, and ten of each kind of the following elongated shot were prepared for the 40-pounder M.L. gun:—

(1) Solid. Hemispherical head.
(2) Solid. Hemispheroidal head (ratio of diameters 1 : 2).
(3) Solid. Ogival, struck with radius equal to a diameter of shot.
(4) Solid. Ogival, struck with radius equal to two diameters of shot.
(5) Hollow. Ogival, having precisely the same external form and length as (3).
(6) Hollow. Ogival, having precisely the same external form and length as (4).

* "In working the instrument it is essential that the current pass only through one target at a time, there being but one coil and one battery no matter how many targets may be used. After the first target is ruptured, the current must be transferred to the succeeding one before the projectile reaches it, and so on throughout the series."—BENET, p. 36. See also the Practical Mechanic's Journal, Oct. 1, 1867, p. 195, to the same effect.

† Tonempfindung, p. 584.

Forms of Shot.



(1) Hemispherical head. (2) Hemispheroidal head. (3) and (5) Ogival head (1 diam.). (4) and (6) Ogival head (2 diams.).

Rounds 1–13 were fired on the 25th September 1866.
14–31 „ „ „ 26th September 1866.
32–43 „ „ „ 27th September 1866.

A barge having anchored in the range, there was no opportunity of firing a single round during the whole of the following day, which interfered with the completeness of the experiment with the solid shot. As a uniform charge of 5 lbs. of powder was used for all the rounds fired, and as the solid were nearly double the weight of the hollow shot, the hollow shot had a much higher initial velocity than the solid. Hence the hollow shot had also a much higher initial angular velocity than the solid shot; and it is probable that the initial angular velocity would be preserved nearly unimpaired throughout the observed range. This is manifested in the greater steadiness of the hollow ogivals.

For the first round, the method of forming the Time-table, and the mode of using it in the calculation of the times of arrival of the shot at each successive screen, are indicated at full length. For further details I must refer the reader to the published description of the chronograph.

The screens were placed 150 feet apart, which distance is denoted by l. The first screen was 75 feet from the gun. t denotes the time occupied by the shot in passing from the first screen to a distance s feet, when $t=as+bs^3$; and $0, t_2, t_3, \ldots t_n$ will denote the time when the shot passes the first, second, third, ... nth screen, or the times corresponding to the particular values $0, l, 2l, \ldots \overline{n-1}\ l$ of s.

(1) Hemispherical-headed Shot. Diameter 4·7 inches.

Round 1. Weight of shot 39·34 lbs.

Clock.					
	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	4·91	0	4·910	+15·910	−70
2	20·82	0	20·820	15·840	67
3	36·65	+·010	36·660	15·773	64
4	52·44	−·007	52·433	15·709	−61
5	68·14	+·002	68·142	+15·648	
6	83·79	0	83·790		

Time-table obtained by interpolation.		
t		
2·5	28·748	
		+1·584
2·6	30·332	
		1·583
2·7	31·915	
		1·582
2·8	33·497	
		1·582
2·9	35·079	
		1·581
3·0	36·660	
		1·580
3·1	38·240	
		1·580
3·2	39·820	
		1·579
3·3	41·399	
		1·578
3·4	42·977	
		1·578
3·5	44·555	
		1·577
3·6	46·132	
		1·576
3·7	47·708	
		1·576
3·8	49·284	
		1·575
3·9	50·859	
		+1·574
4·0	52·433	

Interpolation.			
t		Δ_1	Δ_2
2·0	20·820	+7·928	−16
2·5	28·748	7·912	−17
3·0	36·660	7·895	−17
3·5	44·555	+7·878	
4·0	52·433		

Screens.					
No.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	28·26	0	28·260	+2·002	+28
2	30·26	+·002	30·262	2·030	28
3	32·30	−·008	32·292	2·058	28
4	34·35	0	34·350	2·086	28
5	36·43	+·006	36·436	2·114	27
6	38·54	+·010	38·550	2·141	26
7	40·76	*	40·691	2·167	26
8	42·86	−·002	42·858	2·193	26
9	45·06	−·009	45·051	+2·219	
10	47·26	+·010	47·270		

Having thus obtained the space described by the clock marker at the end of every tenth of a second, from 2″·5 up to 4″·0, we can calculate the times when the screens were passed as follows:—

Screen 1	28·260 passed at $t = 2·5 - \frac{488}{1584} \times ·01 = 2·4692$	t		
		0·0000		
			+1964	
„ 2	30·262 passed at $t = 2·6 - \frac{70}{1584} \times ·01 = 2·5956$		Δ_2	
		·1964		+18
			1282	
„ 3	32·292 passed at $t = 2·7 + \frac{377}{1582} \times ·01 = 2·7238$			19
		·3246	1301	
„ 4	34·350 passed at $t = 2·8 + \frac{853}{1582} \times ·01 = 2·8539$			18
		·3847	1319	
„ 5	36·436 passed at $t = 3·0 - \frac{224}{1581} \times ·01 = 2·9858$			19
		·5166	1338	
„ 6	38·550 passed at $t = 3·1 + \frac{310}{1580} \times ·01 = 3·1196$			18
		·6504	1356	
„ 7	40·691 passed at $t = 3·2 + \frac{871}{1579} \times ·01 = 3·2552$			17
		·7860	1373	
„ 8	42·858 passed at $t = 3·4 - \frac{119}{1578} \times ·01 = 3·3925$			17
		·9233	1390	
„ 9	45·051 passed at $t = 3·5 + \frac{496}{1577} \times ·01 = 3·5315$			17
		1·0623	+1407	
„ 10	47·270 passed at $t = 3·7 - \frac{438}{1576} \times ·01 = 3·6729$	1·2030		

Round 13. Weight of shot 39·33 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ	Screen.	Reading.	Correction.	Corrected reading.	Δ	Δ	Time of passing each screen.	Δ	Δ
											t.		
1	15·51	0	15·510		1	64·69	0	64·690			0·0000		
2	39·56	0	39·560	+24·050	2	67·72	−·005	67·715	+3·025	+36	·1266	+1266	+16
3	63·52	−·007	63·513	23·953	3	70·77	+·006	70·776	3·061	36	·2548	1282	16
4	87·37	0	87·370		4	73·87	+·003	73·873	3·097	34	·3846	1298	14
5	111·13	0	111·130	+23·760	5	77·01	−·006	77·004	3·131	35	·5158	1312	16
					6	80·17	0	80·170	3·166	35	·6486	1328	15
					7	* *	*	83·371	3·201	36	·7829	1343	16
					8	86·61	−·002	86·606	3·237	37	·9188	1359	16
					9	89·89	−·008	89·889	3·274	+37	1·0563	1375	+17
					10	93·19	+·003	93·193	+3·311		1·1955	+1392	

Round 34. Weight of shot 39·34 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ	Δ	Screen.	Reading.	Correction.	Corrected reading.	Δ	Δ	Time of passing each screen.	Δ	Δ
												t.		
4	89·69	0	89·690			1	104·33	0	104·330			0·0000		
5	111·17	−·010	111·160	+21·470	−70	2	107·05	−·002	107·048	+2·718	+33	·1967	+1967	+16
6	132·55	+·010	132·560	21·400	−70	3	109·80	−·001	109·799	2·751	33	·2550	1283	16
7	153·89	0	153·890	+21·330		4	112·58	+·003	112·583	2·784	32	·3849	1299	16
						5	115·40	−·001	115·399	2·816	33	·5164	1315	16
						6	118·24	+·010	118·250	2·851	36	·6495	1331	17
						7	121·16	−·023	121·137	2·887	38	·7843	1348	19
						8	124·06	+·002	124·062	2·925	36	·9210	1367	17
						9	127·02	+·003	127·023	2·961	+36	1·0594	1384	+18
						10	130·02	0	130·020	+2·997		1·1996	+1402	

Round 43. Weight of shot 39·34 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ	Δ	Screen.	Reading.	Correction.	Corrected reading.	Δ	Δ	Time of passing each screen.	Δ	Δ
												t.		
1	24·05	0	24·050			1	75·83	0	75·830			0·0000		
2	45·51	−·005	45·505	+21·455	−70	2	78·50	+·015	78·515	+2·685	+35	·1260	+1260	+16
3	66·88	+·010	66·890	21·385	70	3	81·24	−·005	81·235	2·720	35	·2536	1276	17
4	88·21	−·005	88·205	21·315	−70	4	83·99	0	83·990	2·755	35	·3829	1293	18
5	109·45	0	109·450	+21·245		5	86·78	0	86·780	2·790	37	·5140	1311	18
						6	89·61	−·003	89·607	2·827	36	·6469	1329	17
						7	92·48	−·010	92·470	2·863	37	·7815	1346	18
						8	95·37	0	95·370	2·900	37	·9179	1364	18
						9	98·31	−·003	98·307	2·937	+36	1·0561	1382	+18
						10	101·28	0	101·280	+2·973		1·1961	+1400	

Summary (1). Hemispherical-headed Shot.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
1	0·0000	0·1264	0·2546	0·3847	0·5166	0·6504	0·7860	0·9233	1·0623	1·2030
13	0·0000	·1266	·2548	·3846	·5158	·6486	·7829	·9188	1·0563	1·1955
34	0·0000	·1267	·2550	·3849	·5164	·6495	·7843	·9210	1·0594	1·1996
43	0·0000	·1260	·2536	·3829	·5140	·6469	·7815	·9179	1·0561	1·1961

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.
1	1187	1170	1153	1137	1121	1106	1092	1079	1066
13	1185	1170	1156	1143	1130	1117	1104	1091	1078
34	1184	1169	1155	1141	1127	1113	1097	1084	1070
43	1190	1175	1160	1144	1129	1114	1100	1085	1071

No. of round.	Weight of shot.	Value of δP.	Difference from mean.
	lbs.		
1	39·34	·00091	+·00007
13	39·33	·00078	−·00006
34	39·34	·00089	−·00002
43	39·34	·00085	+·00001
Means	39·34	·00084	·00004

(2) Hemispheroidal-headed Shot (Axes 1 : 2), Solid.

Round 2. Weight of shot 38·72 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t		
2	24·44	0	24·440			1	103·28	+·005	103·285			0·0000		
3	48·19	0	48·190	+23·750	−44	2	106·28	+·002	106·282	+2·997	+24	·1267	+1267	+11
4	71·90	−·004	71·896	23·706	39	3	109·31	−·007	109·303	3·021	29	·2545	1278	13
5	95·56	+·003	95·563	23·667	34	4	112·35	+·002	112·352	3·049	31	·3836	1291	19
6	119·20	−·004	119·196	23·633	−29	5	115·43	+·002	115·432	3·080	32	·5139	1303	15
7	142·80	0	142·800	+23·604		6	118·55	−·006	118·544	3·112	34	·6457	1318	14
						7	121·70	−·010	121·690	3·146	+35	·7789	1332	+15
						8	124·87	+·001	124·871	+3·181		·9136	+1347	

Round 7. Weight of shot 38·69 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t		
3	60·24	0	60·240			1	82·59	+·003	82·593			0·0000		
4	81·88	0	81·880	+21·640	−80	2	85·34	+·001	85·341	+2·748	+28	·1373	+1373	+13
5	103·45	−·010	103·440	21·560	80	3	88·13	−·013	88·117	2·776	28	·2559	1286	14
6	124·91	+·010	124·920	21·480	−80	4	90·92	+·001	90·921	2·804	30	·3859	1300	14
7	146·32	0	146·320	+21·400		5	93·75	+·005	93·755	2·834	30	·5173	1314	15
						6	96·62	−·001	96·619	2·864	27	·6502	1329	13
						7	99·51	0	99·510	2·891	28	·7844	1342	14
						8	102·43	−·001	102·429	2·919	+27	·9200	1356	+13
						9	105·35	+·025	105·375	+2·946		1·0569	+1369	

Round 35. Weight of shot 38·69 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t		
3	52·97	0	52·970			1	68·09	0	68·090			0·0000		
4	71·20	0	71·200	+18·230	−63	2	**	*	70·399	+2·309	+20	·1268	+1268	+12
5	89·37	−·003	89·367	18·167	−64	3	**	*	72·728	2·329	23	·2548	1280	13
6	107·47	0	107·470	+18·103		4	**	*	75·080	2·352	23	·3841	1293	13
						5	**	*	77·455	2·375	23	·5147	1306	13
						6	79·85	+·003	79·853	2·398	23	·6466	1319	14
						7	82·29	−·016	82·274	2·421	24	·7799	1333	14
						8	84·72	−·001	84·719	2·445	24	·9146	1347	14
						9	87·19	−·002	87·188	2·469	+23	1·0507	1361	+13
						10	89·68	0	89·680	+2·492		1·1881	+1374	

Round 40. Weight of shot 38·69 lbs..

Clock.	Reading.	Correction.	Corrected reading.			Screen.	Reading.	Correction.	Corrected reading.			Time of passing each screen.		
				Δ_1	Δ_2					Δ_1	Δ_2	*t*	Δ_1	Δ_2
4	87·30	0	87·300	+19·750	−50	1	113·53	0	113·530	+2·491	+26	0·0000	+1·264	+14
5	107·05	0	107·050	19·700	−50	2	116·02	+·001	116·021	2·517	26	·1264	1278	13
6	126·75	0	126·750	+19·650		3	118·55	−·012	118·538	2·543	26	·2542	1291	14
7	146·40	0	146·400			4	121·08	+·001	121·081	2·569	26	·3833	1305	14
						5	123·65	0	123·650	2·596	27	·5138	1319	14
						6	126·23	+·016	126·246	2·596	27	·6457	1333	14
						7	128·87	−·001	128·869	2·623	26	·7790	1347	14
						8	131·52	−·002	131·518	2·649	27	·9137	1361	14
						9	134·19	+·004	134·194	2·676	+26	1·0498	+1·375	+14
						10	136·90	−·004	136·896	+2·702		1·1873		

Summary (2). Hemispheroidal-headed Shot.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
	t	*t*	*t*	*t*	*t*	*t*	*t*	*t*	*t*	*t*
2	0·0000	0·1267	0·2545	0·3836	0·5139	0·6457	0·7789	0·9136	**	**
7	0·0000	·1273	·2559	·3859	·5173	·6502	·7844	·9200	1·0569	**
35	0·0000	·1268	·2548	·3841	·5147	·6466	·7799	·9146	1·0507	1·1881
40	0·0000	·1264	·2543	·3833	·5138	·6457	·7790	·9137	1·0498	1·1873

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
2	1184	1174	1162	1151	1138	1126	1114	**	**
7	1178	1166	1154	1142	1129	1118	1106	1096	**
35	1183	1172	1160	1149	1137	1125	1114	1102	1092
40	1187	1174	1162	1149	1137	1125	1114	1102	1091

No. of round.	Weight of shot.	Value of δP.	Difference from mean value.
	lbs.		
2	38·72	·00064	−·00003
7	38·69	·00069	+·00002
35	38·69	·00065	−·00002
40	38·69	·00069	+·00002
Means	38·70	·00067	·00002

(3) Ogival-headed Shot (one diameter), Solid.

Round 3. Weight of shot 39·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.			Screen.	Reading.	Correction.	Corrected reading.			Time of passing each screen.		
				Δ_1	Δ_2					Δ_1	Δ_2	*t*	Δ_1	Δ_2
1	21·00	0	21·000	+25·850	−100	1	114·12	0	114·120	+3·310	+35	0·0000	+1·293	+14
2	46·94	0	46·940	25·750	79	2	117·43	0	117·430	3·345	43	·1293	1307	18
3	72·69	0	72·690	25·671	62	3	120·78	−·005	120·775	3·388	48	·2600	1325	18
4	98·36	+·001	98·361	25·609	− 49	4	124·15	+·013	124·163	3·436	49	·3925	1343	20
5	123·97	0	123·970	+25·560		5	127·60	−·001	127·599	3·485	46	·5268	1363	18
6	149·53	0	149·530			6	131·09	−·006	131·084	3·531	+39	·6631	1381	18
						7	134·62	−·005	134·615	+3·570		·8012	+1·397	+16
						8	138·18	+·005	138·185			·9409		

Round 36. Weight of shot 39·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_v	Δ_v
3	52·79	0	52·790	+19·700	− 105
4	72·49	0	72·490	19·595	100
5	92·08	+·005	92·085	19·495	− 95
6	111·58	0	111·580	+19·400	
7	130·98	0	130·980		

Screen.	Reading.	Correction.	Corrected reading.	Δ_v	Δ_v
1	99·25	0	99·250		
2	101·29	*	101·716	+2·466	+27
3	104·21	−·001	104·209	2·483	26
4	106·73	−·002	106·728	2·519	25
5	109·27	+·002	109·272	2·544	25
6	111·84	+·001	111·841	2·569	25
7	114·44	−·005	114·435	2·594	26
8	117·05	+·005	117·055	2·620	+25
9	119·70	0	119·700	+2·645	

Time of passing each screen.	Δ_v	Δ_v
t.		
0·0000		
·1964	+1964	+15
·3543	1979	15
·3837	1994	13
·5144	1307	13
·6464	1320	13
·7799	1335	15
·9148	1349	14
1·0510	+1362	+13

Round 41. Weight of shot 39·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_v	Δ_v
2	53·16	−·006	53·154	+23·494	−90
3	76·64	+·008	76·648	23·404	90
4	100·06	−·008	100·052	23·314	−90
5	123·36	+·006	123·366	+23·224	
6	146·59	0	146·590		

Screen.	Reading.	Correction.	Corrected reading.	Δ_v	Δ_v
1	123·37	0	123·370		
2	126·30	+·001	126·301	+2·931	+31
3	129·26	+·003	129·263	2·962	31
4	132·26	−·004	132·256	3·023	30
5	135·27	+·009	135·279	3·052	29
6	138·33	+·001	138·331	3·081	29
7	141·41	+·002	141·412	3·110	29
8	144·51	+·012	144·522	3·139	+29
9	147·65	+·011	147·661	+3·168	
10	150·84	−·011	150·829		

Time of passing each screen.	Δ_v	Δ_v
t.		
0·0000		
·1260	+1260	+14
·2534	1274	14
·3822	1288	13
·5123	1301	13
·6437	1314	14
·7765	1328	13
·9106	1341	13
1·0460	1354	+13
1·1827	+1367	

Summary (3). Ogival-headed Shot (one diameter), Solid.

Time occupied by shot in passing from the first to each of the other screens.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
	t.	t.	t.	t.	t.	t.	t.	t.	t.	t.
3	0·0000	0·1293	0·2600	0·3925	0·5268	0·6631	0·8012	0·9409	*	*
36	0·0000	·1264	·2543	·3837	·5144	·6464	·7799	·9148	1·0510	*
41	0·0000	·1260	·2534	·3822	·5123	·6437	·7765	·9106	1·0460	1·1827

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.
3	1160	1147	1132	1117	1101	1086	1074	*	*
36	1187	1173	1159	1148	1136	1124	1112	1101	*
41	1190	1177	1165	1153	1142	1130	1119	1108	1097

No. of round.	Weight of shot.	Values of kl^3.
	lbs.	
3	39·56	·00085
36	39·56	·00071 }
41	39·56	·00068 }
Mean...	39·56	Mean of 2 rounds ·00070

(4) Ogival-headed Shot (two diameters), Solid.

Round 4. Weight of shot 38·56 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
"					
1	17·50	0	17·500		
				+23·115	−80
2	40·62	−·005	40·615		
				23·035	
3	63·64	+·010	63·650		67
				22·968	
4	86·63	−·012	86·618		−56
				+22·912	
5	109·53	0	109·530		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	63·78	−·001	63·779		
				+2·910	+33
2	66·67	+·019	66·689		
				2·943	33
3	69·63	+·002	69·632		
				2·976	33
4	72·61	−·002	72·608		
				3·008	32
5	75·62	−·004	75·616		
				3·040	32
6	78·65	+·006	78·656		
				3·071	31
7	81·73	−·003	81·727		
				3·099	28
8	84·83	−·004	84·826		
				3·128	29
9	87·95	+·004	87·954		+28
				+3·156	
10	91·11	0	91·110		

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
	+1966	+14
·1966		
	1280	15
·3246		
	1295	15
·4541		
	1310	15
·5851		
	1324	13
·7175		
	1337	14
·8512		
	1351	13
·9863		
	1364	+12
1·0527		
	+1376	
1·1903		

Round 37. Weight of shot 38·48 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
"					
4	51·40	0	51·400		
				+16·640	−80
5	68·04	0	68·040		
				16·560	−80
6	84·60	0	84·600		
				+16·480	
7	101·08	0	101·080		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	71·99	0	71·990		
				+2·113	+23
2	74·10	+·003	74·103		
				2·136	21
3	76·24	−·001	76·239		
				2·157	21
4	78·40	−·004	78·396		
				2·178	+18
5	80·57	+·004	80·574		
				+2·196	
6	82·77	0	82·770		

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
	+1975	+15
·1975		
	1290	13
·3265		
	1303	13
·3868		
	1316	+13
·5184		
	+1339	
·6513		

Round 42. Weight of shot 38·47 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
"					
4	69·77	0	69·770		
				+14·960	−80
5	84·74	−·010	84·730		
				14·880	−80
6	99·61	0	99·610		
				+14·800	
7	114·41	0	114·410		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	81·34	−·004	81·336		
				+1·836	+24
2	83·17	+·002	83·172		
				1·860	23
3	85·03	+·002	85·032		
				1·883	23
4	86·91	+·005	86·915		
				1·906	23
5	88·82	+·001	88·821		
				1·929	23
6	90·75	0	90·750		
				1·952	23
7	92·71	−·008	92·702		
				1·974	22
8	94·68	−·004	94·676		
				1·996	22
9	96·66	+·012	96·672		+22
				+2·018	
10	98·69	+ 0	98·690		

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
	+1230	+16
·1230		
	1246	17
·2476		
	1263	16
·3739		
	1279	16
·5018		
	1295	16
·6313		
	1312	17
·7625		
	1327	15
·8952		
	1343	16
1·0295		
	+1359	+16
1·1654		

Summary (4). Ogival-headed Shot (two diameters), Solid.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
	"	"	"	"	"	"	"	"	"	"
4	0·0000	0·1966	0·3246	0·3841	0·5151	0·6475	0·7812	0·9163	1·0527	1·1903
37	0·0000	·1975	·3265	·3868	·5184	·6513	*	*	*	*
42	0·0000	·1230	·2476	·3739	·5018	·6313	·7625	·8952	1·0295	1·1654

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.
4	1185	1172	1158	1145	1133	1122	1110	1100	1090
37	1176	1163	1151	1140	1129	*	*	*	*
42	1220	1204	1188	1173	1158	1143	1130	1117	1104

No. of round.	Weight of shot.	Values of $b'P$.
	lbs.	
4	38·56	·00071 }
37	38·48	·00069 }
42	38·47	·00081
Mean...	38·52	Mean of 2 rounds ·00070

(5) Ogival-headed Shot (one diameter), Hollow.

Round 14. Weight of shot 21·78 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	27·48	−·001	27·479		
				+21·079	−80
2	48·56	−·002	48·558		
				20·999	−78
3	69·55	+·007	69·557		
				20·921	80
4	90·47	+·008	90·478		
				20·841	−80
5	111·33	−·011	111·319		
				+20·761	
6	132·08	0	132·080		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	96·37	+·002	96·372		
				+2·087	+52
2	98·46	−·001	98·459		
				2·139	52
3	100·61	−·012	100·598		
				2·191	52
4	102·79	−·001	102·789		
				2·243	52
5	105·02	+·012	105·032		
				2·295	51
6	107·32	+·007	107·327		
				2·346	50
7	109·68	−·007	109·673		
				2·396	51
8	112·09	−·021	112·069		
				2·447	+51
9	114·52	−·004	114·516		
				+2·498	
10	117·01	+·004	117·014		

Time of passing each screen.	Δ_1.	Δ_2.
t.		
0·0000		
	+1001	+25
·1001		
	1026	25
·2027		
	1051	26
·3078		
	1077	26
·4155		
	1103	24
·5258		
	1127	25
·6385		
	1152	24
·7537		
	1176	+22
·8713		
	+1198	
·9911		

Round 16. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	5·14	0	5·140		
				+26·490	−50
2	31·63	0	31·630		
				26·440	62
3	58·07	0	58·070		
				26·378	−76
4	84·45	−·002	84·448		
				+26·302	
5	110·75	0	110·750		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	58·75	+·005	58·755		
				+2·603	+55
2	61·37	−·012	61·358		
				2·662	62
3	64·02	0	64·020		
				2·724	66
4	66·74	+·004	66·744		
				2·790	69
5	69·52	+·014	69·534		
				2·859	68
6	72·39	+·003	72·393		
				2·927	68
7	75·33	−·010	75·320		
				2·995	68
8	78·33	−·015	78·315		
				3·063	+68
9	81·37	+·008	81·378		
				+3·131	
10	84·50	+·009	84·509		

Time of passing each screen.	Δ_1.	Δ_2.
t.		
0·0000		
	+986	+22
·0986		
	1008	24
·1994		
	1032	26
·3026		
	1058	26
·4084		
	1084	26
·5168		
	1110	26
·6278		
	1136	26
·7414		
	1162	+26
·8576		
	+1188	
·9764		

Round 18. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	28·12	−·005	28·115		
				+21·864	−65
2	49·99	−·011	49·979		
				21·799	60
3	71·77	+·008	71·778		
				21·739	55
4	93·51	+·007	93·517		
				21·684	−50
5	115·21	−·009	115·201		
				+21·634	
6	136·84	−·005	136·835		

Screen.	Reading.	Correction.	Corrected reading.	Δ_1.	Δ_2.
1	101·10	−·002	101·098		
				+2·153	+55
2	103·23	+·021	103·251		
				2·208	55
3	105·46	−·001	105·459		
				2·263	54
4	107·73	−·008	107·722		
				2·317	54
5	110·03	+·009	110·039		
				2·371	54
6	112·41	0	112·410		
				2·425	54
7	114·86	−·025	114·835		
				2·479	54
8	117·32	−·006	117·314		
				2·533	+54
9	119·85	−·003	119·847		
				+2·587	
10	122·43	+·004	122·434		

Time of passing each screen.	Δ_1.	Δ_2.
t.		
0·0000		
	+993	+25
·0993		
	1018	26
·2011		
	1044	25
·3055		
	1069	25
·4124		
	1094	26
·5218		
	1120	25
·6338		
	1145	25
·7483		
	1170	+25
·8653		
	+1195	
·9848		

Round 20. Weight of shot 21·83 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	11·95	0	11·950		
2	33·74	0	33·740	+21·790	−70
3	55·45	+·010	55·460	21·720	70
4	77·11	0	77·110	21·650	70
5	98·68	+·010	98·690	21·580	70
6	120·20	0	120·200	21·510	−70
7	141·65	−·010	141·640	+21·440	

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	94·50	0	94·500		
2	96·64	−·005	96·635	+2·135	+56
3	98·82	+·006	98·826	2·191	56
4	101·06	−·007	101·072	2·247	54
5	103·37	+·004	103·374	2·301	52
6	105·73	−·003	105·727	2·353	51
7	108·15	−·019	108·131	2·404	50
8	110·59	−·005	110·585	2·454	50
9	113·08	+·009	113·089	2·504	+50
10	115·64	+·003	115·643	+2·554	

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
·0990	+ 990	+27
·2007	1017	26
·3050	1043	26
·4119	1069	24
·5212	1093	24
·6329	1117	24
·7470	1141	24
·8635	1165	+23
·9823	+1188	

Round 22. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
3	66·77	0	66·770		
4	87·23	−·005	87·225	+20·455	−72
5	107·60	+·008	107·608	20·383	−71
6	127·92	0	127·920	+20·312	

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	97·44	0	97·440		
2	99·45	+·006	99·456	+2·016	+47
3	101·52	−·001	101·519	2·063	47
4	103·63	−·001	103·629	2·110	48
5	105·78	+·007	105·787	2·158	49
6	107·99	+·004	107·994	2·207	49
7	110·26	−·010	110·250	2·256	49
8	112·55	+·005	112·555	2·305	50
9	114·90	+·010	114·910	2·355	+50
10	117·32	−·005	117·315	+2·405	

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
·0990	+ 990	+22
·2002	1012	24
·3038	1036	24
·4098	1060	25
·5183	1085	24
·6292	1109	24
·7425	1133	26
·8584	1159	+25
·9768	+1184	

Round 24. Weight of shot 21·83 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
2	36·69	0	36·690		
3	61·65	−·010	61·640	+24·950	−98
4	86·49	+·002	86·492	24·852	106
5	111·24	−·002	111·238	24·746	−112
6	135·87	+·002	135·872	+24·634	

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	71·78	0	71·780		
2	74·26	−·006	74·254	+2·474	+61
3	76·79	−·001	76·789	2·536	61
4	79·39	−·005	79·385	2·658	62
5	82·04	+·002	82·043	2·730	63
6	84·75	+·013	84·763	2·793	63
7	87·56	−·015	87·545	2·845	63
8	90·39	0	90·390	2·908	+63
9	93·30	−·002	93·298	+2·971	
10	96·27	−·001	96·269		

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
·0995	+ 995	+26
·2016	1021	24
·3061	1045	26
·4132	1071	25
·5228	1096	26
·6350	1122	25
·7497	1147	27
·8671	1174	+25
·9870	+1199	

Round 26. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
3	38·40	+·001	38·401		
4	56·51	−·003	56·507	+18·106	−69
5	74·54	+·004	74·544	18·037	62
6	92·59	−·001	92·519	17·975	−54
7	110·44	0	110·440	+17·921	

Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2
1	78·16	−·005	78·155		
2	79·94	+·010	79·950	+1·795	+45
3	81·78	+·010	81·790	1·840	45
4	83·68	−·005	83·675	1·885	44
5	85·59	+·014	85·604	1·929	43
6	87·57	+·006	87·576	1·972	43
7	89·60	−·009	89·591	2·015	42
8	91·65	−·002	91·648	2·057	42
9	93·74	+·007	93·747	2·099	+42
10	95·89	−·002	95·888	+2·141	

Time of passing each screen.	Δ_1	Δ_2
t.		
0·0000		
·0997	+ 997	+27
·2021	1024	24
·3069	1048	25
·4142	1073	24
·5239	1097	26
·6362	1123	23
·7508	1146	23
·8677	1169	+24
·9870	+1193	

Round 28. Weight of shot 21·83 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
1	27·41	+·010	27·420	+20·505		1	77·15	+·010	77·160	+2·020		0·0000		
2	47·93	−·005	47·925		−·52	2	79·19	−·010	79·180		+·50	·0991	+991	+23
3	68·37	+·008	68·378	20·453	51	3	81·25	0	81·250	2·070	50	·2005	1014	25
4	88·78	0	88·780	20·402	−52	4	83·37	0	83·370	2·120	50	·3044	1039	26
5	109·13	0	109·130	+20·350		5	85·54	0	85·540	2·170	50	·4109	1065	26
						6	85·76	0	87·760	2·220	50	·5198	1089	24
						7	90·05	−·020	90·030	2·270	50	·6312	1114	25
						8	92·35	0	92·350	2·320	50	·7452	1140	26
						9	94·72	0	94·720	2·370	+50	·8616	1164	24
						10	97·14	0	97·140	+2·420		·9804	+1188	+24

Round 30. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
1	27·54	0	27·540	+22·065		1	91·67	0	91·670	+2·173		0·0000		
2	49·61	−·005	49·605		−90	2	93·83	+·013	93·843		+54	·0995	+995	+24
3	71·58	0	71·580	21·890	85	3	96·07	0	96·070	2·227	55	·2014	1019	26
4	93·47	0	93·470	+21·810	−80	4	98·35	+·002	98·352	2·282	55	·3059	1045	26
5	115·28	0	115·280			5	100·69	−·001	100·689	2·337	54	·4130	1071	25
						6	103·08	0	103·080	2·391	55	·5226	1096	25
						7	105·53	−·004	105·526	2·446	54	·6347	1121	26
						8	108·04	−·014	108·026	2·500	53	·7494	1147	24
						9	110·59	−·011	110·579	2·553	+52	·8665	1171	+27
						10	113·18	+·004	113·184	+2·605		·9863	+1198	

Round 32†. Weight of shot 21·81 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
3	44·45	0	44·450	+21·630		1	65·19	0	65·190	+2·239		0·0000		
4	66·08	0	66·080		−27	2	**	*	67·429		+54	·1036	+1036	+25
5	87·68	+·003	87·683	21·603	−26	3	69·72	+·002	69·722	2·293	54	·2097	1061	25
6	109·26	0	109·260	+21·577		4	72·07	−·001	72·069	2·347	49	·3183	1086	23
						5	74·46	+·005	74·465	2·396	50	·4292	1109	23
						6	76·91	+·001	76·911	2·446	52	·5424	1132	25
						7	79·41	−·001	79·409	2·498	57	·6581	1157	25
						8	81·97	−·006	81·964	2·555	59	·7764	1183	26
						9	84·58	−·002	84·578	2·614	+58	·8974	1210	27
						10	87·25	0	87·250	+2·672		1·0211	+1237	+27

Summary (5). Ogival-headed Shot (one diameter), Hollow.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
	t.	t.	t.	t.	t.	t.	t.	t.	t.	t.
14	0·0000	0·1001	0·2027	0·3078	0·4155	0·5258	0·6385	0·7537	0·8713	0·9911
16	0·0000	·0986	·1994	·3026	·4084	·5168	·6278	·7414	·8576	·9764
18	0·0000	·0993	·2011	·3055	·4124	·5218	·6338	·7483	·8653	·9848
20	0·0000	·0990	·2007	·3050	·4119	·5212	·6329	·7470	·8635	·9833
22	0·0000	·0990	·2002	·3038	·4098	·5183	·6292	·7435	·8584	·9768
24	0·0000	·0995	·2016	·3061	·4132	·5228	·6350	·7497	·8671	·9870
26	0·0000	·0997	·2021	·3069	·4142	·5239	·6362	·7508	·8677	·9870
28	0·0000	·0991	·2005	·3044	·4109	·5198	·6312	·7452	·8616	·9804
30	0·0000	·0995	·2014	·3059	·4130	·5226	·6347	·7494	·8665	·9863
32†	0·0000	·1036	·2097	·3183	·4292	·5424	·6581	·7764	·8974	1·0211

† Gun loaded all night. The initial velocity is therefore reduced.

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*	*f.s.*
14	1499	1462	1427	1393	1360	1331	1302	1276	1252
16	1521	1488	1453	1418	1384	1351	1320	1291	1263
18	1511	1473	1437	1403	1371	1339	1310	1282	1255
20	1515	1475	1438	1402	1372	1343	1315	1288	1263
22	1515	1482	1448	1415	1382	1353	1324	1294	1267
24	1508	1469	1435	1401	1369	1337	1308	1278	1251
26	1505	1465	1431	1398	1367	1336	1309	1283	1257
28	1514	1479	1444	1408	1377	1347	1316	1289	1263
30	1508	1472	1435	1401	1369	1338	1308	1281	1252
32	1448	1414	1381	1353	1325	1296	1268	1240	1213

No. of round.	Weight of shot.	Values of δP.	Difference from mean value.
	lbs.		
14	21·78	·00125	·00000
16	21·81	·00124	−·00001
18	21·81	·00127	+·00002
20	21·83	·00127	+·00002
22	21·81	·00119	−·00006
24	21·83	·00127	+·00002
26	21·81	·00125	·00000
28	21·83	·00123	−·00002
30	21·81	·00126	+·00001
32	21·81	·00123	−·00002
Means	21·81	·00125	·00002

(6) Ogival-headed Shot (two diameters), Hollow.

Round 15. Weight of shot 21·92 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
0″	3·55	0	3·550	+22·450		1	44·59	−·003	44·587	+2·221		0″·0000		
1	26·00	0	26·000		−60	2	46·80	+·008	46·808		+55	·0993	+993	
2	48·39	0	48·390	22·390		3	49·08	+·004	49·084	2·276		·2012	1019	+26
3	70·69	0	70·690	22·300	90	4	51·43	−·015	51·415	2·331	55	·3055	1043	24
4	92·87	0	92·870	+22·180	−120	5	53·79	+·011	53·801	2·386	55	·4194	1069	26
5						6	56·25	−·009	56·241	2·440	54	·5217	1093	24
						7	58·74	−·006	58·734	2·493	53	·6334	1117	24
						8	61·27	+·008	61·278	2·544	51	·7475	1141	24
						9	63·87	+·002	63·872	+2·594	+50	·8640	+1165	+24

Round 21. Weight of shot 21·89 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
0″	4·24	0	4·240	+20·420		1	59·66	−·005	59·655	+2·018		0″·0000		
1	24·66	0	24·660		−69	2	61·66	+·013	61·673		+47	·0994	+994	
2	45·02	−·009	45·011	20·351	65	3	63·74	−·002	63·738	2·065	48	·2014	1020	+26
3	65·29	+·007	65·297	20·286	61	4	65·85	+·001	65·851	2·113	48	·3058	1044	24
4	85·53	−·008	85·522	20·225	−57	5	68·00	+·012	68·012	2·161	48	·4125	1067	23
5	105·69	0	105·690	+20·168		6	70·23	−·009	70·221	2·209	48	·5216	1091	24
						7	72·48	−·003	72·477	2·256	47	·6331	1115	24
						8	74·78	−·001	74·779	2·302	46	·7469	1138	23
						9	77·13	−·003	77·127	2·348	46	·8630	1161	23
						10	79·52	0	79·520	+2·393	+45	·9815	+1185	+24

Round 23. Weight of shot 21·94 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
1	21·66	0	21·660	+23·340	−50	1	92·55	0	92·550	+2·288	+53	0·0000	+ 986	+25
2	45·00	0	45·000	23·290	50	2	94·84	−·002	94·838	2·341	53	·0986	1009	24
3	68·28	+·010	68·290	23·240	50	3	97·18	−·001	97·179	2·394	55	·1995	1033	23
4	91·53	0	91·530	23·190	−50	4	99·57	+·003	99·573	2·449	56	·3028	1056	24
5	114·73	−·010	114·720	+23·140		5	102·01	+·012	102·022	2·505	56	·4084	1080	24
6	137·85	+·010	137·860			6	104·53	−·003	104·527	2·560	55	·5164	1104	24
						7	107·11	−·023	107·087	2·616	56	·6268	1129	25
						8	109·71	−·007	109·703	+2·672	+56	·7397	+1152	+23
						9	112·37	+·005	112·375			·8549		

Round 25. Weight of shot 21·97 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
1	28·14	0	28·140	+21·315	−102	1	97·88	−·002	97·878	+2·087	+47	0·0000	+ 993	+22
2	49·45	+·005	49·455	21·213	101	2	99·95	+·015	99·965	2·134	47	·0993	1015	24
3	70·67	−·002	70·668	21·112	101	3	102·10	−·001	102·099	2·181	47	·2008	1039	24
4	91·78	0	91·780	21·010	102	4	104·28	0	104·280	2·227	46	·3047	1061	22
5	112·78	+·010	112·790	+20·910	−100	5	106·50	+·007	106·507	2·274	47	·4108	1084	23
6	133·70	0	133·700			6	108·78	+·001	108·781	2·321	47	·5192	1106	22
						7	111·11	−·008	111·102	2·368	47	·6298	1129	23
						8	113·46	+·010	113·470	2·416	48	·7427	1153	24
						9	115·88	+·006	115·886	+2·464	+48	·8580	+1177	+24
						10	118·35	0	118·350			·9757		

Round 27. Weight of shot 21·97 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
1	1·40	0	1·400	+19·430	−40	1	45·61	0	45·610	+1·916	+46	0·0000	+ 989	+25
2	20·84	−·010	20·830	19·390	40	2	47·52	+·006	47·526	1·962	46	·0989	1014	24
3	40·21	+·010	40·220	19·350	−40	3	49·46	+·028	49·488	2·008	46	·2003	1038	24
4	59·57	0	59·570	+19·310		4	51·50	−·004	51·496	2·052	44	·3041	1061	23
5	78·88	0	78·880			5	53·54	+·008	53·548	2·095	43	·4102	1083	22
						6	55·65	−·007	55·643	2·139	44	·5185	1106	23
						7	57·78	+·002	57·782	2·182	43	·6291	1129	23
						8	59·96	+·004	59·964	2·226	44	·7420	1152	23
						9	62·19	0	62·190	+2·270	+44	·8572	+1175	+23
						10	64·46	0	64·460			·9747		

Round 29. Weight of shot 21·97 lbs.

Clock.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Screen.	Reading.	Correction.	Corrected reading.	Δ_1	Δ_2	Time of passing each screen.	Δ_1	Δ_2
												t.		
1	16·26	0	16·260	+23·270	−80	1	62·14	0	62·140	+2·291	+56	0·0000	+ 989	+24
2	39·53	0	39·530	23·210	73	2	64·44	−·009	64·431	2·347	56	·0989	1013	25
3	62·73	+·010	62·740	23·137	−85	3	66·77	+·008	66·778	2·403	54	·2002	1038	23
4	85·88	−·003	85·877	+23·052		4	69·20	−·019	69·181	2·457	53	·3040	1061	23
5	108·93	−·001	108·929			5	71·64	−·002	71·638	2·510	53	·4101	1085	24
						6	74·15	−·002	74·148	2·563	53	·5186	1108	23
						7	76·81	−·099	76·711	2·615	52	·6294	1131	23
						8	79·32	+·006	79·326	2·666	51	·7425	1153	22
						9	* *	*	81·992	+2·718	+52	·8578	+1176	+23
						10	84·71	0	84·710			·9754		

Round 31. Weight of shot 21·91 lbs.

Clock.	Reading.	Correction.	Corrected reading.			Screen.	Reading.	Correction.	Corrected reading.			Time of passing each screen.		
				Δ1	Δ2					Δ1	Δ2	t	Δ1	Δ2
1	13·22	−·010	13·210	+19·260		1	57·06	0	57·060	+1·897		0·0000	+·963	
2	32·46	+·010	32·470	19·180	−80	2	58·96	−·003	58·957	1·938	+41	·0993	1015	+52
3	51·66	−·010	51·650	19·090	−90	3	60·89	+·005	60·895	1·979	41	·2008	1036	21
4	70·73	+·010	70·740	18·990	−100	4	62·87	+·004	62·874	2·022	43	·3044	1060	24
5	89·73	0	89·730	+18·990		5	64·89	+·006	64·896	2·066	44	·4104	1084	24
						6	66·95	+·012	66·962	2·112	46	·5188	1108	24
						7	69·07	+·004	69·074	2·158	46	·6296	1133	25
						8	71·24	−·006	71·232	2·204	46	·7429	1158	25
						9	73·44	−·004	73·436	+2·250	+46	·8587	+1183	+25
						10	75·68	+·006	75·686			·9770		

Round 33†. Weight of shot 21·94 lbs.

Clock.	Reading.	Correction.	Corrected reading.			Screen.	Reading.	Correction.	Corrected reading.			Time of passing each screen.		
				Δ1	Δ2					Δ1	Δ2	t	Δ1	Δ2
3	45·67	0	45·670	+18·580		1	59·32	0	59·320	+1·893		0·0000	+1018	
4	64·25	0	64·250	+18·570		2	61·21	+·003	61·213	1·933	+40	·1018	1040	+22
5	82·82	0	82·820			3	63·14	+·006	63·146	1·973	42	·2058	1062	23
						4	65·13	−·009	65·121	2·019	44	·3121	1086	23
						5	67·14	0	67·140	2·065	46	·4207	1112	26
						6	69·21	−·005	69·205	2·110	45	·5319	1137	25
						7	71·33	−·015	71·315	2·155	45	·6456	1161	24
						8	73·47	0	73·470	+2·202	+47	·7617	+1187	+26
						9	75·67	+·002	75·672			·8804		

Summary (6). Ogival-headed Shot (two diameters), Hollow.

Time occupied by shot in passing from the first to each succeeding screen.

No. of round.	Screen 1.	Screen 2.	Screen 3.	Screen 4.	Screen 5.	Screen 6.	Screen 7.	Screen 8.	Screen 9.	Screen 10.
	t	t	t	t	t	t	t	t	t	t
15	0·0000	0·0993	0·2012	0·3055	0·4124	0·5217	0·6334	0·7475	0·8640	**
21	0·0000	·0994	·2014	·3058	·4125	·5216	·6331	·7469	·8630	·9815
23	0·0000	·0986	·1995	·3028	·4084	·5164	·6268	·7397	·8549	**
25	0·0000	·0993	·2008	·3047	·4108	·5192	·6298	·7437	·8580	·9757
27	0·0000	·0989	·2003	·3041	·4102	·5185	·6291	·7430	·8572	·9747
29	0·0000	·0989	·2002	·3040	·4101	·5186	·6294	·7435	·8578	·9754
31	0·0000	·0993	·2008	·3044	·4104	·5188	·6296	·7429	·8587	·9770
33	0·0000	·1018	·2058	·3121	·4207	·5319	·6456	·7617	·8804	**

Velocities at the following distances from the gun.

No. of round.	150 ft.	300 ft.	450 ft.	600 ft.	750 ft.	900 ft.	1050 ft.	1200 ft.	1350 ft.
	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.	f.s.
15	1511	1479	1438	1403	1372	1343	1315	1288	**
21	1509	1471	1437	1406	1375	1345	1318	1292	1266
23	1521	1487	1452	1420	1389	1359	1329	1302	**
25	1511	1478	1444	1414	1384	1356	1329	1301	1274
27	1517	1479	1445	1414	1385	1356	1329	1302	1277
29	1517	1481	1445	1414	1382	1354	1326	1301	1276
31	1511	1478	1448	1415	1384	1354	1324	1295	1268
33	1473	1442	1411	1381	1349	1319	1292	1264	**

† Round 32 had remained in the gun all night. This probably accounts for the reduced initial velocity of Round 33.

No. of round.	Weight of shot.	Value of bl^2.	Difference from mean.
	lbs.		
15	21·92	·00124	+·00006
21	21·89	·00121	+·00003
23	21·94	·00118	·00000
25	21·97	·00114	−·00004
27	21·97	·00118	·00000
29	21·97	·00119	+·00001
31	21·91	·00116	−·00002
33	21·94	·00118	·00000
Means	21·94	·00118	·00002

Since the second differences of $0, t_2, t_3 \ldots\ldots t_{10}$ are nearly constant in the preceding experiments, we are led to the equation

$$t = as + bs^2, \text{ connecting space and time.}$$

This shows that if v be the velocity of the shot at time t, and f the retarding force, then

$$v = \frac{ds}{dt} = \frac{1}{a+2bs};$$

and if $v = V$ when $s = 0$, we have $V = \frac{1}{a}$;

$$\therefore \quad v = \frac{1}{\frac{1}{V} + 2bs},$$

and

$$f = \frac{d^2s}{dt^2} = -2bv^3.$$

The values of bl^2 for each experiment were obtained as follows:—

Since $t = as + bs^2$

If $s = l$, then $t_1 = al + bl^2$,

$s = 2l$, „ $t_2 = 2al + 4bl^2$

. . . . . .

$s = \overline{n-1}\,l$, „ $t_n = \overline{n-1}\,al + \overline{n-1}^2 bl^2$,

or

$$\frac{t_1}{1} = al + bl^2,$$

$$\frac{t_2}{2} = al + 2bl^2,$$

. . . . . .

$$\frac{t_n}{n-1} = al + \overline{n-1}\,bl^2.$$

Finding, then, the numerical values of

$$\frac{t_1}{1}, \frac{t_2}{2}, \ldots\ldots \frac{t_n}{n-1}$$

for each experiment, and taking the difference of two of these quantities, we find

$$\frac{t_n}{n-1} - \frac{t_2}{2} = \overline{n-3}\,bl^2,$$

where $l=150$ feet, and the mean value of bl^2 so determined from each round may be taken to correspond to the mean velocity of that round.

Since the retarding force is $f=-2bv^2$, acting upon an experimental shot of weight W′ lbs., the resistance of the air measured in pounds

$$=-2bv^2\frac{W'}{g};$$

and the resistance will always be the same against a shot of the same external form when moving with the velocity v. If W be the weight of a different shot, then the retarding force

$$f'=-2bv^2\frac{W'}{g}\times\frac{g}{W}=-2\left(b\frac{W'}{W}\right)v^2,$$

or b varies inversely as the weight of a shot of given external form. If we vary the diameter of the shot, it is usual to assume that the resistance of the air varies as the square of the diameter for similar forms of heads of shot. The values of $2b$ are given below in terms of R, the radius of the shot in feet, and W, its weight in pounds. But inasmuch as the diameters of shot are generally given in inches, the same values of $2b$ have been expressed in terms of d, the diameter of the shot in inches. Unfortunately there are only three successful rounds for each of the solid ogivals, and one value of bl^2 in each case is manifestly too large. This must have arisen from unsteadiness of the shot in its flight. If we reject these two, the values of bl^2 derived from the solid ogivals agree very well with those derived from the hollow shot of the like forms.

For the hemispherical head we have $bl^2=\cdot00084$ for shot of mean weight 39·34 lbs., and 4·7 inches in diameter. To find the value of $2b$ adapted for hemispherical-headed shot of weight W lbs., and diameter$=d$ inches$=2$R feet, we have

$$2b=\frac{2\times\cdot00084}{(150)^2}\times\left(\frac{d}{4\cdot7}\right)^2\left(\frac{39\cdot34}{W}\right)=\cdot0000001329\,\frac{d^2}{W},$$

or

$$=\frac{2\times\cdot00084}{150}\left(\frac{24R}{4\cdot7}\right)^2\frac{39\cdot34}{W}=\cdot000077\,\frac{R^2}{W},\text{ and so on for the rest.}$$

Table of values of $2b$ for differently formed heads adapted for elongated shot of weight W lbs. and radius R feet, or diameter d inches.

Form of head.	Experimental value of bl^2.	Mean weight of experimental shot.	Value of $2b$ when diameter $=2$R feet.	Value of $2b$ when diameter $=d$ inches.
		lbs.		
(1) Hemispherical	·00084	39·34	$\cdot000077\,\frac{R^2}{W}$	$\cdot0000001329\,\frac{d^2}{W}$
(2) Hemispheroidal.....	·00067	38·70	$\cdot000060\,\frac{R^2}{W}$	$\cdot0000001043\,\frac{d^2}{W}$
(3) Ogival (1 diameter)	·00070	39·56	$\cdot000064\,\frac{R^2}{W}$	$\cdot0000001114\,\frac{d^2}{W}$
(4) Ogival (2 diameters).....	·00070	38·52	$\cdot000062\,\frac{R^2}{W}$	$\cdot0000001085\,\frac{d^2}{W}$
(5) Ogival (1 diameter)	·00125	21·81	$\cdot000063\,\frac{R^2}{W}$	$\cdot0000001097\,\frac{d^2}{W}$
(6) Ogival (2 diameters) ...	·00118	21·94	$\cdot000060\,\frac{R^2}{W}$	$\cdot0000001049\,\frac{d^2}{W}$

Although the motion of a shot may be well represented by supposing a retarding force $=-2bv^2$ to act through a range of 1400 feet, there is reason to suppose that for velocities ranging from 1500 to 900 feet per second the value of b will be less for the lower velocities with equal degrees of steadiness. It unfortunately happens, however, that the angular velocity imparted to a shot, which most probably remains little changed during the time of flight, depends directly upon the initial velocity of the shot. Hence, when shot are fired with low initial velocities with a view to determine the value of b for low velocities, the steadiness of the shot is diminished, and therefore there is an increase of the resistance of the air on this account. The only way to meet the difficulty is to place screens near the gun to find the initial velocity, and others at a distance of 2000 yards or more, and so compare theory and experiment.

It is worthy of notice that if a body move in a straight line under the action of a force varying as the velocity cubed, the mean velocity obtained by dividing space by time is exactly the actual velocity at the middle point of that space. Thus

$$\frac{\text{space } 2s}{\text{time of describing space } 2s}=\frac{2s}{2as+4bs^3}=\frac{1}{a+2bs}=\text{velocity at distance } s.$$

The date of the Report of the above experiments was October 23, 1866.

I have long been aware that Major Otto had made trial of various laws of the resistance of the air in a work published in 1855. The law of the cube of the velocity was tried, but without any definite result*. It was in April 1867 that I first learnt that M. Hélie† had proposed the law of the cube of the velocity as the law of the resistance of the air to elongated projectiles in a work dated 1865, which law he had deduced from experiments made at Gâvre in 1859, 1860, and 1861‡. It will be convenient to quote his own statement of the best series of experiments made at Gâvre in 1859, in order to show the nature of the work done, and the perfect independence of my own methods and numerical results. M. Hélie used one of the electro-ballistic pendulums to measure his velocities, but he does not state distinctly which it was.

"Si la résistance de l'air est réellement proportionnelle au cube de la vitesse, on doit avoir un résultat sensiblement constant en substituant, dans l'expression $\frac{v'-v''}{v'v''x}$, les valeurs de v', v'' et x correspondantes à la même charge.

* Hilfsmittel für ballistische Rechnungen, 1855, p. 12.

† Traité de Balistique, 1865.

‡ [In a Memoir, "Études de Balistique expérimentale," presented to the Belgian Academy by Captain P. C. Boulengé, June 12, 1867, the author, having deduced the cubic law of resistance of the air from his experiments, proceeds to remark:—

"Ce résultat est en accord complet avec les travaux les plus récents faits en France; en effet, les expériences exécutées par la commission des principes du tir, en 1856 et 1857, ont conduit M. le capitaine Welter, professeur à l'École d'application de l'artillerie et du génie de Metz, à reconnaître que la résistance de l'air sur les projectiles sphériques est simplement proportionnelle au cube de la vitesse.

"Cette loi, admise depuis 1862, comme base des études balistiques à cette école, a fourni des formules très-simples et très-facilement calculables sans l'intervention de tables, se prêtant à des recherches que les anciennes formules balistiques ne permettaient pas d'aborder, et donnant des résultats plus conformes à la pratique" (p. 84). —Aug. 1, 1868.]

"Résultats moyens des expériences.

(Chaque vitesse est déduite de 30 coups.)

Charge du canon.	Première vitesse, v', à 33 mètres du canon.	Dernière vitesse, v''.	Intervalle des points d'observation, x.	Valeur de $\frac{v'-v''}{v'v''x}$ ou c.
kilog.	mètr.	mètr.	mètr.	
1·5	225·1	215·6	464	0·000000422
2·0	263·7	252·5	467	0·000000380
2·5	291·9	275·9	467	0·000000425
3·0	309·6	291·8	467	0·000000422
3·5	326·9	306·4	467	0·000000438

"Sauf l'anomalie que présente la charge de $2^k{\cdot}0$ la valeur de $\frac{v'-v''}{v'v''x}$ se montre sensiblement constante. La formule $r=cv^2$ est donc suffisamment justifiée"*. The diameter of the shot was $0^m{\cdot}1623=6^{in}{\cdot}39$, its weight $30^k=66^{lbs}{\cdot}14$, and its form hollow ogival with head struck with a radius of nearly two diameters.

In order to facilitate the comparison of this experiment with my own, I have converted the French into English measures.

Charge.	v'.	v''.	x.	$c=2b$.
lbs.	*f.s.*	*f.s.*	ft.	
3·31	738·5	707·4	1522·4	0·0000000391
4·41	865·1	828·4	1532·2	0·0000000334
5·51	957·7	905·2	1532·2	0·0000000395
6·61	1015·8	957·4	1532·2	0·0000000392
7·72	1079·5	1005·3	1532·2	0·0000000407
		Mean value of c or $2b=$		0·0000000384
			$=$ ·0000358 $\frac{R^2}{W}$	if R ft. = radius of shot.
			$=$ ·000000069 $\frac{d^3}{W}$	if d in. = diameter of shot.

The values of $2b$ given by my own experiments for shot of the same form were

$$\cdot 000060\,\frac{R^2}{W} \text{ or } \cdot 0000001042\,\frac{d^3}{W}$$

for velocities ranging from 1520 to 1270 *f.s.*; so that the value of $2b$ deduced from my own experiments might be expected to be sensibly greater than its value deduced from M. Hélie's experiments made with lower velocities. There are three or four other tabular statements of less complete experiments given by M. Hélie. In the present state of the question it is impossible to make any more exact comparison of the two systems of experimenting†.

* Hélie, Traité, pp. 407, 408.

† [An extended series of experiments just completed at Shoeburyness, with 3, 5, 7, and 9-inch elongated shot, has shown conclusively that, although the motion of a shot may be well represented by supposing the resistance of the air to vary as the cube of the velocity, and to be equal to $-2bv^3$ for a range of 1200 or 1300 feet, still

Meteorological Register, Shoeburyness.

Date.	Hour.		Barometer.		Thermometer.		Wind.		Weather and Remarks.
	A.M.	P.M.	Reading.	Temperature.	Dry.	Wet.	Direction.	Force.	
1866.									
Sept. 25th	10	...	30·05	61	61	54	S.W. by S.	2	Clear.
	...	3	30·05	61	61	55	S.W. by W.	2	Clear.
Sept. 26th	10	...	30·05	61	61	59	S.S.W.	3	Hazy.
	...	3	30·00	61	60	59	S.W. by S.	3	Ditto.
Sept. 27th	10	...	29·95	59	56	56	N.W. by W.	2	Ditto with rain.
	...	3	29·95	60	59	58	W.	2	Hazy.

Direction of firing from the North to South.

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b is subject to considerable variations for large variations in v. For ogival heads struck with a radius of $1\frac{1}{2}$ diameter of the shot, fired with great steadiness, the following values of v and b have been found.

v.	$2b\frac{W}{d^3}$.
f.s.	
900	0·0000000804
950	0·0000000840
1000	0·0000000720
1050	0·0000000858
1100	0·0000001005
1200	0·0000001085
1300	0·0000001046
1400	0·0000000977
1500	0·0000000956
1600	0·0000000925

It thus appears that M. Hélie's value of $2b$ or c is true only for velocities about 950 *f.s.*—Aug. 1, 1868.]

XV. *On the Law of the Resistance of the Air to Rifled Projectiles.*

By CHARLES W. MERRIFIELD, *F.R.S., Principal of the Royal School of Naval Architecture.*

Received March 19,—Read April 23, 1868.

AT the beginning of this month Lieut.-Colonel H. R. HALFORD applied to me to obtain for him the law of atmospheric resistance resulting from his experiments in shooting with METFORD'S match-rifle, a small-bore with increasing pitch. I found the resistance to vary as the cube of the velocity.

As the calculations and experiments were all made without any notion of the resulting law, and without any knowledge of the work already done by Professor HÉLIE and Professor BASHFORTH, they afford a remarkable confirmation of the results obtained by those gentlemen. This is the more worthy of notice, as their data belong to pieces of large calibre, and mine to small-bore bullets.

The experimental data originally submitted to me were simply the elevations found necessary to give the different ranges of every 100 yards from 100 to 1100. These were as follows:—

Yards range.	Elevation.
	° ′ ″
100	9 30
200	20 —
300	31 30
400	44 —
500	57 30
600	1 12 —
700	1 27 30
800	1 44 —
900	2 1 30
1000	2 20 —
1100	2 39 30

I was informed that these were the means derived from a great number of selected trials, bad shots being of course discarded.

The laws of the motion were arrived at tentatively. I began the calculations from an assumed velocity of 1360 feet per second, which is seen by the shorter ranges to be very nearly true, of necessity. I then found that the law of the squares did not fit the results at all, but that the law of the cubes very nearly did so, and a slight alteration in the assumed velocity made them do so with exactness.

The method used was to determine the coefficient of resistance m, in $\frac{dv}{dt} = -mv^3$, which

gives $m=\frac{2(tv_1\cos\theta-s)}{s^2v_1\cos\theta}$, v_1 being the initial velocity, s the actual range, and θ the inclination. The time t was that of the range *in vacuo* due to the supposed initial velocity and given elevation. It was assumed, *first*, that the vacuum time might be taken to represent the real time, and, *second*, that the horizontal resistance might be taken to vary as the cube of the horizontal velocity. The result is given in the following Table.

Calculation of Resistance, supposed to vary as the Cube of the Velocity.

Initial velocity taken as 1358 feet per second.

Angle of elevation.	Actual range, in feet.	Range *in vacuo*, in feet.	Reduction of range by resistance, in feet.	Time of flight *in vacuo*, in seconds.	Coefficient of resistance proportionate to	Departure from mean value.	Final velocity, in feet per second.	Velocity lost in each 100 yards, in feet per second.	Actual range, in yards.
° ′ ″									
9 30	300	316·62	16·62	0·23316	16·620	−0·018	1222·4	135·6	100
20	600	666·57	66·57	0·49085	16·643	+0·005	1111·5	110·9	200
31 30	900	1049·81	149·81	0·77309	16·647	+0·009	1018·8	92·7	300
44	1200	1466·32	266·32	1·07985	16·647	+0·009	940·6	78·2	400
57 30	1500	1916·02	416·02	1·41115	16·643	+0·005	873·4	67·2	500
1 12	1800	2399·00	599·00	1·76695	16·654	+0·016	815·2	58·2	600
1 27 30	2100	2915·04	815·04	2·14732	16·644	+0·006	764·3	50·9	700
1 44	2400	3464·11	1064·11	2·55206	16·634	−0·004	719·4	44·9	800
2 1 30	2700	4046·12	1346·12	2·98133	16·639	+0·001	679·4	40·0	900
2 20	3000	4660·92	1660·92	3·43504	16·633	−0·005	643·7	35·7	1000
2 39 30	3300	5308·37	2008·37	3·91317	16·616	−0·022	611·5	32·2	1100
These two columns contain the data.				Mean	16·638				

Resulting Coefficient of resistance $=0\cdot00000\quad02723$.

The coefficient of resistance, so determined for each range, has no sensible variation, thus proving that the facts are exactly met by the law that the resistance varies according to the cube of the velocity, with a coefficient of resistance, for this particular bullet, of 0·00000 02723. This is subject to the assumptions previously mentioned. It is further either involved or assumed that the bullet presents an invariable aspect in a horizontal direction.

These assumptions are sufficiently right for such a very low trajectory as is given by 2° 40′ for 1100 yards, giving a vacuum height of 64 feet. I got out a graphical solution of the resisted time of a bullet thrown with a velocity of 64 feet per second (which corresponds to that height) and with a coefficient of 0·000001, and I found the variation in time too small to detect in fine curves carefully drawn with ordinates of nearly 2 feet in length. I consider this as amounting to proof that the difference between real and vacuum time is insignificant in this problem. The second assumption, as to the horizontal resistance varying as the cube of the horizontal velocity, is justified by the low trajectory. On the question of aspect, my present knowledge does not authorize me to form any decided opinion.

The graphical method mentioned above was devised to evade the arithmetical difficulties of the question; and I hope shortly to be able to publish an account of it*.

Colonel HALFORD has since been so obliging as to communicate to me the following notes of the experimental data. These are as follows:—

"*The means by which the angles of* METFORD'S *match-rifles were arrived at.*

"The rifles were shot at $12\frac{1}{2}$ yards to get the zero on the tangent-scale.

"This distance is taken because, with a radius of $12\frac{1}{2}$ yards, $\frac{1}{4}$ of an inch is one minute of angular space, an easy quantity to deal with. Corrections made for height of fore sight above axis of bore, and for fall of bullet, at $12\frac{1}{2}$ yards, taken as one minute.

"The tangent-scale is then carefully cut for radius between the sights to read to minutes with a vernier. It can be read to 15″ by careful inspection. The Table of angles is then made from careful registers of shooting in all weathers, but especially calm weather, at different distances, but chiefly at 200 yards, 500, 600, 800, 900, and 1100 yards. Nine times out of ten the angles will be found to be perfectly correct in calm weather. Difference of barometrical pressure will make a slight difference. The Table furnished [*i.e.* of elevations corresponding to ranges] was drawn out by Mr. METFORD, and has been very heavily tested by myself, and tallies exactly with my experience. The angles have been compared with those used by other men using similar rifles, and have been found to tally, making slight allowance for the little differences of speed which different barrels will give—usually not more than is due to 0° 2′ of elevation in 1000 yards. I must have fired from 8000 to 9000 shots. I have most of my shots registered. I should say that the Tables have been worked out from results given by at least 30,000 shots.

Ranges.	Elevations, Table No. 1.	Elevations, Table No. 2.	Elevations, Table No. 3.
Yards.	° ′ ″	° ′ ″	° ′ ″
100	9 15	9 30	10 30
200	19 30	20	20 30
300	30 45	31 30	32 30
400	43	44	45 15
500	56 15	57 30	59
600	1 10 30	1 12	1 14
700	1 25 45	1 27 30	1 29 30
800	1 42	1 44	1 46 30
900	1 59 30	2 1 30	2 4 45
1000	2 17 30	2 20	2 23
1100	2 36 45	2 39 30	2 42 30

"No. 3 Table is not perfect in itself; it is a Table used occasionally with a head wind, and has not been so carefully worked out as the others; a head wind is so disturbing in its effects that, practically, it is almost impossible to verify it. The other Tables are reliable within very small fractions.

* [This has since appeared in the Philosophical Magazine for June 1868.]

"Details of bullet, &c.

"METFORD'S match-rifle: *calibre* 0·4605 inch.

"*Charge*: 90 grains, CURTIS and HARVEY'S No. 5 powder.

"*Length of bullet* . . . 1·45 inch.

"*Diameter* at base . . . 0·451 „

$\frac{1}{2}$ inch from base 0·45 „

$\frac{1}{4}$ inch from base 0·447 „

"*Weight*: 530 grains.

"*Material*: lead hardened with alloy.

"*Shape*: cup-shaped in rear, nearly cylindrical for the first inch, and gradually rounding off from there to a spherical or paraboloidal head, not pointed."

These notes were not given to me until after my calculations had been made. It will be observed that the data originally supplied, and from which I worked, are those of Table 2. This appears to be the most reliable, being very near the mean of the other two.

On the whole, I consider myself singularly fortunate in having had such a very good series of experiments from which to calculate.

It will not fail to be noticed that the velocity of sound is intermediate between the initial and final velocities of the bullets at the longer ranges.

Note added 3rd August 1868.—For the sake of comparison with other results, I give the reduction of this coefficient of resistance to the standard of one pound weight avoirdupois, and one inch transverse section, on the (doubtful) assumption that the coefficient varies inversely as the weight, and directly as the transverse section.

Coefficient reduced to 1 lb. weight and to one square inch section 0·00000 01267,

Ditto, and to one circular inch section . . . . . 0·00000 00995.

XVI. *On the Communication of Vibration from a Vibrating Body to a surrounding Gas.* By G. G. Stokes, *M.A., D.C.L., Sec. R.S., Fellow of Pembroke College, and Lucasian Professor of Mathematics in the University of Cambridge.*

Received June 18,—Read June 18, 1868.

In the first volume of the Transactions of the Cambridge Philosophical Society is a short paper by Professor John Leslie, "On Sounds excited in Hydrogen Gas," in which the author mentions some remarkable experiments indicating the singular incapacity of hydrogen for becoming the vehicle of the transmission of sound when a bell is struck in that gas, either pure or mixed with air. With reference to the most striking of his experiments the author observes (p. 267), "The most remarkable fact is, that the admixture of hydrogen gas with atmospheric air has a predominant influence in blunting or stifling sound. If one half of the volume of atmospheric air be extracted [from the receiver of the air-pump], and hydrogen gas be admitted to fill the vacant space, the sound will now become scarcely audible."

No definite explanation of the results is given, but with reference to the feebleness of sound in hydrogen the author observes, "These facts, I think, depend partly on the tenuity of hydrogen gas, and partly on the rapidity with which the pulsations of sound are conveyed through this very elastic medium;" and he states that, according to his view, he "should expect the intensity of sound to be diminished 100 times, or in the compound ratio of its tenuity and of the square of the velocity with which it conveys the vibratory impressions." With reference to the effect of the admixture of hydrogen with air he says, "When hydrogen gas is mixed with common air, it probably does not intimately combine, but dissipates the pulsatory impressions before the sound is vigorously formed."

In referring to Leslie's experiment in which a half-exhausted receiver is filled up with hydrogen, Sir John Herschel suggests a possible explanation founded on Dalton's hypothesis that every gas acts as a vacuum towards every other*. According to this view there is a constant tendency for sound-waves to be propagated with different velocities in the air and hydrogen of which the mixture consists, but this tendency is constantly checked by the resistance which one gas opposes to the passage of another, calling into play something analogous to internal friction, whereby the sound-vibration though at first produced is rapidly stifled. Air itself indeed is a mixture; but the velocities of propagation of sound in nitrogen and oxygen are so nearly equal that the effect is supposed not to be sensible in this case.

* Encyclopædia Metropolitana, vol. iv. Art. Sound, § 108.

This explanation never satisfied me, believing, as I always have done, for reasons which it would take too long here to explain, that for purely hydrodynamical phenomena (such as those of sound) an intimate mixture of gases was equivalent to a single homogeneous medium. I had some idea of repeating the experiment, thinking that possibly LESLIE might not have allowed sufficient time for the gases to be perfectly mixed, though that did not appear likely, when another explanation occurred to me, which immediately struck me as being in all probability the true one.

In reading some years ago an investigation of Mr. EARNSHAW's, in which a certain result relating to the propagation of sound in a straight tube was expressed in terms among other things of the velocity of propagation, the idea occurred to me that the high velocity of propagation of sound in hydrogen would account for the result of LESLIE's experiment, though in a manner altogether different from anything relating to the propagation of sound in one dimension only.

Suppose a person to move his hand to and fro through a small space. The motion which is occasioned in the air is almost exactly the same as it would have been if the air had been an incompressible fluid. There is a mere local reciprocating motion, in which the air immediately in front is pushed forwards, and that immediately behind impelled after the moving body, while in the anterior space generally the air recedes from the encroachment of the moving body, and in the posterior space generally flows in from all sides, to supply the vacuum which tends to be created; so that in lateral directions the motion of the fluid is backwards, a portion of the excess of fluid in the front going to supply the deficiency behind. Now conceive the periodic time of the motion to be continually diminished. Gradually the alternation of movement becomes too rapid to permit of the full establishment of the merely local reciprocating flow; the air is sensibly compressed and rarefied, and a sensible sound-wave (or wave of the same nature, in case the periodic time be beyond the limits suitable to hearing) is propagated to a distance. The same takes place in any gas; and the more rapid be the propagation of condensations and rarefactions in the gas, the more nearly will it approach, in relation to the motions we have under consideration, to the condition of an incompressible fluid; the more nearly will the conditions of the displacement of the gas at the surface of the solid be satisfied by a merely local reciprocating flow.

This explanation when once it suggested itself seemed so simple and obvious that I could not doubt that it afforded the true mode of accounting for the phenomenon. It remained only to test the accuracy of the assigned cause by actual numerical calculation in some case or cases sufficiently simple to permit of precise analytical determination. The result of calculations of the kind applied to a sphere proved that the assigned cause was abundantly sufficient to account for the observed result. I have not hitherto published these results; but as the phenomenon has not to my knowledge been satisfactorily explained by others, I venture to hope that the explanation I have to offer, simple as it is in principle, may not be unworthy of the notice of the Royal Society.

For the purpose of exact analytical investigation I have taken the two cases of a vibra-

ting sphere and a long vibrating cylinder, the motion of the fluid in the latter case being supposed to be in two dimensions. The sphere is chosen as the best representative of a bell, among the few geometrical forms of body for which the problem can be solved. The cylinder is chosen as the representative of a vibrating string. In the case of the sphere the problem is identical with that solved by POISSON in his memoir "Sur les movements simultanés d'un pendule et de l'air environnant"*, but the solution is discussed with a totally different object in view, and is obtained from the beginning, to avoid the needless complexity introduced by taking account of the initial circumstances, instead of supposing the motion already going on.

A. *Solution of the Problem in the case of a Vibrating Sphere.*

Suppose an elastic solid, spherical externally in its undisturbed position, to vibrate in the manner of a bell, the amplitude of vibration being very small. Suppose it surrounded by a homogeneous gas, which is at rest except in so far as it is set in motion by the sphere; and let it be required to determine the motion of the gas in terms of that of the sphere supposed given. We may evidently for the purposes of the present problem suppose the gas not to be subject to the action of external forces.

Let the gas be referred to the rectangular axes of x, y, z, and let u, v, w be the components of the velocity. Since the gas is at rest except as to the disturbance communicated to it from the sphere, u, v, w are by a well-known theorem the partial differential coefficients with respect to x, y, z of a function φ of the coordinates; and if a^2 be the constant expressing the ratio of the small variations of pressure to the corresponding small variations of density, we must have

$$\frac{d^2\varphi}{dt^2}=a^2\left(\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}\right); \quad \ldots\ldots\ldots \quad (1)$$

and if s be the small condensation,

$$s=-\frac{1}{a^2}\frac{d\varphi}{dt}.$$

As we have to deal with a sphere, it will be convenient to refer the gas to polar coordinates r, θ, ω, the origin being in the centre. In terms of these coordinates, equation (1) becomes

$$\frac{d^2\varphi}{dt^2}=a^2\left\{\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}+\frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\varphi}{d\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{d^2\varphi}{d\omega^2}\right\}; \quad \ldots\ldots \quad (2)$$

and if u', v', w' be the components of the velocity along the radius vector and in two directions perpendicular to the radius vector, the first in and the second perpendicular to the plane in which θ is measured,

$$u'=\frac{d\varphi}{dr}, \quad v'=\frac{1}{r}\frac{d\varphi}{d\theta}, \quad w'=\frac{1}{r\sin\theta}\frac{d\varphi}{d\omega}. \quad \ldots\ldots\ldots \quad (3)$$

Let c be the radius of the sphere, and V the velocity of any point of its surface

* Mémoires de l'Académie des Sciences, t. xi. p. 521.

resolved in a direction normal to the surface, V being a given function of t, θ, and ω; then we must have

$$\frac{d\varphi}{dr}=\mathrm{V}, \text{ when } r=c. \quad \ldots \ldots \ldots \ldots \quad (4)$$

Another condition, arising from what takes place at a great distance from the sphere, will be considered presently.

The sphere vibrating under the action of its elastic forces, its motion will be periodic, expressed so far as the time is concerned partly by the sine and partly by the cosine of an angle proportional to the time, suppose mat. Actually the vibrations will slowly die away, in consequence partly of the imperfect elasticity of the sphere, partly of communication of motion to the gas, but for our present purpose this need not be taken into account. Moreover there will in general be a series of periodic disturbances coexisting, corresponding to different periodic times, but these may be considered separately. We might therefore assume

$$\mathrm{V}=\mathrm{U}\sin mat+\mathrm{U}'\cos mat,$$

but it will materially shorten the investigation to employ an imaginary exponential instead of circular functions. If we take

$$\mathrm{V}=\mathrm{U}e^{imat}, \quad \ldots \ldots \ldots \ldots \quad (5)$$

where i is an abbreviation for $\sqrt{-1}$, and determine φ by the conditions of the problem, the real and imaginary parts of φ and V must satisfy all those conditions separately; and therefore we may take the real parts alone, or the coefficients of i or $\sqrt{-1}$ in the imaginary parts, or any linear combination of these even after having changed the arbitrary constants which enter into the expression of the motion of the sphere, as the solution of the problem, according to the way in which we conceive the given quantity V expressed.

The function φ will be periodic in a similar manner to V, so that we may take

$$\varphi=\psi e^{imat}. \quad \ldots \ldots \ldots \ldots \quad (6)$$

As regards the periodicity merely, we might have had a term involving e^{-imat} as well as that written above; but it will be readily seen that in order to satisfy the equation of condition (4) the sign of the index of the exponential in φ must be the same as in V.

On substituting in (2) the expression for φ given by (6), the factor involving t will divide out, and we shall get for the determination of ψ a partial differential equation free from t. Now ψ may be expanded in a series of LAPLACE'S Functions so that

$$\psi=\psi_0+\psi_1+\psi_2+\ldots \quad \ldots \ldots \ldots \quad (7)$$

Substituting in the differential equation just mentioned, taking account of the fundamental equation

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\psi_n}{d\theta}\right)+\frac{1}{\sin^2\theta}\frac{d^2\psi_n}{d\omega^2}=-n(n+1)\psi_n,$$

and equating to zero the sum of the LAPLACE'S Functions of the same order, we find

$$\frac{d^2\psi_n}{dr^2}+\frac{2}{r}\frac{d\psi_n}{dr}-\frac{n(n+1)}{r^2}\psi_n+m^2\psi_n=0.$$

This equation belongs to a known integrable form. The integral is

$$r\psi_n = u_n e^{-imr}\left\{1+\frac{n(n+1)}{2.imr}+\frac{(n-1)n(n+1)(n+2)}{2.4(imr)^2}+\dots\right\}$$
$$+u'_n e^{imr}\left\{1-\frac{n(n+1)}{2.imr}+\frac{(n-1)n(n+1)(n+2)}{2.4(imr)^2}-\dots\right\},$$

u_n and u'_n being evidently LAPLACE's Functions of the order n, since that is the case with ψ_n.

It will be convenient to take next the condition which has to be satisfied at a great distance from the sphere. When r is very large the series within braces may be reduced to their first terms 1, and we shall have

$$r\varphi = e^{im(at-r)}\Sigma u_n + e^{im(at+r)}\Sigma u'_n.$$

The first of these terms denotes a disturbance travelling outwards from the centre, the second a disturbance travelling towards the centre, the amplitude of vibration in both cases, for a given phase, varying inversely as the distance from the centre. In the problem before us there is no disturbance travelling towards the centre, and therefore $\Sigma u'_n = 0$, which requires that each function u'_n should separately be equal to zero. We have therefore simply

$$r\psi_n = u_n e^{-imr}\left\{1+\frac{n(n+1)}{2.imr}+\frac{(n-1)\dots(n+2)}{2.4(imr)^2}+\dots+\frac{1.2.3\dots 2n}{2.4.6\dots 2n(imr)^n}\right\}, \quad (8)$$

or, if we choose to reverse the series,

$$r\psi_n = u_n e^{-imr}\frac{1.3.5\dots(2n-1)}{(imr)^n}\left\{1+\frac{2n}{1.2n}imr+\frac{(2n-2)2n}{1.2(2n-1)2n}(imr)^2\dots+\frac{2.4.6\dots 2n}{1.2.3\dots 2n}(imr)^n\right\}. \quad (9)$$

Putting for shortness $f_n(r)$ for the multiplier of $u_n e^{-imr}$ on the right-hand member of (8) or (9), we shall have

$$\varphi = \frac{1}{r}e^{im(at-r)}\Sigma u_n f_n(r).$$

It remains to satisfy the equation of condition (4). Put for shortness

$$\frac{d}{dr}\left\{\frac{1}{r}e^{-imr}f_n(r)\right\} = -\frac{1}{r^2}e^{-imr}F_n(r),$$

so that

$$F_n(r) = (1+imr)f_n(r) - rf'_n(r), \quad (10)$$

and suppose U expanded in a series of LAPLACE's Functions,

$$U_0 + U_1 + U_2 + \dots;$$

then substituting and equating the functions of the same order on the two sides of the equation, we have

$$U_n = -\frac{1}{c^2}e^{-imr}F_n(c)u_n,$$

and therefore

$$\varphi = -\frac{c^2}{r}e^{im(at-r+c)}\Sigma\frac{U_n}{F_n(c)}f_n(r). \quad (11)$$

This expression contains the solution of the problem, and it remains only to discuss it.

At a great distance from the sphere the function $f_n(r)$ becomes ultimately equal to 1, and we have

$$\phi = -\frac{c^2}{r} e^{i\kappa(at-r+c)} \Sigma \frac{U_n}{F_n(c)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

It appears from (3) that the component of the velocity along the radius vector is of the order r^{-1}, and that in any direction perpendicular to the radius vector of the order r^{-2}, so that the lateral motion may be disregarded except in the neighbourhood of the sphere.

In order to examine the influence of the lateral motion in the neighbourhood of the sphere, let us compare the actual disturbance at a great distance with what it would have been if all lateral motion had been prevented, suppose by infinitely thin conical partitions dividing the fluid into elementary canals, each bounded by a conical surface having its vertex at the centre.

On this supposition the motion in any canal would evidently be the same as it would be in all directions if the sphere vibrated by contraction and expansion of the surface, the same all round, and such that the normal velocity of the surface was the same as it is at the particular point at which the canal in question abuts on the surface. Now if U were constant the expansion of U would be reduced to its first term U_0, and seeing that $f_0(r)=1$ we should have from (11)

$$\phi = -\frac{c^2}{r} e^{i\kappa(at-r+c)} \frac{U_0}{F_0(c)}.$$

This expression will apply to any particular canal if we take U_0 to denote the normal velocity at the sphere's surface for that particular canal; and therefore to obtain an expression applicable at once to all the canals we have merely to write U for U_0. To facilitate a comparison with (11) and (12) I shall, however, write ΣU_n for U. We have then

$$\phi = -\frac{c^2}{r} e^{i\kappa(at-r+c)} \frac{\Sigma U_n}{F_0(c)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

It must be remembered that this is merely an expression applicable at once to all the canals, the motion in each of which takes place wholly along the radius vector, and accordingly the expression is not to be differentiated with respect to θ or ω with the view of applying the formulæ (3).

On comparing (13) with the expression for the function ϕ in the actual motion at a great distance from the sphere (12), we see that the two are identical with the exception that U_n is divided by two different constants, namely $F_0(c)$ in the former case and $F_n(c)$ in the latter. The same will be true of the leading terms (or those of the order r^{-1}) in the expressions for the condensation and velocity*. Hence if the mode of vibration of

* Of course it would be true if the *complete* differential coefficients with respect to r of the right-hand members of (12) and (13) were taken, but then the former does not give the velocity w' except as to its leading term, since (12) has been deduced from the exact expression (11) by reducing $f_n(r)$ to its first term 1; nor again is it true, except as to terms of the order r^{-1}, of the actual motion of the unimpeded fluid that the whole velocity is in the direction of the radius vector.

the sphere is such that the normal velocity of its surface is expressed by a LAPLACE'S Function of any one order, the disturbance at a great distance from the sphere will vary from one direction to another according to the same law as if lateral motion had been prevented, the amplitude of excursion at a given distance from the centre varying in both cases as the amplitude of excursion, in a normal direction, of the surface of the sphere itself. The only difference is that expressed by the symbolic ratio $F_n(c) : F_0(c)$. If we suppose $F_n(c)$ reduced to the form $\mu_n(\cos\alpha_n + \sqrt{-1}\sin\alpha_n)$, the amplitude of vibration in the actual case will be to that in the supposed case as μ_0 to μ_n, and the phases in the two cases will differ by $\alpha_0 - \alpha_n$.

If the normal velocity of the surface of the sphere be not expressible by a single LAPLACE'S Function, but only by a series, finite or infinite, of such functions, the disturbance at a given great distance from the centre will no longer vary from one direction to another according to the same law as the normal velocity of the surface of the sphere, since the modulus μ_n and likewise the amplitude α_n of the imaginary quantity $F_n(c)$ vary with the order of the function.

Let us now suppose the disturbance expressed by a LAPLACE'S Function of some one order, and seek the numerical value of the alteration of intensity at a distance, produced by the lateral motion which actually exists.

The intensity will be measured by the *vis viva* produced in a given time, and consequently will vary as the density multiplied by the velocity of propagation multiplied by the square of the amplitude of vibration. It is the last factor alone that is different from what it would have been if there had been no lateral motion. The amplitude is altered in the proportion of μ_0 to μ_n, so that if

$$\frac{\mu_n^2}{\mu_0^2} = I_n,$$

I_n is the quantity by which the intensity which would have existed if the fluid had been hindered from lateral motion has to be divided.

For the first five orders the values of the function $F_n(c)$ are as follows:—

$$F_0(c) = imc + 1,$$

$$F_1(c) = imc + 2 + \frac{2}{imc},$$

$$F_2(c) = imc + 4 + \frac{9}{imc} + \frac{9}{(imc)^2},$$

$$F_3(c) = imc + 7 + \frac{27}{imc} + \frac{60}{(imc)^2} + \frac{60}{(imc)^3},$$

$$F_4(c) = imc + 11 + \frac{65}{imc} + \frac{240}{(imc)^2} + \frac{525}{(imc)^3} + \frac{525}{(imc)^4}.$$

If λ be the length of the sound-wave corresponding to the period of the vibration, $m = \frac{2\pi}{\lambda}$, so that mc is the ratio of the circumference of the sphere to the length of a

wave. If we suppose the gas to be air and λ to be 2 feet, which would correspond to about 550 vibrations in a second, and the circumference $2\pi c$ to be 1 foot (a size and pitch which would correspond with the case of a common house bell), we shall have $mc=\frac{1}{2}$. The following Table gives the values of the square of the modulus and of the ratio I_n for the functions $F_n(c)$ of the first five orders, for each of the values 4, 2, 1, $\frac{1}{2}$, and $\frac{1}{4}$ of mc. It will presently appear why the Table has been extended further in the direction of values greater than $\frac{1}{2}$ than it has in the opposite direction. Five significant figures at least are retained.

mc.	$n=0$.	$n=1$.	$n=2$.	$n=3$.	$n=4$.	
4	17	16·25	14·879	13·848	20·177	Values of μ_n^2.
2	5	5	9·3125	80	1495·8	
1	2	5	89	3965	300137	
0·5	1·25	16·25	1330·2	236191	72086371	
0·25	1·0625	64·062	20878	14837899	18160×10^4	
4	1	0·95588	0·87523	0·81459	1·1869	Values of I_n.
2	1	1	1·8625	16	299·16	
1	1	2·5	44·5	1982·5	150068	
0·5	1	13	1064·2	188953	57669097	
0·25	1	60·294	19650	13965×10^3	17092×10^4	

When $mc=\infty$ we get from the analytical expressions $I_n=1$. We see from the Table that when mc is somewhat large I_n is liable to be *a little* less than 1, and consequently the sound to be *a little* more intense than if lateral motion had been prevented. The possibility of this is explained by considering that the waves of condensation spreading from those compartments of the sphere which at a given moment are vibrating positively, *i. e.* outwards, after the lapse of a half period may have spread over the neighbouring compartments, which are now in their turn vibrating positively, so that these latter compartments in their outward motion work against a somewhat greater pressure than if each compartment had opposite to it only the vibration of the gas which it had itself occasioned; and the same explanation applies *mutatis mutandis* to the waves of rarefaction. However, the increase of sound thus occasioned by the existence of lateral motion is but small in any case, whereas when mc is somewhat small I_n increases enormously, and the sound becomes a mere nothing compared with what it would have been had lateral motion been prevented.

The higher be the order of the function, the greater will be the number of compartments, alternately positive and negative as to their mode of vibration at a given moment, into which the surface of the sphere will be divided. We see from the Table that for a given periodic time as well as radius the value of I_n becomes considerable when n is somewhat high. However practically vibrations of this kind are produced when the elastic sphere executes, not its principal, but one of its subordinate vibrations, the pitch corresponding to which rises with the order of the vibration, so that m increases with that order. It was for this reason that the Table was extended from

$mc=0{\cdot}5$ further in the direction of high pitch than low pitch, namely, to three octaves higher and only one octave lower.

When the sphere vibrates symmetrically about the centre, *i. e.* so that any two opposite points of the surface are at a given moment moving with equal velocities in opposite directions, or more generally when the mode of vibration is such that there is no change of position of the centre of gravity of the volume, there is no term of the order 1. For a sphere vibrating in the manner of a bell the principal vibration is that expressed by a term of the order 2, to which I shall now more particularly attend.

Putting, for shortness, $m^3c^3=q$, we have

$$\mu_0^3=q+1,\ \mu_2^3=(q^4-9q^{-1})^3+\left(4-\frac{9}{q}\right)^3=q-2+\frac{9}{q}+\frac{81}{q^3},$$

$$I_2=\frac{q^3-2q^2+9q+81}{q^2(q+1)}.$$

The minimum value of I_2 is determined by

$$q^3-6q^2-84q-54=0,$$

giving approximately

$$q=12{\cdot}859,\ mc=3{\cdot}586,\ \mu_0^3=13{\cdot}859,\ \mu_2^3=12{\cdot}049,\ I_2={\cdot}86941;$$

so that the utmost increase of sound produced by lateral motion amounts to about 15 per cent.

I come now more particularly to Leslie's experiments. Nothing is stated as to the form, size, or pitch of his bell; and even if these had been accurately described, there would have been a good deal of guesswork in fixing on the size of the sphere which should be considered the best representative of the bell. Hence all we can do is to choose such values for m and c as are comparable with the probable conditions of the experiment.

I possess a bell, belonging to an old bell-in-air apparatus, which may probably be somewhat similar to that used by Leslie. It is nearly hemispherical, the diameter is 1·96 inch, and the pitch an octave above the middle C of a piano. Taking the number of vibrations 1056 per second, and the velocity of sound in air 1100 feet per second, we have $\lambda=12{\cdot}5$ inches. To represent the bell by a sphere of the same radius would be very greatly to underrate the influence of local circulation, since near the mouth the gas has but a little way to get round from the outside to the inside, or the reverse. To represent it by a sphere of half the radius would still apparently be to underrate the effect. Nevertheless for the sake of rather underestimating than exaggerating the influence of the cause here investigated, I will make these two suppositions successively, giving respectively $c={\cdot}98$ and $c={\cdot}49$, $mc={\cdot}4926$, and $mc={\cdot}2463$ for air.

If it were not for lateral motion the intensity would vary from gas to gas in the proportion of the density into the velocity of propagation, and therefore as the pressure into the square root of the density under a standard pressure, if we take the factor depending on the development of heat as sensibly the same for the gases and gaseous mixtures with which we have to deal. In the following Table the first column gives the

gas, the second the pressure p, in atmospheres, the third the density D under the pressure p, referred to the density of air at the atmospheric pressure as unity, the fourth, Q_r, what would have been the intensity had the motion been wholly radial, referred to the intensity in air at atmospheric pressure as unity, or, in other words, a quantity varying as $p \times$ (the density at pressure 1)$^{\frac{1}{2}}$. Then follow the values of q, I_s, and Q, the last being the actual intensity referring to air as before.

Gas.	p.	D.	Q_r.	$c=\cdot98$.			$c=\cdot49$.		
				q.	I_s.	Q.	q.	I_s.	Q.
Air	1	1	1	·2427	1136	1	·06067	20890	1
Hydrogen	1	·0690	·2627	·01674	284700	·001048	·004186	4604000	·001191
Air, rarefied	·01	·01	·01	·2427	1136	·01	·06067	20890	·01
The same filled with H	1	·0783	·2798	·01900	220600	·001440	·004751	3572000	·001637
Air of same density	·0783	·0783	·0783	·2427	1136	·0783	·06067	20890	·0783
Air rarefied $\frac{1}{2}$	·5	·5	·5	·2427	1136	·5	·06067	20890	·5
The same filled with H	1	·5345	·7311	·1297	4322	·1921	·0324	74890	·2039

An inspection of the numbers contained in the columns headed Q will show that the cause here investigated is amply sufficient to account for the facts mentioned by Leslie.

It may be noticed that while q is 4 times smaller, and I_s is 16 or 18 times larger, for $c=\cdot49$ than for $c=\cdot98$, there is no great difference in the values of Q in the two cases for hydrogen and mixtures of hydrogen and air in given proportions. This arises from the circumstance that q is sufficiently small to make the last terms in μ_0^2 and μ_2^2, namely, 1 and $81q^{-2}$, the most important, so that I_s does does not greatly differ from $81q^{-2}$. If this result had been exact instead of approximate, the intensity in different gases, supposed for simplicity to be at a common pressure, would have varied as $D^{\frac{1}{2}}$; and it will be found that for the cases in which $p=1$ the values of Q in the above Table, especially those in the last column, do not greatly deviate from this proportion. But the simplicity of this result depends on two things. First, the vibration must be expressed by a Laplace's function of the order 2; for a different order the power of D would have been different; and this is just one of the points respecting which we cannot infer what would be true of a bell of the ordinary shape from what we have proved for a sphere. Secondly, the radius must be sufficiently small, or the pitch sufficiently low, to make q small; at the other extremity of the scale, in which c is supposed to be very large, or λ very small, Q varies nearly as $D^{\frac{1}{2}}$ instead of $D^{\frac{1}{2}}$, whatever be the order of the Laplace's function. Hence no simple relation can be expected between the numbers furnished by experiment and the numerical constants of the gas in such experiments as those of M. Perolle*, in which the same bell was rung in succession in different gases.

B. *Solution of the Problem in the case of a Vibrating Cylinder.*

I will here suppose the motion to be in two dimensions only. In the case of a vibrating string, which I have mainly in view, it is true that the amplitude of excursion

* Mémoires de l'Académie des Sciences de Turin, t. iii. (1786–7); Mém. des Correspondans, p. 1.

of the string varies sensibly on proceeding even a moderate distance along it, and that the propagation of the sound-wave produced by no means takes place in two dimensions only. But the question how far a sound-wave is produced at all, and how far the displacement of the gas by the cylinder merely produces a local motion to and fro, is decided by what takes place in the immediate neighbourhood of the string, such as within a distance of a few diameters; and though the sound-wave, when once produced, in its subsequent progress diverges in three dimensions, the same takes place with the hypothetical sound-wave which would be produced if lateral motion were prevented, so that the comparison which it is the object of the investigation to institute is not affected thereby.

Assuming, then, the motion to be in two dimensions, and referring the fluid to polar coordinates, r, θ, r being measured from the axis of the undisturbed cylinder, we shall have for the fundamental equation derived from (1)

$$\frac{d^2\varphi}{dt^2}=a^2\left\{\frac{d^2\varphi}{dr^2}+\frac{1}{r}\frac{d\varphi}{dr}+\frac{1}{r^2}\frac{d^2\varphi}{d\theta^2}\right\}; \quad \ldots\ldots\ldots \quad (14)$$

and if u', v' be the components of the velocity along and perpendicular to the radius vector,

$$u'=\frac{d\varphi}{dr}, \qquad v'=\frac{1}{r}\frac{d\varphi}{d\theta}.$$

If c be the radius of the cylinder, and V the normal component of the velocity of the surface of the cylinder, we must have

$$\frac{d\varphi}{dr}=\mathrm{V}, \text{ when } r=c.$$

As before, I will suppose the motion of the cylinder, and consequently of the fluid, to be regularly periodic, but instead of using circular functions directly I will employ the imaginary exponential e^{imat}, i denoting as before $\sqrt{-1}$, and will put accordingly $\mathrm{V}=e^{imat}\mathrm{U}$, U being a given function of θ, and $\varphi=\psi e^{imat}$. For a given value of r, ψ may by a known theorem be developed in a series of sines and cosines of θ and its multiples, and therefore for general values of r can be so developed, the coefficients being functions of r. If ψ_n be the coefficient of $\cos n\theta$ or $\sin n\theta$, we find

$$\frac{d^2\psi_n}{dr^2}+\frac{1}{r}\frac{d\psi_n}{dr}-\frac{n^2}{r^2}\psi_n+m^2\psi_n=0. \quad \ldots\ldots\ldots \quad (15)$$

If we suppose the normal velocity of the surface of the cylinder to vary in a given manner from one generating line to another, so that U is a given function of θ, we may expand U in a series of the form

$$\mathrm{U}=\mathrm{U}_0+\mathrm{U}_1\cos\theta+\mathrm{U}_2\cos 2\theta+\ldots$$
$$+\mathrm{U}'_1\sin\theta+\mathrm{U}'_2\sin 2\theta+\ldots$$

On applying now the equation of condition which has to be satisfied at the surface of the cylinder, we see that a term $\mathrm{U}_n\cos n\theta$ or $\mathrm{U}'_n\sin n\theta$ of the nth order in the expression

for U will introduce a function ψ_n of the same order in the general expression for ϕ. Now the only case of interest relating to an infinite cylinder is that of a vibrating string, in which the cylinder moves as a whole. The vibration may be regarded as compounded of the vibrations in any two rectangular planes passing through the axis, the phases of the component vibrations, it may be, being different. These component vibrations may be treated separately, and thus it will suffice to suppose the vibration confined to one plane, which we may take to be that from which θ is measured. We shall accordingly have

$$U = U_1 \cos\theta,$$

U_1 being a given constant, and the only function ψ_n which will appear in the general expression for ϕ will be that of the order 1. Besides this we shall have to investigate, for the sake of comparison, an ideal vibration in which the cylinder alternately contracts and expands in all directions alike, and for which accordingly U is a constant U_0. Hence the equation (15) need only be considered for the values 0 and 1 for n.

For general values of n the equation (15) is easily integrated in the form of infinite series according to ascending powers of r. The result is

$$\left.\begin{aligned}\psi_n = {} & Ar^n\left\{1-\frac{m^2r^2}{2(2+2n)}+\frac{m^4r^4}{2.4(2+2n)(4+2n)}-\dots\right\}\\ & +Br^{-n}\left\{1-\frac{m^2r^2}{2(2-2n)}+\frac{m^4r^4}{2.4(2-2n)(4-2n)}-\dots\right\}\end{aligned}\right\} \quad \dots\dots \quad (16)$$

When n is any integer the integral as it stands becomes illusory; but the complete integral, which in this case assumes a special form, is readily obtained as a limiting case of the complete integral for general values of n.

The series in (16) are convergent for any value of r however great, but they give us no information of what becomes of the functions for very large values of r.

When r is very large, the equation (15) becomes approximately

$$\frac{d^2\psi_n}{dr^2}+m^2\psi_n=0,$$

the integral of which is $\psi_n = Re^{-imr}+R'e^{imr}$, where R and R′ are constant. This suggests putting the complete integral of (15) under the same form, R and R′ being now functions of r, which, when r is large, vary but slowly, that is, remain nearly constant when r is altered by only a small multiple of λ. Assuming for R and R′ series of the form $Ar^\alpha+Br^\beta+Cr^\gamma\dots$, where $\alpha, \beta, \gamma\dots$ are in decreasing order algebraically, and determining the indices and coefficients so as to satisfy (15), we get for another form of the complete integral

$$\left.\begin{aligned}\psi_n = {} & C(imr)^{-\frac{1}{2}}e^{-imr}\left\{1-\frac{1^2-4n^2}{1.8imr}+\frac{(1^2-4n^2)(3^2-4n^2)}{1.2(8imr)^2}-\frac{(1^2-4n^2)(3^2-4n^2)(5^2-4n^2)}{1.2.3(8im)^3}+\dots\right\}\\ & +D(imr)^{-\frac{1}{2}}e^{imr}\left\{1+\frac{(1^2-4n^2)}{1.8imr}+\frac{(1^2-4n^2)(3^2-4n^2)}{1.2(8imr)^2}+\frac{(1^2-4n^2)(3^2-4n^2)(5^2-4n^2)}{1.2.3(8imr)^3}+\dots\right\}\end{aligned}\right\} \quad (17)$$

These series, though ultimately divergent, begin by converging rapidly when r is large, and may be employed with great advantage when r is large, if we confine ourselves to the converging part. Moreover we have at once D=0 as the condition to be satisfied at a great distance from the cylinder. If mc were large we might employ the second form of integral in satisfying the condition at the surface of the cylinder, and the problem would present no further difficulty. But practically in the case of vibrating strings mc is a very small fraction; the series (16) are rapidly convergent, and the series (17) cannot be employed. To complete the solution of the problem therefore it is essential to express the constants A and B in terms of C and D, or at any rate to find the relation between A and B imposed by the condition D=0.

This may be effected by means of the complete integral of (15) expressed in the form of a definite integral. For $n=0$ we know that

$$\psi_0=\int_0^{\frac{\pi}{2}}\{E+F\log(r\sin^2\zeta)\}\cos(mr\cos\zeta)d\zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

is a third form of the integral of (15). It is not difficult to deduce from this the integral of (15) in a similar form for any integral value of n. Assuming

$$\psi_n=r^n\int r^n\chi_n dr,$$

and substituting in (15), we have

$$r^{n+\beta}\frac{d\chi_n}{dr}+(2\alpha+\beta+1)r^{n+\beta-1}\chi_n+(\alpha^2-n^2)r^{n-2}\int r^n\chi_n dr+m^2r^n\int r^n\chi_n dr=0.$$

Assume

$$\alpha^2-n^2=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

divide the equation by r^n, differentiate with respect to r, and then divide by r^β. The result is

$$\frac{d^2\chi_n}{dr^2}+\frac{2\alpha+2\beta+1}{r}\frac{d\chi_n}{dr}+(2\alpha+\beta+1)(\beta-1)\frac{\chi_n}{r^2}+m^2\chi_n=0.$$

This equation will be of the same form as (15) provided,

$$\alpha+\beta=0,$$

which reduces the coefficient of the last term but one to $-(\alpha+1)^2$. In order that this coefficient may be increased we must choose the positive root of (19), namely n, which I will suppose positive. Hence

$$\psi_n=r^n\int r^{-n}\chi_n dr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

gives

$$\frac{d^2\chi_n}{dr^2}+\frac{1}{r}\frac{d\chi_n}{dr}-\frac{(n+1)^2}{r^2}\chi_n+m^2\chi_n=0,$$

the same equation as that for the determination of ψ_{n+1}. Hence expressing χ_n in terms of ψ_n from (20), writing $n-1$ for n, and replacing χ_{n-1} by ψ_n, we have

$$\psi_n=r^{n-1}\frac{d}{dr}r^{-(n-1)}\psi_{n-1},$$

a formula of reduction which when n is integral serves to express ψ_n in terms of ψ_0. We have

$$\psi_n = r^n \left(\frac{1}{r}\frac{d}{dr}\right)^n \psi_0, \quad \ldots \quad (21)$$

an equation which when applied to (18) gives the complete integral of (15) for integral values of n in the form of a definite integral.

Let us attend now more particularly to the case of $n=0$. The equation (16) is of the form $\psi_n = Af(n)+Bf(-n)$, $f(n)$ containing r as well as n. Expanding by MACLAURIN'S Theorem, we have

$$\psi_n = (A+B)f(0)+(A-B)f'(0)n+(A+B)f''(0)\frac{n^2}{1.2}+\ldots$$

Writing A for $A+B$, $n^{-1}B$ for $A-B$, and then making n vanish, we have

$$\psi_0 = Af(0)+Bf'(0),$$

or

$$\left.\begin{aligned}\psi_0 = (A+B\log r)\left(1-\frac{m^2r^2}{2^2}+\frac{m^4r^4}{2^2 4^2}-\frac{m^6r^6}{2^2 4^2 6^2}+\ldots\right)\\ +B\left(\frac{m^2r^2}{2^2}S_1-\frac{m^4r^4}{2^2 4^2}S_2+\frac{m^6r^6}{2^2 4^2 6^2}S_3-\ldots\right),\end{aligned}\right\} \quad \ldots \quad (22)$$

where

$$S_n = 1^{-1}+2^{-1}+3^{-1}\ldots+n^{-1}.$$

The integral in the form (17) assumes no peculiar shape when n is integral, and we have at once

$$\left.\begin{aligned}\psi_n = C(imr)^{-\frac{1}{2}}e^{-imr}\left\{1-\frac{1^2}{1.8imr}+\frac{1^2 3^2}{1.2(8imr)^2}-\frac{1^2 3^2 5^2}{1.2.3(8imr)^3}+\ldots\right\}\\ +D(imr)^{-\frac{1}{2}}e^{imr}\left\{1+\frac{1^2}{1.8imr}+\frac{1^2 3^2}{1.2(8imr)^2}-\frac{1^2 3^2 5^2}{1.2.3(8imr)^3}+\ldots\right\}\end{aligned}\right\} \quad \ldots \quad (23)$$

I have explained at length the mode of dealing with such functions, and especially of connecting the arbitrary constants in the ascending and descending series, in two papers published in the Transactions of the Cambridge Philosophical Society*, in the second of which the connexion of the constants is worked out in this very example. To these I will refer, merely observing that while it is perfectly easy to connect A, B with E, F, the connexion of C, D with E, F involves some extremely curious points of analysis. The result of eliminating E, F between the two equations connecting A, B with E, F and the two connecting C, D with E, F is given, except as to notation, in the two equations (41) of my second paper. To render the notation identical with that of the former paper, it will be sufficient to write $A-B\log(im)+B\log(imr)$ for $A+B\log r$, and x for imr. The equations referred to may be simplified by the introduction of EULER'S constant γ, the value of which is ·57721566 &c., since it is known that

$$x^{-\frac{1}{2}}\Gamma(\tfrac{1}{2})+\log 4+\gamma = 0,$$

* "On the Numerical Calculation of a Class of Definite Integrals and Infinite Series," vol. ix. p. 166, and "On the Discontinuity of Arbitrary Constants which appear in Divergent Developments," vol. x. p. 105. A supplement to the latter paper has recently been read before the Cambridge Philosophical Society.

$\Gamma'(x)$ denoting the derivative of the function $\Gamma(n)$. Putting

$$A-B\log im=A',$$

we have by equations (41) of the second paper referred to

$$C=(2\pi)^{-\frac{1}{2}}\{iA'+[(\log 2-\gamma)i-\pi]B\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$D=(2\pi)^{-\frac{1}{2}}\{A'+(\log 2-\gamma)B\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

i being written for $\sqrt{-1}$. It is shown in that paper that these values of C, D hold good when the amplitude of the imaginary variable x lies between the limits 0 and π, or that of r (supposed to be imaginary) between the limits $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, but in crossing either of these limits one or other of the constants C, D is changed. In the investigation of the present paper r is of course real.

We have now

$$A'=A-B\log im=(\gamma-\log 2)B$$

for the relation between A and B arising from the condition that the motion is propagated outwards from the cylinder; and substituting in (22), we have for the value of ψ_0 subject to this condition

$$\left.\begin{aligned}\psi_0=&B\left(\gamma+\log\frac{imr}{2}\right)\left(1-\frac{m^2r^2}{2^2}+\frac{m^4r^4}{2^2.4^2}-\ldots\right)\\&+B\left(\frac{m^2r^2}{2^2}S_1-\frac{m^4r^4}{2^2.4^2}S_2+\ldots\right);\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

or expressed by means of the descending series,

$$\psi_0=-B\left(\frac{\pi}{2imr}\right)e^{-imr}\left\{1-\frac{1^2}{1.8imr}+\frac{1^2.3^2}{1.2(8imr)^2}-\frac{1^2.3^2.5^2}{1.2.3(8imr)^3}+\ldots\right\} \quad . \quad . \quad (27)$$

We have from (21)

$$\psi_1=\frac{d\psi_0}{dr},$$

from which the complete integral of (15) for $n=1$ may be got from that for $n=0$. In the form (17) of the integral the parts arising from differentiation of the parts containing e^{-imr} and e^{imr} respectively will contain those same exponentials, and therefore the complete integral of (15) for $n=1$, subject to the condition that the part containing e^{imr} shall disappear, will be got by differentiating the complete integral for $n=0$ subject to that same condition. The *form* of the integral in the shape of a descending series is given at once by (17). Hence we get by differentiating (26) and (27), and at the same time changing the arbitrary constant by writing B_1m^{-1} for B,

$$\left.\begin{aligned}\psi_1=&\frac{B_1}{mr}\left\{1-\frac{m^2r^2}{2^2}+\frac{m^4r^4}{2^2.4^2}-\ldots\right\}\\&-B_1\left(\gamma+\log\frac{imr}{2}\right)\left(\frac{mr}{2}-\frac{m^3r^3}{2^2.4}+\frac{m^5r^5}{2^2.4^2.6}-\ldots\right)\\&+B_1\left(\frac{mr}{2}S_1-\frac{m^3r^3}{2^2.4}S_2+\frac{m^5r^5}{2^2.4^2.6}S_3-\ldots\right)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\psi_1=B_1\left(\frac{\pi i}{2mr}\right)^{\frac{1}{2}}e^{-imr}\left\{1-\frac{-1.3}{1.8mr}+\frac{-1.1.3.5}{1.2(8mr)^2}-\frac{-1.1.3^2.5.7}{1.2.3(8mr)^3}+\ldots\right\} \quad . \quad . \quad (29)$$

To determine the arbitrary constants B_1 and B, the first belonging to the actual motion, the second to the motion which would take place if the fluid were confined by an infinite number of planes passing through the axis, we must have, as before, for $r=c$,

$$\frac{d\psi_0}{dr}=U_1, \qquad \frac{d\psi_1}{dr}=U_1,$$

whence

$$\left.\begin{aligned}\frac{cU_1}{B}=1-\frac{m^2c^2}{2^2}+\frac{m^4c^4}{2^2.4^2}-\ldots-\left(\gamma+\log\frac{mc}{2}+i\frac{\pi}{2}\right)\left(\frac{m^2c^2}{2}-\frac{m^4c^4}{2^2.4}+\ldots\right)\\ +\frac{m^2c^2}{2}S_1-\frac{m^4c^4}{2^2.4}S_2+\frac{m^6c^6}{2^2.4^2.6}S_3-\ldots\\ =F_0(mc)+i\frac{\pi}{2}f_0(mc), \text{ suppose};\end{aligned}\right\} \quad \ldots \quad (30)$$

$$\left.\begin{aligned}\frac{cU_1}{B_1}=-\frac{1}{mc}-\frac{3mc}{2^2}+\frac{7m^3c^3}{2^2.4^2}-\frac{11m^5c^5}{2^2.4^2.6^2}+\ldots\\ -\left(\gamma+\log\frac{mc}{2}+i\frac{\pi}{2}\right)\left(\frac{mc}{2}-\frac{3m^3c^3}{2^2.4}+\ldots\right)\\ +\frac{mc}{2}S_1-\frac{3m^3c^3}{2^2.4}S_2+\frac{5m^5c^5}{2^2.4^2.6}S_3-\ldots\\ =\frac{1}{mc}\left\{F_1(mc)+i\frac{\pi}{2}f_1(mc)\right\}, \text{ suppose.}\end{aligned}\right\} \quad \ldots \quad (31)$$

If I be the ratio of the intensities at a distance in the supposed and in the actual case, we see from (30) and (31) that I will be equal to the ratio of the squares of the moduli of B and B_1, and we shall therefore have

$$I=\frac{\{4F_1(mc)\}^2+\pi^2\{f_1(mc)\}^2}{m^2c^2\left[4\{F_0(mc)\}^2+\pi^2\{f_0(mc)\}^2\right]}. \quad \ldots \quad (32)$$

For a piano string corresponding to the middle C, c may be about ·02 inch, and λ is about 25 inches. This gives $mc=$·005027. For such small values of mc I does not sensibly differ from $(mc)^{-2}$, which in the present case is 39571, so that the sound is nearly 40000 times weaker than it would have been if the motion of the particles of air had taken place in planes passing through the axis of the string. This shows the vital importance of sounding-boards in stringed instruments. Although the amplitude of vibration of the particles of the sounding-board is extremely small compared with that of the particles of the string, yet as it presents a broad surface to the air it is able to excite loud sonorous vibrations, whereas were the string supported in an absolutely rigid manner, the vibrations which it could excite directly in the air would be so small as to be almost or altogether inaudible.

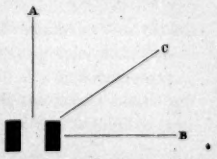
I may here mention a phenomenon which fell under my notice, and which is readily explained by the principles laid down in this paper. As I was walking one windy day on a road near Cambridge, on the other side of which ran a line of telegraph, my attention was attracted by a peculiar sound of extremely high pitch, which seemed to come from the opposite side of the road. On going over to ascertain the cause, I found that it came directly through air from the telegraph wires. On standing near a telegraph

post, the ordinary comparatively bass sound with which we are so familiar was heard, appearing to emanate from the post. On receding from the post the bass sound became feebler, and midway between two posts was quite inaudible. Nothing was then heard but the peculiar high-pitched sound, which appeared to emanate from the wires overhead. It had a peculiar metallic ring about it which the ear distinguished from the whistling of the wind in the twigs of a bush. Although the telegraph ran for miles, it was only at one spot that the peculiar sound was noticed, and even there only in certain states of the wind. The wires seemed to be less curved than usual at the place in question, from which it may be inferred that they were there subject to an unusually great tension.

The explanation of the phenomenon is easy after what precedes. The wires were thrown into vibration by the wind, and a number of different vibrations, having different periodic times, coexisted. As regards the vibrations of comparatively long period, the air around the wires behaved nearly like an incompressible fluid, and no sonorous vibrations of sensible amount were produced. These vibrations of the wires were, however, communicated to the posts, which being broad acted as sounding-boards, and thus sonorous vibrations of corresponding period were indirectly excited in the air. But as regards the vibrations of extremely short periodic time, the wires in spite of their narrowness were able by acting directly on the air to produce condensations and rarefactions of sensible amount.

The diameter of the telegraph wire was about ·166 inch; and if we take the C below the middle C of a piano for the representative of the pitch of the lower note, and a note five octaves higher for that of the higher, we have in the first case $\lambda=50$ inches nearly, and in the second $\lambda=50\times 2^{-5}$, giving in the former case $mc=\cdot 01043$, and in the latter $mc=\cdot 3338$ The former of these values is so small that we may take $I=(mc)^{-2}$; in the latter case the formula (32) gives for I a value a little less than $(mc)^{-2}$. We find in the two cases I=9192 and I=7·202 respectively, so that in the former case the sound is more than 9000 times feebler than that corresponding to the amplitude of vibration of the wire on the supposition of the absence of lateral motion, whereas in the latter case the actual intensity is nearly one-seventh of the full intensity corresponding to the amplitude.

The increase of sound produced by the stoppage of lateral motion may be prettily exhibited by a very simple experiment. Take a tuning-fork, and holding it in the fingers after it has been made to vibrate, place a sheet of paper or the blade of a broad knife with its edge parallel to the axis of the fork, and as near to the fork as conveniently may be without touching. If the plane of the obstacle coincide with either of the planes of symmetry of the fork, as represented in section at A or B, no effect is produced; but if it be placed in an intermediate position, such as C, the sound becomes much stronger.



XVII. *Comparison of Magnetic Disturbances recorded by the Self-registering Magnetometers at the Royal Observatory, Greenwich, with Magnetic Disturbances deduced from the corresponding Terrestrial Galvanic Currents recorded by the Self-registering Galvanometers of the Royal Observatory.* By GEORGE BIDDELL AIRY, *Astronomer Royal.*

Received December 20, 1867,—Read February 6, 1868.

IT would be difficult, even at this early stage in the history of the treatment of Spontaneous Terrestrial Galvanic Currents, to say how the science began*. As far as relates to my own acquaintance with it, it originated in some general knowledge of the existence, on special days, of currents disturbing practical telegraphic operations, which were soon found to synchronize with magnetic storms, and in communications with Mr. EDWIN CLARK and Mr. LATIMER CLARK, both of whom volunteered their assistance, but more particularly in conversations, and subsequently in correspondence, with CHARLES V. WALKER, Esq. On several occasions, tabular numerical statements were transmitted by Mr. WALKER to me, and copies of the Greenwich magnetic photograms were transmitted by me to Mr. WALKER; Mr. GLAISHER also went to Tunbridge several times, to discuss the subject with Mr. WALKER. It is not necessary here further to allude to the important papers in the Philosophical Transactions by Mr. W. H. BARLOW (1849) and by Mr. WALKER (1861 and 1862), or to the subsequent observations of Dr. LAMONT, except for the purpose of remarking that these philosophers possessed no means of maintaining a constant record by self-registering apparatus.

The history of the establishment of apparatus at the Royal Observatory is as follows.

In my Annual Report to the Board of Visitors of the Royal Observatory, 1860, June 2, I submitted to the consideration of the Board, "It may be worth considering whether it would ever be desirable to establish in two directions at right angles to each other (for instance, along the Brighton Railway and along the North Kent Railway) wires which would photographically register in the Royal Observatory the currents that pass in these directions, exhibiting their indications by photographic curves in close juxtaposition with the registers of the magnetic elements."

The subject having been thus formally opened, and various communications having passed with Mr. WALKER as Superintendent and Engineer of the Telegraphs of the South-Eastern Railway (in which capacity Mr. WALKER has rendered very important services to the Royal Observatory), I again, on 1861, June 1, brought the matter before the Board of Visitors, in these words: "I alluded in my last Report to the possible esta-

* Allusion will be made hereafter to the paper by the late Professor BARLOW in the Philosophical Transactions, 1831, of which, however, the subject is different from that treated here.

blishment of galvanic wires which would record, perhaps upon the same photographic sheet which bears the declination and horizontal force, the magnitude and direction of earth-currents in two directions. I conceive that this may be justly regarded as an important physical experiment; and I hope to be able shortly to lay before the Visitors some details of plan, and to ask their opinion in a more precise form." The Board, after consideration of my proposals, resolved "that it be recommended to the Board of Admiralty that two lines of insulated wires be established from the Royal Observatory to the stations near Dartford and Croydon, with suitable apparatus at the Royal Observatory for the purpose of registering the intensity and direction of terrestrial galvanic currents dependent on natural causes at present obscure."

Mr. WALKER, in his paper in the Philosophical Transactions, 1861, alluded to the matter as having come before him, and as having been the subject of his Report and Estimate; and added, "I have, to a certain small extent, made progress in anticipation."

On 1861, August 14, I submitted a Memorial to the Lords Commissioners of the Admiralty on the advantage of erecting the necessary wires and recording apparatus, at an estimated expense of £250; and on August 19 I received Their Lordships' sanction to my placing that sum on the Navy Estimates for the Financial Year 1862–1863. On August 24 I made formal application to the Directors of the South-Eastern Railway for permission to place the wires on the poles of their Telegraph, and on December 20 I received their assent. The terms on which this assent was granted were most liberal; no stipulation being made except for the nominal annual rent of five shillings for the use of the poles and five shillings for the maintenance of the wires in a state of repair. And I take this opportunity of adding that, when it was found that, by the adoption of insulators of a high class, the expense was made to exceed the estimate, Messrs. SILVER, the contractors for the insulators, at once resigned all profit. The whole of the wire-work has been done at cost price.

The wires were completely established under the superintending care of Mr. WALKER about the autumn of 1862. The wire is that technically called No. 8, $\frac{1}{64}$ inch in diameter, annealed iron wire, coated with zinc by galvanic deposit. The insulators consist each of a double ebonite cup with outer porcelain cup. From the Observatory to the Railway, the wires at different times have been led by different channels, but are now led underground. The connexions with earth have been made in various ways, but at present they are made at each of the three stations by soldering the wires to water-pipes.

The plan for forming, on a revolving cylinder (covered with photographic paper), a spot of light whose position should depend on the galvanic current passing through the wires, was arranged at the beginning of 1863. Each wire communicates with the coil of a galvanometer. It may be mentioned here that the current has proved far stronger than was anticipated. At first, a nearly astatic combination of needles was provided; it was speedily found necessary to turn the red* ends of the needles in the same direction;

* By the *red* end of a needle, I mean that end which is charged with austral magnetism, and which when the needle is freely suspended turns to the north magnetic meridian.

and subsequently the coils were reduced. At present, the entire number of turns of the wire about the acting needle of each galvanometer is 200, and the length of the acting needle is a little more than 1 inch. The needle is suspended by a human hair. A stalk below carries a small plane mirror, whose surface is about $\frac{1}{2}$ inch by 1 inch. With the assistance of cylindrical lenses, a very neat spot of light is thrown upon the revolving barrel by the following arrangement. The beam of light from the lamp falls upon a fixed vertical cylindrical lens, and is made to converge in the horizontal plane. In this state it falls on the plane mirror, converging in the horizontal plane, diverging in the vertical plane. It leaves the plane mirror in the same state, only having received the change of direction corresponding to the play of the galvanometer-magnet; and it then falls on a fixed horizontal lens, which makes it converge in the vertical plane. (This is the plane of a transverse section of the revolving cylinder, which defines the direction of the time-abscissa of the photographic curve.) A single lamp gives light for the recording apparatus of both wires.

The revolving barrels, which had been used for many years in conjunction with the self-registering magnetometers of the Royal Observatory, were of glass, selected with great care, first by Mr. Brooke, afterwards by Mr. Glaisher, from the extensive stock of a large manufacturer; and their circularity and cylindricity of form were almost perfect. I was desirous, however, of using for the earth-current-apparatus a barrel turned in the lathe. After endeavouring in vain to obtain glass so turned, I succeeded in procuring a barrel of ebonite, manufactured by Messrs. Silver. The difficulty of securing this occasioned nearly a year's delay in the completion of the apparatus. Another year was added by the excavation of the Magnetic Basement in which the apparatus was to be placed, and by the caution necessary to be used while the walls were damp. Finally, the apparatus was brought into action and the records were made as a matter of routine, on 1865, April 1, and from that time the registers are almost perfect; the principal interruption being that produced by the great snowstorm of 1866, January.

The instrumental elements on which it appeared desirable to maintain a constant check are the following: the freedom of the barrel from end-shake, and the uniformity of its rotation; the zero of time on the line of time-abscissa; and the zero of galvanic measure for the ordinates. And the checks are thus made.

In regard to end-shake of the barrel, a small lamp throws a light through a definite hole in the cover of the barrel, which, as the barrel revolves, forms a photographic line parallel to the line of time-abscissa, usually called the "photographic base-line."

In regard to the uniformity of rotation, it has been the practice, since self-registering apparatus was established, to produce an interruption in the register, by stopping off the light for a few minutes, three times every day; the observer registering from the clock-dial the times of stopping and readmitting the light. For the test of circularity, this has been done on a limited number of days, six times in the day, at moments extending with approximate uniformity of intervals through the twenty-four hours.

The operation to which I have alluded, even if performed only three times in a day, when used in conjunction with pasteboard scales carefully adjusted to exhibit 24^h for

the revolution of the barrel, gives ample information whereby a time-scale can be laid down applying to every part of the photographic curve.

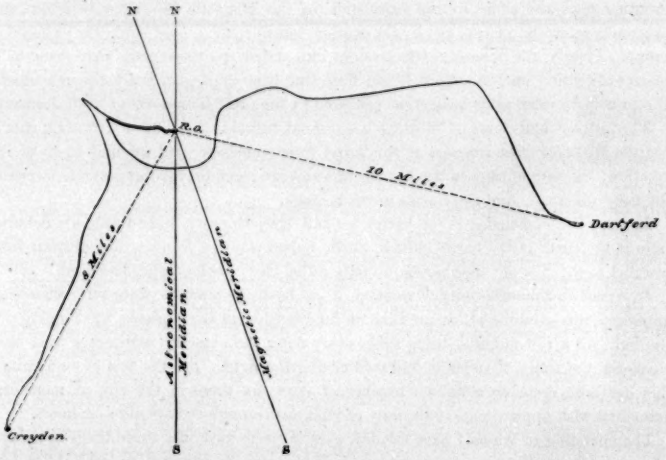
The zero of galvanic measure is obtained by simply interrupting the wire-circuit.

I desire to state that great attention has been given to every one of these adjustments, more especially to those which relate to time. At an early epoch in the comparison of the two classes of measures, it became obvious that questions of simultaneity would arise. Great care has been taken by Mr. GLAISHER in the formation and application of all the time-scales, and I am confident that I can answer for the simultaneity or non-simultaneity of the phenomena, to one or two minutes of time.

Before entering into details of our observations, I will remark that it is perfectly certain that atmospheric influences have produced no effect on our records. This appears in two ways. First, that the Croydon currents (which convey almost the whole of the terrestrial galvanism) have, at all times, an aspect totally different from that of the Dartford currents; secondly, that whenever a wire is broken, at whatever place on the wire, the galvanometer-needle then remains motionless at the zero point. In one or two instances the wire has been broken at a considerable distance from the Observatory, and has fallen so that its end lodged in the ground; and then it has conveyed a very good terrestrial current.

We thus obtain simultaneous galvanic indications from the two instruments, measurable by the parts of scales of sensibly equal value. I proceed to explain the method by which they have been treated.

From the Map which accompanies this communication, it appears that the azimuth of



the Dartford station as viewed from the Royal Observatory, measured from the magnetic

north in the direction N., E., S., W., is $122\frac{1}{4}^{\circ}$; and that of the Croydon station is $229\frac{1}{4}^{\circ}$. Let C be the measure of a galvanic current from Croydon, D that of a current of the same kind (as referred to a battery-pole) from Dartford; and assume these to be caused by a galvanic current N of the same kind towards the north, and a galvanic current W of the same kind towards the west; and that these north and west currents admit of being resolved according to the rules of resolution of mechanical statical force. Put α for the azimuth of Croydon, α' for that of Dartford. Then it is easily found that

$$W = C \cdot \frac{\cos \alpha'}{\sin (\alpha - \alpha')} - D \cdot \frac{\cos \alpha}{\sin (\alpha - \alpha')};$$

$$N = C \cdot \frac{\sin \alpha'}{\sin (\alpha - \alpha')} - D \cdot \frac{\sin \alpha}{\sin (\alpha - \alpha')};$$

which, converted into numbers, give

$$W = -C \times 0{\cdot}55800 + D \times 0{\cdot}68259,$$

$$N = +C \times 0{\cdot}88437 + D \times 0{\cdot}79218.$$

Now, upon interposing a battery in the course of each of the wires, with its graphite or copper pole towards Greenwich, it was found that, on the Croydon wire, it gave an ordinate upon the photographic sheet in the direction which we are accustomed to measure as positive; and on the Dartford wire it gave an ordinate upon the photographic sheet in the negative direction. Let C′ and D′ be the measured quantities; then $C' = +C$, $D' = -D$; and

$$\text{Graphite current towards } W = -C' \times 0{\cdot}55800 - D' \times 0{\cdot}68259,$$

$$\text{Graphite current towards } N = +C' \times 0{\cdot}88437 - D' \times 0{\cdot}79218.$$

Conceiving this graphite current, if it is really a terrestrial galvanic current acting on our magnetometers, to be below them, it will deflect any part of a magnet possessing red or austral magnetism to the right, estimated with reference to the direction of the current's course. Hence we obtain

$$\text{Magnetic tendency to } N = -C' \times 0{\cdot}55800 - D' \times 0{\cdot}68259,$$

$$\text{Magnetic tendency to } W = -C' \times 0{\cdot}88437 + D' \times 0{\cdot}79218.$$

It was found that when the ordinates as measured from the photographic sheet were multiplied by these factors, they gave results larger than those on the magnetometer sheets with which I desired to compare them; and, in order to diminish their magnitude, they were multiplied by 0·5437. Hence, putting $e = 0{\cdot}68259 \times 0{\cdot}5437$, $f = 0{\cdot}55800 \times 0{\cdot}5437$, $g = 0{\cdot}79218 \times 0{\cdot}5437$, $h = 0{\cdot}88437 \times 0{\cdot}5437$ (the order of C′ and D′ having been accidentally inverted),

$$\text{Magnetic tendency to } N = -D' \times e - C' \times f,$$

$$\text{Magnetic tendency to } W = +D' \times g - C' \times h.$$

It now became important to consider how, from the photographic curves of the Earth-currents, these Magnetic Tendencies could be most conveniently obtained. For my own

preliminary trials, I had four proportional compasses constructed by Mr. SIMMS, by means of which I took the proportional part (as expressed by the factor e, or f, &c.) of the ordinate to which that factor applied, laid it down on a new ordinate, and laid, at the top or bottom of the ordinate, the other part which was to be added or subtracted, and thus formed curves for Magnetic Tendencies without the use of any numerals whatever. But this method is not suited to a junior computer. After having satisfied myself that there was good ground for proceeding further, I arranged, for the work of the young assistants, the following process. Four pasteboard scales were prepared, with different graduations:—

"Scale E for Dartford," with graduation $=\frac{1}{e}\times$ graduation of Horizontal-Force-Magnetometer, measuring negatively;

"Scale F for Croydon," with graduation $=\frac{1}{f}\times$ graduation of Horizontal-Force-Magnetometer, measuring negatively;

"Scale G for Dartford," with graduation $=\frac{1}{g}\times$ graduation of Horizontal-Force-Magnetometer, measuring positively;

"Scale H for Croydon," with graduation $=\frac{1}{h}\times$ graduation of Horizontal-Force-Magnetometer, measuring negatively.

Then, the times for which the measures were to be taken having been previously prepared, the ordinate D′ was measured with scale E, and the ordinate C′ was measured with scale F; the numbers were set down in parallel columns and added together, and the sum was used to form a new ordinate of Magnetic Tendency to N. Similarly D′ was measured with scale G, and C′ with scale H, and their numbers were added to form a new ordinate of Magnetic Tendency to W. At every point where the zero of ordinate had been obtained by breaking the wire-circuit, the ordinate was measured in the same way.

In this manner were formed all the curves, exhibiting Magnetic Tendency to N. and W. inferred from Galvanic Measures, which I now exhibit to the Society. (Plates XXV. to XXX.)

For the other curves, that which exhibits Magnetic Force to the North is simply a copy, on an altered scale, of the record of the Horizontal-Force-Magnetometer; and that which exhibits Magnetic Force to the West is derived from the record of the Declination-Magnetometer, the angular deviations being converted into proportionate parts of horizontal force, and laid down on the same scale as the ordinates of the Horizontal-Force-Magnetometer.

The days were selected as exhibiting rather bold curves. I was prevented from taking several which I should have thought preferable, from some scruples (which afterwards proved to be entirely unfounded) about the accuracy of the time-scale.

I now proceed to remark on the comparison of the two systems of curves.

It is impossible, I think, to see these curves without being struck with their general

agreement. In particular, those which exhibit the irregularities of Northerly Magnetic Force are extremely accordant. Yet there are some minor points of discordance, both general and special, which merit most careful attention; and to these I now call the attention of the Society.

The remark of most extensive application is, that the irregularities of the curves derived from the Galvanic Currents are more numerous than those derived from the Magnetometers. I know not whether this is inherent in the nature or origin of the currents, or whether the Magnetic apparatus is less sensitive than the Galvanic apparatus.

The next remark is, that the irregularities derived from the Galvanic Currents almost always precede those derived from the Magnetometers. These intervals of time, as I have already remarked, are perfectly certain. There are a few instances (as West Force, 1867, April 8, 14^h, and June 1, $15^h\ 50^m$; and North Force, 1867, May 28, $7^h\ 20^m$, and June 1, 12^h and 13^h) in which the Galvanic irregularity appears to follow the Magnetometer irregularity; and a greater number, especially in the North Force, in which their coincidence of time is sensibly perfect; but in very far the greatest number, the Galvanic irregularities precede the Magnetometer irregularities. In the instance of West Force, 1866, October 4, $9^h\ 40^m$, and probably in some others, the advance of the Galvanic irregularity is nearly half an hour.

In several instances, the proportions of consecutive rise and fall are not the same in the two systems of curves. Thus on 1866, October 4, 5^h to 6^h, the proportions are unequal for the West Force, and, in a minor degree, for the North Force. So also for the West Force, 1867, May 28, 7^h to 8^h, and June 1, 10^h to 11^h. In this element, as in the times, the accordance is better for the North Force.

On 1867, June 7, 3^h, there is a remarkable salience of the Galvanometer curve for the West Force. Mr. GLAISHER suggests an explanation of this which I believe to be correct. The circuits had been interrupted in order to make time-marks, and the wires had probably taken a galvanic charge, which apparently burst out with great force when the circuits were restored.

On 1867, April 4, West Force, there is discordance which is completely explained by ordinary acceleration of Galvanic results and inequality in the proportions of rise and fall.

On 1867, June 2, West Force, there is discordance of which it is possible the explanation may be different. The day was the Sunday following the Visitation, and it is possible that the instruments may have been disturbed, and that the disturbances may not have been corrected. [It is now believed that the movement of the Declination Magnetometer was impeded by contact with its box.]

After all attention has been given to these points, I think that, on repeatedly examining the agreements of the two systems of curves, it is impossible to avoid the conclusion that the magnetic disturbances are produced by terrestrial galvanic currents below the magnets. Yet there remain some points to be explained before we can state that galvanic currents, as we observe them, will account for all that we observe in the magnetometer records.

One point, of considerable importance, is the relation of the new curves to their zeros. I have stated that the readings by means of Scale E, Scale F, &c. have been taken in the same way for the zeros of galvanic ordinates as for the points of the curves; and, by means of these, zero-lines have been laid down on the diagrams. From general inspection of these, it would seem that, on the days of magnetic storms, the North Force, as shown by the galvanic currents, is on the whole increased. But I believe that in all examinations of magnetometer results for North Force during magnetic storms, and certainly for those in the Royal Observatory (see the Analysis of 177 Magnetic Storms, Philosophical Transactions, 1863) it is found that the North Force is diminished. I cannot at present explain this discordance.

The other discordances, both of time and measure, may not improbably depend on the following circumstance. The south-western (Croydon) wire is entirely south-west of the Royal Observatory; and the south-eastern (Dartford) wire is entirely south-east of the Royal Observatory. It is probable therefore that the times and magnitudes of the terrestrial currents which pass through these wires, correspond rather to places near the middle of the straight lines connecting the Royal Observatory with Croydon and Dartford than to the locality of the Royal Observatory itself.

It might be a question whether, in continuing these observations, the length of wires might not be considerably reduced, and whether an attempt might not be made to give such terminations to each that the recording apparatus should be nearly in the middle of the length of each. In the actual topographical circumstances, it may not be easy to secure the latter condition.

At present we are unable to say whether the records of the galvanic currents throw any light on the origin of the diurnal variations of the magnetic elements.

On one point, however, we seem to be able to speak with confidence; they do not explain the existence of the principal part of terrestrial magnetism. The hypothesis of Professor Barlow in the Philosophical Transactions, 1831, plausible as it undoubtedly was, is not supported by the discovery of galvanic currents competent to explain the earth's general magnetism.

XVIII. *On the Relation of Form and Dimensions to Weight of Material in the Construction of Iron-clad Ships.* *By* E. J. Reed, *Chief Constructor of the Navy. Communicated by Professor* G. G. Stokes, *Sec. R.S.*

Received March 3,—Read March 19, 1868.

The object of the present paper is to show that the proportion of length to breadth in a ship, and the form of her water-lines, should be made in a very great degree dependent upon the weight of the material of which her hull is to be constructed—that an armour-plated ship, for example, should be made of very different proportions and form from those of a ship without armour, and that as the extent and thickness of the armour to be carried by a ship are increased the proportion of length to breadth should be diminished, and the water-lines increased in fulness.

It is highly desirable that this subject should receive the attention of men of science, not only because it bears most directly upon both the cost and the efficiency of future iron-clad fleets, but also because it opens up a theoretical question which has hitherto, I believe, received absolutely no consideration from scientific writers upon the forms and resistances of ships, viz. the manner in which the weight of the material composing the hull should influence the form.

Prior to the design of the 'Bellerophon,' the forms of ships were determined in complete disregard of this consideration; and even the most recent works upon the subject incite the naval architect to aim always at approaching the form of least resistance. The following investigations will show, however, that the adoption of a form of least resistance, or of small comparative resistance, may, in fact, lead to a lavish outlay upon our ships, and to a great sacrifice of efficiency; while, on the other hand, the adoption of a form of greater resistance would contribute in certain classes of ships to great economy and to superior efficiency.

In order to indicate clearly, but approximately only, the purpose in view, I will first assume that figs. 1 & 2 roughly represent a long and a shorter ship respectively, both being prismatic in a vertical sense. The length of fig. 1 is seven times its breadth; that of fig. 2 five times its breadth, the middle portion of the latter being parallel for two-fifths of its length. I will further assume that the ship (fig. 1) will give a constant of 600, and fig. 2 a constant of 500 in the Admiralty formula

$$\frac{\text{speed}^3 \times \text{midship section}}{\text{indicated horse-power}} = \text{constant}.$$

The draught of water is in each case 25 feet, and the total depth 50 feet.

I will take it for granted that the form fig. 1 has been found satisfactory for a ship of such scantlings that we may consider her to be built of iron of a uniform thickness

Fig. 1.

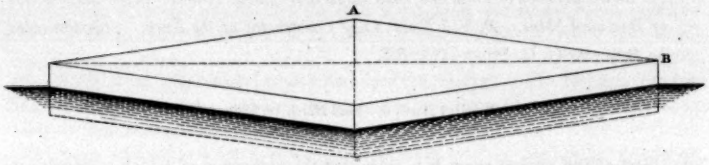
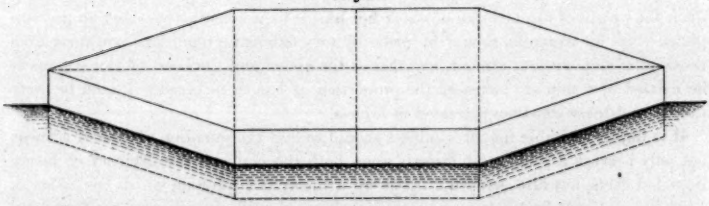


Fig. 2.



of 6 inches, the top and bottom being weightless, and I will take fourteen knots as the speed at which the constant 600 was obtained.

Now, let it be required to design a ship of equal speed, draught of water, and depth, but of such increased scantlings (whether of hull proper or of armour) that the weight shall be equivalent to a uniform thickness of 12-inch iron, the top and bottom being weightless as before. First, we will give to this new ship the proportions and form of fig. 1; secondly, we will give her the form and proportions of fig. 2.

In each case the ship shall be engined with engines developing seven times their nominal horse-power, and weighing (with boilers, water, &c.) 1 ton per nominal H.P.

In order that both ships may steam the same distance at the same speed, we will give them a coal-supply equal in each case to the weight of the engines; but as the smaller ship will require less men and provisions, and a less weight of other stores, we will require the larger ship to carry 2000 tons, and the smaller but 1500 tons of such additional weights.

Taking, first, the long ship fig. 1, and assuming $2x$ to be the breadth, we have

$$\text{I.H.P.} = \frac{14^3 \times 50x}{600}$$

$$= 228x.$$

Therefore weight of engines in tons $= \frac{228x}{7} = 32{\cdot}5x$ tons.

$$\text{Weight of hull} = 4.\overline{AB} \times 50 \times 12 \times \tfrac{42}{3140} \text{ tons}$$
$$= 4\sqrt{50}x \times \tfrac{7}{5}$$
$$= 214\sqrt{2}x.$$

Therefore, weight of hull, engines, and coal

$$= 214\sqrt{2}x + 2 \times 32{\cdot}5x$$
$$= (65 + 214\sqrt{2})x,$$

and displacement, or total weight,

$$= 2000 + (65 + 214\sqrt{2})x.$$

But displacement is also equal to

$$\frac{14x \times 2x}{2} \times 25 \times \tfrac{1}{35}$$
$$= 10x^3;$$
$$\therefore\ 10x^3 = 2000 + (65 + 214\sqrt{2})x,$$

whence

$$x = 41{\cdot}5.$$

The following, therefore, are the dimensions &c. of the new ship of the fig. 1 type, viz.—

Length	$=14x$	$=$	581 feet.
Breadth	$=2x$	$=$	83 „
Nominal horse-power	$=32{\cdot}5x$	$=$	1350 H.P.
Indicated horse-power	$=228x$	$=$	9450 „
Weight of hull	$=214\sqrt{2}x$	$=$	12570 tons.
Weight of engines	$=32{\cdot}5x$	$=$	1350 „
Weight of coals	$=32{\cdot}5x$	$=$	1350 „
Weight carried		$=$	2000 „
Total displacement . .			17270 tons.

Taking next the shorter ship of fig. 2 type, and assuming $2x$ to be the breadth, as before, and therefore $10x$ the length, and $4x$ the length of the parallel portion, we have

$$\text{I.H.P.} = \frac{14^3 \times 50x}{500}$$
$$= 274x.$$

Pursuing the same process as before, and equating again the two values of the displacement, we shall have

$$10x^3 = 1500 + 299x$$

and

$$x = 34{\cdot}28.$$

The following, therefore, are the dimensions &c. of the new ship of the shorter type, viz.—

Length	$= 10x =$	342 feet.
Breadth	$= 2x =$	$68\frac{1}{2}$ „
Nominal horse-power	$= \frac{274x}{7} =$	1337 H.P.
Indicated horse-power	$= 274x =$	9359 „
Weight of hull . .	$= 221x =$	7576 tons.
Weight of engines .	$= 39x =$	1337 „
Weight of coals . .	$= 39x =$	1337 „
Weight carried	$=$	1500 „
Total displacement . .	$=$	11750 „

It will therefore be seen that, by adopting in the new design the proportions and form of fig. 2, a ship of the required scantlings and speed will be obtained on a length of 342 feet, and a breadth of $68\frac{1}{2}$ feet; whereas if the proportions and form of fig. 1 are adopted, the ship, although of the same scantlings and speed only, will require to be 581 feet long and 83 feet broad, the steam-power being in both cases as nearly as possible the same.

It was by considerations of the above nature, worked out more fully and corrected by the results of the trials of many large ships of war, that in designing the 'Bellerophon' I was led to depart so considerably from the proportions and form of the 'Warrior,' 'Minotaur,' and other long iron-clads.

We are now in possession of the actual performances of these ships, and it is upon these as a basis that I propose to continue the present investigation.

From the drawings of the 'Minotaur' and of the 'Bellerophon,' and from calculations based upon them, also from the official reports of the measured mile trials made with those ships rigged for sea, the following particulars are obtained:—

	'Minotaur.'	'Bellerophon.'
Mean displacement	9492 tons.	6808 tons.
Mean midship section	1235 sq. ft.	1127 sq. ft.
Mean of mean draughts of water	24′ 11″	22′ $10\frac{1}{2}$″
Weight of hull	5092 tons.	3477 tons.
Weight of backing to armour	335 „	165 „
Weight of armour	1776 „	1218 „
Weight of engines, coals, and stores	2289 „	1948 „

It follows from the above that the areas of the mean horizontal sections, which must be equal to the mean displacement in cubic feet divided by the mean draught of water, are

'Minotaur.' $\frac{9492 \times 35}{24\frac{11}{12}} = 13333$ sq. ft. 'Bellerophon.' $\frac{6808 \times 35}{22\frac{1}{2}} = 10407$ sq. ft.

The draughts of water which are found to correspond with water-line areas of these sections are,

'Minotaur' . . . 9·6 feet.
'Bellerophon' . . 8·3 feet.

Supposing the mean sections at these depths to be parallel to the construction water-lines, we find from the drawings that their dimensions are—

	'Minotaur.'	'Bellerophon.'
Half breadth extreme	26·8 ft.	26 ft.
Length measured along the side . . .	394 ft.	305·5 ft.

We also find from the drawings that the mean lengths of the horizontal sections of the armoured sides are—

'Minotaur,' 420 ft. 'Bellerophon,' 320 ft.

Let us now suppose that the weight of the armour and backing is uniformly distributed in each case over vertical prismatic sides of the dimensions of the armoured sides, and that the weight of the hull is similarly distributed over vertical prismatic sides of the dimensions below water of the mean horizontal section, and above water of the armoured side, observing that the total depth of the armour and backing on the 'Minotaur' is 22 feet, and on the 'Bellerophon' 20·3 feet, and that the 'Minotaur's' armour descends $5\frac{3}{4}$ feet below water, and the 'Bellerophon's' 6 feet.

Then we shall have

	'Minotaur.'	Tons.	'Bellerophon.'	Tons.
Weight per sq. ft. of armour and backing .	$\frac{2111}{2\times420\times22}$	$=\cdot1142$;	$\frac{1383}{2\times320\times20\cdot3}$	$=\cdot1064$.
Weight of hull	$\frac{5092}{2\times394\times24\frac{1}{2}+2\times420\times16\frac{1}{4}}$	$=\cdot153$;	$\frac{3477}{2\times305\cdot5\times22\frac{7}{8}+2\times320\times14\cdot3}$	$=\cdot1503$.

In these two classes of ships, therefore, we have very similar conditions as respects weight per sq. ft. both of hull and of defensive material carried (armour and backing).

The 'Minotaur' has slightly the heavier weight per sq. ft. for both, but as the 'Bellerophon' has a large reserve of strength compared with 'Minotaur,' we may without error take the mean weights per sq. ft. both of hull and of armour and backing to be the same. We shall thus have in round numbers for both ships—

Weight per sq. ft. of hull . . . . . = ·152 ton.
" " armour and backing = ·11 "

Here, then, we have two types of ship (the 'Minotaur' 400 feet long between perpendiculars and 59 feet $3\frac{1}{2}$ inches broad, and the 'Bellerophon' 300 feet long and 56 feet broad), both steaming on the measured mile at fourteen knots, and both having equal weights per sq. ft. of hull and of defensive armour and backing; and it has been ascertained that on their measured mile trials, which were conducted in precisely the same way, and with both ships fully rigged, they have given the following mean constants of performance calculated from the formula

$$C=\frac{\text{mid. sec.}\times\text{speed}^3}{\text{I.H.P.}}, \text{ viz.}$$

	'Minotaur.'	'Bellerophon.'
Mean constant. . . .	572·9	516·5

The questions I now wish to consider are these: presuming it to be necessary to build another ship, which shall steam at the same speed, carry the same proportionate supply of coal to engine-power and proportionate quantities of stores, but shall have her armour and backing of double the weight of the armour and backing of the 'Bellerophon' and 'Minotaur,' then (1) what will be the size, engine-power, and cost of the new ship if built of the 'Minotaur' type, and having the same mean draught and depth of armour? and (2) what will be her size, engine-power, and cost if built of the 'Bellerophon' type, and having her mean draught and depth of armour?—this condition implying, of course, that the same constants of performance as before will be realized in each case. Looking to the great disproportion of size between the two types of ship, it is obvious that the smaller one will require a much less weight of stores, crew, &c. than the larger vessel; we will therefore take the weight to be carried by the latter, in addition to the weight of engines, boilers, and coals, as 1000 tons, and that to be carried by the former 700 tons. The engines in each case are to weigh 1 ton per nominal H.P., and to work up to seven times that power. The coal is to be equal to the weight of the engines in each case. Taking first the case of the new ship of the 'Minotaur' type, and calling the half breadth of her mean level section b, we shall have (since for similar curves the lengths are proportional to a dimension, and the areas to the square of a dimension)

Length along the side of the mean level section $= \frac{394}{26{\cdot}8}\ b = 14{\cdot}7b.$

Area of the mean level section $= \frac{13333}{26{\cdot}8\vert^2}\ b^2 = 18{\cdot}56b^2.$

Length of armoured side $= \frac{420}{26{\cdot}8}\ b = 15{\cdot}67b.$

Length between perpendiculars $= \frac{400}{26{\cdot}8}\ b = 14{\cdot}9b.$

Area (approximate) of immersed mid. sec. $= \frac{1235}{26{\cdot}8}\ b = 46{\cdot}08b.$

Area of armoured sides $= 2 \times 15{\cdot}67b \times 22 = 689{\cdot}5b.$

Area of surface for hull $= 2\{14{\cdot}7b \times 24\tfrac{11}{12} + 15{\cdot}67b \times 16\tfrac{1}{4}\} = 1241{\cdot}8b.$

Weight of armour and backing $= 689{\cdot}5b \times {\cdot}22$ tons $= 151{\cdot}69b$ tons.

Weight of hull $= 1241{\cdot}8b \times {\cdot}152$ tons $= 188{\cdot}75b$ . . . (1)

Also we have, for the new ship,

$$\text{I.H.P.} = \frac{14^3 \times \text{mid. sec.}}{572{\cdot}9} = \frac{2744 \times 46{\cdot}08b}{572{\cdot}9} = 220{\cdot}7b.$$

Therefore

$$\text{Nominal horse-power} = \tfrac{1}{7}\text{I.H.P.} = 31{\cdot}53b,$$

and

$$\text{Weight of engines and coals} = 2 \text{ nom. H.P.} = 63{\cdot}06b \text{ tons.}$$

Now,

$$\text{Displacement in tons} = \frac{\text{area of mean level sec.} \times \text{draught}}{35} = \frac{18{\cdot}56b^2 \times 24\frac{1}{12}}{35} = 13{\cdot}2b^3.$$

And displacement is also equal to weight of hull + weight of armour and backing + weight of engines and coals + 1000 tons. Whence we have the equation

$$13{\cdot}2b^3 = 188{\cdot}75b + 151{\cdot}69b + 63{\cdot}06b + 1000; \quad \ldots\ldots\ldots \quad (A)$$

$$\therefore\ 13{\cdot}2b^3 - 403{\cdot}5b = 1000,$$

and

$$b = 32{\cdot}87.$$

From this value of b we get the following dimensions and particulars of the new ship of the 'Minotaur' type with thick armour, viz.—

Length between perpendiculars	$=14{\cdot}9b$	$=489{\cdot}7$ feet.
Breadth extreme	$=\frac{489{\cdot}7}{400} \times 59\frac{7}{24} =$	72·5 „
Displacement	$=13{\cdot}2b^3$	=14253 tons.
Indicated horse-power	$=220{\cdot}7b$	=7254 H.P.
Nominal horse-power	$=\frac{220{\cdot}7}{7}b$	$=1036\frac{1}{2}$ „
Weight of hull	$=188{\cdot}75b$	=6194 tons.
Weight of armour and backing	$=151{\cdot}69b$	=4986 „
Weight of engines and coals	$=63{\cdot}06b$	=2073 „

Taking now the new ship of the 'Bellerophon' type, and calling the half breadth of her mean level section b', we shall have, by a similar process to the preceding,

Length along the side of the mean level section	$=\frac{305{\cdot}5}{26}b'$	$=11{\cdot}75b'$.
Area of mean level section	$=\frac{10407}{26^2}b'^2$	$=15{\cdot}4b'^2$.
Length of armoured side	$=\frac{320}{26}b'$	$=12{\cdot}3b'$.
Length between perpendiculars	$=\frac{300}{26}b'$	$=11{\cdot}54b'$.
Area (approximate) of immersed mid. sec.	$=\frac{1127}{26}b'$	$=43{\cdot}3b'$.
Area of armoured sides	$=2\times12{\cdot}3b'\times20{\cdot}3$	$=499{\cdot}38b'$.
Area of surface for hull	$=2(11{\cdot}75b'\times22\frac{7}{8}+12{\cdot}3b'\times14{\cdot}3)$	$=889{\cdot}54b'$.
Weight of armour and backing	$=499{\cdot}38b'\times{\cdot}22$ ton	$=109{\cdot}86b'$.
Weight of hull	$=889{\cdot}54b'\times{\cdot}152$ ton	$=135{\cdot}21b'$.

Also we have

$$\text{I.H.P.} = \frac{14^3 \times \text{mid. sec.}}{516{\cdot}5} = \frac{2744 \times 43{\cdot}3b'}{516{\cdot}5} = 230{\cdot}3b'.$$

Therefore

$$\text{Nominal H.P.} = \tfrac{1}{7}\,\text{I.H.P.} \quad = 32{\cdot}9b',$$

and

$$\text{Weight of engines and coals} = 65{\cdot}8b'.$$

Equating the two values of the displacement, as in the former case, we have

$$10{\cdot}06b'^3 = 135{\cdot}21b' + 109{\cdot}86b' + 65{\cdot}8b' + 700;$$

$$\therefore\ 10{\cdot}06b'^3 - 310{\cdot}87b' = 700$$

and

$$b' = 33.$$

From this value of b', substituted in former expressions, we get the following dimensions and particulars of the new ship of the 'Bellerophon' type with thick armour, viz.—

Length between the perpendiculars	= 380·8 feet.
Breadth extreme	= 71·16 „
Displacement	= 10958 tons.
Indicated horse-power	= 7599 H.P.
Nominal horse-power	= 1085½ „
Weight of hull	= 4462 tons.
Weight of armour and backing	= 3625 „
Weight of engines and coals	= 2171 „

The foregoing investigation requires a correction which can with advantage be made now. It will be observed that the length of the new ship of the 'Bellerophon' type is less than that of the present 'Minotaur,' and her displacement is not much greater than the actual displacement of the present 'Minotaur' when on sea service. The strength of the present 'Bellerophon,' which is greater than that of the present 'Minotaur,' will therefore suffice for the new ship of the 'Bellerophon' type. But the present strength, and therefore weight of hull, would not suffice for a ship of the dimensions and proportions of the new 'Minotaur,' which is to be nearly 500 feet long, of more than 14000 tons displacement, and heavily armour-plated upon long fine lines. Consequently the strength and weight of hull of the new ship of the 'Minotaur' type must obviously undergo a considerable increase, which it would be unsafe to make less than 10 per cent. upon the present weight of hull.

Making this correction, (1) will become

$$\text{Weight of hull} = 1{\cdot}1 \times 188{\cdot}75b$$

$$= 207{\cdot}625b,$$

and the equation (A) being modified accordingly leads to the quadratic

$$13{\cdot}2b^3 - 422{\cdot}37b = 1000,$$

whence

$$b = 34{\cdot}21,$$

and the dimensions and particulars of the new ship of the 'Minotaur' type become

Length between perpendiculars	=509·7 feet.
Breadth extreme	= 75·5 „
Displacement	=15447 tons.
Indicated horse-power	=7550 H.P.
Nominal horse-power	=1078 „
Weight of hull	=7100 tons.
Weight of armour and backing	=5190 „
Weight of engines and coals	=2157 „

Putting the results into round numbers, writing them down for the two ships side by side and adding the tonnage, we shall have

	New ship of 'Minotaur' type.	New ship of 'Bellerophon' type.
Length	510 feet.	380 feet.
Breadth	75 „	71 „
Tonnage	13770 tons.	8620 tons.
Nominal horse-power	1080 H.P.	1080 H.P.
Indicated horse-power	7560 „	7560 „
Weight of hull	7100 tons.	4460 tons.
Weight of armour and backing	5190 „	3630 „
Weight of engines and coals	2160 „	2160 „
Weight of stores carried	1000 „	700 „
Displacement	15450 tons.	10950 tons.

Here, then, we find that with the same thickness of armour to be carried, and the speed of 14 knots to be attained, the differences resulting from the adoption of the 'Bellerophon' type in preference to the 'Minotaur' type would be a decrease in length of 130 feet, in breadth of 4 feet, in tonnage of 5150 tons, in quantity of armour and backing of 1560 tons, in hull of 2640 tons, and in total weight of ship, or displacement, of 4500 tons. The average cost of the hull of armour-plated ships is about £55 per ton of tonnage, so that the saving in the present ship would amount to £283,250, or considerably more than a quarter of a million sterling.

The engines and coal would cost the same in both cases.

When it is borne in mind that the dimensions of the 'Minotaur' and a tonnage of 6643 tons were attained in that type of ship with almost the earliest armour backing and skin plating, while in the 'Hercules' much greater weights per sq. ft. of surface are carried on a tonnage of 5226 tons (1417 tons less), it will be seen that there is nothing contrary to actual practice in the above estimate of a saving of more than a quarter of a million, large as it undoubtedly is for a single ship. And it must in fairness be added that the ship of the 'Bellerophon' type which would be thus econo-

mical in her first cost, would cost much less than the other for use, for maintenance, and for repairs during the whole period of her existence, and would possess the inestimable advantage of greatly superior handiness in action.

I will now pass from the consideration of existing ships, and take a more abstract case.

Let us presume that we have a ship with prismatic sides vertically, as before, but with curves of sines for horizontal sections.

The weight of hull is to be distributed equally over the sides and bottom.

And let us in the first instance take the proportion of length to breadth to be 7 to 1. It will be found by construction that the length along the curve is to the breadth as 7·1 to 1.

Let b = half breadth extreme.

w = weight per sq. ft. of surface for hull.

$2w'$ = weight per sq. ft. of armour and backing.

d' = depth of armoured side.

$\frac{d'}{4}$ = depth of lower edge of armour below water.

d = draught of water.

$3W$ = weight of equipment (exclusive of engines, boilers, and coal)

s = required speed in knots.

Surface for weight of hull . . $=4\times7\cdot1b(d+\frac{3}{4}d')+14b^2$
$=28\cdot4b(d+\frac{3}{4}d')+14b^2.$

Surface for armour and backing $=4\times7\cdot1b\times d'$
$=28\cdot4bd'.$

Let us assume in this case that Professor RANKINE'S rule for the calculation of horse-power and speed holds, viz. that the indicated horse-power equals the "augmented surface" multiplied by the cube of the speed and divided by a certain "coefficient of propulsion;" where the "augmented surface" is the immersed surface multiplied by a coefficient of augmentation, which is equal to $1+4$ (sine of greatest obliquity of water-line)$^3+$ the 4th power of the same sine. The coefficient of propulsion for a clean iron ship of good form is 20000.

In the present case,

Angle of maximum obliquity of water-line curve$=\frac{180^\circ}{2}\times\frac{1}{7}=12\frac{6}{7}^\circ.$

Therefore coefficient of augmentation $=1+4\overline{\sin 12\frac{6}{7}^\circ}^3+\overline{\sin 12\frac{6}{7}^\circ}^4$
$=1+4\times\overline{\cdot2225}^3+\overline{\cdot2225}^4$
$=1+4\times\cdot0495+\cdot0024$
$=1\cdot2.$

And augmented surface . . . . $=14b(2d+b)\times1\cdot2$
$=16\cdot8b(2d+b),$

and I.H.P. . . . . $=\frac{16\cdot8b(2d+b)s^3}{20000},$

and nom. H.P . . . . . $=\frac{2\cdot4b(2d+b)s^3}{20000}.$

And supposing the engines (including boilers, &c.) to weigh 1 ton per nom. H.P., and the coal to weigh as much as the engines and boilers, we have

$$\text{Weight of engines and coal} = \frac{2{\cdot}4b(2d+b)s^3}{10000}.$$

Equating the two expressions for the displacement, we shall have

$$\frac{14b^3d}{35} = \{14b^2 + 28{\cdot}4b(d+\tfrac{3}{4}d')\}w + 28{\cdot}4\,.\,b\,.\,d'\,.\,w' \times 2 + \frac{2{\cdot}4b(2d+b)s^3}{10000} + 3W. \quad . \quad (I)$$

In the new ship let $s=14$ knots, $d=25$ ft., $d'=24$ ft., $W=450$ tons; and while w remains the same as in 'Bellerophon,' say, one-tenth of a ton, let w' be doubled, say, $\frac{6}{28}$ ton; the equation (I) then becomes

$$10b^3 = (14b^2 + 28{\cdot}4b \times 43) \times {\cdot}1 + 28{\cdot}4b \times 24 \times \frac{6}{28} + \frac{2{\cdot}4b(50+b)2744}{10000} + 1350,$$

or $$7{\cdot}94b^3 = 301b = 1350,$$

whence $$b = 41{\cdot}95;$$

consequently

Length	$=14b=$	587·3 feet.
Breadth	$=2b=$	83·9 „
I.H.P.	$=$	8890 H.P.
Weight of hull	$=$	7586·7 tons.
Weight of armour and backing .	$=$	6124·7 „
Weight of engines and coals . .	$=$	2541·3 „
Weight of equipment	$=$	1350 „
Displacement	$=$	17602·7 tons.

The area of the immersed surface of this ship . . . . . $=14b^2+28{\cdot}4b\times d.$
$=54422$ sq. ft.

and area of immersed mid. sec. . . . . $=83{\cdot}9\times 25$
$=2097{\cdot}5$ sq. ft.

Next, let us take the case of a curve-of-sines ship having a proportion of length to breadth of 5 to 1. It will be found by construction that the length along the curve will be to breadth as 5·12 to 1. All the previous notation and values will be retained, except that b' will be substituted for b, and 2W for 3W, the latter change conditioning that this vessel shall carry two-thirds the dead weight of the former vessel.

Surface for weight of hull . . . . $=20{\cdot}48b'(d+\frac{3}{4}d')+10b'^2.$
Surface for armour and backing . . $=20{\cdot}48b'd'.$

Taking Professor RANKINE'S rule as before,

Angle of maximum obliquity of water-line curve . . $=\frac{18^\circ}{2}\times\frac{1}{3}=18^\circ.$
Therefore coefficient of augmentation . . . . . $=1+4\times{\cdot}0669+{\cdot}0049$
$=1{\cdot}273,$
and augmented surface . . . . . $=10b'(2d+b')\times 1{\cdot}273,$

and I.H.P. . . . . $= \frac{12{\cdot}73b'(2d+b')s^3}{20000}$,

and nominal H.P. . . . . $= \frac{1{\cdot}82b'(2d+b')s^3}{20000}$.

Weight of engines and coal . . . . . $= \frac{1{\cdot}82b'(2d+b')s^3}{10000}$.

Hence we have, equating the two expressions for the displacement, as before,

$$\frac{10b^2d}{35} = \{10b'^2 + 20{\cdot}48b'(d+\tfrac{3}{4}d')\}w + 20{\cdot}48 \cdot b' \cdot d'w' \times 2 + \frac{1{\cdot}82b'(2d+b')s^3}{10000} + 900 \text{ tons.}$$

Substituting the numerical values, we have . . $5{\cdot}64b'^2 - 218{\cdot}35b' = 900$,

whence . . . . . $b' = 42{\cdot}45$,

consequently

Length	$= 10b' =$	424·5 ft.
Breadth	$= 2b' =$	84·9 „
I.H.P.	=	6859·6 H.P.
Weight of hull	=	5540·3 tons.
Weight of armour and backing	=	4470·8 „
Weight of engines and coals .	=	1959 „
Weight of equipment . . .	=	900 „
Displacement	=	12870·1 tons.

The area of the immersed surface of this ship . . . . $= 10b'^2 + 20{\cdot}48b'd$
$= 39753$ sq. ft.

and area of immersed midship section . . . . . $= 84{\cdot}9 \times 25$
$= 2122{\cdot}5$ sq. ft.

Collecting the results for the two ships, and writing them down in round numbers side by side, we have

	Ship seven times her breadth.	Ship five times her breadth.
Length	585 ft.	425 ft.
Breadth	84 ft.	85 ft.
Indicated horse-power	8890 H.P.	6860 H.P.
Nominal horse-power	1270 „	980 „
Weight of hull	7586 tons.	5540 tons.
Weight of armour and backing .	6124 „	4470 „
Weight of engines and coals . .	2540 „	1960 „
Weight of equipment	1350 „	900 „
Displacement	17600 tons.	12870 tons.

These results are very different in detail from those obtained in the cases based on the actual ships 'Bellerophon' and 'Minotaur,' and on their trials, the shorter ship requiring in this case 2000 indicated horse-power less than the longer ship to drive her at the

given speed of fourteen knots, while the breadth of the short ship slightly exceeds that of the long ship. But these differences in results are not greater than might have been anticipated from the adoption of such different forms of ships, and from the application to them of so different a rule for connecting speed with power.

At the same time the investigation serves to show conclusively, that whether we adopt the theoretical best form of ship, and apply the most recent rule in our calculations, or whether we guide ourselves by the practical trials of existing iron-clad ships, we in either case find that the speed of fourteen knots can be obtained in the short type of ship with surprisingly less size and cost than the long type of ship requires.

It is easy to see how the difference between the horse-powers of the two curve-of-sines ships arises. The immersed surface of the longer vessel exceeds that of the shorter by 14669 sq. ft. At the speed of fourteen knots the frictional resistance per sq. ft. of surface $=1{\cdot}96$ lb., and consequently the excess of this resistance upon the larger ship is $14669 \times 1{\cdot}96 = 28810$ lbs. This resistence has to be overcome through a distance of fourteen knots per hour; and taking the effective work of one I.H.P. to be equal to 200-knot lbs. per hour (which is the quantity used by Professor RANKINE in his calculation), we obtain

I.H.P. expended in overcoming the excess of frictional resistance upon the larger ship

$$= \frac{28810 \times 14}{200}$$

$$= 2016{\cdot}7 \text{ H.P.},$$

which corresponds very nearly to the difference between the engine-power of the two ships. As the immersed midship sections differ so little in the two cases, the difference of horse-power due to this course would obviously be very slight.

It must be observed that the weight per sq. ft. of hull has been taken the same for both curve-of-sines ships, whereas an increase would be necessary for the larger ship.

I omit this correction as it would lead to a cubic equation; but it would obviously enhance the size and cost of the larger ship, and thus exhibit the advantage of the shorter ship in a still clearer light.

XIX. *On the Amount and Changes of the Polar Magnetism at certain positions in Her Majesty's Iron-built and Armour-plated Ship 'Northumberland.'* By FREDERICK JOHN EVANS, *Staff Captain, R.N., F.R.S., Chief Naval Assistant, Hydrographic Department, Admiralty.*

Received March 5,—Read March 26, 1868.

IN the year 1860 an official report made by me on the Deviations of the Compass observed in all the Iron-built Ships, and in a selection of Wood-built Ships in Her Majesty's Navy, and in the Iron Steam-ship 'Great Eastern,' was communicated by Captain WASHINGTON, R.N., F.R.S., the Hydrographer of the Admiralty, to the Royal Society, and was published in the Philosophical Transactions for 1860, p. 337.

In the year 1865, with the sanction of the Lords Commissioners of the Admiralty, and in conjunction with ARCHIBALD SMITH, Esq., F.R S., I presented to the Society a paper on the Magnetic Character of the Armour-plated Ships of the Royal Navy, which was published in the Philosophical Transactions for 1865, p. 263.

These papers contained a reduction and discussion of observations of the deviations of the compass, and of the horizontal and vertical magnetic force made on board a large number of iron-built ships at different positions in the ship, at different times, and in different geographical positions; and comprised almost all the results of any value or importance regarding the deviations of the compass which up to the time of that publication had been obtained in the classes of ships to which they related.

The system pursued in the Compass Department of the Royal Navy of making careful observations whenever the occasion offers, of the deviations and horizontal and vertical force in the ships of the Royal Navy, and of reducing such observations so as to obtain the magnetic coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, λ, μ, will, I hope, enable me hereafter to lay before the Society a continuation of the former papers, in which I trust one of the deficiencies, viz. the want of a variety of observations made in the same ship in different geographical positions, will be removed by the zeal of the Navigating Officers now serving in several of Her Majesty's ships on foreign stations.

In the meantime I take the opportunity of an unusually detailed set of observations having been made in the latest and largest of the armour-plated ships, the 'Northumberland*,' to lay before the Society, with the sanction of the Lords Commissioners of the

* The observations discussed in this paper were made, and the coefficients computed, by Staff Commander WILLIAM MAYES, my successor as Superintendent of Compasses of Her Majesty's Navy; and I am happy to have an opportunity of bearing testimony to the care and zeal with which he has discharged the duties of his office. I

Admiralty, some results as to the amount and changes of the magnetic forces at several positions in that ship.

These results may be thought to have some special interest from the circumstance that the 'Northumberland' was made the subject of an attempt to "depolarize" her, which created some interest and expectation not only in the general public, but even in the Naval profession*.

The 'Northumberland' is an iron-built ship of 6621 tons, 400 feet long, 59¼ broad, 26¼ feet draught of water, 1350 horse-power, screw engines, armour-plated completely round, with plates of an average of 6 inches in thickness. She has further three complete iron decks supported by iron beams and iron uprights, and a poop-deck of wood, but supported by iron beams and iron uprights. The five lower masts are also of iron. The weight of iron employed in the construction of the hull was about 4250 tons, and in the armour-plating about 1550 tons.

The ship was built at Millwall on the river Thames; the direction of her head in building being N. 39½° E. magnetic. Contrary to the usual practice with ships built on a slip, the armour-plating was completed previous to launching; the latter operation it may be recollected was performed with great difficulty and occupied several days. The launch was completed on the 17th April, 1866, when the ship was anchored in the river, and allowed to swing with the tide. On the 18th April she was towed to the Victoria Docks, where, on the recommendation of the Compass department, she was placed in a direction as nearly opposite to that of building as could be conveniently arranged (viz. S. 22° W. magnetic).

Unfortunately two large iron-plated ships (one iron-built) lay close alongside, and no doubt affected considerably the magnetic phenomena observed in the 'Northumberland.' This circumstance prevents the observations made during her stay in the Victoria Docks being strictly comparable with those made before and afterwards; and this must be borne in mind in looking at the Tables. It has, as far as possible, been allowed for in the assumed value of λ. Some irregularities were also caused by the introduction and movement of the large masses of iron constituting the steam-boilers and engines.

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have also to acknowledge my obligations to Mr. Archibald Smith, F.R.S., for the assistance which he has given me in the discussion of the observations, especially in their mathematical and graphical treatment.

* I would refer especially to two papers read before the Royal United Service Institution, London, and the discussions thereon, as published in the Journal of the Institution,—the first paper on "Terrestrial Magnetism with reference to the Compasses of Iron Ships; their Deviation and Remedies," read January 29th, 1866; the second paper on "The Demagnetization of Iron Ships, and of the Iron beams, &c. of Wooden vessels, to prevent the deviation of the Compasses, experimentally shown by means of a model," read May 6th, 1867, both papers by Evan Hopkins, C.E., F.G.S. The latter paper was also read at the Salle des Conferences, Champ de Mars, Paris (in connexion with the International Exhibition), on the 22nd June 1867, by Captain F. A. B. Craufurd, R.N.

At the end of December 1866, the ship was completed in her equipment for temporary service and steamed to Sheerness, where she remained swinging to the wind and tide till the early part of March 1867; when she steamed to Devonport, at which place, with the exception of two days' trial at sea to test the machinery in the middle of May, she has remained in a dry dock with her head directed S. 84° E. magnetic till the present time.

The positions of the compasses at which observations were made were the following:—

Standard Compass.—The 'Northumberland' having been built with her head nearly N.E., the magnetism principally developed was in the upper part of the stern and starboard quarter, and it was therefore desirable that this compass should be as far forward as possible. It was accordingly placed 172 feet from the stern, and $8\frac{3}{4}$ feet above the iron deck.

Steering Compasses.—The upper deck steering-wheel is 52 feet from the stern, under the fore part of the poop wooden deck; two compasses were placed close in front of it 6 feet apart, each 4 feet above the iron deck, and 3 feet 8 inches below the iron beams supporting the poop-deck.

Poop Compass.—This compass was placed on the fore extreme of the poop-deck, and 9 inches before the line joining the steering compasses, and 4 feet above the poop-deck. In the selection of a place for these latter compasses there was no room for choice; the arrangements of the architect and the requirements of the seaman could be alone consulted.

The results of the observations will be found in the Table appended to this paper, and will include a few made at temporary positions not necessary to describe in detail.

For a complete explanation of the meaning of the quantities tabulated, and the method of obtaining them by observation, I must refer to the last of the two papers mentioned above. Here it may suffice to say, that if

ζ represents the magnetic azimuth of the ship's head,

ζ' the azimuth by disturbed compass, $\delta = \zeta - \zeta'$ the deviation, then

$$\sin\delta = \mathfrak{A}\cos\delta + \mathfrak{B}\sin\zeta' + \mathfrak{C}\cos\zeta' + \mathfrak{D}\sin(2\zeta' + \delta) + \mathfrak{E}\cos(2\zeta' + \delta) \text{ exactly,}$$

or

$$\delta = A + B\sin\zeta' + C\cos\zeta' + D\sin 2\zeta' + E\cos 2\zeta' \text{ approximately.}$$

Of these coefficients $\mathfrak{A}$, $\mathfrak{D}$, $\mathfrak{E}$ (or A, D, E) depend solely on the transient magnetism induced in the soft iron, and therefore cannot be affected by any artificial magnetization, or demagnetization.

$\mathfrak{B}$ (or B) depends partly on the magnetism induced in soft iron by the earth's vertical force, partly on the permanent or subpermanent magnetism of the hard iron. $\mathfrak{C}$ (or C) depends on the last. It is therefore to the changes in $\mathfrak{B}$ and $\mathfrak{C}$ only that we are to look for the effects of polarization or depolarization.

λ is a factor almost always less than unity, representing the mean force, to north, as affected by the soft iron in the ship.

$\lambda\mathfrak{B}$ is the mean force, or, in other words, the polar force of the ship, to head.

$\lambda\mathfrak{C}$ is the mean or polar force to starboard, $\lambda\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ the mean or polar horizontal force of the ship; each in terms of the earth's horizontal force as unit.

$\frac{\mathfrak{C}}{\mathfrak{B}}$ is the tangent of the angle which the direction of the ship's polar horizontal force makes with the line drawn to the ship's head, or the "starboard angle."

μ is the mean or polar force downwards of earth and ship, in terms of the earth's vertical force as unit; and depends partly on the subpermanent force of the hard iron, partly on vertical induction.

$\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta\times1^\circ$ is the heeling coefficient to windward, and represents the deviation to windward caused by an inclination of the ship of 1°, when her head is North or South by compass.

The values of these coefficients, and also of a and e, the coefficients of horizontal induction, headward and to starboard, for the several compasses are given in the General Table appended to this paper.

The character of the deviations of the standard, steering and poop compasses and of their changes, may be thus generally described.

In each, the $\mathfrak{B}$ has originally a large negative value, caused by the ship having been built nearly head North, (N. 39°½ E.). This gradually diminishes as she lies in the Victoria Docks with her head to the South, but, as is usually found, shows a tendency to return to its original value when the ship is allowed to swing. $\mathfrak{C}$, which has originally a large positive value caused by the starboard side having been to the South, decreases, and even changes its sign in Victoria Docks, but returns to its original sign and nearly to its original value when the ship swings at Sheerness.

In the poop and steering compasses down to 1st January 1867, and in the Standard compass throughout,—except for a short period while a magnet was applied to reduce the deviation—there are no changes except what may be considered to be due to the ship's position, and to the other circumstances adverted to; but as regards the poop and steering compasses between 1st and 26th January, the case is different; the causes of the difference, and the inferences to be drawn from it, it is proposed now to consider.

Early in 1866, Mr. Evan Hopkins*, C.E. applied for a patent for "An improved method of correcting the Deviation of Compasses in Iron Ships." In the provisional specification, dated 23rd January 1866, the method is described as "destroying the

* Mr. Evan Hopkins was the author of a work entitled "On the Connexion of Geology with Terrestrial Magnetism," 1844, 2nd edition, in which many singular opinions are propounded on Astronomy, Magnetism, and general Physics. I regret to have to speak of Mr. Hopkins in the past tense. He died in the middle of 1867.

polarity acquired while the ship is building, by passing electro-currents through the hull." In the final specification, dated 23rd July 1866, it is described as "moving an electro-magnet from end to end over, and in contact with the main plates of the ship." There is some obscurity in both specifications, from the patentee not distinguishing between electric currents and lines of magnetic force; but whatever may be the meaning of the method described in the provisional specification, it is certain that that method would be utterly inadequate to produce any sensible effect in a large ship.

The method described in the final specification would no doubt produce, wherever applied, some local effect; but the effect produced by the local application of magnetic force of high and rapidly varying intensity must necessarily be wholly different from that which has arisen from the general application of a force of low and uniform intensity, and the former cannot possibly produce any general destruction of the latter. The process is in fact not one of general demagnetization, but of partial counter magnetization. The result will be an irregular distribution of magnetism of very variable intensity, necessarily very unstable, and producing, wherever effective, a rapidly varying field of force. The justice of these remarks will, I think, be shown in the sequel.

In April 1866 Mr. Hopkins applied to the Admiralty for permission to experiment on the 'Northumberland,' the largest and most heavily armoured ship in the Royal Navy. The application was in the usual course referred to the Magnetic department, and on a report that no injury was to be apprehended, the required permission was granted*.

The first trials were made on the 4th August 1866, and are thus described by Mr. Hopkins in a report dated 10th August, and received at the Admiralty a few days afterwards. "After having ascertained the actual magnetic condition of the ship, I applied two of Grove's batteries of five cells each, with the electro-magnets to the main plates at the stern and bow, and in a few hours the polarity of the hull was destroyed"†.

The results of the observations shown in the General Table, I think entitle me to say that there is no foundation whatever for the statement that the polarity of the hull was destroyed; there is in fact no evidence of its having been affected in even the slightest degree. To facilitate the examination I subjoin the values of the semicircular deviation of the two compasses which were most continuously examined in the period comprising the 4th August, viz. the poop and starboard steering compasses; the position of the Standard compass being at that time occupied by machinery.

* That report, for which I am responsible, was made on the supposition that the process was one to be applied to the hull of the ship generally, according to the specification of the patent. Had I understood that the iron in the immediate vicinity of any compass was to be magnetized to a high degree of intensity, I should certainly have reported differently; and it will be seen in the sequel that from the results of Mr. Hopkins's experiments, I was obliged to submit that no such experiments for the future be permitted within 20 feet of any compass placed for the navigation of the ship.

† This statement was subsequently repeated at a Meeting of the British Association for the Advancement of Science held at Nottingham in 1866, in a paper "On the Depolarization of Iron Ships, to prevent the Deviation of the Compass," by Mr. Evan Hopkins, C.E. See Athenæum of September 8th, 1866.

	Maximum of Semicircular Deviation.	
	Poop compass.	Starboard Steering compass.
1866.	°	°
May 29th . . .	48¾	39½
July 12th . . .	64½	50
August 10th . . .	55¾	46½

In each case it will be seen there is a slight decrease in the interval comprising the 4th August, but the decrease does not exceed half the increase in the preceeding interval; and seems only the recovery from an anomalous increase in July, probably attributable to some external disturbing cause of the nature before mentioned, viz. the proximity of the two armour-plated ships in the Victoria Docks.

The next and final trial was made between the 1st and 26th January 1867, at Sheerness: ten or twelve days were occupied in passing large electro-magnets along the outside of the ship from the water-line to the top sides. Subsequently, and apparently as an after thought,—as no mention is made in either specification of internal demagnetization on so large a scale,—about five days more were occupied in applying the electro-magnets to the transverse iron beams of the poop and upper deck nearest the poop and steering compasses, and to two adjacent vertical iron stanchions supporting the upper deck. As no appreciable effect was produced on the Standard compass, or on any compasses except those in the immediate vicinity of the beams and stanchions operated on, we may have confidence in attributing the change which took place entirely to the latter operations.

The following Tables, derived from the General Table, furnish the means of deciding this question.

TABLE I.

At position of the undermentioned compasses.	(I.) B Part of semicircular deviation from Headward force.	(II.) C Part of semicircular deviation from Transverse force.	(III.) $\sin^{-1}\sqrt{B^2+C^2}$ Maximum of semicircular deviation.	(IV.) $\tan^{-1}\frac{C}{B}$ Starboard angle.	(V.) μ Mean vertical force of earth and ship, in terms of earth's horizontal force as unit.	(VI.) Heeling error to windward for 1° of heel.	(VII.)
	°	°	°	°		° ′	
1st January 1867. Standard	−37	+ 4¼	40¼	173¾	1·223	+1 18	
Starboard steering	−39½	+15½	46½	161½	1·056	+1 4	
Poop	−48¾	+15½	56½	164½	1·469	+2 1	
Port steering	−43	+19¾	51	157½			

TABLE II.

26th January 1867. Standard	−36½	+ 7	40¼	170½	1·271	+1 27	
Starboard steering	− 2½	+38½	38¾	94½	1·250	+1 45	
Poop	− 3½	+12½	12½	104½	·986	+0 37	
Port steering	− 4½	−11	11	246			

TABLE III.—Amount and direction of Magnetic Forces on 1st January 1867.

	$\lambda \mathfrak{B}$ Ship's force to head.	$\lambda \mathfrak{C}$ Ship's force to starboard.	$(\mu-1)\tan\theta$ Ship's force downwards.	$\lambda\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ Horizontal force of ship.	$\sqrt{\text{Col. III}^2+\text{IV}^2}$. Total force of ship.	$\tan^{-1}\frac{\mathfrak{C}}{\mathfrak{B}}$ Starboard angle of ship's force.	$\tan^{-1}\frac{\text{Col. III.}}{\text{Col. IV.}}$ Dip of ship's force.
						°	° ′
Standard	−·562	+·062	+ ·553	·566	·791	173¾	+44 24
Starboard steering	−·546	+·186	+ ·139	·578	·595	161⅛	+13 32
Poop	−·691	+·196	+1·163	·720	1·368	164½	+52 46
Port steering	−·580	+·242	.....	·628	.....	157¼	

TABLE IV.—Amount and direction of Magnetic Forces on 26th January 1867.

Standard	−·554	+·094	+·672	·562	·876	170½	+50 5
Starboard steering	−·037	+·477	+·620	·478	·783	94½	+52 25
Poop	−·051	+·175	+·035	·181	·184	104¾	−10 57
Port steering	−·062	−·141	.....	·153	.....	246	

TABLE V.—Amount and direction of additional forces introduced between January 1st and 26th, 1867.

Standard	+·008	+·032	+ ·119				
Starboard steering	+·509	+·291	+ ·481	·586	·758	+30	+38 25
Poop	+·648	−·021	−1·198	·648	1·362	− 2	−61 35
Port steering	+·518	−·383	.....	·644	.....	−36¼	

The values of B and C in Table I. show the remarkable amount of accordance in the deviations of the *four* compasses; a similarity which clearly indicates that the cause of the deviations is to be sought for, not in the iron in the immediate vicinity of those compasses, but in iron at such a distance that the distance between the compasses does not materially affect its action on them.

The values of B and C in Table II. show an important change in *three* of the compasses. The value of B for the starboard steering, poop, and port steering are nearly reduced to zero, showing the introduction of a powerful force attracting to the bow of the ship, or repelling from the stern. With C the case is very different: in the port compass a deviation to port is produced, in the starboard compassa deviation to starboard; indicating the introduction of a repelling force between the two. The same conclusion may be drawn, and perhaps with greater facility, from a comparison of the value of the quantities in columns III. and IV.

The quantities in columns V. and VI. show that the change produced is an upward force on the poop compass, and downward on the starboard steering compass; pointing to a repelling force emanating from a point or region at a height intermediate between the height of the two compasses. These several comparisons show that the change was really caused by a repelling force (a north pole) being introduced in the iron of the poop-deck a little abaft the poop compass.

The precise amount and direction of the force so introduced, and the changes it caused in the previously existing forces, will be seen distinctly by the mathematician from Tables III., IV., V.

The action of the several forces may perhaps be more clearly apprehended when they are represented graphically as in Plate XXXI. In this, fig. 1 represents the projection of lines indicating the amount and direction of the magnetic forces which act on the three compasses on the horizontal plane. Fig. 2 represents the projection of the same lines on the fore and aft vertical plane. The figures are drawn to a scale in which one-fourth of an inch represents one foot, and also one-tenth of the earth's horizontal force. The lightly dotted parts represent iron.

P, Q, R represent the positions of the port, the poop, and the starboard steering compasses respectively; Pp, Qq, Rr represent the projection of lines representing the magnetic forces of the ship at these positions on the 1st January 1867; Pp', Qq', Rr' the same projection on the 26th January 1867; Pp'', Qq'', Rr'' the projection of the additional forces introduced in the interval.

In fig. 1 the near approach to parallelism and equality in Pp, Qq, Rr indicate a distinct cause of magnetic force. The lines Pp'', Qq'', Rr'' produced backwards nearly meet in a point about 5 feet abaft the poop compass; indicating that the additional force is introduced at or near that point. A similar convergence of the line Qq'', Rr'' in fig. 2 indicates that the point lies in or near the poop deck, and suggests that it arose from the magnetization of the central part of the iron beams of the poop deck, modified possibly by some magnetization of the beams of, or stanchions supporting the upper deck.

On the 28th February, and preparatory to the ship being navigated to Devonport, the deviations and magnetic forces were observed. The deviation of the Standard compass being too great for the safe navigation of the ship, and the deviation of the starboard steering compass being so great as to make it practically useless, it was necessary to reduce their semicircular deviations by the application of fixed magnets. The process employed, and which is that generally employed in the Royal Navy, is identical with one of the two methods described by the Astronomer Royal in his well-known paper on the magnetism of iron-built ships (Philosophical Transactions, 1839, see page 196), and may be described as follows.

The coefficients $\mathfrak{B}$, $\mathfrak{C}$, λ being found by observation, or where necessary $\mathfrak{B}$ and $\mathfrak{C}$ being found by observation, and λ being estimated; we have $\lambda\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ the tangent of the semicircular deviation when the polar force acts to the east or west of the compass, and $\frac{\mathfrak{C}}{\mathfrak{B}}$ the tangent of the "starboard angle." If we desire to correct the semicircular deviation completely, a magnet of suitable size, adequate power, and proved permanence is selected from those in store at the Compass Observatory at Woolwich. The distance above or below the card at which this magnet, when placed East and West, will produce a deviation of which the tangent is $\lambda\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$, is ascertained by actual trial. The magnet is then inserted into the pedestal of the Standard compass at the ascertained distance immediately below the centre of the card and in the direction of the starboard angle, the poles being so placed as to counteract the polar magnetism of the ship.

As in newly built ships the polar force is generally undergoing a process of gradual diminution, it is generally considered best not to correct entirely the semicircular devia-

tion, but to under correct it, leaving about 5° uncorrected. This was the object sought in the application of magnets to the 'Northumberland'; and the success with which it was effected, and the certainty of the process, may be seen by the following comparison of the deviations on the 28th February and the 2nd March.

		A	B	C	D	E
		° ′	° ′	° ′	° ′	° ′
Standard compass	February 28, 1867	+0 01	−36 13	+ 7 06	+7 19	+0 59
	March 2	−0 14	− 4 55	+ 0 14	+7 13	+1 15
Starboard steering-compass	February 28	+0 41	− 5 20	+40 41	+6 28	+2 40
	March 2	−0 18	− 4 15	− 1 04	+6 42	+1 44

In general no attempt is made to correct the quadrantal deviation, that deviation and the residual semicircular deviation being the subject of tabular correction. I may observe that in some cases Mr. AIRY's second or tentative mode of correcting, viz. by one or more fore and aft and transverse magnets, is adopted, but only when special circumstances prevent the first method being applicable. In general by the process first described, all that is necessary or desirable in the way of mechanical correction can be effected.

In the middle of March 1867, the 'Northumberland,' as before stated, was placed in a dry dock at Devonport, and the magnets were removed; her head in dock was S. 84° E. magnetic. Such a position would, from all former experience, and especially in a newly launched ship, be expected to increase considerably the force to starboard, or the value of +𝔊, but without much alteration in the fore and aft force, or the value of 𝔅, in those compasses in which the 𝔅 and 𝔊 were caused by the general magnetism of the hull. We have continuous observations to the end of 1867 strictly comparable as regards the Standard compass; but as regards the other three compasses allowance must be made for the introduction between August and December 1867 of five iron beams* to extend the light poop deck before the poop compass.

The changes in the Standard compass while the ship lay in dock with her head S. 84° E. are shown in the following Table.

	[1867].	𝔅.	𝔊.
Swinging at anchors	January 26th	−·637	+·108
	February 28th	−·637	+·110
Head S. 84° E. magnetic from middle of March	June 26th	−·576	+·277
	August 29th	−·569	+·256
	December 10th	−·593	+·329

* The effect of the introduction of the five new poop-deck beams when the ship's head was S. 84° E. magnetic would be to increase 𝔊 and 𝔇 and diminish λ. The exact value of the altered 𝔊 cannot be computed without knowing the altered values of 𝔇 and λ; and these cannot be ascertained till the ship is again swung. I think, however, it is certain that the change in 𝔇 cannot exceed ·017 (or 1° in 𝔇), and the change in λ cannot exceed ·020, and I have accordingly inserted in the General Table, under date 10th December 1867, the values of 𝔅 and 𝔊 computed as well with the original values of 𝔇 and λ, as with values allowing these amounts. It may be safely assumed that the true values lie between the two as limits.

These results are clearly attributable to the change in the general magnetism of the ship due to position; the fore and aft force has been little affected, but there is a decided increase in the transverse force, $= +\cdot219$.

With respect to the compasses affected by Mr. HOPKINS's process, the results are widely different:—in seven months (end of January to end of August), as will be seen by the following Table, at the starboard steering $-\mathfrak{B}$ has increased ·183, $\mathfrak{C}$ having scarcely changed in value; at the poop compass $-\mathfrak{B}$ has increased ·281, and $+\mathfrak{C}$ ·162; at the port steering compass $-\mathfrak{B}$ has increased ·098, while the $\mathfrak{C}$ has increased ·320; so that the increase of the $\mathfrak{C}$ in the poop compass is almost exactly the mean of the increase in the port and starboard compasses.

1867.	Port steering compass.		Poop compass.		Starboard steering compass.	
	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{B}$	$\mathfrak{C}$
January 26	−·078	−·176	−·059	+·203	−·046	+·596
February 28	−·071	−·128	−·070	+·230	−·078	+·625
June 26			−·153	+·360		
August 29	−·176	+·150	−·340	+·365	−·229	+·637
December 10	−·212 −·218	+·508 +·481	−·215 −·221	+·541 +·516	−·136 −·141	+·641 +·617

If we confine ourselves to the practical question of the amount of the semicircular deviation, and also of the direction of the ship's force [starboard angle], or, in other words, the direction of the neutral or zero-points of the semicircular deviation, the comparison is as follows:—

1867.	Standard.		Port steering.		Poop.		Starboard steering.	
	Maximum semi-circular deviation.	Starboard angle.	Maximum semi-circular deviation.	Starboard angle.	Maximum semi-circular deviation.	Starboard angle.	Maximum semi-circular deviation.	Starboard angle.
	°	°	°	°	°	°	°	°
January 1	37½	173½	47¼	157	51	164½	42½	161½
„ 26	37	170½	12	246	12½	104½	38½	94½
February 28	37	170	8½	241	14½	107	41¼	97
June 26	39½	155	...	...	23	112		
August 29	38½	156	13½	139½	30	133	42½	110
December 10	42½	151	33½	112½	35½	111½	40	102

From this comparison it will be seen how short lived is the apparent benefit derived from the process we have been considering.

The nature of the changes in the polar force will perhaps be more clearly apprehended if we represent them graphically, as in Plate XXXII., by taking the position of each compass as origin and laying down the points, of which $\lambda\mathfrak{B}$ and $\lambda\mathfrak{C}$ are the coordinates, as derived from the observations on January 1st and 26th, February 28th, August 29th, and December 10th, 1867.

As regards each compass, a line drawn from the origin to one of these points represents in amount and direction the polar force acting on that compass at that epoch; and a line drawn from one point to the next succeeding in order of date represents, in amount and direction, the additional polar forces acting on that compass introduced in

the interval. To avoid confusion in the figure, the lines from the origin are only drawn for the earliest date. The *joints* added to these lines, therefore, represent the additional forces introduced in each interval.

This Plate shows strikingly the contrast between the comparatively small and regular changes in the polar magnetism of the Standard compass, and the large and irregular changes in the polar magnetism of the other compasses; but at the same time shows strikingly the general analogy in the changes in these three compasses.

An attentive study of it will also show, that while the changes in the polar magnetism of the Standard compass are almost entirely from an increasing attraction to the south side of the ship, in the other compasses this is combined with an increasing attraction to, or rather a diminishing repulsion from, the point abaft the poop compass to which a powerful north pole had been communicated.

In the interval between the 28th August and the 10th December, a general headward force on the three stern compasses is also added. What was the cause of this force I am unable say; it may possibly be connected with the introduction of the five iron beams in front of these compasses, and the general hammering to which that part of the ship was consequently exposed.

The views which I have taken of the effect of the application of the electro-magnets are, I think, strongly confirmed by observations made on the 10th December 1867, at four points surrounding the position of the supposed north pole produced by them.

Of these, the position of the poop compass was one; the second was 18 feet abaft the poop compass; the third, 7 feet 3 inches abaft the poop compass, and 4 feet to port of the midship line; the fourth, 7 feet 3 inches abaft the poop compass, and 4 feet to starboard of the midship line. The values of $\mathfrak{B}$ and $\mathfrak{C}$ for these compasses were—

	$\mathfrak{B}$.	$\mathfrak{C}$.
No. 1. Poop compass	−·215	+·541
2. 18 feet abaft Poop compass	−·641	+·482
3. $7\frac{1}{4}$ feet abaft Poop compass, 4 feet to port	−·457	+·260
4. $7\frac{1}{4}$ feet abaft Poop compass, 4 feet to starboard	−·458	+·692

An inspection of these values will show that they indicate a generally attractive force to the stern and starboard sides; but modified by a repulsive force emanating from a point nearly centrically situated with respect to the four compasses. We may, in fact, I think, infer that the mean of these values, or a $\mathfrak{B}$ of −·443, and a $\mathfrak{C}$ of +·494, is due to the general magnetism of the ship, and as regards $\mathfrak{C}$ partly to the new iron beams, and that the difference from these values as to each compass arises from local magnetization. These differences—which are also multiplied by λ(·860) to express them in terms of the earth's magnetic force as unit—will be

	$\mathfrak{B}$.	$\mathfrak{C}$.	$\lambda\mathfrak{B}$.	$\lambda\mathfrak{C}$.
No. 1. Station (Poop)	+·228	+·047	+·196	+·040
2. „	−·198	−·012	−·170	−·010
3. „	−·014	−·234	−·012	−·201
4. „	−·015	+·198	−·013	+·170

The interpretation of which Table is that, besides the general magnetism of the ship, there is a horizontal force nearly equal to one-fifth of the earth's horizontal force, repelling the north end of each needle from a point situated nearly in the centre of the compasses. This I conceive to be the remains of the strong north pole to which I have referred, which it will be remembered affected the poop compass with a repelling force =·648, and now reduced to ·200; so that more than two-thirds of the force introduced in the operations of January 1867 seem to have disappeared in the course of eleven months; and the general result of these operations may be described as the introduction at a point in the poop-deck, a few feet abaft the poop compass, of a north pole acting on the compass with a force of nearly two-thirds of the earth's horizontal force, and which force in the course of eleven months diminished to about one-fifth of the earth's force, or to less than one-third of its original amount.

The effect of the forces introduced by the operations of January 1867, and of the gradual decay of these forces in the interval between January and December, is no less obvious in the heeling-errors of the different compasses. This error, it may be remembered, is expressed by the deviation to windward produced by an inclination of the ship of 1°, when its head is North or South by the disturbed compass.

In the interval between the 1st and 26th January 1867, there is a diminution of the heeling-error of the poop compass from 2° 1′ to 0° 37′, but an increase of the heeling-error of the starboard compass from 1° 4′ to 1° 45′. The diminution is caused by the upward force on the poop compass; the increase by the downward force on the starboard steering compass introduced in the interval.

The changes so introduced in both compasses diminish as time passes. By the 10th December that of the poop compass had risen to 1° 14′, that of the starboard steering diminished to 0° 56′. During the whole of this period the heeling-error of the Standard compass is hardly altered; the slight apparent changes being not greater than can be accounted for by unavoidable errors of observation.

We are now in a position to form an opinion as to the real nature of the changes effected by the operations of January 1867, and the advantages and disadvantages of these changes.

The process was in no sense of the word one of "depolarization," either of the whole ship or of any part of it. It was, on the contrary, the "polarization" to a high degree of intensity of a particular portion of the iron in the neighbourhood of three of the compasses. The iron so magnetized was iron capable of receiving only subpermanent magnetism, and which from its forming part of the structure of the vessel was subject to strains and concussions from which detached magnets are wholly free. The magnetism so communicated was therefore necessarily unstable and transient, and from its liability to change suddenly and unexpectedly, was a source of danger to the vessel. So strongly was I impressed with the danger that in an official report to the Admiralty of 31st January 1867, after a careful reduction of the observations the results of which are already given, I expressed the hope that, should further experiments be permitted in

Her Majesty's ships for depolarizing their hulls, the so-called "depolarization" should not be allowed "within 20 feet of any compass that may be placed for the navigation of the ship."

Nor could the effect produced, even if it had been much more permanent than it proved to be, be considered an advantage. In two out of the three compasses to which it was applied the semicircular deviation was reduced within the limits which make tabular corrections possible, not within those which allow it to be dispensed with. In the third—the starboard steering compass—the effect, though considerable, was rather a change in the direction, than a reduction of the amount, of the semicircular deviation; for it exceeded 36°, and a reduction by a magnet was still necessary. In the two compasses in which the process was effective to the extent we have mentioned, the requisite reduction might have been effected with infinitely greater ease and certainty, as well as permanency, by the application of a single magnet to each compass.

In dismissing this subject it may seem that some apology is necessary for occupying the time of the Society with the details of a process which had so little to recommend it, and which has proved injurious, not beneficial; it is, however, a process to which many persons looked with hope, and from which no one apprehended danger; both were mistaken, and in both respects it is desirable that the results of the trials should be known.

The Tables and the discussion will also I hope be accepted as an interesting example of the method now constantly practised in the Royal Navy of supplementing the observations made in "swinging" a ship, by observations of deviation and horizontal and vertical force made on one azimuth.

As regards the observations generally, the conclusions to be drawn from them, or rather which, having been already drawn from numerous other observations, are supported by those made in the 'Northumberland,' seem to be—

1. That in an iron-built ship, and in that part of her within which the Standard compass is generally placed, the polar force is that from the magnetism of the whole body of the ship, and is nearly uniform.

2. That we cannot escape from the action of that force by any care in the selection of a place for the compass.

3. That though positions may be found where from the magnetism of particular masses of iron counteracting that of the ship, the deviation will be small, yet that such positions are in general to be avoided, as the change of magnetic force in such positions will probably be larger and less regular than when the compass is only acted on by the general magnetism of the whole ship:—any attempt to produce this counteraction by magnetizing artificially masses of iron in the vicinity of a compass is to be deprecated.

4. That in iron-built ships, as at present constructed, the ship's polar force is generally so great as to make it necessary to employ magnets to equalize the directive force on different azimuths of the ship's head, even at the most carefully selected position; but that the use of correcting magnets does not dispense with the necessity of ascertaining from time to time by observations, the amount of the remaining deviations.

General Table.—Her Majesty's Ship Northumberland.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. H	B	C	D	E
				° ′	° ′	° ′	° ′	° ′					
Northumberland.	Standard.	On Stocks	Mar. and Apr. 1866	*By deviation, horizontal and vertical forces on one point*						−·157	+·427	+·133a	.....
		Off Deptford	Apr. 19, 1866							−·004	+·416	+·133a	.....
(6621 tons),		Sheerness	Jan. 1, 1867	+0 50	−36 53	+ 4 35	+7 02	−0 24	+·015	−·646	+·071	+·122	−·007
26 guns, 1350 horse-power. Iron hull, iron-plated.		Sheerness	Jan. 26, 1867	+1 12	−36 13	+ 7 09	+7 22	−0 35	+·021	−·637	+·108	+·128	−·010
		Sheerness	Feb. 28, 1867	+0 01	−36 13	+ 7 09	+7 19	+0 59	+·001	−·637	+·110	+·127	+·017
		Sheerness	**March 2, 1867**	−0 14	−4 55	+ 0 14	+7 13	+1 15	−·004	−·091	+·004	+·126	+·022
Armour bulk-heads. Rifle-tower on quarter-deck.		Devonport	June 26, 1867	*By deviation, horizontal and vertical forces on one point. Ship in dock, head S. 84° E. magnetic*						−·576	+·277	+·123a	.....
		Devonport	Aug. 29, 1867							−·569	+·256	+·120a	.....
		Devonport	Dec. 10, 1867							−·593	+·329	+·120a	.....
Built at Millwall, River Thames; head N. 39½° E. magnetic.	Poop.	On Stocks	Mar. and Apr. 1866	*By deviation and forces on one point*						−·788	+·358	+·090a	.....
										−·801	+·361	+·104a	.....
Plated on Stocks.		Off Deptford	Apr. 19, 1866	*From swinging through a Quadrant*						−·877	+·296	+·090a	.....
										−·890	+·291	+·104a	.....
Launched April 17, 1866.		In Victoria Docks.	Apr. 21, 1866	*By deviation and forces on one point (head S. 22° W. magnetic). Two iron-plated ships alongside. Steam machinery, iron masts, &c., placed on board during this interval*						−·800	+·135	+·090a	.....
			May 29, 1866							−·743	+·129	,,	.....
			July 12, 1866							−·900	+·075	,,	.....
			Aug. 10, 1866							−·821	+·099	,,	.....
			Sept. 1, 1866							−·842	+·076	,,	.....
			Oct. 24, 1866							−·814	+·003	,,	.....
			Dec. 17, 1866							−·825	+·025	,,	.....
		Sheerness	Jan. 1, 1867	+1 15	−48 43	+15 13	+5 55	−0 35	+·022	−·808	+·228	+·103	−·010
		Sheerness	*Jan. 26, 1867*	+2 14	− 3 16	+12 20	+5 39	−0 43	+·039	−·059	+·203	+·098	−·014
		Sheerness	*Feb. 28, 1867*	+0 35	− 3 51	+13 59	+6 07	+0 29	+·010	−·070	+·230	+·107	+·008
		Devonport	*June 26, 1867*	*By deviation, horizontal and vertical forces on one point. Ship in dock, S. 84° E. magnetic*						−·153	+·360	+·100a	.....
		Devonport	*Aug. 29, 1867*							−·340	+·365	,,	.....
		Devonport	*Dec. 10, 1867*							−·215	+·541	,,	.....
		Devonport	*Dec. 10, 1867*							−·221	+·516	+·117a	.....

Note.—The coefficients marked (a) are assumed values.

General Table (continued).—Her Majesty's Ship Northumberland.

Maximum of semicircular deviation. Mean horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$. Amount.	Direction.	Mean force to North, λ †	$\frac{1}{\lambda}$	Coefficients of horizontal induction. Headward. a †	To starboard. e ‡	Part of $\mathfrak{D}$ from Fore and aft induction.	Transverse induction.	Mean Vertical force, μ §	Heeling coefficient to windward.	Heeling coefficient for Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	Variable part of Vertical force. $\frac{g}{\tan\theta}$ §	g ‡
°	°					° ′	° ′		° ′	° ′	° ′		
27½ ·461	110	·860a	.....	.....	.....	.....	.....	1·241	+1 25	.....	.....	+·007a	
24½ ·416	90½	·860a	.....	.....	.....	.....	.....	1·164	+1 11				
40½ ·650	173½	·869	1·151	−·025	−·237	−0 48	+7 50	1·223	+1 18	+0 40	+0 38	+·007	+·017
40½ ·646	170½	·870	1·149	−·019	−·241	−0 34	+7 56	1·271	+1 27	+0 41	+0 46		
40½ ·646	170												
5½ ·091	177½	·878	1·139	−·012	−·232	−0 21	+7 34	1·168	+1 07	+0 39	+0 28	+·013	+·032
39½ ·639	154½	·885a	.....	.....	.....	.....	.....	1·272	+1 24	.....	.....	+·010a	
38½ ·623	156	·885a	.....	.....	.....	.....	.....	1·232	+1 17	.....	.....	,,	
49½ ·678	151	·885a	.....	.....	.....	.....	.....	1·266	+1 22	.....	.....		
59½	155½	·800	.....	.....	.....	.....	.....	1·637	+2 50	.....	.....	+·070a	
61½ ·880	155½	·840	.....	.....	.....	.....	.....	1·691	+2 48	.....	.....	·000	
67½	161½	·800	.....	.....	.....	.....	.....	1·646	+2 52	.....	.....	+·070a	
69½ ·936	161½	·840	.....	.....	.....	.....	.....	1·596	+2 30	.....	.....	·000	
54½ ·811	170½	·800a	.....	.....	.....	.....	.....	1·529	+2 30	.....	.....	+·070a	
48½ ·753	170	,,	.....	.....	.....	.....	.....	1·531	+2 06	.....	.....	,,	
64½ ·902	175½	,,	.....	.....	.....	.....	.....	1·383	+2 03	.....	.....	,,	
55½ ·827	173½	,,	.....	.....	.....	.....	.....	1·411	+2 08	.....	.....	,,	
57½ ·847	174½	,,	.....	.....	.....	.....	.....	1·449	+2 15	.....	.....	,,	
54½ ·814	180	,,	.....	.....	.....	.....	.....	1·442	+2 14	.....	.....	,,	
55½ ·825	178	,,	.....	.....	.....	.....	.....	1·355	+1 58	.....	.....	,,	
56½ ·834	164½	·860	1·163	−·052	−·228	−1 44	+7 30	1·469	+2 01	+0 39	+1 22	−·001	−·003
13½ ·211	104½	·854	1·171	−·062	−·230	−2 04	+7 43	·986	+0 37	+0 40	−0 03		
14 ·240	107	·894	1·118	−·010	−·202	−0 21	+6 28	·939	+0 22	+0 34	−0 12	+·033	+·082
23 ·391	112	·880a	.....	.....	.....	.....	.....	·996	+0 35	.....	.....	+·030a	
30 ·499	133	,,	.....	.....	.....	.....	.....	1·044	+0 43	.....	.....	,,	
35½ ·580	111½	,,	.....	.....	.....	.....	.....	1·230	+1 14	.....	.....	,,	
34 ·561	113	·860a											

$$a = \lambda\mathfrak{D} - (1-\lambda).$$
$$e = -\lambda\mathfrak{D} - (1-\lambda).$$

Part of $\mathfrak{D}$ from { Fore and aft induction $= \frac{a}{2\lambda} = \frac{\mathfrak{D}}{2} - \left(\frac{1}{\lambda}-1\right)$. Transverse induction $= -\frac{e}{2\lambda} = \frac{\mathfrak{D}}{2} + \left(\frac{1}{\lambda}-1\right)$.

Heeling coefficient to *windward* $= \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1\right) \tan\theta$.

Part of heeling coefficient. { from Vertical induction in transverse iron ..... $= \left(\mathfrak{D} + \frac{1}{\lambda} - 1\right) \tan\theta$. from Vertical force and Vertical induction in Vertical iron..... $= \left(\frac{\mu}{\lambda} - \frac{1}{\lambda}\right) \tan\theta$.

† Mean force to North (λH) being unit. ‡ Earth's Horizontal force (H) being unit. § Earth's Vertical force (Z) being unit.

GENERAL TABLE (continued).—Her Majesty's Ship Northumberland.

Ship.	Compass.	Place.	Date.	Approximate coefficients. A	B	C	D	E	Exact coefficients. A	B	C	D	E
NORTHUMBERLAND (*continued*).	Starboard steering.	On stocks	Mar. and Apr. 1866	° ′	° ′	° ′	° ′	° ′		−·846	+·499	+·105a	.....
										−·855	+·491	+·117a	.....
		Off Deptford	April 19, 1866	*By deviation and forces on one point*						−·882	+·455	+·105a	.....
										−·874	+·447	+·117a	.....
		In Victoria Docks.	April 21, 1866						.....	−·761	+·150	+·105a	.....
			May 29, 1866	*By deviation and force on one point (head S. 22° W. magnetic).* *Two iron-plated ships alongside steam machinery, iron masts, &c. placed on board during this interval.*						−·635	+·018	,,	.....
			July 12, 1866							−·760	−·108	,,	.....
			Aug. 10, 1866							−·726	+·022	,,	.....
			Sept. 1, 1866							−·740	−·071	,,	.....
			Oct. 24, 1866							−·668	−·107	,,	.....
			Dec. 17, 1866							−·651	−·089	,,	.....
		Sheerness	Jan. 1, 1867	+0 09	−39 50	+15 18	+ 6 39	+ 0 42	+·003	−·683	+·232	+·116	+·012
		Sheerness	Jan. 26, 1867	−0 24	− 2 37	+38 19	+ 6 24	− 0 20	−·007	−·046	+·596	+·112	−·006
		Sheerness	Feb. 28, 1867	+0 41	− 5 20	+40 41	+ 6 28	+ 2 40	+·012	−·098	+·625	+·113	+·046
			Mar. 2, 1867	**−0 18**	**−4 15**	**−1 04**	**+ 6 42**	**+1 44**	**−·005**	**−·078**	**−·018**	**+·117**	**+·030**
		Devonport	*Aug. 29, 1867*	*By deviation and forces on one point*						−·229	+·637	+·110a	.....
		Devonport	*Dec. 10, 1867*	*Ship in dock (head S. 84° E. magnetic)*						−·136	+·641	+·110a	.....
		Devonport	*Dec. 10, 1867*							−·141	+·617	+·127a	.....
	Port steering.	Sheerness	Jan. 1, 1867	+1 13	+43 0	+19 48	+ 6 18	− 0 28	+·021	−·725	+·302	+·110	−·008
		Sheerness	*Jan. 26, 1867*	−0 39	− 4 13	−10 52	+ 7 40	− 2 11	−·011	−·078	−·176	+·133	−·038
		Sheerness	*Feb. 28, 1867*	+0 53	− 3 55	− 7 50	+ 6 54	− 0 12	+·015	−·071	−·128	+·120	−·003
		Devonport	*Aug. 29, 1867*	*By deviation and forces on one point*						−·176	+·150	+·110a	
		Devonport	*Dec. 10, 1867*	*Ship in dock (head S. 84° E. magnetic)*						−·212	+·508	.....	
		Devonport	*Dec. 10, 1867*							−·218	+·481	+·127	
	Lower deck.	Sheerness	Feb. 28, 1867	−0 11	− 3 23	+11 05	+13 02	+ 0 32	−·003	−·065	+·170	+·228	+·009
		Devonport	Aug. 29, 1867	*By deviation and forces on one point*						−·025	+·333	+·220	
		Devonport	Dec. 10, 1867	*Ship in dock (head S. 84° E. magnetic)*						−·082	+·412	+·220	

Note.—The coefficients marked (a) are assumed values.

General Table (continued).—Her Majesty's Ship Northumberland.

Maximum of semicircular deviation: Mean Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$†: Amount.	Maximum of semicircular deviation: Mean Horizontal force of ship $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$†: Direction.	Mean force to North, λ ‡	$\frac{1}{\lambda}$	Coefficients of horizontal induction: Headward. a ‡	Coefficients of horizontal induction: To starboard. e ‡	Part of $\mathfrak{D}$ from: Fore-and-aft induction.	Part of $\mathfrak{D}$ from: Transverse induction.	Mean vertical force, μ §	Heeling coefficient to *windward*.	Heeling coefficient for: Vertical induction in transverse iron.	Heeling coefficient for: Vertical force and induction in vertical iron.	Variable part of vertical force. $\frac{g}{\tan\theta}$ §	Variable part of vertical force. g ‡
°	°					° ′	° ′		° ′	° ′	° ′		
77½	149½							1·186	+1 28	.....	.....	+·100a	
79½	150½	·800a	.....	.....	.....	.....	.....	1·259	+1 43	.....	.....	−·005a	
84½	152½							1·296	+1 49	.....	.....	+·100a	
78½	153	·800a	.....	.....	.....	.....	.....	1·291	+1 36	.....	.....	−·005a	
51	168½	·800a	.....	.....	.....	.....	.....	1·126	+1 17	.....	.....	+·100a	
39½	178½	,,	.....	.....	.....	.....	.....	1·163	+1 24	.....	.....	,,	
50	188½	,,	.....	.....	.....	.....	.....	·996	+0 58	.....	.....	,,	
46½	178½	,,	.....	.....	.....	.....	.....	·891	+0 37	.....	.....	,,	
48½	185½	,,	.....	.....	.....	.....	.....	·935	+0 40	.....	.....	,,	
42½	189	,,	.....	.....	.....	.....	.....	·949	+0 43	.....	.....	,,	
41	188	,,	.....	.....	.....	.....	.....	·924	+0 39	.....	.....	,,	
46½	161½	·802	1·247	−·105	−·291	−3 45	+10 24	1·056	+1 04	+0 54	+0 10	−·005	−·012
36½	94½	·783	1·277	−·105	−·304	−4 43	+11 12	1·250	+1 45	+0 57	+0 47		
39½	97												
44	193	·841	1·189	−·064	−·254	−2 11	+ 8 39	1·285	+1 35	+0 45	+0 50	+·041	+·102
42½	110	·820a	.....	.....	.....	.....	.....	1·114	+1 10	.....	.....	,,	
40	102	,,		.....	.....	.....	.....	1·036	+0 56	.....	.....	,,	
39½	103	·800a											
51	157½												
11	246												
8½	241												
13½	139½	·820a	.....	.....	.....	.....	.....	1·255	+1 36	.....	.....	,,	
33½	112½	,,	.....	.....	.....	.....	.....	1·102	+1 08	.....	.....	,,	
31½	114	·800a											
10½	111	·729	1·371	−·105	−·437	−4 05	+17 24						
19½	94	·740a											
24½	101	,,											

$$a = \lambda\mathfrak{D} - (1-\lambda).$$
$$e = -\lambda\mathfrak{D} - (1-\lambda).$$

Part of $\mathfrak{D}$ from { Fore-and-aft induction $= \frac{a}{2\lambda} = \frac{\mathfrak{D}}{2} - \left(\frac{1}{\lambda} - 1\right)$. Transverse induction $= -\frac{e}{2\lambda} = \frac{\mathfrak{D}}{2} + \left(\frac{1}{\lambda} - 1\right)$.

Heeling coefficient to *windward* $= \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1\right)\tan\theta$.

Part of heeling coefficient. { from Vertical induction in transverse iron ..... } $= \left(\mathfrak{D} + \frac{1}{\lambda} - 1\right)\tan\theta$. { from Vertical force and Vertical induction in Vertical iron ..... } $= \left(\frac{\mu}{\lambda} - \frac{1}{\lambda}\right)\tan\theta$.

† Mean force to North (λH) being unit. ‡ Earth's Horizontal force (H being unit). § Earth's Vertical force (Z) being unit.

XX. *Contributions towards determining the Weight of the Brain in different Races of Man.* By Joseph Barnard Davis, *M.D.*, *&c.* *Communicated by* J. Marshall, *Esq.*, *F.R.S.*

Received November 30, 1867,—Read January 23, 1868.

The Brain being the most essential and characteristic portion of the human organization, as connected with intelligence and mental manifestations, it would naturally be expected to have absorbed great attention. Still, comparatively little has been done to ascertain its relative magnitude in the different races of mankind. Opportunities for the examination of exotic brains are very rare; and it is only by gauging the capacities of trustworthy skulls of different races, and thence deducing the volume of the encephalon that extended and reliable data are to be obtained. That an accumulation of observations of this kind is required for any results deserving confidence, ensues from the manifest diversity of volume and weight which pervades all individual organs of the body. These skulls are but seldom met with in such variety and such number as to yield satisfactory data. Hence it seems to be very desirable to place on record the averages calculated for a considerable collection of human crania, embracing most of the chief divisions of mankind.

It may be supposed that this method is inferior to that of ascertaining directly the weight of the brain. This, however, is itself subject to considerable fluctuations, dependent upon sex, age, the kind of disease with which the person has been affected, and the condition of the organ at the period of death*. These all influence the determination, and complicate the deduction of any average weight and volume of the brain when subjected to the manipulation of the observer. It has been asserted "that the actual weights of human brains can alone form just data for conclusions; and that it seems scarcely possible that any method of ascertaining the size of the brain from examination of the skull can be free from fallacy"†. Without undervaluing the views of so excellent an observer as Dr. Peacock, it may be said that, if this be allowed to be correct in reference to any individual case, since the disease which terminates the life of a person itself alters the relation of the solid to the fluid contents of the cranium, it may yet with confidence be affirmed that the examination of a large series of skulls in ascertaining their

* Chronic diseases tend to change the proportions between the weight of the brain and the fluids of the cranium. These fluids are abundant at an early period of life, diminish from birth to a mature age, from twenty to thirty years of age, after which they again increase. Dr. A. Weisbach, in his careful observations, found the fluids more abundant in the brains of men than in those of women.

† Dr. Thomas B. Peacock, 'Tables of the Weights of the Brain and of some other Organs of the Human Body,' 1861, p. 22. Reprinted from the 'Monthly Journal of Medical Science,' vol. vii. 1847.

capacities and deducing from those capacities the average volume of the brain, affords, in some respects, more available data for determining this relative volume for any particular race than the weighing of the brain itself. It might be less easy in this way to fix the exact weight of any individual encephalon, which might be much changed by some lingering and wasting disease (a large portion of Dr. PEACOCK's cases died of Phthisis); but, practically, this method is more sure to yield an accurate average size of the organ, because we have it in our power to use an unchangeable substance with which to gauge the capacity of the skull. And we thus arrive at conclusions the same in result as if we had the brain in all skulls at a uniform density, which, in fact, is the true basis of comparison.

The method followed in the present researches has been to fill each skull, as uniformly as possible, with clean and dry Calais sand of a definite specific weight, 1425 *. Then to pour out the sand, and weigh it carefully. The great difficulty which has always stood at the threshold of ascertaining the volume of the brain from the mere internal capacity of a skull, has arisen from the necessity for, and the variable mode of, making allowance for the other contents of the cranium besides the encephalon, and the uncertainty of the proper amount of such allowance in each special case. This difficulty being once overcome and a uniform rate of allowance being fixed upon a reliable basis, it will be almost as satisfactory in all investigations of this kind to obtain the weight of the brain by gauging the capacity of the skull as by weighing the encephalon itself.

In order to get the proper allowance, or *tare*, to be deducted from the weight of sand which filled the cranium, as a compensation for the weight of the dura mater, the fluids of the membranes and ventricles, and also of the blood contained in the large vessels (after much inquiry and deliberation and consulting the best information to be got upon the absolute weight of the membranes and fluids of the human brain), it has been decided to allow a proportionate or percentage deduction as a tare upon the capacity of the skull and its contents; and this deduction has been fixed at 15 per centum. By this means the allowance will vary in exact proportion to the size of the brain itself, increasing as this increases. And when we come to compare the brain-weights acquired by different observers from actual metrological experiments with the results obtained by such a uniform deduction of 15 per cent. they will be found to agree upon the whole accurately †.

The weight of the sand, after this deduction of 15 per cent., has then to be converted into its equivalent of cerebral matter of the specific gravity of 1040, which is the nearest average datum carefully determined by different competent observers ‡.

* The weight of a given bulk of dry Calais sand, moderately shaken down, is, to distilled water, as 1425 to 1000.

† In the 'Crania Britannica,' p. 224, Note *, it was proposed to allow 5 ounces av. in the skulls of men, and 4·5 oz. in those of women. This was the result of some observations made by Dr. THURNAM.

‡ In a subsequent part of this memoir a reference will be made to actual metrological experiments on individual brains, where the cranial capacity has likewise been determined, when it will be seen that the results

It has already been stated that the 15 per centum tare is intended to cover the weight of the dura mater and pia mater, the arachnoid, the fluids of these membranes and of the ventricles, and also of the blood contained in the large vessels. This weight altogether is considerable. Professor MARSHALL has made the following explanatory remarks. "If the brain, in its natural state, filled the cranial cavity as completely as water will afterwards, it would be easy, by taking the specific gravity of nervous substance as compared with water, to estimate the quantity of brain which once occupied any given skull; but the fact that this is not the case, especially in regard to the base of the brain, and the difficulty in determining the weight of the membranes, the amount of blood which the vessels may contain, and the quantity of cerebro-spinal fluid which fills the ventricles and all otherwise unoccupied spaces, render it impossible thus to arrive at so definite an estimate as in the other way," *i. e.* by weighing the encephalon*.

The present may be regarded as the first attempt to deduce with accuracy the weight of the brain from the capacity of the skull, on any commensurate scale. Professor TIEDEMANN'S and Professor MORTON'S crude observations are of small value in this respect, and the attempts of others have been very limited and uncertain. It is remarkable that neither of these two accomplished men made any allowance in their observations for the membranes and fluids. This might not be deemed necessary in the comparison of the relative size of the cranial cavity in different races, aimed at by the former, who, when he subjected the brain to the weighing process, divested it of the arachnoid and the pia mater. The latter observer might seem to have overlooked the need of any allowance; at all events, in his Memoir†, he tells us that by his process of gauging the cavity of the skull with leaden shot, "I thus obtain the absolute capacity of the cranium, or bulk of the brain, in cubic inches;" and he describes his great Table as "showing the size of the brain in cubic inches"—whereas, in fact, it merely gives the capacity of 623 crania in cubic inches. It is quite needless to endeavour to prove that an allowance for membranes and fluids is absolutely necessary.

Attempts have been made by different observers to determine the proportion in weight by which the brains of men exceed those of women. This appears to be variable, and possibly the variation may be in relation to particular races; but to decide this question would require materials of a very exact nature. The proportion differs from about 5 per cent. in our Australians, to more than 10 per cent. in our Tasmanians. It may be said generally in our series to range from less than 10 per cent. to something more than 12½ per cent., so that the proposal of Professor WELCKER to regard it as 10 per cent. may for the present be as safe as any other.

When Professor TIEDEMANN considered as the result of his investigation, that he was

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closely coincide. For reasons already given, it is apparent that no rule can be formed which shall agree with all individual cases.

* JOHN MARSHALL, F.R.S., "On the Brain of a Bushwoman," &c., Philosophical Transactions, 1864, p. 506.

† Proceedings of the Academy of Natural Sciences of Philadelphia, October 1849.

justified in making the assertion, which he said could not be regarded as a hypothesis, that Nature, inasmuch as a certain size and mass of brain is a necessary condition for the exercise of the faculties of the mind, hath furnished the people of all human races herewith in an equal degree, he was compelled to have recourse to a series of secondary influences to account for the diverse intellectual state of mankind. It was the hypothetical error of his system that led him to insist on the physical unity of man; his intellectual inequality could not be denied*. On the other hand, Professor HUSCHKE affirmed that a difference in the weight of the brain in different races cannot be mistaken, with which the stature may correspond. He adds, thus the German brain exceeds in the mean 1400 grammes, the French has been by many observers specified to be only 1300 grammes, and that of the small Hindoos and indigenes of Bombay amounts to only 1000 to 1100 grammes†. It will be easier to test the correctness of these and various other statements, after our Tables have been examined.

The importance of the average stature and weight of the body, so variable in different races, in their relations to the size and weight of the brain, although these relations have not yet been properly investigated, has not been overlooked. The large Germans and the small Hindoos are obvious instances. Up to the present time, exact observations upon the stature and weight of the races of man are almost wholly wanting. The notes to the Tables will include some information upon these points, in the cases in which it can be obtained.

The weight of the brain is much influenced by sex and by age. In the following Tables the sexes of the crania have been determined, as accurately as may be from their examination, where they have not been otherwise known. They are always distinguished in their relative numbers. This was omitted in MORTON'S Table, as may be mentioned with regret. The observations have been confined to the skulls of *adults*. Any more definite rule respecting age it was not easy to lay down. The brain-weights have been arranged in six Tables, corresponding very much with the races of the six great divisions of the globe. In all cases, the estimated weight of the heaviest brain of the series is given first, then that of the lightest, and last the average weight of the whole. By the insertion of the averages of each sex and of the whole series, as much is done as may be to correct the preponderance of those individual exceptional cases of large brains, *megalocephali*, and small brains, *microcephali*, which occur in probably all races of man.

In general it has not been thought necessary to exclude from our estimates either synostotic or artificially distorted skulls. The effects of synostosis are almost always much confined to limiting the development of the cranial cavity *in one direction*, whilst it has received a corresponding expansion in another compensatory direction; so that there is no material diminution in the mass of the cerebral matter contained in the skull. In the cranium of the Stettin Weaver, the most remarkable example of syno-

* Professor TIEDEMANN'S Memoir was read before the Royal Society in 1835, and appears in the Philosophical Transactions, vol. cxxvii. The reference is here made to the German edition, Das Hirn des Negers mit dem des Europäers und Orang-Outangs verglichen, 1837, S. 47.

† Schædel, Hirn und Seele des Menschen und der Thiere, 1854, S. 60.

stotic deformation recorded, the internal capacity is not materially interfered with*. There are exceptional cases, as those in which most of the sutures are simultaneously ossified at a very early period of life, producing a limited growth of the brain in the whole mass, and constrained microcephalism†. Artificial distortion of the skull also acts in a similar manner; it arrests development in one direction, which is compensated for by increased growth in another. This mainly corresponds with the evidence of all the best observers, as MORTON and CATLIN, that individuals with the most frightfully compressed heads, speaking in ordinary terms, are in every respect equal in intellectual power to those whose heads have undergone no distortion. But this must not be allowed to affect the question, whether those with distorted crania, either from synostosis or by art, are not more prone to moral perversions, and more frequently the subjects of mental aberration than others‡.

In the Notes to the following Tables it is intended to add and to collate observed weights of brains, where they have been ascertained, with the weights deduced, so as to compare, authenticate, and correct the computed results.

As already said, the *heaviest* brain-weights are given first, the *lightest* next, and then the *average*, which is obtained by adding the special weights of all the examples of either sex together, and dividing by the number of individual specimens. The *mean of the sexes* is obtained by adding the average of the males and that of the females together, and dividing by two. The *mean of the series* is obtained by adding the particular weights of all the examples of both males and females together, and dividing by the number of specimens. This is a convenient mean, but must be taken subject to some variation, according to the relative proportion of the examples of the one sex to those of the other. In general, the males greatly exceed the females in number; in some cases, as in the French, the Vedahs, the Cingalese, and the Hindoos, the sexes are almost equal; in the Esquimaux of Greenland and in the Caribs, they are exactly so; and in the Irish and Guanches there is a preponderance in the number of the skulls of women§.

* On Synostotic Crania among Aboriginal Races of Man. J. BARNARD DAVIS, 1865. Plates 9, 10, and 11. Transactions of the Dutch Society of Sciences of Haarlem.

† Ibid. p. 21.

‡ Ibid. p. 22.

§ There are many other important queries suggested by these Tables, such as the exact origin of the skulls themselves, what particular portions of the different countries they are derived from, their authenticity, and other questions which cannot be introduced here. They are all more or less fully illustrated in an octavo volume just issued, entitled 'Thesaurus Craniorum. Catalogue of the Skulls of the Various Races of Man in the Collection of JOSEPH BARNARD DAVIS, M.D.' London 1867.

Tables of the Computed Weights of the Brains contained in Skulls of People of different Races.

(Expressed in Avoirdupois Ounces, and in Grammes.)

Table I.—EUROPEAN RACES.

Races.	1.	Brain-weights of the Skulls of Men.						8.	Brain-weights of the Skulls of Women.						Mean of Sexes.		Mean of Series.		19.
		2.	3.	4.	5.	6.	7.		9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	
	Number.	Heaviest.		Lightest.		Average.		Number.	Heaviest.		Lightest.		Average.						Mean internal capacity.
		oz. av.	grms.	oz. av.	grms.	oz. av.	grms.		oz. av.	grms.	oz. av.	grms.	oz. av.	grms.	oz. av.	grms.	oz. av.	grms.	cub. inch.
1. Ancient Britons	56	57·07	1617	40·94	1160	52·54	1489	10	45·32	1285	39·70	1125	43·27	1226	47·90	1357	45·83	1299	89·8
2. Ancient Scottish	3	48·16	1365	38·82	1100	43·30	1227	2	44·66	1266	42·18	1195	43·42	1230	43·36	1229	43·42	1230	85·1
3. Ancient Romano-Britons	44	63·49	1795	39·70	1125	47·80	1355	12	45·68	1295	31·67	897	41·45	1175	44·62	1264	45·97	1303	90·1
4. Anglo-Saxons	39	54·59	1547	40·95	1160	47·07	1334	16	47·14	1336	36·63	1038	41·67	1181	45·61	1293	45·46	1288	89·1
5. English	21	57·43	1628	44·08	1249	50·28	1425	13	49·62	1406	37·22	1055	43·13	1222	46·70	1323	47·50	1346	93·1
6. Irish	12	62·03	1758	39·48	1119	49·62	1406	16	52·54	1489	38·46	1090	44·51	1261	47·06	1334	46·70	1323	91·5
7. Merovingian Franks	4	51·89	1471	48·75	1382	50·28	1425	...	.....	.....	.....	.....	.....	.....	.....	.....	50·28	1425	98·5
8. French	9	52·10	1477	42·18	1195	47·21	1338	7	47·14	1336	38·46	1090	42·54	1206	44·87	1272	45·17	1280	80·4
9. Spaniards and Portuguese	7	50·35	1427	45·68	1295	48·31	1369	...	.....	.....	.....	.....	.....	.....	.....	.....	48·31	1369	94·7
10. Ancient Romans of Italy	8	54·37	1541	41·96	1189	47·14	1336	...	.....	.....	.....	.....	.....	.....	.....	.....	47·14	1336	92·4
11. Italians	13	52·10	1477	42·18	1195	48·24	1367	2	45·03	1276	40·06	1135	42·54	1206	45·39	1286	47·58	1348	93·3
12. Lapps	5	54·00	1530	45·68	1295	47·65	1350	2	44·59	1264	44·59	1264	44·59	1264	46·14	1308	46·19	1309	90·5
13. Swedes	9	51·59	1462	47·14	1336	49·11	1392	3	47·14	1336	42·18	1195	44·22	1253	46·66	1322	47·94	1359	94
14. Frisians	3	54·00	1530	45·39	1286	48·60	1377	...	.....	.....	.....	.....	.....	.....	.....	.....	48·60	1377	95·3
15. Dutch	18	54·00	1530	39·70	1125	49·55	1404	7	43·42	1230	40·94	1160	41·96	1189	45·75	1296	47·36	1342	92·8
16. Germans	13	55·83	1582	47·14	1336	52·83	1499	2	41·60	1179	40·35	1143	40·94	1160	46·88	1329	50·28	1425	98·5
17. Russniak	1	52·76	1495	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	52·76	1495	103·4
18. Poles	2	47·14	1336	47·14	1336	47·14	1336	...	.....	.....	.....	.....	.....	.....	.....	.....	47·14	1336	92·4
19. Czechs	2	52·76	1495	41·60	1179	47·14	1336	...	.....	.....	.....	.....	.....	.....	.....	.....	47·14	1336	92·4
20. Magyar	1	45·03	1276	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	45·03	1276	88·5
21. Slovak	1	43·78	1241	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	43·78	1241	85·8
22. Finns	7	50·86	1441	46·56	1319	48·31	1369	...	.....	.....	.....	.....	.....	.....	.....	.....	48·31	1369	94·7
23. Russians	10	57·07	1617	44·66	1266	50·28	1425	...	.....	.....	.....	.....	.....	.....	.....	.....	50·28	1425	98·5
24. Turks	3	50·28	1425	45·90	1301	47·58	1348	...	.....	.....	.....	.....	.....	.....	.....	.....	47·58	1348	93·3
25. Rumanyos	6	52·23	1480	44·66	1266	45·97	1303	...	.....	.....	.....	.....	.....	.....	.....	.....	45·97	1303	90·1
26. Gipsies	2	45·02	1276	42·84	1214	43·93	1245	2	44·44	1259	41·96	1189	43·20	1224	43·56	1234	43·41	1230	85·1
Numbers and Averages	299	52·68	1493	43·61	1236	48·25	1367	94	46·02	1304	39·56	1121	42·49	1204	45·73	1296	47·12	1335	92·3

Table II.—ASIATIC RACES.

1. Vedahs of Ceylon	3	57·58	1632	39·69	1125	45·51	1290	5	45·03	1276	34·75	985	39·33	1115	42·42	1202	41·23	1168	80·8
2. Cingalese	6	52·76	1495	38·45	1090	47·21	1338	5	46·56	1320	36·61	1037	42·98	1218	45·09	1278	45·32	1284	88·6
3. Afghans	4	58·96	1671	40·35	1148	45·32	1285	3	38·46	1090	36·63	1038	37·65	1067	41·48	1175	42·11	1193	82·5
4. Hindoos	35	51·52	1460	36·63	1038	44·22	1253	31	47·80	1355	33·49	949	39·99	1133	42·10	1193	42·11	1193	82·5
5. Musulmans	14	52·76	1495	37·87	1073	44·66	1266	7	46·56	1320	39·11	1108	41·60	1179	43·13	1222	43·57	1235	85·4
6. Khônds	2	38·46	1090	37·22	1015	37·87	1073	...	.....	.....	.....	.....	.....	.....	.....	.....	37·87	1073	74·2
7. Gônd	1	40·72	1159	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	40·72	1159	79·8
8. Lepchas	7	49·62	1406	39·11	1108	45·90	1301	6	49·03	1390	37·87	1073	42·03	1191	43·96	1246	44·08	1249	86·4
9. Bodos	4	50·28	1425	42·18	1195	46·92	1330	7	48·38	1371	42·18	1195	45·32	1284	46·12	1307	45·97	1303	90·1
10. Bhotias	4	53·34	1512	42·18	1195	46·78	1326	9	52·47	1487	37·22	1055	43·85	1243	45·31	1284	45·90	1301	90
11. Mishmees	3	50·28	1425	44·08	1249	47·36	1342	...	.....	.....	.....	.....	.....	.....	.....	.....	47·36	1342	92·8
12. Singpho	1	47·87	1357	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	47·87	1357	93·8
13. Nagas	2	50·28	1425	45·32	1284	47·80	1355	...	.....	.....	.....	.....	.....	.....	.....	.....	47·80	1355	93·7
14. Siamese	4	50·28	1425	47·16	1337	48·95	1387	3	48·38	1371	39·11	1108	44·88	1272	46·91	1329	47·14	1336	92·4
15. Chinese	25	57·07	1617	43·42	1231	47·87	1357	8	49·99	1417	37·87	1073	43·71	1239	45·79	1298	47·00	1332	92·1
16. Burmese	5	50·28	1425	45·90	1301	47·87	1357	...	.....	.....	.....	.....	.....	.....	.....	.....	47·87	1357	93·8
17. Japanese	1	45·25	1282	.....	.....	.....	.....	1	35·39	1003	.....	.....	.....	.....	40·32	1143	40·32	1143	79
18. Ainos	3	47·65	1350	44·08	1249	45·90	1301	1	43·42	1231	.....	.....	.....	.....	44·66	1266	45·83	1299	89·7
Numbers and Averages	124	50·27	1425	41·57	1178	46·00	1304	86	45·95	1302	37·48	1062	42·13	1194	43·94	1245	44·44	1259	87·1

Table III.—AFRICAN RACES.

1. Berbers	...	.....	.....	.....	.....	.....	.....	4	48·38	1371	40·35	1148	43·42	1231	.....	.....	43·42	1231	85·1
2. Guanches	3	48·38	1371	42·18	1195	45·32	1284	18	49·62	1406	39·11	1108	43·13	1222	44·22	1253	43·49	1233	85·2
3. Negroes of Tribes unknown	12	54·00	1530	40·94	1160	44·30	1255	4	45·32	1284	37·00	1048	43·49	1230	43·89	1244	44·08	1249	86·4
4. Joloff	1	46·92	1330	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	46·92	1330	92
5. Foulah	1	43·42	1231	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	43·42	1231	85·1
6. Mandingoes	1	47·14	1336	.....	.....	.....	.....	1	34·15	968	.....	.....	.....	.....	40·64	1152	40·64	1152	79·6
7. Serryia	1	46·92	1330	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	46·92	1330	92
8. Fantees	1	41·60	1179	.....	.....	.....	.....	1	39·11	1108	.....	.....	.....	.....	40·35	1143	40·35	1143	79·1
9. Ashantees	3	47·00	1332	39·70	1125	42·91	1216	...	.....	.....	.....	.....	.....	.....	.....	.....	42·91	1216	84·1
10. Dahomans	9	50·28	1425	40·94	1160	46·63	1322	3	47·80	1355	42·18	1195	44·08	1249	45·35	1285	46·34	1313	90·8
11. Eboes	...	.....	.....	.....	.....	.....	.....	4	43·42	1231	40·35	1143	41·74	1183	.....	.....	41·74	1183	81·8
12. Yorubans	...	.....	.....	.....	.....	.....	.....	4	49·04	1390	40·35	1143	45·46	1288	.....	.....	45·46	1288	89·1
13. Akassa	1	44·08	1249	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	44·08	1249	86·4
14. Bakeles	2	52·76	1495	48·39	1371	50·57	1433	5	51·23	1452	40·94	1160	46·08	1306	48·32	1369	46·41	1315	91
15. Osyekanis	1	48·16	1365	.....	.....	.....	.....	5	45·32	1284	35·76	1013	37·58	1065	42·86	1215	41·89	1187	82·1
16. M'Pongwe	1	46·56	1319	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	46·56	1319	91·2
17. Asango	...	.....	.....	.....	.....	.....	.....	1	34·15	968	.....	.....	.....	.....	.....	.....	34·15	968	66·9
18. Asira	1	45·90	1301	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	45·91	1301	90
19. M'Fans	...	.....	.....	.....	.....	.....	.....	3	45·90	1301	42·18	1195	43·85	1243	.....	.....	43·85	1243	85·9
20. Congos	2	40·35	1143	39·18	1110	39·76	1127	...	.....	.....	.....	.....	.....	.....	.....	.....	39·76	1127	77·9
21. Manganja	1	52·76	1495	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	52·76	1495	103·4
22. Hottentots	...	.....	.....	.....	.....	.....	.....	3	42·84	1216	39·69	1125	41·86	1187	.....	.....	41·86	1187	82
23. Kafirs	7	53·34	1512	39·70	1130	49·04	1390	1	41·60	1179	.....	.....	.....	.....	47·47	1345	48·16	1365	94·4
24. Zulus	4	50·28	1424	44·52	1262	46·56	1319	...	.....	.....	.....	.....	.....	.....	.....	.....	46·56	1319	91·2
25. Bushmans	1	40·35	1143	.....	.....	.....	.....	3	40·78	1156	37·22	1055	39·48	1119	39·91	1131	39·70	1125	77·8
Numbers and Averages	53	47·37	1342	41·93	1188	45·63	1293	60	43·91	1244	39·59	1122	42·74	1211	43·66	1237	43·89	1244	86

Table IV.—AMERICAN RACES.

Races.	1.	Brain-weights of the Skulls of Men. 2. 3. 4. 5. 6. 7.						8.	Brain-weights of the Skulls of Women. 9. 10. 11. 12. 13. 14.						Mean of Sexes. 15. 16.		Mean of Series. 17. 18.		19.
	Number.	Heaviest.		Lightest.		Average.		Number.	Heaviest.		Lightest.		Average.						Mean internal capacity.
		oz. av.	grms.	oz. av.	grms.	oz. av.	grms.		oz. av.	grms.	oz. av.	grms.	oz. av.	grms.	oz. av.	grms.	oz. av.	grms.	cub. inch.
1. Esquimaux of Greenland	5	57·72	1636	47·14	1336	49·25	1396	5	49·03	1390	37·42	1055	44·00	1247	46·62	1321			
2. Esquimaux, Eastern	4	49·62	1406	42·85	1214	46·23	1310	1	47·14	1336	.....	.....	.....	.....	46·68	1323	46·56	1319	91·2
3. Esquimaux, Western	4	47·14	1336	44·15	1251	45·39	1286	2	43·42	1230	41·30	1170	42·36	1200	43·87	1243			
4. Iroquois	1	45·90	1301	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	45·90	1301	90
5. Mississaga	...	.....	.....	.....	.....	.....	.....	1	44·66	1266	.....	.....	.....	.....	.....	.....	44·66	1266	87·5
6. Athabasca	...	.....	.....	.....	.....	.....	.....	1	44·08	1249	.....	.....	.....	.....	.....	.....	44·08	1249	86·4
7. Shushwaps	1	50·53	1432	.....	.....	.....	.....	1	46·92	1330	.....	.....	.....	.....	48·72	1382	48·72	1382	95·5
8. Chemesyans	2	50·86	1441	41·60	1179	46·23	1310	1	42·47	1208	.....	.....	.....	.....	44·35	1257	46·23	1310	90·6
9. Selipsh	1	44·08	1249	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	44·08	1249	86·4
10. Quatsimas	1	51·52	1460	.....	.....	.....	.....	2	47·14	1336	38·46	1090	42·80	1213	47·16	1336	45·70	1295	89·6
11. Songass	1	44·65	1265	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	44·65	1265	87·5
12. Bilhoolas	2	48·16	1365	47·14	1336	47·65	1350	...	.....	.....	.....	.....	.....	.....	.....	.....	47·65	1350	93·4
13. Lenni Lenape	1	47·80	1355	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	47·80	1355	93·8
14. Illinois	1	48·75	1382	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	48·75	1382	95·5
15. Comanche	1	43·42	1230	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	43·42	1230	85·1
16. Caribs	2	44·81	1270	42·18	1195	43·49	1232	2	42·18	1195	40·94	1160	41·56	1178	42·52	1205	42·32	1199	82·9
17. Muiscas	8	51·52	1460	39·70	1125	44·88	1272	4	44·66	1266	40·94	1160	42·84	1214	43·86	1243	44·92	1273	88
18. Yunca	...	.....	.....	.....	.....	.....	.....	1	44·65	1265	.....	.....	.....	.....	.....	.....	44·65	1265	87·5
19. Quichuas	3	45·02	1276	43·42	1230	44·15	1251	2	41·60	1179	38·46	1090	40·03	1134	42·00	1193	42·54	1206	83·4
20. Chauca	...	.....	.....	.....	.....	.....	.....	1	42·84	1214	.....	.....	.....	.....	.....	.....	42·84	1214	84
21. Aymaras	5	50·86	1441	44·08	1249	47·22	1338	...	.....	.....	.....	.....	.....	.....	.....	.....	47·22	1338	92·5
22. Colla	...	.....	.....	.....	.....	.....	.....	1	37·87	1073	.....	.....	.....	.....	.....	.....	37·87	1073	74·2
23. Charcas	2	44·66	1266	43·42	1230	44·08	1249	1	41·60	1179	.....	.....	.....	.....	42·84	1214	43·22	1225	84·7
24. Araucanians	5	58·96	1671	42·84	1214	49·33	1398	1	41·60	1179	.....	.....	.....	.....	45·46	1288	48·02	1361	94·1
25. Paraguay Indian	1	41·45	1175	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	41·45	1175	81·2
26. Gran Chaco Indian	1	51·52	1460	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	51·52	1460	101
27. Arawack	...	.....	.....	.....	.....	.....	.....	1	41·60	1179	.....	.....	.....	.....	.....	.....	41·60	1179	81·5
28. Caribi	...	.....	.....	.....	.....	.....	.....	1	37·22	1055	.....	.....	.....	.....	.....	.....	37·22	1055	72·9
29. Tarumas	...	.....	.....	.....	.....	.....	.....	2	42·84	1214	36·63	1038	39·70	1125	.....	.....	39·70	1125	77·8
30. Macusi	1	47·80	1355	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	47·80	1355	93·8
31. Wapisiana	1	41·30	1170	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	41·30	1170	80·9
32. Orinoco Indian	1	47·80	1355	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	47·80	1355	93·8
Numbers and Averages	52	48·16	1360	43·50	1233	46·17	1308	31	45·44	1288	39·13	1109	41·89	1187	44·92	1273	44·64	1265	87·5

Table V.—AUSTRALIAN RACES.

1. Australians	17	53·35	1512	36·71	1040	42·25	1197	7	44·08	1249	34·15	968	39·62	1123	40·93	1160	41·38	1173	81·1
2. Tasmanians	7	48·38	1371	37·22	1055	43·42	1230	4	41·89	1187	35·39	1003	38·82	1100	41·12	1165	42·25	1197	82·8
Numbers and Averages	24	50·86	1441	36·96	1047	42·83	1214	11	42·98	1218	34·77	985	39·22	1111	41·02	1162	41·81	1185	81·9

Table VI.—OCEANIC RACES.

1. Nicobarian	1	40·35	1143	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	40·35	1143	79·1
2. Nians	2	44·44	1259	38·23	1083	41·33	1171	1	44·66	1266	.....	.....	.....	.....	42·99	1218	43·47	1232	85·2
3. Sumatrans	7	51·52	1460	41·60	1179	45·81	1301	...	.....	.....	.....	.....	.....	.....	.....	.....	45·81	1301	89·8
4. Malays	6	54·00	1530	45·32	1313	50·13	1421	2	38·46	1090	37·58	1065	38·02	1077	44·07	1249	47·07	1334	92·3
5. Bankans	2	53·37	1513	53·35	1512	53·36	1512	...	.....	.....	.....	.....	.....	.....	.....	.....	53·36	1512	104·6
6. Javans	30	54·59	1547	40·94	1160	47·21	1338	6	45·90	1301	39 11	1108	41·45	1175	44·33	1256	44·66	1266	87·5
7. Madurans	10	56·48	1601	46·56	1319	50·79	1439	...	.....	.....	.....	.....	.....	.....	.....	.....	50·79	1439	99·5
8. Balinese	3	50·86	1441	40·94	1160	44·44	1259	...	.....	.....	.....	.....	.....	.....	.....	.....	44·44	1259	87·1
9. Bimanese	1	49·62	1406	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	49·62	1406	97·3
10. Timorese	1	47·14	1336	.....	.....	.....	.....	1	42·18	1195	.....	.....	.....	.....	44·66	1266	44·66	1266	87·5
11. Amboynese	4	53·35	1512	45·90	1301	48·38	1371	1	37·58	1065	.....	.....	.....	.....	42·98	1218	46·19	1309	90·5
12. Tidorese	2	52·76	1495	47·83	1355	50·28	1425	...	.....	.....	.....	.....	.....	.....	.....	.....	50·28	1425	98·5
13. Alfourous	2	47·14	1336	45·03	1276	46·05	1305	1	44·08	1249	.....	.....	.....	.....	45·06	1277	45·41	1287	89
14. Celebeans	8	51·52	1460	41·60	1179	45·97	1303	2	47·51	1346	45·25	1282	46·38	1314	46·17	1308	46·51	1318	91·2
15. Dayaks	12	50·28	1425	37·22	1015	44·80	1270	4	43·42	1230	40·94	1160	41·81	1185	43·30	1227	44·08	1249	86·4
16. Bisayans	5	48·38	1371	43·42	1230	44·73	1268	3	43·78	1241	42·18	1195	43·20	1224	43·96	1246	44·08	1249	86·4
17. Negritos	2	43·20	1224	37·87	1073	40·53	1149	...	.....	.....	.....	.....	.....	.....	.....	.....	40·53	1149	79·4
18. Oolean	1	42·13	1194	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	42·13	1194	82·6
19. Papuans	3	49·62	1406	45·32	1284	46·78	1326	1	43·20	1224	.....	.....	.....	.....	44·99	1275	45·76	1297	89·7
20. Salomon Islanders	3	45·20	1281	41·30	1170	44·15	1251	1	44·08	1249	.....	.....	.....	.....	44·11	1250	44·15	1251	86·5
21. New Caledonians	4	49·62	1406	45·32	1284	47·14	1336	1	40·35	1152	.....	.....	.....	.....	43·74	1240	45·97	1303	90·1
22. Loyalty Islanders	1	45·68	1295	.....	.....	.....	.....	2	50·86	1441	43·42	1230	47·14	1336	46·41	1315	46·65	1322	91·4
23. New Hebrideans	9	49·04	1390	40·94	1160	44·66	1266	1	44·08	1249	.....	.....	.....	.....	44·37	1257	43·92	1245	86·1
24. Feejeeans	...	.....	.....	.....	.....	.....	.....	2	42·84	1214	41·30	1170	42·07	1192	.....	.....	42·07	1192	82·4
25. Maoris	5	48·39	1371	42·18	1195	45·19	1281	...	.....	.....	.....	.....	.....	.....	.....	.....	45·19	1281	88·6
26. Eimeoan	1	47·14	1336	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	47·14	1336	92·4
27. Bolabolan	1	45·90	1301	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	45·90	1301	90
28. Gambier Islander	1	46·56	1319	.....	.....	.....	.....	...	.....	.....	.....	.....	.....	.....	.....	.....	46·56	1319	91·3
29. Marquesan Islanders	16	54·59	1547	42·18	1195	47·80	1355	11	50·64	1435	41·10	1165	43·20	1224	45·50	1289	46·56	1319	91·3
30. Kanakas	67	55·61	1576	37·87	1073	47·89	1357	54	52·76	1495	38·24	1084	43·78	1241	45·83	1299	45·97	1303	90·1
Numbers and Averages	210	49·25	1396	42·90	1216	46·54	1319	95	44·61	1264	41·01	1162	43·00	1219	44·52	1272	45·63	1293	89·4

SUMMARY OF THE SIX TABLES.

I. European Races	299	52·68	1493	43·61	1236	48·25	1367	94	46·02	1304	39·56	1121	42·49	1204	45·73	1296	47·12	1335	92·3
II. Asiatic Races	124	50·27	1425	41·57	1178	46·00	1304	86	45·95	1302	37·48	1062	42·13	1194	43·94	1245	44·44	1259	87·1
III. African Races	53	47·37	1342	41·93	1188	45·63	1293	60	43·91	1244	39·59	1122	42·74	1211	43·66	1237	43·89	1244	86
IV. American Races	52	48·16	1365	43·50	1233	46·17	1308	31	45·44	1288	39·13	1109	41·89	1187	44·92	1273	44·64	1265	87·5
V. Australian Races	24	50·86	1441	36·96	1047	42·83	1214	11	42·98	1218	34·77	985	39·22	1111	41·02	1162	41·81	1185	81·9
VI. Oceanic Races	210	49·25	1396	42·90	1216	46·54	1319	95	44·61	1264	41·01	1162	43·00	1219	44·52	1272	45·63	1293	89·4
Numbers and Averages	762	49·76	1410	41·74	1183	45·90	1301	377	44·81	1270	38·59	1094	41·91	1188	43·96	1246	44·58	1263	87·3

NOTES TO TABLE I.—European Races.

1. *Ancient Britons.*—The average weight of the brain in 56 skulls of men is 52·54 oz., or 1489 grms.; that of 10 women 43·27 oz., or 1226 grms. The mean of the sexes is 47·90 oz., or 1357 grms.; the mean of the series gives a brain-weight of 45·83 oz., or 1299 grms.

2. *Ancient Scottish.*—The mean of these 5 skulls, 3 of men and 2 of women, is 43·42 oz., or 1230 grms.

3. *Ancient Romano-Britons.*—The mean weight of brain deduced from 56 crania, 44 of which are those of males and 12 of females, is 45·97 oz., or 1303 grms.

4. *Anglo-Saxons.*—The average brain-weight of about the same number of Anglo-Saxon crania, in which the males and females are nearly in the same proportion, is very nearly the same, viz. 45·46 oz., or 1288 grms.

5. *English.*—The average brain-weight of 21 English men is 50·28 oz., or 1425 grms.; that of 13 English women 43·13 oz., or 1222 grms. That of the entire series, of which three-fifths are those of men and two-fifths those of women, is 47·50 oz., or 1346 grms. It should here be observed that the great design kept in view in forming the collection of skulls from which our data are derived was to acquire *exotic* crania. Not a single English example has been sought for, and some of those which have accidentally fallen into our hands may be poor specimens; so that the average weight of the English brain may be a little higher than is here set down. Still, a comparison with actually tested weights of brains shows that there cannot be any material error.

Dr. T. B. PEACOCK has made two series of metrological observations on the weight of the brain. The first was conducted at the Edinburgh Infirmary, and therefore refers to the brains of the *Scotch*. The second was conducted in London, and related to *English* brains. Dr. THURNAM has analyzed Dr. PEACOCK's Tables, and gives as the result that he found the Scotch adult male brain to weigh on the average 50 oz. av., and that of the English 49 oz.*

In the former series of observations by Dr. PEACOCK upon the Scotch brains, he states as his first conclusion that "The encephalon in the adult male weighs, on an average, 50 oz. 3·25 dr., or 3 lb. 2 oz. and $3\frac{34}{151}$ drachms avoirdupois, and exceeds in weight that of the female by 5 oz. 4·95 dr., the latter weighing on an average 44 oz. and 14·3 dr., or 2 lb. 12 oz. $14\frac{33}{74}$ dr."†. To express Dr. PEACOCK's conclusion in our mode, it is as nearly as may be that the male brain weighs 50·34 oz., and the female 44·15 oz. Our results give 50·28 oz. and 43·13 oz. respectively for the English brain.

Dr. PEACOCK's Note deserves to be quoted. It is to the following effect:—"A comparison of these averages, with those deduced by Dr. REID, will show that they correspond very closely, though the numbers on which the calculations are based are considerably extended. They do not differ, also, very greatly from the conclusions of Sir

* "On the Weight of the Brain, and on the Circumstances affecting it, by JOHN THURNAM," Journal of Mental Science, vol. xii. p. 15.

† *Op. cit.* p. 19.

Wilaiam Hamilton, Dr. Sims, and Dr. Clendinning. Sir W. Hamilton estimated the weight of the adult male encephalon at 3 lb. 8 oz. troy, and the female at 3 lb. 4 oz., which are nearly 48 oz. 5 dr. and 43 oz. 15 dr. avoirdupois. On calculating the weights of the brain in the two sexes separately, from the observations published by Dr. Sims, I find the male brain, in 54 persons between 20 and 60 years of age, to average 47 oz. 13 dr.; and the female brain, in 58 persons, 44 oz. and 10 dr. Dr. Clendinning states the male brain, in persons between 21 and 60 years of age, to average 45·85 oz., and the female 41·25 oz. These several averages, together with those deduced by Dr. Reid and myself, range between 45¾ oz. and 50¼ oz. for the male, and 41¼ oz. and nearly 45 oz. for the female"*.

Dr. Robert Boyd states, as the result of his great and continued metrological labours upon the weight of the brain, that the average weight of the encephalon in *insane* males at adult age, varied from 48·17 oz. to 43·87 oz., and in *insane* females from 44·55 oz. to 40·55 oz.; and that in the *sane* at the same period of life, the average varied in the male from 48·2 oz. to 45·34 oz., and in the *sane* female from 43·7 oz. to 39·77 oz. To place this in another form, it may be said that the average weight of the brain in *insane* English men was found to be 46·02 oz., or 1304 grms.; and in *insane* English women 42·55 oz., or 1206 grms. In the *sane* English brains the average for the men was 46·77 oz., or 1325 grms.; and for the women 41·73 oz., or 1183 grms., or, taking both together, the mean would have been 44·25 oz., or 1262 grms.†. It will be seen that this result does not quite agree with the weight as deduced from the capacity of our English skulls, nor with Dr. Peacock's observations with the scales. It is rather more than two ounces less than our mean, and three ounces less than Dr. Peacock's.

The conclusions given by Dr. Thurnam from the examination, by actual weighing, of the brains of 257 insane men and 213 insane women, who died in the Wiltshire County Asylum, agree very closely with those of Dr. Boyd. The average weight of the former was 46·2 oz., or 1309 grms.; that of the latter 41 oz., or 1162 grms., producing a mean of the men and women of 43·6 oz., or 1236 grms., a little less than the mean derived from Dr. Boyd's results.

6. *Irish.*—The Irish brains of these 28 skulls have been slightly larger than those of the English. Still the mean of the series is less, viz. English, 47·50 oz., or 1346 grms.; Irish, 46·70 oz., or 1323 grms.

8. *French.*—Of these there are 16 examples, 9 of men and 7 of women, or nearly an equality of the sexes. The mean of the series is 45·17 oz., or 1280 grms. They are decidedly below the English.

Dr. Parchappe weighed a few French brains, and the result, as stated by Dr. Thurnam‡,

* *Op. cit.* p. 19.

† "Tables of the Weights of the Human Body and Internal Organs in the Sane and Insane of both sexes at various ages, arranged from 2614 post-mortem examinations. By Robert Boyd, M.D.," Philosophical Transactions, 1861, p. 261.

‡ *Loc. cit.* p. 15.

was an average brain-weight of 47·9 oz., or 1358 grms., *i. e.* two ounces more than our mean.

The results derived from our 16 French skulls may be compared with those obtained by Professor Paul Broca from carefully gauging the internal capacities of a large series of French crania of both sexes*. These were 115, derived from a vault in the Cité at Paris, and considered to be of the XII.th century, which gave a mean capacity of 1425 cub. centims.; 117 derived from the Cemetery of the Innocents, having a mean capacity of 1409·031 cub. centims.; and 125 others from the Cemetery de l'Ouest, at Paris, skulls of the XIX.th century, which gave a slightly greater internal capacity, viz. 1461·53 cub. centims. By first of all subjecting these figures to our rule of the deduction of 15 per cent., and then reducing them to the weight of brain they represent, the following results are produced. The 115 French skulls of the XII.th century must have held brains averaging 44·37 oz., or 1257 grms.; those of the Innocents, 43·93 oz., or 1245 grms.; and those of the XIX.th century, 45·46 oz., or 1288 grms. This gives a mean of the whole 357 French crania, which are both those of men and women, the relative number of the two sexes not having been stated, of 44·58 oz., or 1263 grms., only 17 grms. below our reduced general average,—a result which shows a remarkable agreement between the two different series of observations. Besides which it seems to indicate that the rough allowance of 1300 grms. to the French brain, by Professor Huschke, is sufficiently liberal.

11. *Italians.*—15 skulls of Italians, 13 those of men and only 2 those of women, give averages agreeing closely with those deduced from the crania of the two sexes of English. Dr. Weisbach gauged the skulls of 27 Italian men of the Austrian dominions, and his observations, when reduced to our standard, give an average of 46·19 oz., or 1309 grms.†, again closely agreeing with our series, which afford 47·58 oz., or 1348 grms.

12. *Lapps.*—7 skulls of Lapps, of pure blood, exhibit weights of brain very nearly equal to those of our Italians, and the same as the Italian men of Dr. Weisbach‡.

13. *Swedes.*—12 skulls of Swedes, three-fourths of which are those of men, present a general mean closely agreeing with that of the English.

14, 15. *Frisians, Dutch.*—These skulls again agree in brain-weight with the English.

16. *Germans.*—Of these there are 13 males and 2 females, or a great predominance of the skulls of men, *i. e.* about six-sevenths. Hence it might be anticipated that our mean would be high. It is 50·28 oz., or 1425 grms. It is probable that our crania of the men are unusually large, and those of the 2 women seem to be unusually small. The mean of the series of German skulls exceeds our English mean.

The late Professor Huschke weighed a number of brains of Germans, and these are

* Bulletins de la Société d'Anthropologie de Paris, tome iii. p. 106 (1862).

† Beiträge zur Kentniss der Schädelformen österreichischen Völker, 1867, S. 76, I. Abtheilung.

‡ The Lapps are short people. Among 8 Lapp men the mean stature was found to be 5 ft. 0·3 in., among 3 Lapp women the mean was 4 ft. 8·7 in. Notwithstanding this low stature the brain-weight of the Lapps is seen to be considerable.

the averages he obtained*. In the encephala of 40 adult men the mean weight was 1384 grms., of 22 adult women it was 1244 grms., and the mean of the two sexes, in these unequal numbers, was 1314 grms. In comparing these results of Professor HUSCHKE with our Table, it is seen that our German skulls of men are much above, and those of our women below the average size. The mean of our entire series exceeds that of HUSCHKE'S series by 100 grammes. The late Professor RUDOLPH WAGNER weighed the brains of 31 Germans, 18 of men and 13 of women. The results of these metric observations are that the mean weight of the 18 brains of men was 1392 grms., and of the 13 brains of women 1209 grms., which confirms the remarks already made with respect to our male skulls, and our females also. By combining the means of both sexes, and embracing the observations of HUSCHKE, WAGNER, and those of our Table, we obtain a mean of 1314 grms. as the mean weight of the brain in Germans. In a series of investigations by Professor HERMANN WELCKER, he filled 30 normal adult skulls of German men and 30 of women with husked wheat corns, and then poured the grains into a graduated glass measure, so as to obtain the internal capacity of each in cubic centimetres. By the deduction of 15 per cent., and the conversion of his means into their equivalents of brain, we obtain these results. The average weight of brain of the 30 men was 45·17 oz., or 1280 grms., that of the 30 women 40·50 oz., or 1148 grms., and the mean of the two sexes 42·83 oz., or 1214 grms.† It will thus be seen that the brains of WELCKER'S 60 skulls, both those of the men and those of the women, are smaller than the averages obtained from the three former sources. And the same will be made apparent if we refer to the observations of Dr. WEISBACH, who, however, made them upon the German people of Austria, who cannot be regarded as a pure German race. The average of his 50 men is 47·36 oz., or 1342 grms. of brain, and that of 19 women 40·94 oz., or 1160 grms., the mean of the two sexes being 44·15 oz., or 1251 grms.‡ So that there is a considerable probability that the magnitude of the brain among Germans has been overestimated§.

17. *Russniak.* 18. *Poles.*—We here enter upon the decidedly brachycephalic races of eastern and south-eastern Europe. In this series of skulls to the end of the Table, with the exception of the Wallachian Gipsies, it must be expressly noted that we have the skulls of men only, therefore we get the weights of male brains solely, the great reason why the figures are so high. In this division we shall be able to avail ourselves of the elaborate researches of Dr. A. WEISBACH on the skull-forms of Austrian people. The method he employed was that of filling each skull well with groats, and thus

* Schædel, Hirn und Seele des Menschen und der Thiere, Jena, 1854, S. 113.

† Untersuchungen über Wachsthum und Bau des menschlichen Schädels, 1862, S. 35, 130.

‡ *Op. cit.* S. 34, 56.

§ Without supposing that brain-weight stands in *direct* relation to stature, authentic data concerning the latter deserve attention. Dr. A. WEISBACH gives the results of his observations on the stature of the Germans. The measurement of 30 men afforded an average of 1680 millims., or 5 ft. 6·2 in.; of 11 women 1544 millims., or 5 ft. 0·9 in.—Reise der Novara. Anthropologischer Theil. Körpermessungen. Tabelle VII. 1867.

obtaining the cubic contents in centimetres. There will be this advantage, that Dr. WEISBACH's observations were made upon the crania of young *men* only, soldiers dying in the Military Hospitals of Vienna, with which he was connected. His results will be subjected to the usual deduction of 15 per cent., and converted into their equivalents of ounces of brain, and the corresponding grammes. Our two Poles give a mean brain-weight of 47·14 oz., or 1336 grms. Dr. WEISBACH deduces a mean internal capacity from his 25 Polish skulls of 1517·42 cub. centims. By subjecting this volume to our process we obtain a weight of brain of 47·21 oz., or 1338 grms., which is almost identical with the result of our observations, being but 2 grms. more. The result of WEISBACH's gauging of his 15 Russniak skulls, when reduced, is a brain-weight of 47·07 oz., or 1334 grms., showing that ours is an unusually capacious one.

19. *Czechs.*—Two skulls of men yield exactly the same mean brain-weight as the two Poles, viz. 47·14 oz., or 1336 grms. One of our Czech crania is unusually large. Dr. WEISBACH's mean of 28 Bohemian skulls does not exceed 45·32 oz., or 1284 grms.

20. *Magyar.*—One man yielding a brain-weight of 45·03 oz., or 1276 grms. Dr. WEISBACH's mean of 29 men's crania is 44·30 oz. of brain, or 1255 grms.

21. *Slovak.*—This male skull affords a brain-weight of only 43·78 oz., or 1241 grms. The mean deduced from Dr. WEISBACH's 9 Slovak men is also somewhat low, viz. 45·76 oz., or 1297 grms.*

22. *Finns.* 23. *Russians.* 24. *Turks.*—These three classes yield considerable brain-weights, viz. 48·31 oz., or 1369 grms., 50·28 oz., or 1425 grms., and 47·58 oz., or 1348 grms. The Russians, which are the heaviest, are most probably represented by exceptionally large skulls.

25. *Rumanyos*, or Wallachians.—These people are not a branch of the Sclavic family. The 6 male crania yield a brain-weight of 45·97 oz., or 1303 grms.

26. *Gipsies* of Wallachia.—These consist of 2 skulls of men and 2 of women. They afford an average brain-weight of the men of 43·93 oz., or 1245 grms., and of the women of 43·20 oz., or 1224 grms. Thus exhibiting an unusual uniformity in the two sexes, and also a marked diminution of brain-weight when compared to that of the Rumanyos, among whom they live.

The general average of brain-weights of the whole series of 393 skulls of European Races, in unequal numbers as to sexes, about 3 males to 1 female, is 47·12 oz., or 1335 grms., which must be regarded as a high average.

Without attaching any more importance to it than it deserves, the order of the brain-weights of this series of European crania may here be stated, beginning with the highest and descending to the lowest. Russniak, one single example, a megalocephalic skull, Merovingian Franks, Germans, Russians, Frisians, Spaniards and Portuguese, Finns, Swedes, Italians and Turks, English, Dutch, Ancient Romans of Italy, Poles and Czechs, Irish, Lapps, Ancient Romano-Britons and Rumanyos, Ancient Britons, Anglo-Saxons,

* Dr. WEISBACH found the mean stature of 20 men of Slavic races to approach very closely to that of his German men, viz. 1678 millims.

French, Magyar, Slovak, Ancient Scottish, and Gipsies; the Gipsies standing the lowest in the whole series of European peoples in brain-weight, and, so far, supporting their supposed Indian origin.

TABLE II.—Asiatic Races.

In entering upon the Asiatic Races, we shall perceive a striking diminution in the volume of the brain at the commencement of the series.

1. *Vedahs* of Ceylon.—The three skulls of men give an average of 45·51 oz., or 1290 grms., that of five women is only 39·33 oz., or 1115 grms., the former being kept up by a megalocephalic cranium with an unusual brain-weight of 57·58 oz., or 1632 grms. The general mean of the series is nevertheless only 41·23 oz., or 1168 grms.

2. *Cingalese.*—The 6 men have an average of 47·21 oz., or 1338 grms.; the 5 women of 42·98 oz., or 1218 grms.; the mean of the entire series being 45·32 oz., or 1284 grms.

3. *Afghans.*—It is almost the same with the Afghans as with the Vedahs, the average of the 4 men is 45·32 oz., or 1235 grms., notwithstanding one of the series is a megalocephalic skull giving a brain-weight of 58·96 oz., or 1671 grms. The diversity of magnitude in crania occurring in the same race is well exemplified in two of these male specimens, as given in the Table. The mean of the entire series of males and females is only 42·11 oz., or 1193 grms.

4. *Hindoos.*—The 35 male Hindoos give an average brain-weight of 44·22 oz., or 1253 grms.; the 31 women of 39·99 oz., or 1133 grms.; and the mean of the entire series is 42·11 oz., or 1193 grms., which is exactly the same as that of the Afghans. Both general means are so low that they are almost equalled by that of the small Vedahs of Ceylon. This extraordinary difference in the weight of the brains in the races of India from those of Europe seems to be deserving of the serious consideration of the advocates of the Indo-Germanic, or Aryan hypothesis. The mean brain-weight of Hindoos is one-tenth less than that of Germans and English. Dr. MORTON in his Table introduces his "Indostanic Family," which is composed of 8 "Ayras" and 25 Bengalees, mixed of the two sexes, and when his means are reduced to our standard, according to our rules, they exhibit a general mean of 41·74 oz., or 1183 grms., which is a little less, about 10 grms., than our mean. The results we have thus obtained go far to justify, if they do not entirely justify, the depreciatory estimate of Professor HUSCHKE, when he roughly stated the brain-weight of the Hindoos at from 1000 to 1100 grms.

5. *Mussulmans.*—A comparison of the averages of the brain-weights of these skulls with those of the Hindoos, shows a slight advantage in favour of the Mussulmans.

6. *Khonds.*—The 2 skulls of *men* of this race of the unquestioned aborigines, or Hill Tribes, of India, give a very small mean, only 37·87 oz., or 1073 grms., which is a difference of brain-weight amounting to nearly 6 ounces less than the mean of English *women*.

8. *Lepchas.*—As we ascend the Himalayan slope we meet with races having an

increased weight of brain. The 13 Lepchas, which include almost an equal number of men and of women, give a mean of 44·08 oz., or 1249 grms.

9. *Bodos.* 10. *Bhotias.*—These two races agree in having an equally increased weight of the encephalon, the first of 45·97 oz., or 1303 grms., and the second of 45·90 oz., or 1301 grms.

11. *Mishmees.* 12. *Singpho.* 13. *Nagas.*—These three races of Assam, of whom we have male skulls only, agree in presenting a brain-weight in the medium of 47·80 oz., or 1355 grms.

14. *Siamese.*—The people of Siam are represented by 7 crania, 4 of men and 3 of women, and they come slightly below the races of Assam, the mean being 47·14 oz., or 1336 grms.

15. *Chinese.*—The Chinese are well represented by 33 skulls, 25 of men and 8 of women, or about three-fourths those of men. The mean of the series reaches to 47·00 oz., or 1332 grms*.

16. *Burmese.*—These 5 skulls of men prove that the races of Assam, Siam, China, and Burmah agree in the relative volume of brain, which is a mean on a par with the general mean of European races.

17. *Japanese.*—From the examples here given, which, however, are only one skull of each sex, it might be considered that the Japanese suffer a serious diminution of brain-weight from their neighbours the Chinese and others. The mean of the two is only 40·32 oz., or 1143 grms. Further observations will probably correct this conclusion.

18. *Aïnos.*—The European-looking skulls of the Aïnos of Yesso, present a brain-weight decidedly greater than the Japanese of this Collection.

It will be seen that the general average of the 210 skulls of Asiatic Races is 44·44 oz., or 1259 grms. That of the European Races was 47·12 oz., or 1335 grms., which shows a difference of 76 grms. in favour of Europeans.

Table III.—African Races.

In proceeding to the African Races we commence with

1. *Berbers*, and

2. *Guanches*, the old inhabitants of the Island of Teneriffe. These were people with somewhat small brains; the mean of the series of 21 skulls, the great majority of which are those of women, is 43·49 oz., or 1233 grms.†.

* The mean stature of 26 Chinese men was found by Dr. Weisbach to be 1630 millims., or 5 feet 4·2 inches, of 3 Chinese women 1475 millims., or 4 ft. 10 inches.

† The Guanches, unlike the Negroes of the adjoining Continent, were a race of small stature. The mummy at Cambridge measures 4 feet 10 inches, that of the man in the Museum of the Royal College of Surgeons of England is of the same height, and that of another man in the Museum of the Jardin des Plantes is half an inch taller. The skeleton of the woman in this last collection I found to measure only 4 feet 4 inches. It was a singular misconception on the part of Baron Al. von Humboldt, who said the Guanches were famed for their tall stature, being the Patagonians of the Old World.

3. *Negroes of Tribes unknown.*—These continental Negroes slightly exceed the insular Guanches in brain-weight. There are 16 skulls, three-fourths of which are those of men, and the general mean brain-weight is 44·08 oz., or 1249 grms.

The individual crania of different tribes, of which there are many, give different brain-weights, varying among the men from 46·92 oz., or 1330 grms., a Joloff and a Serryia, both Tribes of the Gambia, to 41·60 oz., or 1179 grms., a Fantee. There are but two of women, a Fantee 39·11 oz., or 1108 grms., and a Mandingoe only 34·15 oz., or 968 grms. These brain-weights deserve to be recorded, but are not sufficiently numerous to be of much value.

10. *Dahomans.*—These 12 crania, of which three-fourths are those of men, and the rest belonged to the Amazonian soldiers of the King of Dahomey, present a higher general mean at 46·34 oz., or 1313 grms.

14. *Bakeles.*—A warlike Tribe on the Equator, of whom there are seven specimens, a majority of which are those of women, present a general mean of the series equal to the Dahomans, viz. 46·41 oz., or 1315 grms.

It may be convenient in this place to express the general means of the series of all these Negro Tribes, from 3. *Negroes of Tribes unknown* to 21. *Manganja.* The mean brain-weights of the 38 men is 45·88 oz., or 1300 grms. That of the 31 women is 41·06 oz., or 1164 grms. And the mean of both sexes is 43·47 oz., or 1232 grms. This is a surprisingly small weight of brain, when we call to mind the assertions of Professor TIEDEMANN upon this subject.

The only Negro brain actually weighed by Professor TIEDEMANN appears to have been that of "HONORÉ," who died at Liège, in 1834, whose brain was preserved in spirits of wine and sent to TIEDEMANN. He used Nuremberg apothecaries weight in his experiments, which he seems to have regarded as equal to our troy weight, but this is not the case*. From the fact of the great change undergone by HONORÉ's brain by hardening in spirit, and the confusion of weights, it is not desirable to dwell upon this observation. The errors into which this distinguished physiologist was led by his sympathies for the Negro race, and which have been so often pointed out, render it advisable that we should refer to more reliable authorities.

Dr. PEACOCK has weighed the brains of 5 Negroes and given a Table, into which he has introduced two observations of Professor JOHN REID. The average brain-weight of the men is 44·34 oz., or 1257 grms., that of the women 43·50 oz., or 1233 grms., and the mean of the sexes 43·92 oz., or 1245 grms.†. This result of actual weighing is exactly 13 grms. more than our mean of the sexes. The brain of a Negro weighed by Professor PAUL BROCA was found to be 925 grms.; but this is not to be regarded as a satisfactory

* The Nuremberg apothecaries weight stands in relation to English apothecaries weight as 357854 to 373095. BALBI, "Abrégé de Géographie," 1833, Tableau Comparatif des Monnaies, et des Poids et Mesures, pp. 1355 and 1295.

† "On the Weight of the Brain of the Negro," Memoirs of the Anthropological Society of London, vol. i. pp. 65 and 520.

observation, since the brain had undergone much decomposition*. The brain of a Negro examined by Dr. EDMOND SIMON was found to weigh, *with the membranes*, 1226 grms.†.

As a general conclusion, without analyzing the results of TIEDEMANN's gaugings of Negro skulls, it may be unhesitatingly asserted that the brain-weight of Negroes is positively below that of Europeans.

23. *Kafirs*.—The average brain-weight of Kafirs is considerable, and quite in contrast with that of the true Negroes. The average of 7 Kafir men is no less than 49·04 oz., or 1390 grms., and that of the series 48·16 oz., or 1365 grms.

TIEDEMANN's friends examined for him four skulls of Kafir men, and one that of a woman. The result of these observations, which, for brevity's sake, shall be reduced to our standard, is for the whole 46·05 oz., or 1305 grms. Perhaps our examples may be of a somewhat unusual size.

25. *Bushmans*.—Finally, we have this singular race of people, inhabiting the self-same countries as the Kafirs and other Tribes, and so remarkably different from them all, especially in the development of the brain. These four skulls of Bushmans, one of a man and three of women, yield a mean brain-weight of only 39·70 oz., or 1125 grms., which is nearly a sixth less than the mean of Europeans, no doubt, a people of greater stature, and about in the same proportion less than their fellow aborigines the Kafirs.

In the elaborate and valuable account of the Brain of a Bushwoman, by Professor JOHN MARSHALL‡, there is a careful calculation of its weight in that specimen. The brain reached him in the cranium preserved in spirits of wine, which had also been injected into the carotid arteries. Professor MARSHALL endeavoured to determine by experiment the amount of loss of weight ensuing from this process of hardening in spirit. His conclusion was that it is "from one-third to one-fourth, *i. e.* as a mean seven-twenty-fourths of the original weight"§. Hence, he determines that in a recent state, when deprived of its membranes, the brain of this Bushwoman, who appears to have been rather aged, would have weighed 30·75 oz., or 875 grms. Our estimate of the weight of the smallest brain of the three Bushwomen is 37·22 oz., or 1055 grms. This is not in agreement with Professor MARSHALL's calculated weight, still, it is fully supported by the statement of Messrs. FLOWER and MURIE, as we shall see, and also by Professor MARSHALL's gauging of the capacity of this Bushwoman's skull.

Besides weighing the brain and carefully restoring its original weight by his experiments to ascertain loss from hardening in spirit, Professor MARSHALL gauged the capacity of the skull itself by filling it with water. He thus found the internal capacity of this Bushwoman's skull was equal to 60·64 cubic inches. Here we have both the cubic capacity of the cranium, and also the actual weight of the brain it contained determined metrologically, which might be regarded as a crucial test of the accuracy of the method we have been led to adopt. By our method followed in the Tables of this Memoir, of

* Bulletins de la Société d'Anthropologie de Paris, tome i. p. 54.

† Ibid. tome iv. p. 350.

‡ Philosophical Transactions, 1864.

§ *Loc. cit.*, p. 506.

allowing a tare of 15 per cent. for membranes and fluids, it is found that a skull of 60·64 cubic inches capacity would contain a brain weighing 31·01 oz. Professor MARSHALL weighed this Bushwoman's brain, and determined its weight in the recent state to have been 30·75 oz., or only ·26 oz., *i. e.* a quarter of an ounce less, which is a very close agreement, showing that in this case a deduction of 15½ per cent. might have been allowed, and that our rule is not excessive, and indeed affording an unexpected and critical proof of the *general* accuracy of the method*. That it is, upon the whole, the readiest and most reliable one for ordinary application in determining the true weight of the brain from the internal capacity of the skull, seems to be proved.

The weight of the Bushwomen's brain dissected by Messrs. FLOWER and MURIE was found to be 38 oz., or 1077 grms.†. This is in close approximation to our deduction from the three Bushwoman's skulls, which yield an average of 39·48 oz., or 1119 grms.‡.

The general mean of our African Races, as deduced from 113 skulls, 53 of men and 60 of women, a tolerable equal proportion, is 43·89 oz., or 1244 grms. This is 3·23 oz., or 91 grms. less than our European general mean.

TABLE IV.—American Races.

1, 2, 3. *Esquimaux.*—We have here included the Esquimaux of the whole Arctic Circle, from Greenland westwards. They have a considerable general mean brain-weight, viz. 46·56 oz., or 1319 grms.

4 to 15. *Iroquois* to *Comanche.*—A series of mostly individual skulls amounting in the whole to 18, 12 of men and 6 of women, which affords a general mean of 46·23 oz., or 1310 grms. With these may be compared the 164 crania of the "Barbarous Tribes" of MORTON's "American Family." When the mean cubic contents of these latter are reduced to our terms, with the observation of our rule, they produce a brain-weight of only 42·84 oz., or 1214 grms., which differs by something more than three ounces from our mean.

14. *Caribs.*—These four skulls, two of each sex, produce a general mean of 42·32 oz., or 1199 grms.

Of South American Tribes.

15. *Muiscas.*—Twelve crania, 8 of men and 4 of women, produce a general mean of 44·92 oz., or 1273 grms. This is not far from the average brain-weight of the Tribes until we reach the

22. *Araucanians.*—These 6 skulls, 5 of which are those of men and one of the five a *megalocephalus*, produce a higher mean, viz. 48·02 oz., or 1361 grms.

The general mean of the whole 83 skulls of American Races, of which 52 are those of

* Perhaps in small brains the membranes will be proportionately heavier than in large ones.

† The Journal of Anatomy and Physiology, May 1867, vol. i. p. 286. Account of the Dissection of a Bushwoman, by W. H. FLOWER, F.R.S., and JAMES MURIE, M.D.

‡ Her stature was 4 feet 7½ inches, or 4 inches less than the Hottentot Venus.

men and 31 those of women, is 44·64 oz., or 1265 grms. This is 2·48 oz., or 70 grms., less than the general mean brain-weight among European Races. It is also so near the general mean of the Asiatic and African Races, as to lead to the conclusion that all three may be regarded in general as nearly equal in the weights of their brains.

Table V.—Australian Races.

We come next to the Australian Races, or the Tribes occupying the Great Australian Continent, and Van Diemen's Land. These are highly interesting people, impressed as they are so deeply with marked differences from the races of all the rest of the globe. These differences extend to their average brain-weights.

1. *Australians.*—These 24 crania of Australians, 17 those of men and 7 of women, afford a general mean brain-weight of 41·38 oz., or 1173 grms.*

2. *Tasmanians.*—Of these there are 11 examples, 7 those of men and 4 of women, and they yield a slightly higher brain-weight than the mean of the Australians. This might have been expected, as they are a distinct race, and also decidedly more robust in physical conformation than the gracile Australians.

The investigation of the 35 skulls of Australian Races, 24 of which are those of men and 11 those of women, gives a general mean brain-weight of 41·81 oz., or 1185 grms. In this they stand apart from the people of all the other great divisions of the globe by possessing the smallest brain. They have a brain-weight which is one-ninth less than that of Europeans.

Table VII.—Oceanic Races.

The last great section into which for convenience we have divided human beings. It includes the aboriginal inhabitants of all the Islands of the North and of the South Pacific Oceans. It embraces very diverse peoples, differing materially in physical conformation and every human character. Some of these races are distinguished by considerable development of the brain, whilst others are as remarkable for deficiency of brain-weight. A few brief remarks upon these peculiarities must suffice.

* Dr. Weisbach states the mean stature of two Australian men to have been 1617 millims., or 5 feet 3·7 inches, and that of two Australian women 1552 millims., or 5 feet 1·2 inch. The two Victorian Australians measured by Dr. Ludwig Becker were respectively 5 feet 2 inches and 5 feet 7½ inches. By adding to these the few reliable replies obtained by the Select Committee of the Legislative Council of Victoria, where the data were acquired by actual measurements, we obtain a mean stature of 15 Australia men of 5 feet 6 inches. (Report on the Aborigines, Victoria, 1858–59.) The only two women said to have been positively measured were respectively 4 feet 10½ inches and 4 feet 11 inches. These, together with Dr. Weisbach's examples, afford a mean of 4 feet 11·5 inches. The heights of three skeletons of male Australians are respectively 1514 millims., or 4 feet 11·7 inches, 1540 millims., or 5 feet 0·6 inch, and 1660 millims, or 5 feet 3·1 inches, in the mean 5 feet 1·9 inch. There is no reason to doubt that the stature of Australians in different regions of the continent varied, even that of different Tribes did not agree. These singular and interesting people afford convincing evidence that the weight of the brain is a special race-character.

1. *Nicobarian**. 4. *Malays.*—Eight skulls, six of men and four of women, afford one of the highest means among the Oceanic Races, viz. 47·07 oz., or 1334 grms. of brain-weight. This might have been anticipated for such a bold and enterprising race who have pushed their migrations, chiefly for commercial purposes, over almost the whole Ocean.

5. *Bankans.* 7. *Madurans*†. 12. *Tidorese.*—These three races are represented by skulls of men only, and hence their mean brain-weights are considerable—53·36 oz., or 1512 grms., 50·79 oz., or 1439 grms., and 50·28 oz., or 1425 grms.

6. *Javans*‡.—The Collection which has afforded the materials for this Memoir is rich in crania from the Dutch dominions in the East Indian Archipelago. It should be observed that among these the skulls of men predominate. Hence the higher means which prevail among the races of these islands.

Among the remaining Islands of the Pacific there are two markedly different brain-weights to be observed, of which examples may be noted here.

29. *Marquesan Islanders.* 30. *Kanakas.*—These peoples are represented by a tolerably fair number of each sex. The former in 16 skulls of men give a mean brain-weight of 47·80 oz., or 1355 grms., and among 11 skulls of women, a mean of 43·20 oz., or 1224 grms; the mean of the whole 16 being 46·56 oz., or 1319 grms. Of the Kanakas, or Sandwich Islanders, there are 67 crania of men, affording a mean brain-weight of 47·89 oz., or 1357 grms., and 54 of women, with a mean of 43·78 oz., or 1241 grms.; the mean of the entire series being 45·97 oz., or 1303 grms. If we suppose these to be representatives of the more highly endowed of the islanders of this vast Ocean, others might be selected to exhibit the less favoured races. Among these are—1. *Nicobarian.* 15. *Dayaks.* 17. *Negritos.* 18. *Oolean.* 20. *Salomon Islanders.* 23. *New Hebrideans.* In some of these, as the Nicobarian, the Negritos and the Oolean, male skulls alone produce a very low brain-weight, as 40·35 oz., or 1143 grms. for the first, 40·53 oz., or 1149 grms. for the next, and 42·13 oz., or 1194 grms., for the third. In others a great preponderance of the skulls of men raises the brain-weight to only a low mean; as the Dayaks, where there are three-fourths of the skulls those of males, the general mean is 44·08 oz., or 1249 grms. The Salomon Islanders offer the same proportion of the sexes, and the mean of the series is 44·15 oz., or 1251 grms. Among the New Hebrideans nine out of ten of the skulls are those of men, yet the mean brain-weight of the series only rises to 43·92 oz., or 1245 grms.

* The mean stature of 51 Nicobar Islanders, men, was found by Dr. Weisbach to be 1668 millims., *i. e.* 5 feet 5·7 inches. The brain-weight of the individual in the Table was only 40·35 oz.

† Dr. Weisbach found the stature of 4 Maduran men to be 1625 millims., or 5 feet 4·1 inches. This is an evidence that, although, as a general rule, the brain is found to be small in people of short races, yet it is not by any means uniformly so. In truth, this and other similar facts show that the size of the brain, like most other human peculiarities, is a race-character.

‡ Dr. A. Weisbach found the average stature of 9 Javan men to be 1679 millims., or 5 feet 6 inches, that of 8 Javan women 1461 millims., or 4 feet 9·6 inches.

25. *Maoris**.

In conclusion, a hope may be expressed that this investigation has shown that some confidence may be placed in the method which has been followed; and that there is reason to think this communication may prove a valid contribution to the important and interesting subject of the weight of the brain in the different Races of Man.

Postscript.

[Received July 31, 1868.]

The value of this communication, so far as it turns upon the proposed tare employed in its calculations, has received an important illustration, which must be alluded to here. The great politeness of an eminent Austrian anatomist, Dr. A. Weisbach, of Vienna, has afforded the means of testing this validity in a more complete manner than could be expected to occur in this country. He has taken the encephala out of 115 skulls of males and females, and carefully weighed them; subsequently, when the cranium was prepared, he has gauged its capacity in each case. The data he has thus obtained, he has had the goodness to communicate. They afford the chief materials for the following Table. The capacity is given in cubic centimetres first, then the estimated weight of the brain as deduced from our method, next the actual weight of the brain, without membranes, fluids and *medulla oblongata*, as determined by Dr. Weisbach. In order to compensate for this deficiency of the medulla oblongata, an addition of 14 grms., or half an ounce av., is made to each mean, so as to show in the next column the real weight of the entire brain. By this addition it is believed that we shall compensate as nearly as may be necessary for the absence of the medulla oblongata, which constitutes a portion of the contents of the skull. Professors Quain and Sharpey, quoting from Dr. Reid, state that the pons Varolii and medulla oblongata conjointly weigh in men 15¾ drs. av., in women 1 oz. and a quarter of a dr.†. By Dr. Robert Boyd it is said, "the average weight of the pons Varolii and medulla varied in the males from 1·15 oz. to 1·02 oz., and from 1·05 to ·95 oz. in the females"‡. Considering these observations to prove that these two portions of the contents of the cranium, the pons Varolii, and the medulla oblongata, weigh on the average about an ounce, we may conveniently regard them as each weighing half an ounce. The last column of the Table shows the exact average amount of the deficiency, or of the excess, of our estimated weight of the brain, in relation to that obtained by Dr. Weisbach's experiments.

* The mean stature of 3 New Zealand men was found by Dr. Weisbach to be 1757 millims., or 5 feet 9·2 inches.

† Elements of Anatomy, 5th ed., 1848, vol. ii. p. 672.

‡ Philosophical Transactions, 1861, p. 262.

Number of cases.	Sexes.	Ages.	Capacity of skulls.	Estimated weights of brains.	Actual weights of brains, without membranes, fluids, and medulla oblongata.	With the addition of 14 grms. for the medulla oblongata.	Differences between estimated weights and actual weights. The former more than the latter +, less than the latter −.
			cub. cent.	grms.	grms.	grms.	grms.
5	♂	10 to 19 years	1436·62	1270·06	1209·85	1223·85	46·21 +
75	♂	20 to 29 years	1535·52	1355·11	1327·43	1341·43	13·68 +
9	♂	30 to 59 years	1556·96	1374·95	1316·12	1330·12	44·83 +
11	♂	60 to 90 years	1550·43	1349·44	1227·21	1241·21	108·23 +
100	♂	10 to 90 years	1533·11	1352·27	1310·06	1324·06	28·21 +
15	♀	.....	1318·30	1165·16	1159·65	1173·65	8·49 −
115	♂ and ♀	.....	1505·09	1329·59	1290·44	1304·44	25·15 +

The results of these tests of our rule of a tare of 15 per cent., for so the experiments of Dr. WEISBACH may be regarded, are that in each of the groups into which the ages are divided our estimated weight exceeds the true weight of the brain, except in the case of the brains of the 15 women. In these the estimated weight falls short of the actual weight by an average of 8·49 grms., or nearly ·30 of an ounce. The other groups stand thus, ascending from the lowest to the highest. In 75 men from 20 to 29 years of age, that is, in the vigour of life (and it is to this period of life that the great majority of the skulls of the preceding race-Tables belong), the estimate exceeds the true weight by only 13·68 grms., or about half an ounce. In the whole number of cases taken together, the excess of the estimate is 25·15 grms., or about ·90 of an ounce. In the entire group of males this excess is 28·21 grms., or as near as may be one ounce. In the 9 males from 30 to 59 years of age, when it seems that the brain begins to have an increment in its fluids, it rises to 44·83 grms., or a trifle more than an ounce and a half. Before this organ reaches its maturity, as in the group of males from 10 to 19 years of age, the excess of our estimate is almost the same, viz. 46·21 grms., or somewhat above an ounce and a half, which shows the large amount of serosity in the brain at this early period of life, when the organ is in a state of active growth. But it is in advanced life, when this serosity again abounds, that our estimate is most remote from the actual weight of the cerebral mass itself. Here the discrepancy amounts to 108·23 grms., or approaching to 4 ounces.

It must be recollected that, although our general rule is of great value in estimating the true weight of the brain from the capacity of the skull, where we have a large number of specimens of different sexes and ages before us, and is perhaps as near as any general rule that can be laid down, it is not to be expected that any uniform rate of deduction can be devised which will be correct even in the majority of individual instances, where the causes of deviation are so varied and complex. The complete test to which our rule has been put by these 115 careful experiments on brains of very different ages, shows its value and general correctness in a more satisfactory manner than could have been anticipated. If further investigations should prove that Dr. WEISBACH's observations may be taken to possess an average value, it will probably be desirable to increase our rate of tare to a slight extent.

XXI. *Further Observations on the Spectra of some of the Stars and Nebulæ, with an Attempt to determine therefrom whether these Bodies are moving towards or from the Earth, also Observations on the Spectra of the Sun and of Comet* II., 1868.

By WILLIAM HUGGINS, *F.R.S.*

Received April 23*,—Read May 14, 1868.

§ I. *Introduction.*

IN a paper "On the Spectra of some of the Fixed Stars"† by myself and Dr. W. A. MILLER, Treas. R.S., we gave an account of the method by which we had succeeded during the years 1862 and 1863 in making trustworthy simultaneous comparisons of the bright lines of terrestrial substances with the dark lines in the spectra of some of the fixed stars. We were at the time fully aware that these direct comparisons were not only of value for the more immediate purpose for which they had been undertaken, namely, to obtain information of the chemical constitution of the investing atmospheres of the stars, but that they might also possibly serve to tell us something of the motions of the stars relatively to our system. If the stars were moving towards or from the earth, their motion, compounded with the earth's motion, would alter to an observer on the earth the refrangibility of the light emitted by them, and consequently the lines of terrestrial substances would no longer coincide in position in the spectrum with the dark lines produced by the absorption of the vapours of the same substances existing in the stars.

The apparatus employed by us was furnished with two prisms of dense flint glass, each with a refracting angle of 60°, and permitted the comparisons to be made with so much accuracy that the displacement of a line, or of a group of lines, to an amount smaller even than the interval which separates the components of FRAUNHOFER'S D would have been easily detected. We were, therefore, in possession of the information that none of the stars the lines in the spectra of which we had compared with sufficient care, were moving in the direction of the visual ray with a velocity so great, relatively to that of light, as to shift a line through an interval corresponding to a difference of wave-length equal to that which separates the components of D. To produce an alteration of refrangibility of this amount, a velocity of about 196 miles per second would be required. The following stars, with some others, were observed with the requisite accuracy:—Aldebaran, α Orionis, β Pegasi, Sirius, α Lyræ, Capella, Arcturus, Pollux, Castor.

[* This date of reception applies to the part of the paper containing the observations on the stars and nebulæ. The part containing the observations on the sun was received April 30; that containing the observations on Comet II., 1868, was received July 2, and is printed in continuation of the paper read May 14.—G. G. S.]

† Philosophical Transactions, 1864, p. 413.

It appeared premature at the time to refer to these negative results, as it did not seem to be probable that the stars were moving with velocities sufficiently great to cause a change of refrangibility which could be detected with our instrument. The insufficiency of our apparatus for this very delicate investigation does not, however, diminish the trustworthiness of the results we obtained respecting the chemical constitution of the stars, as the evidence for the existence or otherwise of a terrestrial substance was made to rest upon the coincidence, or want of coincidence in general character as well as position of *several lines*, and not upon that of a single line.

According to the undulatory theory, light is propagated with equal velocity in all directions, whether the luminous body be at rest or in motion. The change of refrangibility is therefore to be looked for from the diminished or increased distance the light would have to traverse if the luminous object and the observer had a rapid motion towards or from each other. The great relative velocity of light to the known planetary velocities, and to the probable motions of the few stars of which the parallax is known, showed that any alterations of position which might be expected from this cause in the lines of the stellar spectra would not exceed a fraction of the interval between the double line D, for that part of the spectrum.

I have devoted much time to the construction and trial of various forms of apparatus with which I hoped to accomplish the detection of so small an amount of change of refrangibility. The difficulties of this investigation I have found to be very great, and it is only after some years that I have succeeded in obtaining a few results which I hope will be acceptable to the Royal Society.

The subject of the influence of the motions of the heavenly bodies on the index of refraction of light had already, at the time of the publication of our paper in 1864, occupied the attention of Mr. J. C. Maxwell, F.R.S., who had made some experiments in an analogous direction. In the spring of last year, at my request, Mr. Maxwell sent to me a statement of his views and of the experiments which he had made. I have his permission to enrich this communication with the clear statement of the subject which is contained in his letter, dated June 10, 1867.

In 1841, Doppler showed that since the impression which is received by the eye or the ear does not depend upon the intrinsic strength and period of the waves of light and of sound, but is determined by the interval of time in which they fall upon the organ of the observer, it follows that the colour and intensity of an impression of light, and the pitch and strength of a sound will be altered by a motion of the source of the light or of the sound, or by a motion of the observer, towards or from each other*.

Doppler endeavoured by this consideration to account for the remarkable differences of colour which some of the binary stars present, and for some other phenomena of the heavenly bodies. That Doppler was not correct in making this application of his theory is obvious from the consideration that even if a star could be conceived to be moving with

* "Ueber das farbige Licht der Doppelsterne und einiger anderer Gestirne des Himmels," Böhm. Gesell. Abh. ii. 1841–42, s. 465.

a velocity sufficient to alter its colour sensibly to the eye, still no change of colour would be perceived, for the reason that beyond the visible spectrum, at both extremities, there exists a store of invisible waves which would be at the same time exalted or degraded into visibility, to take the place of the waves which had been raised or lowered in refrangibility by the star's motion. No change of colour, therefore, could take place until the whole of those invisible waves of force had been expended, which would only be the case when the relative motion of the source of light and the observer was several times greater than that of light.

In 1845, BALLOT published a series of acoustic experiments which support DOPPLER'S theory in the case of sound. In the same paper BALLOT advances several objections to DOPPLER'S application of his theory to the colours of the stars*.

This paper was followed by several papers by DOPPLER in reply to the objections which were brought against his conclusions†.

In 1847 two memoirs were published by SESTINI on the colours of the stars in connexion with DOPPLER'S theory‡.

More recently, in 1866, KLINKERFUES§ published a memoir on the influence of the motion of a source of light upon the refrangibility of its rays, and described therein a series of observations from which he deduces certain amounts of motion, in the case of some of the objects observed by him.

The method employed by KLINKERFUES has been critically discussed by Dr. SOHNCKE‖.

It may be sufficient to state that as KLINKERFUES employs an achromatic prism, it does not seem possible, by his method of observing, to obtain any information of the motion of the stars; for in such a prism the difference of period of the luminous waves would be as far as possible, annulled. It is, however, conceivable that his observations of the light when travelling from E. to W., and from W. to E., might show a difference in the two cases, arising from the earth's motion through the ether.

Father SECCHI has quite recently called attention to this subject¶. In his paper he states that he has not been able to detect any change of refrangibility in the case of certain stars, of an amount equal to the difference between the components of the double line D. These results are in accordance with those obtained by myself and Dr. MILLER in 1863, so far as they refer to the stars which had been examined by us.

Father SECCHI'S method of using an unrefracted image as a fiducial mark with diverging rays passing through the prisms might, it is conceivable, be open to objection.

* "Akustische Versuche auf der Niederländischen Eisenbahn nebst gelegentlichen Bemerkungen zur Theorie des Hrn. Prof. DOPPLER," Pogg. Ann. B. lxvi, s. 321.

† See Pogg. Ann. B. lxxxi. s. 270, and Pogg. Ann. B. lxxxvi. s. 371.

‡ Memoria sopra i colori delle stelle del catalogo de Baily osservati dal P. BAND SESTINI. Roma, 1847.

§ "Fernere Mittheilungen über den Einfluss der Bewegung der Lichtquelle auf die Brechbarkeit eines Strahls, von W. KLINKERFUES," Nachr. K. G. der W. zu Göttingen, No. 4, s. 33.

‖ "Ueber den Einfluss der Bewegung der Lichtquelle auf die Brechung, kritische Bemerkungen zu der Entdeckung des Hrn. Prof. KLINKERFUES. Von Hrn. Dr. SOHNCKE, Astron." Nachr. No. 1646.

¶ Comptes Rendus, 2 Mars 1868, p. 398.

He appears to consider that, to produce a certain alteration of refrangibility, half the velocity would be required in the case of the approach of a star to that which would be necessary if the star were receding. This is not the case, for equal velocities of separation or approach give equal changes of wave-length. It is true that a difference of an octave is produced by a relative velocity of separation equal to that of light, and by a velocity of approach equal to half that of light; but the difference in length of a wave and its octave below (which is twice as long) is in the same proportion greater than the difference between it and the octave above (which is half as long).

The experiments of M. FIZEAU in connexion with this subject are referred to by Mr. MAXWELL in the following statement of his views and experiments, which was received by me on June 12, 1867.

On the Influence of the Motions of the Heavenly Bodies on the Index of Refraction of Light.

Let a source of light be such that it produces n disturbances or vibrations per second, and let it be at such a distance from the earth that the light requires a time T to reach the earth. Let the distance of the source of light from the earth be altered, either by the motion of the source of light, or by that of the earth, so that the light which emanates from the source t seconds afterwards reaches the earth in a time T'.

During the t seconds nt vibrations of the source of light took place, and these reached the earth between the time T and the time $t+\mathrm{T}'$, that is, during $t+\mathrm{T}'-\mathrm{T}$ seconds. The number of vibrations which reached the earth per second was therefore no longer n, but $n\frac{t}{t+\mathrm{T}'-\mathrm{T}}$.

If v is the velocity of separation of the source of light from the earth, and V the velocity of light between the bodies relative to the earth, then $vt=\mathrm{V}(\mathrm{T}'-\mathrm{T})$, and the number of vibrations per second at the earth will be $n\frac{\mathrm{V}}{\mathrm{V}+v}$.

If V_0 is the velocity of propagation of light in the luminiferous medium, and if v_0 is the velocity of the earth,

$$\mathrm{V}=\mathrm{V}_0-v_0,$$

and the wave-length will be increased by a fraction of itself equal to

$$\frac{v}{\mathrm{V}_0-v_0}.$$

Since v_0 only introduces a correction which is small compared even with the alteration of wave-length, it cannot be determined by spectroscopic observations with our present instruments, and it need not be considered in the discussion of our observations.

If, therefore, the light of the star is due to the combustion of sodium, or any other element which gives rise to vibrations of definite period, or if the light of the star is absorbed by sodium vapour, so as to be deficient in vibrations of a definite period, then the light, when it reaches the earth, will have an excess or defect of rays whose period of vibration is to that of the sodium period as $\mathrm{V}+v$ is to V.

As an example, let us suppose the star to be fixed and the earth to be moving directly away from the star with the velocity due to its motion round the sun. The coefficient of aberration indicates that the velocity of light is about 10,000 times that of the earth in its orbit, and it appears from the observations of ÅNGSTRÖM and DITSCHEINER that the wave-length of the less refrangible of the lines forming D exceeds that of the other by about one-thousandth part of itself. Hence, if the lines corresponding to D in the light of the star are due to sodium in the star, these lines in the starlight will be less refrangible than the corresponding lines in a terrestrial sodium-flame by about a tenth part of the difference between D_1 and D_2.

When the earth is moving towards the star, the lines will be more refrangible than the corresponding terrestrial lines by about the same quantity.

The effect of the proper motion of stars would of course have to be compounded with the effect of the earth's own motion, in order to determine the velocity of approach or separation.

To observe these differences of the light from stars, a *spectroscope* is necessary, that is, an instrument for separating the rays of different periods; and it is immaterial in what direction the refraction of the light through the prisms takes place, because the *period* of the light is the thing to be observed by comparison with that of a terrestrial flame.

If, instead of a spectroscope, an achromatic prism were used, which produces an equal deviation on rays of different periods, no difference between the light of different stars could be detected, as the only difference which could exist is that of their period.

If the motion of a luminiferous medium in the place where the experiment is made is different from that of the earth, a difference in the deviation might be expected according to the *direction* of the ray within the prisms, and this difference would be nearly the same whatever the source of the light.

There are therefore two different and independent subjects of experiment. The one is the alteration in the period of vibration of light due to the relative motion of the stars and the earth. The fact of such an alteration is independent of the form under which we accept the theory of undulations, and the possibility of establishing its existence depends on the discovery of lines in the stellar spectra, indicating by their arrangement that their origin is due to the existence of substances in the star having the same properties as substances found on the earth. Any method of observing small differences in the period of vibration of rays, if sufficiently exact, will enable us to verify the theory, and to determine the actual rate of approach or separation between the earth and any star.

The other subject of experiment is the relation between the index of refraction of a ray and the direction in which it traverses the prism. The essentials of this experiment are entirely terrestrial, and independent of the source of light, and depend only on the relative motion of the prism and the luminiferous medium, and on the direction in which the ray passes through the prism.

The theory of this experiment, however, depends on the form in which we accept the theory of undulations. In every form of the theory, the index of refraction depends on

the retardation which a ray experiences on account of having to traverse a dense medium instead of a vacuum. Let us calculate this retardation.

Let there be a transparent medium whose thickness is a, and let it be supposed fixed. Let the luminiferous ether be supposed to move with velocity v in air, and with velocity v' within the medium. Let light be propagated through the ether with velocity V in air and with velocity V′ within the medium. Then the absolute velocity of the light will be $v+\mathrm{V}$ in air and $v'+\mathrm{V}'$ within the medium, and the retardation, or difference of *time* in traversing a thickness a of the medium and an equal thickness of air, will be

$$a\left(\frac{1}{v'+\mathrm{V}'}-\frac{1}{v+\mathrm{V}}\right);$$

and the retardation in *distance* reckoned as at the velocity, V will be

$$a\left\{\frac{\mathrm{V}}{\mathrm{V}'}-1-\frac{v'}{\mathrm{V}}\left(\frac{\mathrm{V}^2}{\mathrm{V}'^2}-\frac{v}{v'}\right)+\frac{v'^2}{\mathrm{V}^2}\left(\frac{\mathrm{V}^3}{\mathrm{V}'^3}-\frac{v^2}{v'^2}\right)-\&\mathrm{c}.\right\}.$$

Now, according to every form of the theory, $\frac{\mathrm{V}}{\mathrm{V}'}=\mu$, the index of refraction, and according to Fresnel's form of the theory, in which the density of the medium varies as μ^2, the equation of continuity requires that $\frac{v}{v'}=\mu^2$. In this case the second term disappears and the retardation is $a(\mu-1)+$terms in $\frac{v'^2}{\mathrm{V}^2}$, which may be neglected, as V is more than 10000 times v.

Hence, on Fresnel's theory, the retardation due to the prism is not sensibly affected by the motion of the earth. The same would be true on the hypothesis that the luminiferous ether near the earth's surface moves along with the earth, whatever the form of the theory of the medium.

Since the deviation of light by the prism depends entirely on the retardation of the rays within the glass, no effect of the earth's motion on the refrangibility of light is to be expected. Professor Stokes (Phil. Mag. 1846, p. 63) has also given a direct proof of this statement, and the experiment of Arago confirms it to a certain degree of exactness.

In order to test the equality of the index of refraction for light moving in opposite directions through a prism, I employed in 1864 the following arrangement.

I made use of a spectroscope constructed by Mr. Becker, and provided with a tube at right angles to the axis of the observing-telescope, carrying a transparent plate of parallel glass placed between the object-glass and its focus, so as to reflect the light which enters the tube along the axis of the telescope towards the object-glass. In this tube is placed a screen with a vertical slit, in the middle of which is a vertical spider-line so arranged that its virtual image formed by the first surface of the glass plate coincides with the crossing of the spider-lines of the telescope at the principal focus of the object-glass. This coincidence is tested by observing the cross lines through the other telescope, with the two telescopes facing each other. The eyepiece of the second telescope is then removed, and a plane mirror is placed at the focus of the object-glass, perpendicular to the axis, and the telescopes are so adjusted that light entering by the side tube is

reflected down the axis of the first telescope, traverses the prisms in succession, enters the second telescope, is reflected by the mirror at its focus, and emerges from the telescope parallel to its direction at incidence; it then traverses the prisms in the reverse order, and is brought to a focus at the cross lines of the first telescope.

If the deviation of the rays in passing through the prisms from east to west differs from that produced during their passage from west to east, the image of the vertical spider-line formed by the rays which have traversed the prisms twice will not coincide with the intersection of the spider-lines as before.

I have found, however, that when the instrument is properly adjusted, the coincidence is so perfect with respect to rays of all refrangibilities, that the image of the vertical spider-line is seen with perfect distinctness, though the rays which form it have passed twice through three prisms of 60°.

If we observe the coincidence of this image with the intersection of the spider-lines at the focus when the rays pass through the prisms first in the direction of the earth's motion and return in the opposite direction, we may then reverse the whole instrument, so that the rays pursue an opposite path with respect to the earth's motion. I have tried this experiment at various times of the year since the year 1864, and have never detected the slightest effect due to the earth's motion. If the image of the spider-line is hid by the intersection of the cross lines in one position, it remains hid in precisely the same way in the other position, though a deviation corresponding to one-twentieth of the distance of the components of the line D could be easily detected.

On the other hand, M. Fizeau* has observed a difference in the rotation of the plane of polarization according as the ray travels in the direction of the earth's motion or in the contrary direction, and M. Ångström has observed a similar difference in phenomena of diffraction. I am not aware that either of these very difficult observations has been confirmed by repetition.

In another experiment of M. Fizeau, which seems entitled to greater confidence, he has observed that the propagation of light in a stream of water takes place with greater velocity in the direction in which the water moves than in the opposite direction, but that the acceleration is less than that which would be due to the actual velocity of the water, and that the phenomenon does not occur when air is substituted for water. This experiment seems rather to verify Fresnel's theory of the ether; but the whole question of the state of luminiferous medium near the earth, and of its connexion with gross matter, is very far as yet from being settled by experiment.

June 10, 1867. James Clerk Maxwell.

§ II. *Description of Apparatus.*

All the experiments were made with my refractor by Alvan Clark, of 8 inches aperture and 10 feet focal length, which is mounted equatorially, and carried very smoothly

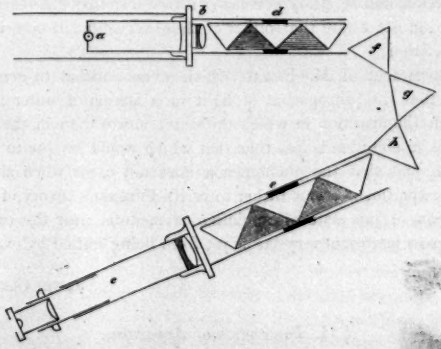
* Ann. de Chimie et de Physique, Feb. 1860.

by a clock-motion. As even on nights of unusual steadiness the lines in the spectra of the stars are necessarily, for several reasons, more difficult of minute discrimination of position than are those of the solar spectrum, it is important that the apparatus employed should give an ample amount of dispersion relatively to the degree of minuteness of observation which it is proposed to attempt.

In 1866 I constructed a spectroscope for the special objects of research described in this paper, which was furnished with three prisms of 60° of very dense flint glass. The solar lines were seen with great distinctness. I found, however, that, in order to obtain a separation of the lines sufficient for my purpose, an eyepiece magnifying ten or twelve diameters was necessary. Under these circumstances the stellar lines were not seen in the continued steady manner which is necessary for the trustworthy determination of the minute differences of position which were to be observed. After devoting to these observations the most favourable nights which occurred during a period of some months, I found that if success was to be obtained, it would probably be with an apparatus in which a larger number of prisms and a smaller magnifying power were employed.

The inconvenience arising from the pencils, after passing through the prisms, crossing those from the collimator when more than three or four prisms are employed, and also, in part, the circumstance that I had in my possession two very fine direct-vision prisms on Amici's principle, which had been made for me by Hofmann of Paris, induced me to attempt to combine in one instrument several simple prisms with one or two compound prisms which give direct vision. An instrument constructed in this way, as will be seen from the following description, possesses several not unimportant advantages*.

a is an adjustible slit; *b* an achromatic collimating lens of 4·5 inches focal length;



e represents the small telescope with which the spectrum is viewed. The train of prisms

* [An apparatus in many respects superior to the one here described has been constructed since.—October 1868.]

consists of two compound prisms, *d* and *e*, and three simple prisms, *f*, *g*, *h*. Each of the compound prisms contains five prisms, cemented together with Canada balsam. The shaded portions of the diagram represent the position of the two prisms of very dense flint glass in each compound prism. The compound prism marked *e* is much larger than the other, and is permanently connected with the telescope *c*, with which it moves. These compound prisms, which were made specially to my order by HOFMANN, are of great perfection, and produced severally a dispersion fully equal to two prisms of ordinary dense flint glass. The prisms *f* and *g* were cut for me from a very fine piece of dense glass of Guinand by Messrs. SIMMS, and have each a refracting angle of 60°. The prism *h* was made by Mr. BROWNING from the dense flint glass manufactured by Messrs. CHANCE, this prism has a refracting angle of 45°. The great excellence of all these prisms is shown by the very great sharpness of definition of the bright lines of the metals when the induction-spark is taken before the slit, even when considerable magnifying power is employed on the small telescope with which the spectrum is viewed. The instrument is provided with a second collimator, of which the object-glass has a focal length of 18 inches.

The compound prism *e* is so fixed that it can be removed at pleasure, when the total dispersive power of the instrument is reduced from about six and a half prisms of 60°, to about four and a half prisms of 60°. The facility of being able to reduce the power of the instrument has been found to be of much service for the observation of faint objects, and also on nights when the state of the atmosphere was not very favourable.

The telescope with which the spectrum is viewed is carried by a micrometer-screw, which, however, has not been employed for taking measures of the spectra, but only for the purpose of setting the telescope to the part of the spectrum which it is intended to observe. This precaution is absolutely necessary when nebulæ are observed which emit light of two or three refrangibilities only.

For the purpose of the simultaneous comparisons of the light of the heavenly bodies with the lines of the terrestrial elements, the slit was provided, in the usual way, with a small prism placed over one half of it, which received the light reflected upon it from a small mirror placed opposite the electrodes. The plan of observation formerly employed, and which is described in the paper "On the Spectra of some of the Fixed Stars," was adopted to ensure perfect accuracy of relative position in the instrument between the star-spectrum and the spectrum to be compared with it, since it is possible by tilting the mirror to alter within narrow limits the position of the spectrum of the terrestrial substance relatively to that of the star. Before commencing an observation, a small alcohol-lamp, in the wick of which bicarbonate of soda was placed, was fixed before the object-glass of the telescope, and then the mirror and the electrodes were so adjusted that the components of the double line D were exactly coincident in both spectra.

This plan was soon found to be very inconvenient, and even in some degree untrustworthy for the more delicate comparisons which were now attempted. An unobserved accidental displacement of the spark, or of the mirror, might cause the two spectra to differ in position by an amount equal to the whole extent of want of coincidence which

it was proposed to seek for in this investigation. The observations of many nights have been rejected, from the uncertainty as to the possible existence of an accidental displacement.

Another inconvenience, so great as even to seem to diminish the hope of ultimate success, was found to arise from the difficulty of bringing the lower margin of the star-spectrum into actual contact with the upper margin of the spectrum of the light reflected into the instrument. The lines in the spectra of the stars are not, on ordinary nights, so steady and distinct as are those of the solar spectrum. Under these difficult circumstances it is very desirable, as an assistance to the eye in its judgment of the absolute identity or otherwise of the position of lines, that the bright lines of comparison should not merely meet the dark lines in the star-spectrum, but that they should overlap them to a small extent. When the two spectra are so arranged as to be in contact, the eye is found to be influenced to some extent by the apparent straightness or otherwise of the compound line formed by the coincident, or nearly coincident lines in the two spectra. Owing to the unavoidable shortness of the collimator the lines in a broad spectrum are slightly curved. From this cause the determination of the identity of lines in spectra which are in contact merely is rendered more difficult, and it may be less trustworthy.

The difficulties of observation which have been referred to were in the first instance sought to be overcome by placing the spark before the object-glass of the telescope. In some respects this method appears to be unexceptionable, but there are disadvantages connected with it. The bright lines, under these circumstances, extend across the star-spectrum, and make the simultaneous observation of dark lines, which are coincident, or nearly so with them, very difficult. When the spark is taken between open electrodes, the consequent disturbance of the air in front of the object-glass is unfavourable to good definition. An important disadvantage arises from the great diminution in the brightness of the spark from the distance (10 feet) at which it is placed from the slit; since in consequence of its nearness to the object-glass, the divergence of the light from it is diminished in a small degree only by that lens. It is obvious that, by means of a lens of short focal length placed between the spark and the object-glass, the light from the spark might be rendered parallel or even convergent; but the adjustments of such a lens, so that the pencils transmitted by it should coincide accurately in direction with the optical axis of the telescope, would be very troublesome. When two Leyden jars, connected as one jar, were interposed, and the spark was taken in air between platinum points, there was visible in the spectroscope only the brightest of the lines of the air-spectrum, namely, the double line belonging to nitrogen, which corresponds to the principal line in the spectra of the gaseous nebulæ. When a vacuum-tube containing hydrogen at a low tension was placed before the object-glass, the line corresponding to F was seen with sufficient distinctness, but the line in the red was visible with difficulty. Some observations, however, have been made with the spark arranged before the object-glass.

The following arrangement for admitting the light from the spark appeared to me to be free from the objections which have been referred to, and to be in all respects adapted

to meet the requirements of the case. In place of the small prism, two pieces of silvered glass were securely fixed before the slit at an angle of 45°. In a direction at right angles to that of the slit, an opening of about $\frac{1}{10}$ inch was left between the pieces of glass for the passage of the pencils from the object-glass. By means of this arrangement the spectrum of a star is seen accompanied by two spectra of comparison, one appearing above, and the other below it. As the reflecting surfaces are about 0·5 inch from the slit, and the rays from the spark are divergent, the light reflected from the pieces of glass will have encroached upon the pencils from the object-glass by the time they reach the slit, and the upper and lower spectra of comparison will appear to overlap to a small extent the spectrum formed by the light from the object-glass. This condition of things is of great assistance to the eye in forming a judgment as to the absolute coincidence or otherwise of lines. For the purpose of avoiding some inconveniences which would arise from glass of the ordinary thickness, pieces of the thin glass used for the covers of microscopic objects were carefully selected, and these were silvered by floating them upon the surface of a silvering solution. In order to ensure that the induction-spark should always preserve the same position relatively to the mirror, a piece of sheet gutta percha was fixed above the silvered glass; in the plate of gutta percha, at the proper place, a small hole was made of about $\frac{1}{10}$ inch in diameter. The ebonite clamp containing the electrodes is so fixed as to permit the point of separation of these to be adjusted exactly over the small hole in the gutta percha. The adjustment of the parts of the apparatus was made by closing the end of the adapting-tube, by which the apparatus is attached to the telescope, with a diaphragm with a small central hole, before which a spirit-lamp was placed. When the lines from the induction-spark, in the two spectra of comparison, were seen to overlap exactly, for a short distance, the lines of sodium from the light of the lamp, the adjustment was considered perfect. The accuracy of adjustment has been confirmed by the exact coincidence of the three lines of magnesium with the component lines of *b* in the spectrum of the moon.

In some cases the spectra produced by the spark are inconveniently bright for comparison with those of the stars and nebulæ. If the spark is reduced in power below a certain point, many of the lines are not then well developed; the plan, therefore, was adopted of diminishing the brightness of the spectrum, by a wedge of neutral-tint glass which can be moved at pleasure between the plate of gutta percha and the silvered mirror.

Two eyepieces were employed with the apparatus, the one magnifying four diameters, and the other six diameters.

The induction-coil was the same which I employed in my former observations. It was excited by a form of bichromate-of-potash battery, which I have found so exceedingly convenient for the occasional work of celestial observations that I will describe it here.

The battery, which was made for me by Mr. Ladd, consists of two large cells of ebonite, each of which contains two plates of graphite, 6 inches by $7\frac{1}{2}$ inches, connected

together with copper bands. These plates are fixed into varnished plates of wood, which form covers to the cells. In each wooden cover is a slit of the length of the width of the carbon plates, by which an amalgamated zinc plate can be inserted between the plates of graphite. An important part of the arrangement is a third large cell of ebonite, which is filled with water acidulated with a few drops of sulphuric acid, and contains at the bottom some mercury. As soon as a plate is removed from the battery after use, it is rubbed clean under running water, and then immersed in the spare cell until it is again required. By this arrangement the plates are always clean and perfectly amalgamated. The solution employed to charge the battery is a saturated solution of bichromate of potash, to which about $\frac{1}{10}$ part of sulphuric acid has been added. The battery is sufficiently powerful and always ready for instant use. For the convenience of varying the power of the battery, three sets of zinc plates are kept in the spare cell; the plates are 2 inches, 4 inches, and 6 inches in width.

§ III. *Observations of Nebulæ.*

For the greater convenience of reference and of comparison, the spectrum of 37 H. IV. Draconis from my paper "On the Spectra of some of the Nebulæ"*, has been added to fig. 2, Plate XXXIII. The spectrum of this nebula may be taken as characteristic, in its general features, of the spectra of all the nebulæ which do not give a continuous spectrum. At present I have determined satisfactorily the general characters of the spectra of about seventy nebulæ. This number forms but a part of the much larger list of nebulæ which I have examined, but in the case of many of these objects their light was found to be too feeble for a satisfactory analysis. Of these seventy nebulæ about one-third give a spectrum of bright lines. The proportion, which is indicated by this examination, of the nebulæ which give a spectrum of bright lines to those of which the spectrum is continuous (namely, as one to two), is probably higher than would result from a wider observation of the objects contained in such catalogues as those of Sir John Herschel and Dr. D'Arrest, since many of the objects which I examined were specially selected, on account of the probability (which was suggested by their form or colour) that they were gaseous in constitution.

All the differences which I have hitherto observed between the spectra of the gaseous nebulæ may be regarded as modifications only of the typical form of spectrum which is represented in the diagram, since they consist of differences of relative intensity, of the deficiency of one or two lines, or of the presence of one or two additional lines. It is worthy of remark that, so far as the nebulæ have been examined, the brightest of the three lines, which agrees in position in the spectrum with the brightest of the lines of the spectrum of nitrogen, is present in *all* the nebulæ which give a spectrum indicative of gaseity. It is a suggestive fact that should not be overlooked, that in no nebula which has a spectrum of bright lines, has any additional line been observed on the less refrangible and brighter side of the line common to all the gaseous nebulæ.

* Philosophical Transactions, 1864, p. 438.

The faint continuous spectrum, which in some cases is also seen, has been traced in certain nebulæ, by its breadth, to a distinct brighter portion of the nebula which it is convenient still to distinguish by the term "nucleus," though at present we know nothing of the true relation of the bright points of the nebulæ to the more diffused surrounding portions.

It must not be forgotten that when gases are rendered luminous there may usually be detected a faintly luminous continuous spectrum. In the case of several of the nebulæ, such as the annular nebula of Lyra and the Dumb-bell nebula, no existence of even a faint continuous spectrum has been yet certainly detected.

The determination of the position in the spectrum of the three bright lines was obtained by simultaneous comparison with the lines of hydrogen, nitrogen, and barium. The instrument which I employed had two prisms, each with a refracting angle of 60°, and the positions of the lines were trustworthy within the limits of about the breadth of the double line D.

The objects which I proposed to myself, in attempting a reexamination of some of the nebulæ with the large instrument described in this paper, were to determine, first, whether any of the nebulæ were possessed of a motion which could be detected by a change of refrangibility; secondly, whether the coincidence which had been observed of the first and the third line with a line of hydrogen and a line of nitrogen would be found to hold good when subjected to the test of a spreading out of the spectrum three or four times greater than that under which the former observations were made. It would not, it seemed, be difficult, in the case of the detection of a want of coincidence, to separate the effects of the two distinct sources referred to, from both of which equally a minute difference of refrangibility between the nebular lines and those of terrestrial substances might arise. The probability is very great indeed that in all the nebulæ which give the kind of spectrum of which I am speaking, the two lines referred to are to be attributed to the same two substances, and that therefore, in all these nebulæ, they were originally of the same degree of refrangibility. On the other hand, it is not to be supposed that nebulæ situated in different positions in the heavens would have a similar motion relatively to the earth. An examination of several nebulæ would therefore show to which of these causes any observed want of coincidence was to be attributed.

The Great Nebula in Orion.—In my description of this nebula* I stated that the light from all the parts of this strangely diversified object, which were bright enough to be observed with my instrument, was resolved into three bright lines similar to those represented in the diagram.

On the present occasion I applied myself in the first place to as careful a comparison as possible of the brightest line with the corresponding line of the spectrum of nitrogen.

My first observations were made with the light from the induction-spark taken in pure nitrogen sealed in a tube at a tension a little less than that of the atmosphere, which

* Proceedings of the Royal Society, vol. xiv. p. 39.

was reflected into the instrument, as in my former series of observations, by means of a mirror and a small prism. The precaution was taken to verify the accuracy of the position of the spectrum of comparison relatively to that of the nebula, by placing a small lamp before the object-glass in the way already described.

The coincidence of the line in the nebula with the brightest of the lines of nitrogen, though now subjected to a much more severe trial, appeared as perfect as it did in my former observations. I expected that I might discover a duplicity in the line in the nebula corresponding to the two component lines of the line of nitrogen, but I was not able, after long and careful scrutiny, to see the line double. The line in the nebula was narrower than the double line of nitrogen; this latter may have appeared broader in consequence of irradiation, as it was much brighter than the line in the nebula.

The following observations are suggestive in connexion with the point under consideration. Electrodes of platinum were placed before the object-glass in the direction of a diameter, so that the spark was as nearly as possible before the centre of the lens. The spark was taken in air. I expected to find the spectrum faint, for the reasons which have been stated in a previous paragraph, but I was surprised to find that only one line was visible in the large spectroscope when adapted to the eye-end of the telescope. This line was the one which agrees in position with the line in the nebula, so that under these circumstances the spectrum of nitrogen appeared precisely similar to the spectra of those nebulæ, of which the light is apparently monochromatic. This resemblance was made more complete by the faintness of the line; from which cause it appeared much narrower, and the separate existence of its two components could no longer be detected. When this line was observed simultaneously with that in the nebula, it was found to appear but a very little broader than that line. When the battery circuit was completed, the line from the spark coincided so accurately in position with the nebular line, that the effect to the eye was as if a sudden increase of brightness in the line of the nebula had taken place. In order to make this observation, and to compare the relative appearance of the lines, the telescope was moved so that the light from the nebula occupied the lower half only of the slit. The line of the spark was now seen to be a very little broader than the line of the nebula, and appeared as a continuation of it in an unbroken straight line. These observations were repeated many times on several nights.

An apparent want of coincidence, which would be represented by 0·02 division of the head of the micrometer-screw, would be about the smallest difference that could be observed under the circumstances under which these observations were made. At the part of the spectrum where this line of nitrogen occurs, the angular interval measured by ·02 division of the micrometer corresponds to a difference of wave-length of ·0460 millionth of a millimetre.

At the time the comparisons were made the earth was receding from the part of the heavens in which the nebula is situated by about half its orbital velocity. If the velocity of light be taken at 185,000 miles per second, and the wave-length of the nitrogen

line at 500·80 millionths of a millimetre, the effect of half the orbital motion would be to degrade the refrangibility of the line by 0·023, an alteration of wave-length which would correspond to about 0·01 of the large micrometer-head, an interval too small to be detected.

We learn from these observations, that if the line be emitted by nitrogen, the nebula is not receding from us with a velocity greater than ten miles per second; for this motion, added to that of the earth's orbital velocity, would have caused a want of coincidence that could be observed. Further, that if the nebula be approaching our system, its velocity may be as much as twenty miles or twenty-five miles per second; for part of its motion of approach would be masked by the effect of the motion of the earth in the contrary direction.

The double line in the nitrogen-spectrum does not consist of sharply defined lines, but each component is nebulous, and remains of a greater width than the image of the slit*. The breadth of these lines appears to be connected with the conditions of tension and of temperature of the gas. PLÜCKER† states that when an induction-spark of great heating-power is employed, the lines expand so as to unite and form an undivided band. Even when the duplicity exists, the eye ceases to have the power to distinguish the component lines, if the intensity of the light be greatly diminished.

Though I have been unable to detect duplicity in the corresponding line in the nebula, it might possibly be found to be double if seen under more favourable conditions; I incline to the belief that it is not double‡.

In my Tables of the lines of the air§ I estimated the brightness of each of the components of the double line in the spectrum of nitrogen at 10, and the components of the double line next in brightness in the orange at 7 and 5, and those of a third double line on the less refrangible side of D at 6 and 4. It was with reference to these two double lines next in apparent brilliancy that I wrote‖, in speaking of the line in the nebula, "If, however, this line were due to nitrogen, we ought to see other lines as well; for there are specially two strong double lines in the spectrum of nitrogen, one at least of which, if they existed in the light of the nebulæ, would be easily visible."

As the disappearance of the whole spectrum of nitrogen, with the exception of the one double line, was unexpected, though, indeed, in accordance with my previous estimations, I examined the spectrum of nitrogen with a spectroscope furnished with one prism with a refracting angle of 60°, in which the whole of the spectrum from C to G is included in the field of view. I then moved between the eye and the little telescope of the spectroscope a wedge of neutral-tint glass corrected for refraction by an inverted similar

* SECCHI states that with his direct spectroscope this line in the annular nebula in Lyra appears double. As the image of the nebula is viewed directly, after elongation by the cylindrical lens, and without a slit, it is probable that the two lines may correspond to the two sides of the elongated annulus of the nebula.

† Philosophical Transactions, 1863, p. 13.

‡ "On the Spectra of the Chemical Elements," Philosophical Transactions, 1864, p. 141.

§ Ibid. ‖ Ibid. p. 443.

wedge of crown glass, and which I had found to be sensibly equal in absorbing power on the different parts of the visible spectrum. As the darker part of the wedge was brought before the eye, the two groups in the orange were quite extinguished, while the lines in the green still remained of considerable brightness. The line which under these circumstances remained longest visible next to the brightest line, was one more refrangible at 2669 of the scale of my map. This observation was made with a narrow slit. When the induction-spark was looked at from a distance of some feet with a direct-vision prism held close to the eye, I was surprised to observe that the double line in the orange appeared to me to be the brightest in the spectrum, and when the neutral-tint wedge was interposed, this line in the orange remained alone visible, all the other lines being extinguished.

When, however, in place of the simple prism a small direct-vision spectroscope provided with a slit was employed, I found it to be possible, by receding from the spark, to find a position in which the double line in the green, with which the line in the nebula coincides, was alone visible, and the spectrum of the spark in nitrogen resembled that of a monochromatic nebula.

It is obvious that if the spectrum of hydrogen were reduced in intensity, the line in the blue, which corresponds to that in the nebula, would remain visible after the line in the red and the lines more refrangible than F had become too feeble to affect the eye.

It therefore becomes a question of much interest whether the one, two, three, or four lines seen in the spectra of these nebulæ represent the whole of the light emitted by these bodies, or whether these lines are the strongest lines only of their spectra which, by reason of their greater intensity, have succeeded in reaching the earth. Since these nebulæ are bodies which have a sensible diameter, and in all probability present a continuous luminous surface, or nearly so, we cannot suppose that any lines have been extinguished by the effect of the distance of these objects from us.

If we had evidence that the other lines which present themselves in the spectra of nitrogen and hydrogen were quenched on their way to us we should have to consider their disappearance as an indication of a power of extinction residing in cosmical space, similar to that which was suggested from theoretical considerations by Cheseaux, and was afterwards supported on other grounds by Olbers and the elder Struve. Further, as the lines which we see in the nebulæ are precisely those which experiment shows would longest resist extinction, at least so far as respects their power of producing an impression on our visual organs, we might conclude that this absorptive property of space is not *elective* in its action on light, but is of the character of a *general* absorption acting equally, or nearly so, on light of every degree of refrangibility. Whatever may be the true state of the case, the result of this reexamination of the spectrum of this nebula appears to give increased probability to the suggestion that followed from my former observations, namely, that the substances hydrogen and nitrogen are the principal constituents of the nebulæ of the class under consideration.

I now pass to observations of the third line of the nebular spectrum, the one which I

found to coincide with the line of hydrogen, which corresponds to FRAUNHOFER'S F. The substance in the nebulæ which is indicated by this line appears to be subject to much greater variation in relative brilliancy, or to be more affected by the conditions under which it emits light; for while the brightest line is always present, the line of which I am speaking seems to be wholly wanting in some nebulæ, and to be of different degrees of relative brightness in some other nebulæ.

In the nebula of Orion this line is relatively stronger than in 37 H. IV. Draconis, and some other nebulæ. I have suspected that the relative brightness of this line varies slightly in different parts of this nebula. It may be estimated perhaps in the nebula of Orion at about the brightness of the second line. The second line suffers in apparent brilliancy from its nearness to the brightest line, and may, without due regard to this circumstance, be estimated as brighter than the third line.

In order to compare the position of the line with that of the corresponding line in the spectrum of hydrogen, I employed a vacuum-tube containing hydrogen at a very small tension, which was placed before the object-glass of the telescope. Under these conditions the line appears narrow, when the slit is narrow, without any sensible nebulosity at the edges. The character of the line is altered, as has been shown by PLÜCKER, when hydrogen at the atmospheric pressure is employed; the line then expands into a nebulous band of considerable width, even with a very narrow slit. Such a condition of the line is obviously unsuitable for the delicate comparisons which it was proposed to attempt.

The narrow, sharply-defined line of hydrogen, when the vacuum-tube was before the slit, was observed to coincide perfectly in position with the third line of the nebula. This observation, which shows the coincidence of these lines with an accuracy three or four times as great as my former observations, increases in the same ratio the probability that the line in the nebula is really due to luminous hydrogen.

I suspect that although the third line in this nebula may impress the eye as strongly as the second line, yet it is not so narrow and well defined as that line. If this suspicion be correct, this condition of the line might indicate that the hydrogen exists at a rather greater tension than that in the so-called vacuum-tubes, but that it is not nearly so dense as would correspond to the atmospheric pressure at the surface of the earth. As, however, the character of the lines of hydrogen is also greatly modified by temperature it is not possible to reason with any certainty as to the state of things in this distant object, the light of which we have now under examination.

I am still unable to find any terrestrial line which corresponds to the middle line. I have made the additional observation that the line in the nebula is in a very slight degree less refrangible than the line of oxygen at 2060 of the scale of my map. It is in a rather larger degree less refrangible than the strong line of barium at 2075 of my scale.

Several other nebulæ have been observed with the large spectroscope, I prefer, however, to re-examine these objects before I publish any observations of them.

§ IV. *Observations of Stars.*

The chief difficulties which I have had to encounter have arisen from the unsteadiness of our atmosphere. There is sufficient light from stars of the first and second magnitude for the large spectroscope described in this paper, and so far as the adjustments of the instrument are concerned, the lines in the spectra of the stars would be well defined. Unless, however, the air is very steady, the lines are seen too fitfully to permit of any certainty in the determination of coincidences of the degree of delicacy which is attempted in the present investigation. I have passed hours in the attempt to determine the position of a single line, and have then not considered that the numerous observations which I had obtained were possessed, even collectively, of sufficient weight to establish with any certainty the coincidence of the line with the one compared with it.

I prefer, therefore, to reject a large number of observations which appear unsatisfactory from this cause, and to give in this place a very few of the most trustworthy of the observations which I have made.

Sirius.—The brilliant light of this star and the great intensity of the four strong lines of its spectrum, make it especially suitable for such an examination. The low altitude of this star in our latitude limits the period in which it can be successfully observed to about one hour on each side of the meridian.

I have confined myself to comparisons of the strong line in the position of F, with the corresponding line of the spectrum of hydrogen. My first trials were made with hydrogen at the ordinary atmospheric pressure; the width of the band of hydrogen, under these circumstances, was greater than the line of Sirius. This line in Sirius, from some cause, is narrower relatively to the length of the spectrum, when considerable dispersion and a narrow slit are employed, than when the image of the star, rendered linear by a cylindrical lens, is observed with a single prism*.

When the large spectroscope was employed I estimated the breadth of the line to be about equal to that of the double line D. In KIRCHHOFF's map the line F of the solar spectrum is represented as a little more than one-fourth of the interval separating the lines D. When the spectroscope attached to the telescope was directed to the moon, the line F appeared even narrower than it is represented in KIRCHHOFF's map; I estimated it at about one-sixth of the apparent breadth of the corresponding line in the spectrum of Sirius. The character of the line agrees precisely with KIRCHHOFF's representation of the solar line F. It appears, as in the diagram, to be equally nebulous at both edges, and agrees in this respect precisely with the line of hydrogen under certain conditions of tension and temperature.

As it was obviously impossible to determine with the required accuracy the coincidence of the line of Sirius when the much broader band of hydrogen at the ordinary pressure was compared with it, I employed a vacuum-tube fixed before the object-glass. In all these observations the slit used was as narrow as possible. The air at the time of the present observations was more favourable than usual, and the line in Sirius was seen with

* See Philosophical Transactions, 1864, p. 42.

great distinctness. The line from the spark appeared, in comparison, very narrow, not more than about one-fifth of the width of the line of Sirius. When the battery circuit was completed, the line of hydrogen could be seen distinctly upon the dark line of Sirius, and extending to some distance on both sides of the spectrum of Sirius. The observation of the comparison of the lines was made many times, and I am certain that the narrow line of hydrogen, though it appeared projected upon the dark line in Sirius, did not coincide with the middle of the line, but crossed it at a distance from the middle, which may be represented by saying that the want of coincidence was *apparently* equal to about one-third or one-fourth of the interval separating the components of the double line D. I was unable to measure directly the distance between the centre of the line of hydrogen and that of the line in the spectrum of Sirius, but several very careful estimations by means of the micrometer give a value for that distance of 0·040 of the micrometer-head. This value is probably not in error by so much as its eighth part.

Comparisons on many other nights were also made, sometimes with the vacuum-tube before the object-glass, and sometimes with the vacuum-tube placed over the small hole in the gutta-percha plate. On all these occasions the numerous comparisons which were made, gave for the line in Sirius a very slightly lower refrangibility than that of the line of hydrogen, but on no one occasion was the air steady enough for a satisfactory determination of the amount of difference of refrangibility.

I have not been able to detect any probable source of error in this result, and it may therefore, I believe, be received as representing a relative motion of recession between Sirius and the earth.

The probability that the substance in Sirius by which this line is produced is really hydrogen, is strengthened almost to certainty by the consideration that there is a strong line in the red part of the spectrum which is also coincident with a strong line of hydrogen. There is a third line more refrangible than F, which appears to coincide with the line of hydrogen in that part of the spectrum.

As the line in Sirius is more expanded than that of the vacuum-tube, it seemed of importance to have proof from experiment that this line of hydrogen, when it becomes broad, expands equally in both directions. I made the comparison of the narrow line of the vacuum-tube with the more expanded band which appears when denser hydrogen is employed. For this purpose the intersection of the wires of the eyepiece was brought, as nearly as could be estimated, upon the middle of the expanded line which is produced by dense hydrogen. The vacuum-tube was then arranged before the slit, when the narrow line which it gives was observed to fall exactly upon the point of intersection of the wire. Under these terrestrial conditions the expansion of the line may be considered to take place to an equal amount in both directions. There is very great probability that a similar equal expansion takes place under the conditions which determine the absorption of light by this gas in the atmosphere of Sirius, for the reason that the nebulosity at the edges of the line in the spectrum of that star is sensibly equal on both sides.

I made some attempts to compare the strong line at C with the corresponding line of hydrogen; but when the large spectroscope was employed, though the lines could be seen with tolerable distinctness, they were not bright enough to admit of a trustworthy determination of their relative position. When one of the compound prisms was removed, the lines were much more easily seen, but under these circumstances the amount of dispersion was insufficient for my present purpose.

The lines of Sirius which, in conjunction with Dr. MILLER, I had compared with those of iron, magnesium, and sodium are not sufficiently well seen in our latitude for comparison, when a powerful train of prisms is employed, such as is necessary for this special inquiry.

From these observations it may, I think, be concluded that the substance in Sirius which produces the strong lines is really hydrogen, as was stated by Dr. MILLER and myself in our former paper. Further, that the aggregate result of the motions of the star and the earth in space, at the time when the observations were made, was to degrade the refrangibility of the line in Sirius by an amount corresponding to 0·040 of the micrometer-screw. Now the value of the wave-lengths of 0·01 division of the micrometer at the position of F is 0·02725 millionth of a millimetre*. The total degradation of refrangibility observed amounts to 0·109 millionth of a millimetre. If the velocity of light be taken at 185,000† miles per second, and the wave-length of F at 486·50 millionths of a millimetre (ÅNGSTRÖM's value is 486·52, DITSCHEINER's 486·49), the observed alteration in period of the line in Sirius will indicate a motion of recession existing between the earth and the star of 41·4 miles per second.

Of this motion a part is due to the earth's motion in space. As the earth moves round the sun in the plane of the ecliptic, it is changing the direction of its motion at every instant. There are two positions, separated by 180°, where the effect of the earth's motion is a maximum, namely, when it is moving in the direction of the visual ray, either towards or from the star. At two other positions in its orbit, at 90° from the former positions, the earth's motion is at right angles to the direction of the light from the star, and therefore has no influence on its refrangibility.

The effect of the earth's motion will be greatest upon the light of a star situated in the plane of the ecliptic, and will decrease as the star's latitude increases, until with

* The value in wave-lengths of the divisions of the micrometer for different parts of the spectrum was determined by the aid of the tables of the wave-lengths corresponding to every tenth line of KIRCHHOFF's map by Dr. WOLCOTT GIBBS (SILLIMAN's Journal, vol. xliii. January 1867). A paper on the same subject by the Astronomer Royal, presented to the Royal Society, is not yet in print. [The Astronomer Royal's paper is contained in the Philosophical Transactions for 1868, Part I. p. 29. The wave-lengths computed by him differ slightly from those assigned to KIRCHHOFF's numbers by Dr. GIBBS at the part of the spectrum under consideration in the text. The difference is due in part to the employment, by the Astronomer Royal, of DITSCHEINER's later measures. These give for F the higher value of 486·87.—October 1868.]

† The new determination of the value of the solar parallax by observations of Mars requires that the usually received velocity of light, 192,000 per second, should be reduced by about the one-twenty-seventh part. The velocity, when diminished in this ratio, agrees nearly with the result obtained by FOUCAULT from direct experiment.

respect to a star situated at the pole of the ecliptic, the earth's motion during the whole of its annual course will be perpendicular to the direction of the light coming to us from it, and will be therefore without influence on its period.

That part of the earth's resolved motion which is in the direction of the visual ray, and which has alone to be considered in this investigation, may be obtained from the following formula:

$$\text{Earth's motion towards star} = v.\cos\lambda.\sin(l - l'),$$

where v is earth's velocity, l the earth's longitude, l' the star's longitude, and λ the star's latitude.

At the time when the estimate of the amount of alteration of period of the line in Sirius was made, the earth was moving from the star with a velocity of about 12 miles per second.

There remains unaccounted for a motion of recession from the earth amounting to 29·4 *miles per second, which we appear to be entitled to attribute to Sirius.*

It may be not unnecessary to state that the solar motion in space, if accepted as a fact, will not materially affect this result, since, according to M. Otto Struve's calculations, the advance of the sun in space takes place with a velocity but little greater than one-fourth of the earth's motion in its orbit. If the apex of the solar motion be situated in Hercules, nearly the whole of it will be from Sirius, and will therefore diminish the velocity to be ascribed to that star.

It is interesting, in connexion with the motion of Sirius deduced from these prismatic observations, to refer to the remarkable inequalities which occur in the rather large proper motion of that star. In 1851 M. Peters* showed that the variable part of the proper motion of Sirius in right ascension might be represented by supposing that Sirius revolves in an elliptic orbit, round some centre of gravity without itself, in a period of 50·093 years. This hypothesis has acquired new interest, and seems indeed to have received confirmation from direct observation by Alvan Clark's discovery of a small companion to Sirius.

Professor Safford† and Dr. Auwers‡ have investigated the periodical variations of the proper motion of Sirius in declination, and they have found that these variations, equally with those in right ascension, would be reconcileable with an elliptic orbital motion round a centre not in Sirius. The close coincidence of the observed positions of the new satellite with those required by theory, seem to show that it may be the hypothetical body suggested by Peters, though we must then suppose it to have a much greater mass relatively to Sirius, than that which its light would indicate.

At the present time the proper motion of Sirius in declination is less than its average

* Astron. Nachrichten, No. 748.

† Proceedings of the American Academy, vol. vi.; also Astron. Notices, Ann. Arbor, No. 28; Monthly Notices, vol. xxii. p. 145.

‡ Astron. Nachrichten, No. 1506; Monthly Notices, vol. xxii. p. 148, and vol. xxv. p. 39.

amount by nearly the whole of that part of it which is variable. May not this smaller apparent motion be interpreted as showing that a part of the motion of the star is *now in the direction of the visual ray*? This circumstance is of much interest in connexion with the result arrived at in this paper.

Independently of the considerations connected with the variable part of the star's proper motion, it must not be forgotten that the whole of the motion which can be directly observed by us is only that portion of its real motion which is at right angles to the visual ray. Now it is precisely the other portion of it, which we could scarcely hope to learn from ordinary observations, which is revealed to us by prismatic investigations. By combining the results of both methods of research we may perhaps expect to obtain some knowledge of the real motions of the brighter stars and nebulæ.

It seems therefore desirable to compare with the result obtained by the prism, the motion of Sirius which corresponds to its assumed constant proper motion. The values adopted by Mr. MAIN*, and inserted by the Astronomer Royal in the Greenwich 'Seven-year Catalogue,' are $-0''{\cdot}035$ in R.A., and $+1''{\cdot}24$ in N.P.D.

The parallax of Sirius from the observations of HENDERSON, corrected by BESSEL, $=0''{\cdot}150$. A recent investigation by Mr. C. ABBE† gives for the parallax the larger value of $0''{\cdot}27$. If the radius of the earth's orbit be taken at its new value of 91,600,000 miles, the assumed annual constant proper motion in N.P.D. of $1''{\cdot}24$ would indicate, with the parallax of HENDERSON, a velocity of Sirius of 24 miles nearly per second, with the larger parallax of Mr. ABBE, a velocity of 43·2 miles per second. It may be that in the case of Sirius we have two distinct motions, one peculiar to the star, and a second motion which it may share in common with a system of which it may form a part.

Observations and comparisons, similar to those on Sirius, have been made on α Canis Minoris, Castor, Betelgeux, Aldebaran, and some other stars. I reserve for the present the results which I have obtained, as I desire to submit these objects to a reexamination. It is seldom that the air is sufficiently favourable for the successful prosecution of this very delicate research.

§ V. *Observations of the Sun.*

Received April 30, 1868.

I have recently applied the large spectroscope described in this paper with some success to the examination of the spectrum of the umbra of a large sun-spot.

Before I describe the results of this examination, I will refer briefly to observations of the sun which I have made on many occasions, since 1864, with three distinct objects in view.

1. I have sought to discover if the spectrum of the light from near the sun's limb differs in any sensible degree from that of the light from the central parts of the disk.

* Memoirs of the Royal Astronomical Society, vol. xix.

† Monthly Notices of the Royal Astronomical Society, vol. xxviii. p. 2.

Since the diminished intensity of the marginal parts of the sun is probably due to the greater depth of solar atmosphere through which it had passed, it was conceivable that by its spectrum the light of these parts of the solar disk might exhibit some indications of the larger amount of absorbing medium which it had traversed.

Two different methods of observation were employed. The telescope, armed with one of the spectroscopes constructed for stellar observations, was directed to the sun. By the aid of the finder the telescope was moved so as to bring in succession upon the slit different parts of the solar disk. Though the detection of any difference depended upon the memory of the observer, still any considerable alteration would certainly have been noticed.

The other plan was to cause the sun's light, after reflection from a plane mirror attached to a heliostat moved by clockwork, to fall upon an achromatic object-glass of 6 feet focal length, by which an image of the sun was formed upon the slit of the large spectroscope which I employed for the examination of the spectra of the chemical elements.

By neither of these methods have I been able to detect any difference in the spectra of different parts of the sun's disk.

2. I have made numerous observations for the purpose of obtaining a view, if possible, of the red prominences which are seen during a solar eclipse. The invisibility of these objects at ordinary times is supposed to arise from the effect of the illumination of our atmosphere. If these bodies are gaseous, their spectra would consist of bright lines. With a powerful spectroscope the light from our atmosphere near the sun's limb would be greatly reduced in intensity by the dispersion of the prisms, while the bright lines of the prominences, if such were present, would remain but little diminished in brilliancy. These observations have been made with different spectroscopes, and also with other contrivances arranged on the same principle, but hitherto without success.

[The observations made in India of the solar eclipse of August 18 have shown that the spectra of the prominences are discontinuous. Lieut. Herschel, R.E., who had charge of the instruments sent out by the Royal Society, determined approximately the position of three bright lines. One in the red about C, one apparently coincident with D, and one near F. On the day after the eclipse, M. Janssen, making use of a method similar to that described above, succeeded in observing the solar protuberances. On October 20, 1868, Mr. Lockyer also succeeded in viewing the spectrum of a protuberance. Now that the positions of the lines are known, one of the "other contrivances" mentioned above will probably succeed. It consisted of screens formed of absorptive media, by which light of all refrangibilities other than those corresponding to the lines of the flames might be absorbed. By this method the flames could be examined and measured.—October 30, 1868.]

3. The third object which I had proposed to myself was to seek to gain from an examination of the spectra of the umbræ and penumbræ of solar spots some information as to the nature of these remarkable phenomena.

I had already made some experiments in this direction when in August 1866 I received

a note from M. FAYE, in which he suggests to me the prismatic examination of solar spots, since, according to his theory of the constitution of the sun, the spectrum of the umbra of a spot should be compound, consisting of a continuous spectrum with dark lines, and a second spectrum of bright lines. My first observations were made with a direct-vision spectroscope of HOFMANN, which was so arranged that the image of the sun was formed upon the slit, after the light had been enfeebled by reflection from a prismatic solar eyepiece.

When, by means of the finder, a spot was brought upon the slit, the feeble light from the umbra appeared as a narrow dark band upon the bright solar spectrum. The lines of FRAUNHOFER appeared stronger and thicker in the spectrum of the umbra.

In October 1866, Mr. LOCKYER, who had independently made similar observations, presented a paper to the Royal Society*, in which he states that he observed the lines of absorption of the solar spectrum to appear thicker where they crossed the spectrum of the spot. He also states that he saw no indication of the presence of bright lines.

It was not until April 15, 1868, that a favourable opportunity occurred to examine a large spot with the new spectroscope described in this paper.

The presence of some haze in the atmosphere permitted the spectroscope to be applied directly to the telescope, and the slit to be placed at the focus of the object-glass. The slit was rotated so that its length was in the direction of the length of the spot, and when the middle of the umbra fell upon the slit, its spectrum appeared as a feebly illuminated band upon the bright solar spectrum. The band appeared divided into two parts by the spectrum of the bright prominence, which extended nearly across the umbra.

The phenomenon of an increase of thickness of the lines of FRAUNHOFER, which I had previously observed, was very marked.

It was obvious that a part only of the light which appeared to form the spectrum of the umbra came from that particular region of the sun. The imperfect transparency of our atmosphere causes it to become strongly illuminated when the sun shines upon the earth; and the brilliant light which is seen to be radiated by it near the sun's limb, is also radiated by that portion of the atmosphere which is between the observer and the sun. It might be, therefore, that the whole of the light which appeared to come from the umbra was really due to the illuminated intervening atmosphere. That such was not the case, and that some part of the light to which the spectrum was due came from the umbra, became evident when the telescope was moved so as to bring the sun's limb across the slit; for then the spectrum of the light from the atmosphere was less bright than the spectrum of the umbra of the spot.

In order to obtain some estimate as to how much of the apparent light from the umbra really came from it, I made use of a graduated wedge of neutral-tint glass. The spectrum of our atmosphere at the sun's limb became so dark that the lines could not be distinguished when the part of the wedge marked 10 was before the eye. To bring

* Proceedings of the Royal Society, vol. xv. p. 256.

the spectrum of the umbra to about the same degree of invisibility the wedge had to be moved until the part marked 20 came before the eye. A photometric examination of the wedge shows that the light intercepted at 10 is to that intercepted at 20 as 1 to 3. It may then be concluded that about three-fourths of the light which formed the spectrum of the umbra was really due to the umbra of the spot.

There were several bright granules on the umbra, but the spectra of these were seen distinct from that of the umbra. Each bright point as it came upon the slit gave a narrow spectrum like a bright thread extending along the dark spectrum of the umbra.

There still remained two sources of uncertainty. 1. In consequence of the mode in which the spectrum is formed, under similar conditions of the instrument, the dark lines should appear rather thicker when the light is feebler. 2. The increased thickness of the lines in the compound spectrum might be due to the light of the umbra, or to that of our atmosphere.

The uncertainty on both these points was removed by observing the feebler spectrum of the illuminated atmosphere near the sun's limb. The lines in this spectrum, though they appeared very slightly stronger, were not so in a degree that could afford an explanation of the very marked increase of strength which most of them presented in the spectrum of the umbra. It seemed, therefore, satisfactorily determined that the light from the umbra had really suffered a more powerful absorption. The term umbra is used to include the cloudy stratum and the darker nucleus into which Mr. Dawes has shown the umbra of a spot may be usually resolved. It is probable that nearly the whole of the light under examination came from the part of the umbra known as the cloudy stratum. It was not possible to distinguish the spectra of these portions of the umbra.

The spectroscope was sufficiently powerful to show all the lines which are given in Kirchhoff's maps.

I carefully examined the spectrum of the umbra with that of the adjoining parts of the solar surface from A to G, but I was not able to detect any line of absorption in the spectrum of the umbra which was not also present in that of the sun's normal surface, or that any ordinary solar line was wanting in that of the umbra.

The increase of thickness, however, did not appear to take place in the same proportion for all lines. The lines C and F, due to hydrogen, appeared increased but very slightly, if indeed they were any thicker than would be due to a spectrum of feebler intensity. I incline to the opinion that these lines are not sensibly altered.

There is a small group of lines a little less refrangible than *b*, at 1601 to 1609, of Kirchhoff's scale, and which in his map are marked as coincident with lines of chromium, which was especially noticeable from increased thickness. That this circumstance was not connected with any peculiarity of the spot under examination is shown by a similar observation having been made on other spots.

Fig. 3, Plate XXXIII. represents the appearance of the double line D in the spec-

trum of the umbra. The line nearly central between the two lines may be due, in part at least, also to sodium*. These lines appeared slightly broader, as if by the addition of a faint and narrowed nebulosity at both edges.

The group of lines at B are stronger, also *b* and E. Many of the lines marked in KIRCHHOFF'S map as coincident with iron appeared much stronger in the spectrum of the spot.

The absence of sensible increase in F was marked in comparison with a line or lines which were very strong, situated at a little distance on the less refrangible side of F—probably those of 2066·2 and 2067·1 of KIRCHHOFF'S map.

It may be well to consider some of the conditions of the solar surface by which the phenomena observed may have been produced. A cooler state of the heated vapours by which the lines of absorption are produced, would diminish the radiation from the gas itself, and so leave more completely uncompensated the absorption by the gas of the light from behind it. This cause would produce increased blackness of the lines, but would not account for more than a slight apparent increase of breadth. The greater breadth of the lines seems to point rather to a condition of the gases in which their power of absorption embraces for each line an increased range of wave-length. That the power of absorption of gases varies in this respect is shown by the increase of breadth which some of the bright lines of some gases assume under altered conditions of tension and temperature. It will be sufficient to refer to the expansion of the lines of hydrogen as the tension increases. A similar increase in the range of its power of absorption on light passing through it should take place under similar conditions of density and temperature.

The phenomena may point to an increase of density in the vapours existing within the umbra. Such a state of things would necessarily exist at a point somewhat nearer the sun's centre; but we do not know through how great a depth of gas below the photosphere we receive the light which comes from the umbra. Our views on this point will be connected with the interpretation we give to Mr. DAWES'S discovery of the existence within the umbræ of spots of a still darker part almost wholly devoid of light. Does this nucleus represent a more complete unveiling of the inner part of the sun? or does it show a still cooler and less luminous part of the down-rushing solar atmosphere? The latter suggestion, which is in accordance with the explanation of sun-spots proposed by Dr. BALFOUR STEWART, would seem to connect a lower temperature with the broader lines of absorption.

Some information might be gained if we could view the spectra of the dark pores of the solar surface, an observation which is perhaps not impossible, since these pores are of varying degrees of darkness, and are probably due to conditions of the solar substance

* The spectrum of the sodium at a high temperature is much more complex than was supposed. In addition to the three double lines besides D and a nebulous band, described in my paper "On the Chemical Elements," Philosophical Transactions, 1864, p. 147, there is a line nearly central between the lines D. It is possible there may be also other lines in the interval between the strong lines D_1 and D_2.

beneath the photosphere, similar to those which exist in the less normal phenomenon of sun-spots.

Mr. LOCKYER's observations and my own would seem to show that probably no considerable part of the light which emanates from the umbra of a spot is due to luminous gas. It does not appear to me that this negative evidence is of great weight as to the complete absence of light in the umbra from such a source. The luminous gas would almost certainly emit light of the same refrangibility as some of the dark lines of the solar spectrum; and if there existed above the spot the vapours of the same substances in a cooler state, the light might be wholly absorbed, and the feebler emanations of the still luminous but cooler vapours might not do more than render somewhat less intense the dark gaps produced by the vapours in the stronger light of all refrangibilities which is evidently present.

What may be the source of the light which gives the continuous spectrum of the umbra we know not. It is not impossible that the dense and intensely heated gases which probably form the inner substance of the sun, may in some cases emit lines so greatly expanded as to form, when numerous spectra are superposed, a sensibly continuous spectrum. In addition to this consideration, Dr. B. STEWART has suggested that, as gases possess a power of *general* absorption of light, a heated mass of gas if sufficiently dense to be opake, or partly so, would give a continuous spectrum as well as the spectrum of bright lines peculiar to it. It may be that, notwithstanding the high temperature, some substances may exist in the liquid state in consequence of the pressure produced by the sun's mass.

§ VI. *Observations of Comet* II., 1868.

Received July 2, 1868.

On June 13 a comet was discovered by Dr. WINNECKE, and also independently the same night by M. BECQUET, Assistant Astronomer at the Observatory of Marseilles.

I was prevented by buildings existing near my observatory from making observations of this comet before June 22. On that evening the comet was much brighter than BRORSEN's comet, a description of the spectrum of which I recently presented to the Royal Society*, and it gave a spectrum sufficiently distinct for measurement and comparison with the spectra of terrestrial substances.

Telescopic appearance of the Comet.—A representation of the comet as it appeared on June 22 at 11 P.M. is given in fig. 1, Plate XXXIII. The comet consisted of a nearly circular coma, which became rather suddenly brighter towards the centre, where there was a nearly round spot of light. The diameter of the coma, including the exterior faint nebulosity, was about 6′ 20″. The tail, which was traced for more than a degree, was sharply defined on the following edge, but faded so gradually away on the opposite side that no limit could be perceived. No connexion was traced between the tail and

* Proceedings of the Royal Society, vol. xvi. p. 386.

the brighter central part of the coma. The circular form of the coma was uninterrupted on the side of the tail, which appeared as an extension of the faint nebulosity which formed the extreme margin of the coma.

The bright roundish spot of light in the centre, when examined with eyepieces magnifying from 200 to 600 diameters, presented merely a nebulous light without a defined form.

Spectrum of the Comet.—When a spectroscope furnished with two prisms of 60° was applied to the telescope, the light of the comet was resolved into three very broad bright bands, which are represented in the diagram.

In the two more refrangible of these bands the light was brightest at the less refrangible end, and gradually diminished towards the other limit of the bands. This gradation of light was not uniform in the middle and brightest band, which continued of nearly equal brilliancy for about one-third of its breadth from the less refrangible end. This band appeared to be commenced at its brightest side by a bright line.

The least refrangible of the three bands did not exhibit a similar marked gradation of brightness. This band, though of nearly uniform brilliancy throughout, was perhaps brightest about the middle of its breadth.

These characters, which are peculiar to the light emitted by the cometary matter, must be distinguished from some appearances which the bands assumed in consequence of the mode of distribution of the light in the coma of the comet. The two more refrangible bands became narrower towards their most refrangible side, as well as diminished in brightness. This appearance was obviously not due to any dissimilarity of the light in the parts of the coma, but to the circumstance that as the light of the coma became brighter towards the centre, it was emitted by a smaller area of the cometary matter. The strong light of the central spot could be traced the whole breadth of the band; but the light surrounding this spot, in proportion as it became fainter and broader, was seen for a shorter distance, so that the light from the faintest parts near the margin of the coma was visible only at the brightest side of the band. Since in the least refrangible band a similar gradation of light did not take place, this band appeared of nearly the same width throughout.

The increasing brightness of the coma up to the brilliant spot in the centre showed itself in this band as a bright axial line fading off gradually in both directions.

On this evening I took repeated measures of the positions of these bands with the micrometer attached to the spectroscope. These measures give the following numbers for the commencement and termination of the three bands on the scale adopted in the diagram.

First band	1094	Second band	1298	Third band	1589
	1196		1440		1700

I could not resolve the bands into lines. When the slit was made narrow the bands became smaller both in breadth and length, from the invisibility of the fainter portions. I suspected, however, the presence of two or three bright lines in the bright central part

of the middle band near its less refrangible limit. This part would consist chiefly of light from the bright central spot.

As has been stated, the middle band commences probably with a bright line; for the limit of the band is here abrupt and distinct. On the contrary the exact point of commencement and termination of the other bands could not be observed with certainty.

I could perceive no other bands, nor light of any kind beyond the three bands, in the parts of the spectrum towards the red and the violet.

When the marginal portions of the coma were brought upon the slit, the three bands of light could still be traced. When, however, the spectrum became very faint, it appeared to me to become continuous; but the light was then so very feeble that it could not be traced beyond the three bands towards the violet or the red.

On this evening I observed the spectrum of the comet in a larger spectroscope, which gives a dispersion equal to about five prisms. In this instrument the middle band was well seen. It retained its nebulous, unresolved character, and the abrupt commencement, as if by a bright line, already mentioned, was distinctly seen.

For convenience of comparison, the spectrum of BRORSEN'S comet, and that of the gaseous nebulæ, have been added to the diagram, fig. 2, Plate XXXIII. The spectrum of BRORSEN'S comet consisted of three bright bands and a faint continuous spectrum. These bands appeared, as represented in the diagram, narrower than those of the comet now under examination. It is not possible to say to what extent this circumstance may be due to the much feebler light of this comet. Though the bands of BRORSEN'S comet fall within the limits of position occupied by the broad bands of Comet II., they do not correspond to the brightest parts of these bands. In the middle band I suspected two bright lines, which appeared shorter than the band, and may be due to the nucleus. BRORSEN'S comet differed from the two small comets which I had previously examined* in the much smaller relative proportion of the light which forms a continuous spectrum. In BRORSEN'S comet the bright middle part of the coma seemed to emit light similar to that of the nucleus, in the other comets the coma appeared to give a continuous spectrum. The three comets resembled each other in the circumstance that the light of the central part was emitted by the cometary matter, while the surrounding nebulosity reflected solar light.

It will be seen in the diagram that the bands of BRORSEN'S comet, and those of Comet II., occupy positions in the spectrum widely removed from those in which the lines of the nebulæ occur. The spectra of the gaseous nebulæ consist of true lines, which become narrow as the slit is made narrower.

The following day I carefully considered these observations of the comet with the hope of a possible identification of its spectrum with that of some terrestrial substance. The spectrum of the comet appeared to me to resemble some of the forms of the spectrum of carbon which I had observed and carefully measured in 1864. On comparing the

* Comet I., 1866. Proceedings, vol. xv. p. 5, and Comet 1867, Monthly Notices of Royal Astronomical Society, vol. xxvii. p. 288.

spectrum of the comet with the diagrams of these spectra of carbon, I was much interested to perceive that the positions of the bands in the spectrum, as well as their general characters and relative brightness, agreed exactly with the spectrum of carbon when the spark is taken in olefiant gas.

These observations on the spectrum of carbon were undertaken in continuation of my researches "On the Spectra of the Chemical Elements"*. I have not presented them to the Royal Society, as they are not so complete as I hope to make them.

Though the essential features of the spectrum of carbon remained unchanged in all the experiments, certain modifications were observed when the spectrum was obtained under different conditions. One of these modifications, which was referred to in my paper "On the Spectra of the Chemical Elements"†, may be mentioned here. One of the strongest of the lines of carbon is a line in the red a little less refrangible than the hydrogen line, which corresponds to FRAUNHOFER'S C. Now this line is not seen when the carbon is subjected to the induction-spark in the presence of hydrogen. Two of the other modifications of the spectrum of carbon are given in fig. 2. The first spectrum represents the appearance of the spectrum of carbon when the induction-spark, with Leyden jars intercalated, was taken between the points of wires of platinum sealed in glass tubes, and placed almost in contact in olive-oil. In this spectrum are seen the principal strong lines which distinguish carbon. The shading of fine lines which accompanies the strong lines cannot be accurately represented on account of the small size of the diagram. A spectrum essentially the same is produced when the spark is taken in a current of cyanogen. It may be mentioned that when the heating-power of the spark was reduced below a certain limit, though the decomposition of the oil still took place, the carbon was not volatilized, and the spectrum was continuous.

The third spectrum in the diagram represents the modification of this typical spectrum when the induction-spark is taken in a current of olefiant gas. The highly heated vapour of carbon emits light of the same refrangibilities as in the case of the oil; but the separate strong lines, with a similar power of spark, were no longer to be distinguished. The shading, when the carbon was obtained from olefiant gas, was not composed of numerous fine lines, but appeared as an unresolved nebulous light.

Of course in all these experiments the lines of the other elements present were also seen, but they were known, and could therefore be disregarded.

In the case of the spark in olefiant gas, the three bands in the diagram constitute the whole spectrum, with the exception of a faint band in the more refrangible part of the spectrum.

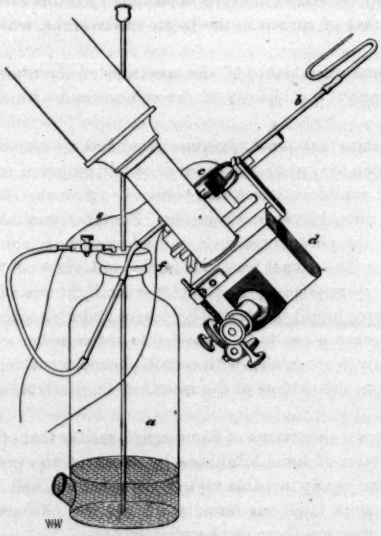
It was with the spectrum of carbon, as thus obtained, that the spectrum of the comet appeared to agree. It seemed, therefore, to be of much importance that the spectrum of the spark in olefiant gas should be compared directly in the spectroscope with the spectrum of the comet. The comparison of the gas with the comet was made the same evening, June 23.

* Philosophical Transactions, 1864, p. 139. † Ibid. p. 145.

My friend, Dr. WILLIAM ALLEN MILLER, visited the observatory on this evening, and kindly took part in the following observations.

The general arrangement of the apparatus with which the comparison was made is shown in the following diagram.

A glass bottle converted into a gas-holder, *a*, contained the olefiant gas. This was connected by means of a flexible tube, with a glass tube *b*, into which were soldered two platinum wires. The part of the tube in front of the points of the wires had been cut away, and the surfaces carefully ground. A small plate of glass closed the opening by being held in its place by a band of vulcanized india-rubber. This tube was arranged in its proper



position before the small mirror of the spectroscope *c*, by which the light of the spark was reflected into the instrument, and its spectrum was seen immediately beneath the spectrum of the comet. The spectroscope employed was furnished with two prisms of 60°.

The brightest end of the middle band of the cometic spectrum was seen to be coincident with the commencement of the corresponding band in the spectrum of the spark. As this limit of the band was well defined in both spectra, the coincidence could be satisfactorily observed up to the power of the spectroscope; and may be considered to be determined within about the distance which separates the components of the double line D. As the limits of the other bands were less distinctly seen, the same amount of certainty of

exact coincidence could not be obtained. We considered these bands to agree precisely in position with the bands corresponding to them in the spectrum of the spark.

The apparent identity of the spectrum of the comet with that of carbon rests not only on the coincidence of position in the spectrum of the bands, but also upon the very remarkable resemblance of the corresponding bands in their general characters, and in their relative brightness. This is very noticeable in the middle band, where the gradation of brightness is not uniform. This band in both spectra remained of nearly equal brightness for the same proportion of its length.

On a subsequent evening, June 25, I repeated these comparisons, when the former observations were fully confirmed in every particular. On this evening I compared the brightest band with that of carbon in the larger spectroscope, which gives a dispersion of about five prisms.

The remarkably close resemblance of the spectrum of the comet to the spectrum of carbon necessarily suggests the identity of the substances by which in both cases the light was emitted.

It may be well to state that some phosphorescent and fluorescent bodies give discontinuous spectra in which the light is restricted to certain ranges of refrangibility. There are, however, several considerations which seem to oppose the idea that the light of comets can be of a phosphorescent character. Phosphorescent bodies are usually so highly reflective that the phosphorescence emitted by them is not seen so long as they are exposed to light. This comet was still in the full glare of the sun, and yet the continuous spectrum corresponding to reflected solar light was of extreme feebleness compared with the three bright bands which we have under consideration. The phenomenon of phosphorescence seems to be restricted to bodies in the solid state, a condition which is not apparently in accordance with certain phenomena which have been observed in large comets, such as the outflow of the matter of the nucleus, and the formation of successive envelopes.

There are, indeed, some phenomena of fluorescence, such as that of a nearly transparent liquid becoming an object of some brightness by means of the property which it possesses of absorbing the nearly invisible rays of the spectrum, and dispersing them in a degraded and much more luminous form, which are less obviously inconsistent with cometary phenomena than are those of phosphorescence.

The violent commotions and internal changes which we witness in comets when near the sun seem, however, to connect the great brightness which they then assume more closely with that part of the solar force we call heat. There is also to be considered the fact of the polarized condition of the light of the tail and some parts of the comæ of comets, which shows that a part of their light is reflected.

The observations of the spectrum of Comet II. contained in this paper, which show that its light was identical with that emitted by highly heated vapour of carbon, appear to be almost decisive of the nature of cometary light. The great fixity of carbon seems indeed to raise some difficulty in the way of accepting the apparently obvious inference

of these prismatic observations. Some comets have approached the sun sufficiently near to acquire a temperature high enough to convert even carbon into vapour*. Indeed for these comets a body of great fixity seems to be necessary. In the case of comets which have been submitted to a less fierce glare of solar heat, it may be suggested that this supposed difficulty is one of degree only; for we do not know of any conditions under which even a gas, permanent at the temperature of the earth, could maintain sufficient heat to emit light, a state of things which appears to exist permanently in the case of the gaseous nebulæ.

If the substance of the comet be taken to be pure carbon, it would appear probable that the nucleus had been condensed from the gaseous state in which it existed at some former period. It would therefore probably consist of carbon in a state of excessively minute division. In such a form it would be able to take in nearly the whole of the sun's energy, and thus acquire more speedily a temperature high enough for its conversion into vapour. In the liquid or gaseous state, or in a continuous solid state, this substance appears, from Dr. TYNDALL'S researches, to be diathermanous. Still, under the most favourable of known conditions, the solar heat, to which the majority of comets are subjected, would seem to be inadequate to the production of luminous vapour of carbon.

It should be stated that olefiant gas when burnt in air may give a similar spectrum of shaded bands. If the gas be ignited at the orifice of the tube from which it issues, the flame is brilliantly white, and gives a continuous spectrum. When a jet of air is directed through the flame it becomes less luminous, and of a greenish-blue colour. The spectrum is now no longer continuous, but exhibits the bands distinctive of carbon. Under these circumstances, for obvious reasons, the bright lines of the hydrogen spectrum are not seen. In this way a spectrum resembling that of the comet may be obtained, with the difference that the fourth more refrangible band, which was not seen in the cometic spectrum, is stronger relatively to the other bands, than is the case when the spark is taken in olefiant gas. If we were to conceive the comet to consist of a compound of carbon and hydrogen, we should diminish in some degree the necessity for the excessively high temperature which pure carbon appears to require for its conversion into luminous vapour; but other difficulties would arise in connexion with the decomposition we must then suppose to take place; for we have no evidence, I believe, that olefiant gas or any other known compound of carbon can furnish this peculiar spectrum of shaded bands without undergoing decomposition. If, indeed, it were allowable to suppose a state of combustion, with oxygen or some other element, set up by the solar heat, we should have an explanation of a possible source of a degree of heat sufficient to render the cometary matter luminous, and which the sun's heat would be directly inadequate to produce.

* The comet of 1843 "approached the luminous surface of the sun within about a seventh part of the sun's radius. The heat to which the comet was subjected (a glare as that of 47,000 suns, such as we experience the warmth of) surpassed that in the focus of PARKER'S great lens in the proportion of 24½ to 1 without, or 3½ to 1 with the concentrating lens. Yet that lens so used melted cornelian, agate, and rock-crystal."—Sir JOHN HERSCHEL, Outlines of Astronomy, 7th edit. p. 401.

There is one observation made by BUNSEN which appears to stand as an exception to the rule that only bodies in the gaseous state give, when luminous, discontinuous spectra. BUNSEN discovered that solid erbia, when heated to incandescence, gives a spectrum containing bright bands. It is therefore conceivable, though all the evidence we possess from experience is opposed to the supposition, that carbon might exist in some form in which it would possess a similar power of giving a discontinuous spectrum without volatilization. There is the further objection to this hypothesis, that the telescopic phenomena observed in comets appear to show that vaporization does usually take place.

However this may be, a state of gas appears to accord with the very small power of reflexion which the matter of the coma of this comet possessed, as was shown by the great faintness of the continuous spectrum.

A remarkable circumstance connected with comets is the great transparency of the bright cometary matter. The most remarkable instance is that of Miss MITCHELL's comet in 1847, which passed centrally over a star of the fifth magnitude. "The star's light appeared in no way enfeebled, yet such a star would be completely obliterated by a moderate fog extending a few yards from the surface of the earth"*. We do not know what amount of transparency is possessed by the vapour of carbon, but the absence of a continuous spectrum seems to show that, as it existed in the comet, it was almost perfectly transparent. The light of a star would suffer, therefore, only that kind and degree of absorption which corresponds with its power of radiation, as shown by its spectrum of bright lines. As these occur in the brightest part of the spectrum, we should expect a noticeable diminution of the star's light, if it were not for the luminous condition of the gas, in consequence of which it would give back to the beam light of precisely the same refrangibilities as it had taken, and so enable the part of the field occupied by the image of the star to appear of its original brightness, or nearly so. This state of things would not prevent an apparent diminution of the star's light from the effect upon the eye of the brightness of the surrounding field. In the case of the tails of comets, the great transparency observed is more probably to be referred to the widely scattered condition of the minute particles of the cometary matter.

I may be permitted to repeat here a paragraph from my paper on the Spectrum of Comet I., 1866†.

"Terrestrial phenomena would suggest that the parts of a comet which are bright by reflecting the sun's light, are probably in the condition of fog or cloud.

"We know, from observation, that the comæ and tails of comets are formed from the matter contained in the nucleus‡.

* Outlines of Astronomy, p. 373.

† Proceedings of the Royal Society, vol. xv. p. 5.

‡ The head of HALLEY's comet in 1835 in a telescope of great power "exhibited the appearance of jets as it were of flame, or rather of luminous smoke, like a gas fan-light. These varied from day to day, as if wavering backwards and forwards, as if they were thrown out of particular parts of the internal nucleus or kernel which shifted round, or to or fro by their recoil like a squib not held fast. The bright smoke of these jets, however, never seemed to be able to get far out towards the sun, but always to be driven back and forced into the tail as if by the action of a violent wind setting against them (always *from* the sun), so as to make it clear that the

"The usual order of the phenomena which attend the formation of a tail appears to be that, as the comet approaches the sun, material is thrown off, at intervals, from the nucleus in the direction towards the sun. This material is not at once driven into the tail, but usually forms in front of the nucleus a dense luminous cloud, into which for a time the bright matter of the nucleus continues to stream. In this way a succession of envelopes may be formed, the material of which afterwards is dissipated in a direction opposite to the sun, and forms the tail. Between these envelopes dark spaces are usually seen.

"If the matter of the nucleus is capable of forming by condensation a cloud-like mass, there must be an intermediate state in which the matter ceases to be self-luminous, but yet retains its gaseous state, and reflects but little light. Such a non-luminous and transparent condition of the cometary matter may possibly be represented by some at least of the dark spaces which, in some comets, separate the cloud-like envelopes from the nucleus and from each other."

Now considerable differences of colour have been remarked in the different parts of some comets. The spectrum of this comet would show that its colour was bluish green. Sir W. HERSCHEL described the head of the Comet of 1811 to be of a greenish or bluish-green colour, while the central point appeared to be of a pale ruddy tint. The representations of HALLEY'S comet at its appearance in 1835, by the elder STRUVE, are coloured bluish green, and the nucleus on October 9 is coloured reddish yellow. He describes the nucleus on that day, thus:—"Der Kern zeigte sich wie eine kleine, etwas ins gelbliche spielende, glühende Kohle von länglicher Form"*. Dr. WINNECKE describes similar colours in the bright comet of 1862. "Die Farbe des Strahls erscheint mir gelbröthlich; die des umgebenden Nebels (vielleicht aus Contrast) mattbläulich." "Die Farbe der Ausströmung erscheint mir gelblich; die Coma hat bläuliches Licht"†.

Now carbon, if incandescent in the solid state, or reflecting, when in a condition of minute division the light of the sun, would afford a light which, in comparison with that emitted by the luminous vapour of carbon, would appear as yellowish or approaching to red.

The views of comets presented in this paper do not, however, afford any clue to the great mystery which surrounds the enormous rapidity with which the tail is often projected to immense distances. There are not any known properties peculiar to carbon, even when in a condition of extremely minute division, which would help to a solution of the enigma of the violent repulsive power from the sun which appears to be exerted upon cometary matter shortly after its expulsion from the nucleus, and upon matter

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tail is neither more or less than the accumulation of this sort of luminous vapour darted off, in the *first instance towards the sun*, as it were something raised up, and, as it were, exploded by the sun's heat out of the kernel, and then immediately and forcibly turned back and *repelled from* the sun."—Sir JOHN HERSCHEL. Familiar Lectures on Scientific Subjects. p. 115.

* Beobachtungen des Halleyschen Cometen, s. 41.

† Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg, tome vii. No. 7.

in this condition only. It may be that this apparent repulsion takes place at the time of the condensation of the gaseous matter of the coma, into the excessively minute solid particles of which the tail probably consists. There is a phenomenon occasionally seen which must not be passed without notice, namely, the formation of faint narrow rays of light, or secondary tails, which start off usually from the brightest side of the principal tail, not far from the head. Sir JOHN HERSCHEL* considers that "they clearly indicate an analysis of the cometic matter by the sun's repulsive action, the matter of the secondary tails being darted off with incomparably greater velocity (indicating an incomparably greater intensity of repulsive energy) than that which went to form the primary one." The important differences which exist between the spectrum of BRORSEN'S comet and that of Comet II., 1868, appear to show that comets may vary in their constitution. If the phenomenon of the secondary tails were observed in a comet which, like Comet II., 1868, appears to consist of carbon, the analytical action supposed by Sir JOHN HERSCHEL might be to separate between particles of carbon in different conditions, or possibly in a state of more or less subdivision. The enormous extent of space, sometimes a hundred millions of miles in length, over which a comparatively minute portion of cometary matter is in this way diffused, would suggest that we have in this phenomenon a remarkable instance of the extreme division of matter. Perhaps it would be too bold a speculation to suggest that, under the circumstances which attend the condensation of the gaseous matter into discrete solid particles, the division may be pushed to its utmost limit, or nearly so. If we could conceive the separate atoms to be removed beyond the sphere of their mutual attraction of cohesion, it might be that they would be affected by the sun's energy in a way altogether different from that of which we have been hitherto the witnesses upon the earth.

Though comets may differ in their constitution, reference may be permitted to the periodical meteors which have been shown to move in orbits identical with those of some comets. If these consist of carbon, we might have some explanation of the appearances presented by these meteors, though their light is doubtless greatly modified by that of the air rendered luminous by their passage, as well as by the degree of temperature to which they are raised. Carbon is abundantly present in some meteorites, but we have no certain evidence at present that the periodical meteors belong to this class of celestial bodies.

NOTE TO PLATE XXXIII.

In fig. 2, the bright line at the beginning of the middle band of the spectrum of olefiant gas is made too strong.

* Familiar Lectures on Scientific Subjects, p. 129.

XXII. *New Researches upon the Dispersion of the Optic Axes in Harmotome and Wöhlerite, proving these Minerals to belong to the Clinorhombic [Oblique] System.* By M. A. L. O. Des Cloizeaux. *Communicated by Professor* W. H. Miller, *For. Sec. R.S.*

Received March 12,—Read April 23, 1868.

We are already acquainted with a considerable number of crystals, natural as well as artificial, the forms of which have only been determined with precision by the examination of their optical properties as doubly refracting bodies. Harmotome and Wöhlerite furnish two fresh examples of this; and they afford all the more important proof of the necessity of appealing to these properties, inasmuch as the crystals of these substances would appear certainly to be derived from a right rhombic prism, so long as we consider only the apparent symmetry of their external forms, or the orientation of the plane containing their optic axes. The different sorts of dispersion which these axes might be capable of presenting* are so feeble, and so difficult of appreciation on account of the slight transparency of Wöhlerite, and the complex structure of the crystals of Harmotome, that the determination of these dispersions has hitherto been too incomplete to allow of any conclusion being drawn as to the crystalline type they might otherwise serve to characterize.

It was a remark of M. Axel Gadolin, to which I shall make further allusion, that induced me to resume the attentive study of the phenomena of dispersion, first in Harmotome, and then in Wöhlerite, and as a consequence to modify the crystallographic type to which these minerals have been in general referred.

Harmotome.

It is now some years† since I showed that simple crystals of Harmotome did not exist, and that those of Strontian in Scotland (Morvenite), considered as such, presented, in fact, a twinning formed by the interpenetration of two principal individuals. The particular orientation of the plane of the optic axes in each of the crystals of which the least complicated of such groups are composed had led me to refer their crystalline form to a right rhombic prism of 124° 47′; and I had been induced to look on this prism as presenting a peculiar sort of hemihedrism, or rather hemimorphism, such that only the half of the fundamental rhombic octahedron existed, that namely formed of four faces parallel two and two, and lying in the same zone. More recently I had established, in

* See my memoir "Sur l'emploi du microscope polarisant," Annales des Mines, 6me série, t. vi. 1864.

† "Sur l'emploi des propriétés optiques biréfringentes pour la détermination des espèces cristallisées," 2nd memoir, Annales des Mines, t. xiv. 1858.

the modifications which heat induces in the position of the optic axes and of their plane*, a phenomenon less compatible with the hypothesis of a primitive rhombic form; but the slight transparency of the plates on which I had operated, the wide separation of their optic axes, which rendered the examination of the two systems of rings almost impossible in air, and finally the almost complete absence of dispersion, had led me to believe in an anomaly as the result of the always highly complex structure of the crystals.

In a memoir published towards the middle of 1867, "Sur la déduction d'un seul principe de tous les systèmes cristallographiques," M. Axel Gadolin suggested the idea that the crystals of Harmotome might probably belong to the oblique system, and that then their forms would no longer present any hemihedrism.

Desirous of verifying the truth of this suggestion, which had been communicated to me by M. Gadolin in the month of June 1867, I had some new plates cut normal to the *positive* acute bisectrix, from very transparent crystals of the Scotch Morvenite, for which I was indebted to the kindness of my friend Mr. Maskelyne; and I have been enabled to establish, both in oil and in air, a very decided twisted dispersion. Thanks to a separation of the optic axes less divergent than that of my former plates, I have been able also to satisfy myself directly that the displacement impressed by heat on the plane containing these axes is a rotary one, quite analogous to that which I have shown to exist in the cases of borax and Heulandite.

It is therefore now beyond doubt that the crystalline type of Harmotome is the oblique rhombic prism, and that the figures 185, 186, and 187 (pls. 31 and 32) of the atlas of my 'Manual of Mineralogy,' ought to be turned so as to make the plane p, which presents the crossed striations, into the plane of symmetry of the new form. The simplest plan, then, is to look on the faces $b\frac{1}{2}$ of the figure as forming the lateral faces m of the new primitive prism, one of the former faces m as the base of this prism, and the other as the plane h^1 truncating its obtuse edge.

The figures 1 to 5 (Plate XXXIV.) which represent the principal combinations of the forms observed have the new symbols, the relations of which with the old ones are as follows:—

New symbols.		Former symbols.
Des Cloizeaux's notation.	Miller's notation.	Des Cloizeaux's notation.
m	110	$b\frac{1}{2}$
h^1	100	m
p	001	m
$h\frac{5}{3}$	410	$b\frac{1}{3}$
$h\frac{7}{3}$	520	,,
g^1	010	p
o^1	101	$h\frac{7}{4}$

* Nouvelles recherches sur les propriétés optiques des cristaux naturels ou artificiels, insérées dans les Mémoires présentés par divers savants à l'Institut Impérial de France, t. xviii. 1867.

The figures 1, 2, and 3 belong to the simple macles which constitute the *Morvenite* of Scotland and the greater part of the crystals of Oberstein; the figures 4 and 5 are the double cross-formed macles so common at Andreasberg. In the first the two individuals interpenetrate one into the other so as to form four sectors, usually very unequal in their magnitude. Theoretically these sectors ought to occur following two planes perpendicular to each other, the one parallel to the base, the other parallel to the very simple modification a^1, the inclination of which upon this base is precisely 90°; but in reality these sectors more frequently are united only in an imperfect manner, and there exist between their sides little channels more or less sinuous (fig. 6). It is easy to show this by passing parallel rays of polarized light through plates cut very thin parallel to the plane of symmetry.

In the cross-formed macles (figs. 4 and 5) two individuals, already composite, penetrate one another so as to have their planes of symmetry perpendicular, but their intercrossing, instead of taking place on the planes inclined at 135° on the base, and very nearly in the plane e^1, usually occurs with surfaces that are irregular and jagged: however, the direction of what is theoretically the plane of assemblage is sometimes indicated by a series of plates excessively thin and which fill the little interval, then sensibly rectilinear, which exists between the two individuals (fig. 7).

It is well known that on crystals of Harmotome it is very difficult to obtain measurements for the angles that are at all precise; nor can it be otherwise; for independently of the crossed striæ the points of junction of which form upon the plane of symmetry g^1 two slightly salient lines perpendicular to each other, the base p is almost always more or less decidedly waved with lines parallel to its intersection with h^1, and the faces m are finely striated in the direction of the edge $\frac{m}{g^1}$; furthermore, the individuals of which the macles are composed are hardly ever united in a manner sufficiently perfect for the four faces h^1 and $_1q$ of a crystal similar to that represented in fig. 1, to be found exactly in the same zone with the two bases p; the same is true of the faces anterior and posterior $m\,h^1\,m$, which ought to form one single vertical zone. Thus it is only an exceptional case, or where one is operating on very small crystals of Morvenite from Strontian, that one can obtain the inclinations $p\,m$ and $h^1\,m$ with any exactitude.

Combining the results formerly known with those which a new and very numerous series of observations have afforded me, the following Table will give the inclinations at which I have now arrived.

$b:h :: 1000:1007{,}00$ D=818,02 d=575,19.

Plane angle of the base . . . =109° 46′ 27″.

Plane angle of the lateral faces . =109° 10′ 50″.

Calculated angles.	Measured angles.
*$m\,m$ 120° 1′	120° 1′ (Köhler).
$m\,h^1$ 150° 0′ 30″	150° 0′ 24″, mean (Dx.); 149° 32′ (Phil.)[a].
$m\,g^1$ 119° 59′ 30″	120° 15′, mean (Dx.); 120° 56′ (Phil.).
$h\frac{5}{3}\,h^1$ 171° 47′	171° 4′ (Phil.).
$h\frac{5}{3}\,g^1$ 98° 13′	,,
$h\frac{7}{3}\,h^1$ 167° 0′	166° 30′ (Dx.)[b].
$m\,h\frac{7}{3}$ 163° 0′	162° 50′ to 163° (Dx.).
$h^1\,g^1$ 90°	,,
$p\,o^1$ 144° 18′	,,
$o^1\,h^1$ 160° 32′	,,
*$p\,h^1$ 124° 50′	124° 50′ just; 124° 53′, mean (Dx.); 125° 5′ (Phil.).
o^1,o 71° 24′ over h^1	,,
h^1,γ adj. 110° 20′	110° 28′, mean (Dx.); 110° 26′ (Phil.); 111° 15′ (Köhler).
h^1,γ 69° 40′ over p	69° 39′, mean (Dx.).
$p\,m$ 119° 39′	119° 5′ approx. (Dx.).
$m\,m$ adj. 120° 42′	120° 45′ to 121° 37′ (Dx.); 121° 6′ (Lévy); 121° 28′ (Köhler).
$m\,m$ 89° 23′ on g^1	89° 2′, mean (Dx.).
*$p\,a^1$ 90°	Twin plane of the simple macles.
$p\,e^1$ 134° 42′	Face nearly corresponding to the twin plane of the crossed macles.

Instead of the prism $h\frac{5}{3}$ of PHILLIPS, I have observed the prism $h\frac{7}{3}$ on many crystals of Morvenite, and instead of o', given by GREG, I have found $o\frac{3}{7}$ with

	Calculated.	Measured.
$p\,o\frac{3}{7}$	131° 49′	131° 30′
$o\frac{3}{7}$	173° 1′	,,

There exist two principal cleavages, the one easy and rather neat parallel to the plane of symmetry g^1, the other less easy parallel to the base p.

The plane of the optic axes and their acute positive bisectrix are perpendicular to the plane of symmetry. The disposition of the coloured rings in plates normal to the two bisectrices had heretofore induced me to look on the mean optic axes as situated exactly in the plane bisecting the angle $p\,h^1$; but I have since assured myself that this was not altogether true. In fact the results from a great number of observations made at 15° C. by means of two Nicol prisms crossed at right angles in the direction in which the maximum of extinction takes place in two neighbouring sectors, taken from many plates of Morvenite parallel to the plane of symmetry, may be stated as—

First. That the plane of the axes corresponding to red rays makes angles of about

25° 42′ with a normal to p,
29° 28′ with a normal to h^1.

Second. That the plane of the axes corresponding with the blue rays makes an angle of about

25° 5′ with a normal to p,
30° 5′ with a normal to h^1.

[a] Phil. = PHILLIPS.

[b] Dx. = DES CLOIZEAUX.

There is then a separation of about 0° 37′ between the plane of the red axes and the plane of the blue axes; and this explains the twisted dispersion which shows itself sufficiently sensibly by the colours exhibited in plates of Harmotome that are sufficiently transparent and are normal to the acute bisectrix, when one examines them by aid of the polarizing microscope.

Among the plates of Morvenite which have allowed, as I before said, of my studying afresh the action of heat on the separation and the orientation of the optic axes, the most perfect has given me in oil for the red rays, and at 24° C., $2H_a = 87° 2'$.

If one compares this number with those which I have published in my 'Manual of Mineralogy,' it will be seen that the optic axes present separations that vary greatly according to the specimens.

The preceding plate, when placed on the heating cell of the microscope in such a manner as to show its optic axes in a horizontal plane perpendicular to the plane of polarization, permits of our seeing, when one inclines it sufficiently to right or left, the greater part of the bar which traverses the central ring of each system. An elevation of the temperature produces an approximation of the axes, and the change in form of each central ring, which have been described in my 'Nouvelles recherches sur les propriétés optiques des cristaux naturels ou artificiels,' while the two transverse bars have a tendency to an inclination in opposite directions and are displaced by the same amount, the one above, the other below the horizontal plane which at first contained them. This displacement, which indicates a rotation of the optic axes, is hardly sensible between 15° and 80° C., but from above 90° it becomes very evident and augments with the temperature.

A peculiar and somewhat abnormal character is imparted to this change by the circumstance observed in one of my experiments in which the temperature had been elevated up to 160° C. In this case the displacement continued on the increase during the cooling of the air-bath from 160° to about 108° C. At the moment of maximum displacement the angle through which the plane of the axes had turned seemed at least equal if not greater than that which I had been enabled to measure in the case of borax. At 92° C. the transverse bars approached again the horizontal position, and began to move in the direction of their first orientation, but they did not exactly attain that orientation until the lapse of about thirty minutes after the thermometer in the air-bath had come back to its original reading. In other experiments, where the temperature was maintained between 50° and 52° C., it took twelve or fifteen minutes for the displacement of the bars to become sensible. In from fifteen to thirty minutes more this displacement had attained its maximum. When the lamp was removed, the return movement of the bars hardly became visible before ten or twelve minutes had elapsed after the thermometer had begun to fall, and thirty to forty-five minutes more subsequently elapsed before they had exactly returned to the position they had occupied at the beginning of the experiment.

Harmotome is up to the present time the only crystallized substance which exhibits

a considerable delay between the moment when certain optical phenomena are manifested, and the moment of the change of temperature. This delay, produced by causes probably very complex, is all the more remarkable from the plate on which I made most of my experiments having dimensions no greater than two millimetres in width by half a millimetre in thickness, dimensions far less than those of the plates which I am in the habit of introducing into my air-bath, and of which the modifications, thermic as well as optical, have always presented themselves to me as nearly as possible simultaneously.

Wöhlerite.

In my 'Manual of Mineralogy' I described the crystals of Wöhlerite as derivable from a prism of very nearly 90°. In the point of view from which I had been induced to look at them, from a consideration of the orientation of their optic axes, they appeared to offer a certain number of homohedral forms, associated with forms that were hemihedral or hemimorphic, analogous to those that I have drawn attention to in Harmotome. Since I have established that this last mineral belonged in reality to the clinorhombic system, I have sought to ascertain whether this was not so also in the case of Wöhlerite, all the forms of which would in that event be homohedral. But in this case a study of the different varieties of dispersion is rendered difficult by the yellow colour, and by the imperfect transparency presented by the substance even when in very thin plates. Besides this, contrary to what one finds in the case of Harmotome, whilst the dispersion belonging to the optic axes is very distinct, the *horizontal and twisted* dispersions, which should be sought for in the plates normal to the two bisectrices, are, on the contrary, but slightly evident. However, on examining in oil some thin plates placed so as to have the plane of their optic axes horizontal and perpendicular to the plane of polarization, I observed in the plates normal to the obtuse *positive* bisectrix, some faint blue and red fringes disposed in contrary directions above and below the bars which traverse the two annular systems. These colours indicate, then, the existence of an appreciable twisted dispersion. In the plates normal to the acute *negative* bisectrix, the transverse bar of each system is bordered on one side by very pale blue, and on the opposite side by an equally pale yellow. The horizontal dispersion is thus feebly indicated. Furthermore, in the former, as in the latter plates, the edges of one system of rings often show a colour decidedly more brilliant than those of the other system; but this apparent dissymmetry is only the result of the imperfection of the plates, and especially of the unequal transparence presented by their different parts in consequence of internal fissures.

The crystals of Wöhlerite ought, then, to be referred to an oblique rhombic prism, in which the plane of symmetry is normal to the plane which contains their optic axes. The primitive form which it seems most convenient to choose is a prism with an angle of very nearly 90°, which presents a cleavage, easy though interrupted, parallel to its plane of symmetry, and cleavages which are more difficult in the directions of the lateral faces m and of the plane h^1, which is parallel to the horizontal diagonals of the base.

The figures 1 to 9 (Plate XXXIV.) represent the actual appearance of the crystals

which carry the different combinations of forms which I have observed. Fig. 10 is the projection upon h^1, of a crystal measured by DAUBER, regarded by him as orthorhombic (POGGENDORFF'S 'Annalen,' t. xcii. p. 242). The spherical projection (fig. 11) made upon the plane of symmetry shows the principal zones in which all the faces are comprised that are hitherto known.

The dimensions of the primitive forms and the calculated inclinations compared with the inclinations as directly measured are given in the following Tables.

$b : h :: 1000 : 487{,}8112 \qquad D = 687{,}8636 \qquad d = 725{,}7450$

Plane angle of the base . . . . . = 86° 56′ 17″·8

Plane angle of the lateral faces . . = 103° 50′ 37″

Calculated.	Measured.
$m\,m$ 90° 14′ in front	90° 3′ to 30′ (Dx.)[a].
$m\,m$ 89° 46′ on g^1	90° approx. (Dx.).
$h^1\,h^{\frac{1}{2}}$ 164° 7′	164° 15′ (Dx.).
$h^{\frac{1}{2}}\,g^1$ 105° 53′	,,
$h^1\,h^2$ 153° 32′	153° 29′, mean (Dx.); 152° 55′ (Dau.)[b].
$h^2\,g^1$ 116° 28′	116° 30′ (Dx.); 116° 51′ (Dau.).
$h^1\,m$ 135° 7′	135° 7′, mean (Dau.); 135° 16′, mean (Dx.).
$m\,g^1$ 134° 53′	134° 54′, mean (Dau.); 134° 59′, mean (Dx.).
$h^1\,g^2$ 116° 39′ 30″	116° 32′, mean (Dx.); 116° 14′ (Dau.).
$g^2\,g^1$ 153° 20′ 30″	153° to 153° 20′ (Dx.); 153° 30′ (Dau.).
$h^1\,g^2$ 108° 30′	108° 58′, mean (Dx.); 107° 39′ (Dau.).
$g^2\,g^1$ 161° 30′	161° 15′ (Dau.).
$m\,g^2$ adj. 161° 32′ 30″	161° 35′ (Dx.).
$m\,g^2$ 108° 13′ 30″ on g^1	108° 40′ (Dx.).
$m\,g^2$ 153° 23′	153° 40′ (Dx.).
$g^2\,g^2$ 171° 50′ 30″	172° (Dx.).
$h^1\,g^1$ 90°	90° (Dx.).
$o^1\,h^1$ 136° 42′	136° 42′ (Dx.); 138° (Dau.).
$o^1\,p$ 152° 33′	152° 32′ (Dx.).
$p\,h^1$ ant. 109° 15′	109° 15′, mean (Dx.); 110° 25′, mean (Dau.).
$p\,a^1$ adj. 140° 49′	140° 48′, mean (Dx.); 140° 15′ (Dau.).
$a^1\,h^1$ adj. 109° 56′	109° 34′, mean (Dx.); 108° 45′ to 109° 40′ (Dau.).
$p\,a_{\frac{1}{2}}$ adj. 113° 41′	113° approx. (Dx.).
$a_{\frac{1}{2}}\,h^1$ adj. 137° 4′	138° 35′? (Dau.).
$p\,h^1$ post. 70° 45′	,,
$p\,e^2$ 161° 30′	162° 45′ (Dau.).
$e^2\,g^1$ 108° 30′	,,
$p\,e^1$ 146° 12′	145° 35′ to 52′; 146° 15′ (Dx.); 146° 14′ (Dau.)
$e^1\,g^1$ 123° 48′	123° 10′; 50′; 124° 15′; 25′ (Dx.); 124° 7′ (Dau.).
$p\,e_{\frac{1}{2}}$ 126° 45′	,,
$e_{\frac{1}{2}}\,g^1$ 143° 15′	143° to 143° 10′ (Dx.).
$p\,g^1$ 90°	90° (Dx.).

[a] Dx. = Des Cloizeaux.

[b] Dau. = Dauber.

Calculated.	Measured.
$p\ d\frac{1}{2}$ 142° 57′	142° 35′; 143°; 143° 30′ (Dx.).
$d\frac{1}{2}\ m$ 140° 34′	140° 35′ (Dx.).
$p\ m$ ant. 103° 31′	103° 15′ approx. (Dx.).
$p\ b\frac{1}{2}$ adj. 130° 10′	129° 35′ to 131° 30′ (Dx.).
$b\frac{1}{2}\ m$ 126° 19′	125° 5′ to 126° (Dx.).
$p\ b\frac{1}{4}$ adj. 106° 48′	"
$b\frac{1}{4}\ m$ 163° 12′	"
$p\ m$ post. 76° 29′	"
$p\ h^3$ ant. 107° 10′	107° 5′ (Dx.).
$p\ x$ adj. 137° 20′	"
$p\ a_3$ adj. 111° 13′	110° approx. (Dx.).
$p\ h^3$ post. 72° 50′	"
$a_3\ h^3$ adj. 141° 37′	"
$p\ y$ 129° 30′	"
$p\ g^3$ ant. 98° 30′	98° 10′ (Dx.)
$p\ \phi$ adj. 117° 43′	"
$p\ g^3$ post. 81° 30′	80° 40′ (Dx.).
$\phi\ g^3$ adj. 143° 47′	143° approx. (Dx.).
$p\ g^2$ ant. 96°	95° 35′ approx. (Dx.).
$e^2\ h^1$ ant. 108° 13′	"
$x\ h^1$ adj. 108° 53′	"
$d\frac{1}{2}\ h^1$ adj. 130° 53′	131° 12′, mean (Dx.); 130° 49′ (Dau.).
$e^1\ h^1$ ant. 105° 54′	106° 10′, mean (Dx.); 106° 54′? (Dau.).
$e^1\ h^1$ post. 74° 6′	73° 20′, mean (Dx.); 73° 31′ (Dau.).
$d\frac{1}{2}\ e^1$ 155° 1′	155° 3′, mean (Dx.).
$e^1\ b\frac{1}{2}$ 147° 37′	147° 30′ (Dx.); 147° 43′ (Dau.).
$b\frac{1}{2}\ a_3$ 155° 15′	155° (Dx.).
$b\frac{1}{2}\ h^1$ 106° 29′ over a_3	106° 25′ to 45′ (Dx.); 105° 44′? (Dau.).
$b\frac{1}{2}\ h^1$ 73° 31′ over e^1	73° 33′, mean (Dx.).
$a_3\ h^1$ adj. 131° 14′	131° 30′ (Dx.); 130° 10′ (Dau.).
$a_3\ d\frac{1}{2}$ 97° 53′ over e^1	97° 40′ (Dx.).
$a_3\ h^1$ 48° 46′ over e^1	49° 8′, mean (Dx.).
$y\ h^1$ adj. 121° 27′	120° 57′ (Dau.).
$e\frac{1}{2}\ h^1$ ant. 101° 23′	101° 45′ to 102° 20′ (Dx.).
$e\frac{1}{2}\ h^1$ post. 78° 37′	78° 18′, mean (Dx.).
$\phi\ h^1$ 101° 48′ over $b\frac{1}{2}$	102° 10′, approx. (Dx.).
$b\frac{1}{2}\ h^1$ adj. 121° 46′	"
$g^1\ y$ 134° 12′	134° 18′ (Dau.).
$g^1\ d\frac{1}{2}$ 115° 56′	115° 18′, mean (Dx.).
$g^1\ o^1$ 90°	"
$g^1\ \varphi$ 143° 8′	143° 15′ to 143° 30′ (Dx.).
$g^1\ b\frac{1}{2}$ 123° 41′	123° 49′, mean (Dx.).
$\varphi\ b\frac{1}{2}$ 160° 33′	160° 30′ (Dx.).
$g^1\ x$ 108° 26′	"
$g^1\ a^1$ 90°	89° 55′ (Dx.).
$b\frac{1}{2}\ a^1$ 146° 19′	145° 40′ (Dx.).
$x\ a^1$ 161° 34′	"

Calculated.	Measured.
$g^1\ b\frac{1}{2}$ 134° 1′	
$g^1\ a_3$ 115° 47′	115° 10′ (Dx.).
$g^1\ a\frac{1}{2}$ 90°	,,
$o^1\ m$ adj. 121° 2′ 30″	121° 55′ (Dx.).
$o^1\ m$ 58° 57′ 30″ over e^3	,,
$e^3\ m$ post. 101° 26′	,,
$\varphi^1\ m$ adj. 135° 12′	135°, approx. (Dx.).
$e^3\ m$ ant. 116° 27′	,,
$y\ m$ adj. 149° 30′	,,
$e^3 m$ ant. 125° 55′	126° 20′, approx. (Dx.).
$a^1\ m$ 76° 1′ over e^1	75° 40′, approx. (Dx.).
$a^1\ m$ 103° 59′ over a_3	103° 40′; 52′ (Dx.)
$a_3\ m$ adj. 140° 43′	140° 2′ to 141° (Dx.).
$a^1\ a_3$ 143° 16′	143° 15′ (Dx.).
$e\frac{1}{2}\ m$ ant. 134° 51′	,,
$b\frac{1}{2}\ m$ 100° 58′ over $e\frac{1}{2}$	,,
$a\frac{1}{2}\ m$ adj. 121° 15′	,,
$o^1\ h^3$ adj. 130° 39′	130° 30′; 131° (Dx.).
$o^1\ e^3$ 147° 18′	,,
$e^3\ h^3$ 93° 14′ over $b\frac{1}{2}$	,,
$b\frac{1}{2}\ h^3$ adj. 120° 5′	,,
$o^1\ g^3$ adj. 109° 4′	109° 6′ (Dx.).
$o^1\ e\frac{1}{2}$ 122° 5′	,,
$e\frac{1}{2}\ g^3$ post. 128° 52′	,,
$d\frac{1}{2}\ g^3$ ant. 133° 12′	132° 10′, approx. (Dx.).
$e^1\ g^3$ ant. 128° 19′	128°; 128° 30′ (Dx.).
$d\frac{1}{2}\ h^3$ ant. 141° 20′	142°, approx. (Dx.).
$e^3\ h^3$ 114° 55′ over $d\frac{1}{2}$	,,
$a^1\ h^3$ 72° 14′ over e^3	72° (Dx.).
$a^1\ h^3$ adj. 107° 46′	,,
$e\frac{1}{2}\ g^3$ ant. 143° 34′	,,
$a^1\ g^3$ 82° 29′ over $e\frac{1}{2}$	,,
$a^1\ g^3$ 97 31′ over $b\frac{1}{4}$	98° 5′ to 20′ (Dx.).
$b\frac{1}{4}\ g^3$ adj. 149° 0′	,,
$e^1\ h^3$ ant. 119° 33′	119° 52′ (Dx.).
$x\ h^3$ 98° 33′ over e^1	,,
$a\frac{1}{3}\ h^3$ 49° 3′ over e^1	,,
$a\frac{1}{3}\ h^3$ adj. 130° 57′	,,
$\varphi\ g^3$ ant. 128° 33′	127° 40′ to 128° (Dx.).
$\varphi\ a\frac{1}{2}$ 122° 16′	,,
$a\frac{1}{2}\ g^3$ adj. 109° 11′	,,
$d\frac{1}{2}\ g^3$ ant. 128° 30′	128° 30′, mean (Dx.).
$e^1\ g^3$ ant. 130° 21′	129°, approx. (Dx.).
$e^1\ g^3$ post. 116° 8′	115° 55′, mean (Dx.).
$e\frac{1}{2}\ m$ post. 115° 11′	114° to 115°? (Dx.).
$b\frac{1}{2}\ g^3$ post. 128° 32′	128° 50′ (Dx.).
$\varphi\ m$ ant. 114° 48′	114° to 115°? (Dx.).
$\varphi\ g^3$ ant. 133° 55′	134° 30′, approx. (Dx.).
$\varphi\ g^3$ post. 145° 26′	145° (Dx.).

$y = (d^1\ b\frac{1}{2}\ g^1)$ $\quad x = (b^1\ b\frac{1}{2}\ h\frac{1}{3})$.

$a_3 = (b^1\ b\frac{1}{2}\ h^1)$ $\quad \varphi = (b^1\ d\frac{1}{2}\ g^1)$.

The forms which are most frequently met with on the crystals which I have had at my disposal are h^1, h^2, m, g^3, g^2, g^1, o^1, p, a^1, e^1, $d^{\frac{1}{3}}$, $b^{\frac{1}{2}}$, a_3. The forms $h^{\frac{2}{3}}$, $a^{\frac{1}{2}}$ (fig. 7), $e^{\frac{1}{2}}$ (fig. 8), φ (fig. 9) are, on the contrary, very rare; e^2 and y have only been announced by DAUBER (fig. 10). I have supposed the existence of x and $b^{\frac{1}{4}}$ corresponding to these two last faces by analogy with what I have observed upon the crystals I myself examined, on which there occur both in front and in rear planes such as p and a^1, o^1 and $a^{\frac{1}{2}}$, e^1 and $b^{\frac{1}{2}}$, $d^{\frac{1}{3}}$ and a_3, $e^{\frac{1}{2}}$ and φ, which make respectively almost equal angles with h^1. The plane of the mean optic axes is very nearly parallel to o^1. In a macle artificially formed of two plates cleaved in the direction g^1 and twinned after the plane h^1, the maximum extinction takes place at the same moment for both plates; these plates therefore have their optic axes situated in planes perpendicular to one another, and each of them inclined at 45° to the face of union h^1. As the yellow colour of the substance does not permit of any certain results being obtained with monochromatic glasses of red or blue, it is impossible to determine directly the angle which the plane of the axes corresponding to the red rays makes with the plane of the axes which correspond to the blue rays; this angle must, however, be very small, in consequence of the weak horizontal and twisted dispersion which is exhibited in oil, and to which I have alluded above. The bisectrix of the acute angle formed by the optic axes is negative and normal to the horizontal diagonal of the base. The separation of these axes varies with the crystals, and even with different portions of one and the same crystal. The following are the actual results that I obtained from three different crystals, each of which contributed a couple of plates normal to the acute and obtuse bisectrices:—

First crystal cited in my 'Manual.'

$2H_a = 89° 34'$, $2H_o = 128° 6'$, whence $2V = 76° 10'$, $\beta = 1{\cdot}67$ for the red rays.

$2H_a = 90° 54'$, $2H_o = 127° 6'$, whence $2V = 77° 2'$, $\beta = 1{\cdot}69$ for the blue rays.

Second crystal recently examined.

$2H_a = 85° 41'$, $2H_o = 139° 3'$, whence $2V = 71° 56'$, $\beta = 1{\cdot}69$ for the red rays.

$2H_a = 86° 12'$, $2H_o = 138° 32'$, whence $2V = 72° 18'$, $\beta = 1{\cdot}71$ for the blue rays.

Third crystal recently examined.

$2H_a = 86° 24'$, $2H_o = 144° 24'$, whence $2V = 71° 26'$, $\beta = 1{\cdot}72$ for the red rays.

$2H_a = 87° 30'$, $2H_o = 144° 8'$, whence $2V = 72° 1'$, $\beta = 1{\cdot}74$ for the blue rays.

Plates cut normally to the acute bisectrix in three other crystals gave for the red rays

$$2H_a = \overset{\text{I.}}{141° 17'};\ \overset{\text{II.}}{138° 42'};\ \overbrace{140° 32', \text{ first portion};\ 138° 48', \text{ second portion.}}^{\text{III.}}$$

The dispersion belonging to the optic axes, weak in oil and in the interior of the crystals with $\rho < v$ is, on the contrary, very considerable in the air; for on the second

crystal the red axes undergo an apparent separation in air equal to 170° 53′, while the blue axes undergo a total reflexion. If we cast our eyes over the Table of the inclinations of the planes, it is easy to see that the crystals of Wöhlerite, besides the faces indicated above, as making angles almost equal in front and in rear with the vertical axis, present, furthermore, modifications with inclinations which are very nearly identical in two zones at right angles to each other. These crystals being rarely sufficiently good to furnish very precise measurements, it is almost impossible to distinguish crystallographically between the one and the other of the forms m, o^1, and $a\frac{1}{2}$; g^3, p, and a^1; $d\frac{1}{2}$ and a_2; e^1 and $b\frac{1}{2}$; $e\frac{1}{2}$ and φ: one can only do this by looking out for the facile cleavage that runs parallel to g^1 and the orientation of the plane of the optic axes. At the time of my first publications on the subject of Wöhlerite*, it was almost impossible to employ these two methods, in consequence of the scarcity of isolated crystals. This scarcity having become less felt in consequence of Professor Nordenskiöld having been so kind as to send me some specimens, and from my having sacrificed one fine specimen upon its gangue, I have been enabled to obtain the precise interpretation of all the crystals I have figured in Plate XXXIV. accompanying my new investigation. The greater part of these crystals have been described in my two former memoirs and in my 'Manual'†; and as there has been confusion between the zones $h^1\,m\,g^1$ and $h^1\,o^1\,p$, on several of them, it may be useful that I should show here how my new symbols correspond with those of Dauber and with those which are introduced in my 'Manual,' or in the figures 234, 235, 236, plate 40 of my 'Atlas.'

New symbols.	Dauber.	Manual.	Fig. 234, pl. 40.	Fig. 235, pl. 40.	Fig. 236, pl. 40.
h^1	a	m	m	.....	m central.
g^1	b	m	p	m	m lateral.
$h^{\frac{5}{3}}$	,,	$b^{\frac{1}{3}}_{\frac{1}{0}}$	,,	,,	,,
$h^{\frac{3}{2}}$	n	$b^{\frac{3}{2}}_{\frac{1}{0}}$	$b^{\frac{3}{2}}_{\frac{1}{0}}$	h^2	h^2 and g^3
m	m	$b^{\frac{3}{1}}_{\frac{1}{0}}$	$b^{\frac{3}{1}}_{\frac{1}{0}}$	h^1 and g^1	h^1 and g^1
g^2	g	$b^{\frac{2}{1}}_{\frac{1}{0}}$	,,	h^2 and g^2	h^2 and g^2
g^3	h	b^3	b^1	h^3 and g^3	h^3 and g^3
p and a^1	k	b^1	h^3 and g^3	,,	b^1
o^1 and $a\frac{1}{2}$	d	$b\frac{1}{3}$	h^1 and g^1	,,	$b\frac{1}{3}$
e^2 and $x=(b^1\,b\frac{1}{2}\,h\frac{1}{2})$	x	a^1 and e^1	,,	,,	a^1 and e^1
e^1 and $b\frac{1}{2}$	o	a_2 and e_2	,,	a_2 and e_2	a_2 and e_2
$e\frac{1}{2}$ and $\varphi=(b^1\,d\frac{1}{2}\,g^1)$	,,	a_4 and e_4	,,	e_4	,,
$d\frac{1}{2}$ and a_2	p	k and χ	,,	,,	k and χ
$y=(d^1\,b\frac{1}{2}\,g^1)$ and $b\frac{1}{4}$	i	s and σ	,,	,,	s and σ

The faces formerly (v and φ) were erroneously placed upon my figure 236; they belong to the zone $g^1\,e^1\,p$ (new symbols), and are the same as a_4 and e_4. The former prism, $h^{\frac{7}{4}}\ g^{\frac{7}{4}}$, having only been met with once and having yielded but one, and that a very imperfect measurement, must, it seems to me, be abandoned.

* Annales de Chimie et de Physique, 3rd series, vol. xl.

† Annales des Mines, 5th series, t. xvi. 1859. Manuel de Minéralogie, vol. i. 1862.

XXIII. *On the Conditions for the existence of Three Equal Roots, or of Two Pairs of Equal Roots, of a Binary Quartic or Quintic.* By A. Cayley, *F.R.S.*

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In considering the conditions for the existence of given systems of equalities between the roots of an equation, we obtain some very interesting examples of the composition of relations. A relation is either onefold, expressed by a single equation $U=0$, or it is, say k-fold, expressed by a system of k or more equations. Of course, as regards onefold relations, the theory of the composition is well known: the relation $UV=0$ is a relation compounded of the relations $U=0$, $V=0$; that is, it is a relation satisfied if, and not satisfied unless one or the other of the two component relations is satisfied. The like notion of composition applies to relations in general; viz., the compound relation is a relation satisfied if, and not satisfied unless one or the other of the two component relations is satisfied. I purposely refrain at present from any further discussion of the theory of composition. I say that the conditions for the existence of given systems of equalities between the roots of an equation furnish instances of such composition; in fact, if we express that the function $(*\,(x, y)^n$, and its first-derived function in regard to x, or, what is the same thing, that the first-derived functions in regard to x, y respectively, have a common quadric factor, we obtain between the coefficients a certain twofold relation, which implies either that the equation $(*\,(x, y)^n=0$ has three equal roots, or else that it has two pairs of equal roots; that is, the relation in question is satisfied if, and it is not satisfied unless there is satisfied either the relation for the existence of three equal roots, or else the relation for the existence of two pairs of equal roots; or the relation for the existence of the quadric factor is compounded of the last-mentioned two relations. The relation for the quadric factor, for any value whatever of n, is at once seen to be expressible by means of an oblong matrix, giving rise to a series of determinants which are each to be put $=0$; the relation for three equal roots and that for two pairs of equal roots, in the particular cases $n=4$ and $n=5$, are given in my "Memoir on the Conditions for the existence of given Systems of Equalities between the roots of an Equation," Phil. Trans. vol. cxlvii. (1857), pp. 727–731; and I propose in the present Memoir to exhibit, for the cases in question $n=4$ and $n=5$, the connexion between the compound relation for the quadric factor with the component relations for the three equal roots and for the two pairs of equal roots respectively.

Article Nos. 1 to 8, the Quartic.

1. For the quartic function

$$(a, b, c, d, e\,(x, y)^4,$$

the condition for three equal roots, or, say, for a root system 31, is that the quadrinvariant and the cubinvariant each of them vanish, viz. we must have

$$I = ae - 4bd + 3c^2 = 0,$$

$$J = ace - ad^2 - b^2e + 2bcd - c^3 = 0.$$

2. The condition for two pairs of equal roots, or for a root system 22, is that the cubicovariant vanishes identically, viz. representing this by

$$(A, B, 5C, 10D, 5E, F, G)(x, y)^6 = 0,$$

we must have

$$\begin{aligned}
A &= a^2d - 3abc + 2b^3 &&= 0,\\
B &= a^2e + 2abd - 9ac^2 + 6b^2c &&= 0,\\
C &= abe - 3acd + 2b^2d &&= 0,\\
D &= -ad^2 + b^2e &&= 0,\\
E &= -ade + 3bce - 2bd^2 &&= 0,\\
F &= -ae^2 - 2bde + 9c^2e - 6cd^2 &&= 0,\\
G &= -be^2 + 3cde - 2d^3 &&= 0.
\end{aligned}$$

3. But the condition for the common quadric factor is

$$\begin{vmatrix}
 & a, & 3b, & 3c, & d & \\
 & b, & 3c, & 3d, & e & \\
a, & 3b, & 3c, & d & & \\
b, & 3c, & 3d, & e & &
\end{vmatrix} = 0,$$

and the determinants formed out of this matrix must therefore vanish for $(I, J) = 0$, and also for $(A, B, C, D, E, F, G) = 0$, that is, the determinants in question must be syzygetically related to the functions (I, J), and also to the functions (A, B, C, D, E, F, G).

4. The values of the determinants are—

$1234 = 3\times$	$1235 = 3\times$	$1245 =$	$1345 = 3\times$	$2345 = 3\times$
$+1\ a^2ce$	$-1\ a^2de$	$-1\ a^2e^2$	$-1\ abe^2$	$+1\ ace^2$
$-3\ a^2d^2$	$+4\ abce$	$+2\ abde$	$+4\ acde$	$-1\ ad^2e$
$-1\ ab^2e$	$+1\ abd^2$	$+9\ ac^2e$	$-3\ ad^3$	$-3\ b^2e^2$
$+14\ abcd$	$-3\ ac^2d$	$-9\ acd^2$	$+1\ b^2de$	$+14\ bcde$
$-9\ ac^3$	$-3\ b^2e$	$-9\ b^2ce$	$-3\ bc^2e$	$-8\ bd^3$
$-8\ b^2d$	$+2\ b^2cd$	$+8\ b^2d^2$	$+2\ bcd^2$	$-9\ c^3e$
$+6\ b^3e^2$				$+6\ c^2d^2$

5. The syzygetic relation with (I, J) is given by means of the identical equation

$$\begin{vmatrix}
y^4, & -4xy^3, & 6x^2y^2, & -4x^3y, & x^4 & \\
 & a, & 3b, & 3c, & d & \\
 & b, & 3c, & 3d, & e & \\
a, & 3b, & 3c, & d, & & \\
b, & 3c, & 3d, & e, & &
\end{vmatrix} = -6I.\tilde{H}U + 9J.U,$$

or, as this may be written,

$$(1234,\ 1235,\ 1245,\ 1345,\ 2345)(x, y)^4 = -6\mathrm{I}.\tilde{\mathrm{H}}\mathrm{U} + 9\mathrm{J}.\mathrm{U},$$

where $\tilde{\mathrm{H}}\mathrm{U}$ is the Hessian of U,

$$= \begin{vmatrix} ac & 2\,ad & ae & 2\,be & ce \\ -b^2 & -2\,bc & +2\,bd & -2\,cd & -d^2 \\ & & -3\,c^2 & & \end{vmatrix} (x, y)^4.$$

6. That is, we have

$$\begin{aligned} 1234 &= (\ ac - b^2\ ,\ a)(-6\mathrm{I}, 9\mathrm{J}), \\ 1235 &= (2ad - 2bc\ ,\ 4b)(-6\mathrm{I}, 9\mathrm{J}), \\ 1245 &= (\ ae + 2bd - 3c^2,\ 6c)(-6\mathrm{I}, 9\mathrm{J}), \\ 1345 &= (2be - 2cd\ ,\ 4d)(-6\mathrm{I}, 9\mathrm{J}), \\ 2345 &= (\ ce - d^2\ ,\ e)(-6\mathrm{I}, 9\mathrm{J}). \end{aligned}$$

7. The determinants thus vanish if $(\mathrm{I}, \mathrm{J}) = 0$, that is, for the root system 31; they will also vanish without this being so, if only

$$\left(\frac{3\mathrm{J}}{2\mathrm{I}} = \right) \frac{ac - b^2}{a} = \frac{ad - bc}{2b} = \frac{ae + 2bd - 3c^2}{6c} = \frac{be - cd}{2d} = \frac{ce - d^2}{e};$$

and we may omit the first member $\left(\frac{3\mathrm{J}}{2\mathrm{I}} = \right)$, since if the remaining terms are equal to each other they will also be $= \frac{3\mathrm{J}}{2\mathrm{I}}$. The equations may then be written

$$\left\| \begin{matrix} ac - b^2, & ad - bc, & ae + 2bd - 3c^2, & be - cd, & ce - d^2 \\ a\ , & 2b\ , & 6c\ , & 2d\ , & e \end{matrix} \right\| = 0,$$

and the ten equations of this system reduce themselves (as it is very easy to show) to the seven equations

$$(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}) = 0,$$

which, as above mentioned, are the conditions for the root system 22.

8. It may be added that we have

	A	B	C	D	E	F	G
$\frac{1}{a}.1234 =$		c	$-4b$	$+3a$			
$\frac{1}{2}.1235 =$			c	$-3b$	$+\ a$		
$0\ =$		d	$-3c$		$+\ a$		
$1245 =$		$-e$	$+4d$	$-3c$			
$0\ =$		$-e$		$+6c$		$-a$	
$0\ =$			$-\ d$	$+3c$	$-\ b$		
$\frac{1}{2}.1345 =$			$-\ e$	$+3d$	$-\ c$		
$0\ =$			$-\ e$		$+3c$	$-b$	
$\frac{1}{3}.2345 =$				$-3e$	$+4d$	$-c$	

where it is to be noticed that the four equations having the left-hand side $= 0$, give

B:C:D:E:F proportional to the determinants of the matrix

$$\begin{vmatrix} d, & -3c, & , & a \\ -e, & . , & 6c, & ., & -a \\ & -d , & 3c, & - b, \\ & -e , & . , & +3c, & -b \end{vmatrix};$$

the determinants in question contain each the factor c, and omitting this factor, the system shows that B, C, D, E, F are proportional to their before-mentioned actual values.

Article Nos. 9 to 15, the Quintic.

9. For the quintic function

$$(a, b, c, d, e, f\chi x, y)^5,$$

the condition of a root system 41 is that the covariant, Table No. 14, shall vanish, viz. we must have

$$\begin{aligned} A&=2(ae-4bd+3c^2)=0, \\ B&=\ af-3be+2cd=0, \\ C&=2(bf-4ce+3d^2)=0. \end{aligned}$$

10. The condition of a root system 32 is that the following covariant, viz.

$$3\ (\text{No. } 13)^2(\text{No. } 14)-25(\text{No. } 15)^3,$$

shall vanish, where

No. 13 $= (a, b, c, d, e, f\chi x, y)^5$, the quintic itself.

No. 14 $= ($

ae	af	bf
$-4\ bd$	$-3\ be$	$-4\ ce$
$+3\ c^2$	$+2\ cd$	$+3\ d^2$

$\chi x, y)^3$.

No. 15 $= ($

ac	$3\ ad$	$3\ ae$	af	$3\ bf$	$3\ cf$	df
$-b^2$	$-3\ bc$	$+3\ bd$	$+7\ be$	$+3\ ce$	$-3\ de$	$-e^2$
		$-6\ c^2$	$-8\ cd$	$-6\ d^2$		

$\chi x, y)^6$.

11. The developed expression of the foregoing function is as follows:—

A	B	C	D	E	F	G	H	I	J	K	L	M
$+3a^5e$	$+3a^4f$	$+33a^3bf$	$+10a^3cf$	$-90a^3df$	$-114a^2ef$	$-19a^2f^3$	$-114abf^3$	$-90acf^3$	$+10adf^3$	$+33aef^3$	$+3af^4$	$+3bf^4$
$-12a^5bd$	$+21a^3be$	$-102a^3ce$	$-390a^2de$	$-195a^2e^2$	$-264abdf$	$-608abef$	$-264acef$	$+360adef$	$+155ae^2f$	$-102bdf^3$	$+21bef^3$	$-12cef^3$
$-16a^5c^2$	$-144a^2cd$	$-216a^2d^2$	$+155ab^2f$	$+360abcf$	$-990abe^2$	$+537acdf$	$+468ad^2f$	$+225ae^3$	$-390bef^3$	$+135be^2f$	$-144cdf^3$	$-16d^2f^3$
$+50ab^2c$	$+30ab^2d$	$+135ab^2e$	$+100abce$	$-1500abde$	$+468ac^2f$	$-245ace^2$	$+900ade^2$	$-195b^2f^3$	$+100bdef$	$-216c^2f^3$	$+30ce^2f$	$+50de^2f$
$-25b^4$	$+240abc^2$	$+120abcd$	$-600abd^2$	$+900ac^2e$	$+1320acde$	$+1740ad^2e$	$-990b^2ef$	$-1500bcef$	$+125be^3$	$+120cdef$	$+240d^2ef$	$-25e^4$
	$-150b^2c$	$+480ac^2$	$+1600ac^2d$	$+1800acd^2$	$+1080ad^3$	$-245b^2df$	$+1320bdef$	$+900bd^2f$	$-600c^2ef$	$-150ce^3$	$-150de^3$	
		$-150b^2d$	$+125b^2e$	$+225b^2f$	$+900b^2cf$	$-1700b^2e^2$	$-2700bce^2$	$+1800c^2df$	$+1600cd^2f$	$+480d^2f$		
		$-300b^2c^2$	$-1000b^2cd$	$-1500b^2d^2$	$-2700b^2de$	$+1740bc^2f$	$+900bd^2e$	$-1500c^2e^2$	$-1000cde^2$	$-300d^2e^2$		
					$+900bc^2e$	$-2000bcde$	$+1080c^3f$					
					$-600bcd^2$	$+600bd^3$	$-600c^3de$					
						$+600c^3e$						
						$-400c^3d^2$						

$(x, y)^{12}$

12. The conditions for the common quadric factor are

$$\left\| \begin{matrix} & a, & 4b, & 6c, & 4d, & e \\ a, & 4b, & 6c, & 4d, & e & \\ & b, & 4c, & 6d, & 4e, & f \\ b, & 4c, & 6d, & 4e, & f & \end{matrix} \right\| = 0,$$

the several determinants whereof are given in Table No. 27 of my "Third Memoir on Quantics," Philosophical Transactions, vol. cxlvi. (1856), pp. 627–647.

13. These determinants must therefore vanish, for (A, B, C)=0, and also for $(\mathfrak{A}, \mathfrak{B}, \ldots \mathfrak{L}, \mathfrak{M})=0$, that is, they must be syzygetically connected with (A, B, C), and also with $(\mathfrak{A}, \mathfrak{B}, \ldots \mathfrak{L}, \mathfrak{M})$. The relation to (A, B, C) is in fact given in the Table appended to Table No. 27, viz. this is

	C×	+B×	+A×
1234=	$+6a^2$	$-12ab$	$+16ac-10b^2$
1235=	$+6ab$	$-2ac-10b^2$	$+6ad$
1236=	$-2ac+8b^2$	$+6ad-18bc$	$-2df+8e^2$
1245=	$+18ac$	$-6ad-30bc$	$+8ae+10bd$
1246=	$+12bc$	$+4ae-4bd-24c^2$	$+4be+8cd$
1345=	$+24ad$	$-8ae-40bd$	$+4af+20be$
1256=	$-1ae+4bd+3c^2$	$+1af+5be-18cd$	$-1bf+4ce+3d^2$
2345=	$+20ae+40bd-30c^2$	$-80be+20cd$	$+20bf+40ce-30d^2$
1346=	$+4ae+8bd+6c^2$	$-36cd$	$+4bf+8ce+6d^2$
2346=	$+4af+20be$	$-8bf-4ce$	$+24cf$
1356=	$+4be+8cd$	$+4bf-4ce-24d^2$	$+12de$
2356=	$+8bf+10ce$	$-6cf-30de$	$+18df$
1456=	$+6ce$	$+6cf-18de$	$-2df+8e^2$
2456=	$+6cf$	$-2df-10e^2$	$+6ef$
3456=	$+16df-10e^2$	$-12ef$	$+6f^2$

14. Between the expressions $\mathfrak{A}$, $\mathfrak{B}$, &c., and 1234, 1235, &c., there exist relations the form of which is indicated by the following Table:

A	B	C	D	E	F	G	H	I	J	K	L	M	1234	1235	1236	1245	1246	1256	1345	1346	2345	2346	2356	1456	2456	3456
c	b	a											a													
d	c	b	a										b	a												
e	d	c	b	a									c	b	a											
f	e	d	c	b	a								d	c	b	a										
	f	e	d	c	b	a							e	d	c	b	a									
		f	e	d	c	b	a						f	e	d	c	b	a								
			f	e	d	c	b	a						f	e	d	c	b	a							
				f	e	d	c	b	a						f	e	d	c	b	a						
					f	e	d	c	b	a						f	e	d	c	b	a					
						f	e	d	c	b	a						f	e	d	c	b	a				
							f	e	d	c	b	a						f	e	d	c	b	a			
								f	e	d	c	b							f	e	d	c	b	a		
									f	e	d	c								f	e	d	c	b	a	
										f	e	d									f	e	d	c	b	a
											f	e										f	e	d	c	b
												f											f	e	d	c
																								f	e	d
																									f	e
																										f

viz. these relations are of the form

$$(\ )\mathfrak{A}+(\ )\mathfrak{B}+(\ )\mathfrak{C} \qquad +(\ ).1234 \qquad =0,$$

$$(\ )\mathfrak{A}+(\ )\mathfrak{B}+(\ )\mathfrak{C}+(\ )\mathfrak{D} \qquad +(\ ).1234+(\ ).1235=0,$$

&c.

where the brackets ( ) denote numerical coefficients, determinate as to their ratios.

15. Assuming the existence of these relations, we have for the determination of the numerical coefficients in each relation a set of linear equations, which are shown by the following Tables, viz. referring to the Table headed $c\mathfrak{A}$, $b\mathfrak{B}$, $a\mathfrak{C}$, $a.1234$, if the multipliers of terms respectively be A, B, C, X, then the Table denotes the system of linear equations

$$\begin{aligned} &0\ \mathrm{A} + 3\,\mathrm{B} + 33\,\mathrm{C} + 0\,\mathrm{X} = 0,\\ &3\ \mathrm{A} + 0\,\mathrm{B} - 102\,\mathrm{C} - 16\,\mathrm{X} = 0,\\ &\&\mathrm{c.}, \end{aligned}$$

that is, nine equations to be satisfied by the ratios of the coefficients A, B, C, X, and which are in fact satisfied by the values at the foot of the Table, viz.

$$\mathrm{A} : \mathrm{B} : \mathrm{C} : \mathrm{X} = 66 : -11 : +1 : +6.$$

There would be in all fourteen Tables, but as those for the second seven would be at once deducible by symmetry from the first seven, I have only written down the seven Tables; the solutions for the first and second Tables were obtained without difficulty, but that for the third Table was so laborious to calculate, and contains such extraordinarily high numbers, that I did not proceed with the calculation, and it is accordingly only the first, second, and third Tables which have at the foot of them respectively the solutions of the linear equations.

16. The results given by these three Tables are, of course,

$$66c\mathfrak{A} - 11b\mathfrak{B} + 1a\mathfrak{C} + 6a.1234 = 0,$$
$$330d\mathfrak{A} + 110c\mathfrak{B} - 55b\mathfrak{C} + 9a\mathfrak{D} - 105\,a.1235 = 0,$$

$$\begin{aligned} &+266478575\ e\mathfrak{A}\\ &-617359490\ d\mathfrak{B}\\ &+144200810\ c\mathfrak{C}\\ &+\ 9656911\ b\mathfrak{D}\\ &+\ 9090785\ a\mathfrak{E}\\ &-721004050\ c.1234\\ &+\ 90914175\ b.1235\\ &-160758675\ a.1245\\ &+\ 11559295\ a.1236 = 0. \end{aligned}$$

It is to be noticed that the nine coefficients of this last equation were obtained from, and that they actually satisfy, a system of fourteen linear equations; so that the correctness of the result is hereby verified.

17. The seven Tables are

First Table.

	$c\mathfrak{A}$	$b\mathfrak{B}$	$a\mathfrak{C}$	$a.1234$
a^3bf		+ 3	+ 33	
a^3ce	+ 3		−102	− 16
a^3d^2			−216	+ 36
a^2be^2		+ 21	+135	+ 16
a^2bcd	−12	−144	+120	−152
a^2c^3	−16		+480	+ 96
ab^3d		+ 30	−150	+ 80
ab^2c^2	+50	+240	−300	− 60
b^4c	−25	−150		
	+66	−11	+1	+ 6

Second Table.

	$d\mathfrak{A}$	$c\mathfrak{B}$	$b\mathfrak{C}$	$a\mathfrak{D}$	$a.1235$	$b.1234$
a^2cf		+ 3		+ 10	− 4	
a^2de	+ 3			− 390	+24	
a^2b^2f			+ 33	+ 155	+ 4	
a^2bce		+ 21	−102	+ 100	−84	− 16
a^2bd^2	−12		−216	− 600	−24	+ 36
a^2c^2d	−16	−144		+1600	+64	+ 16
ab^3e			+135	+ 125	+60	
ab^2cd	+50	+ 30	+120	−1000	−40	−152
abc^3		+240	+480			+ 96
b^4d	−25		−150			+ 80
b^3c^2		−150	−300			− 60
	+330	+110	−55	+9	−105	0

Third Table.

	$e\mathfrak{A}$	$d\mathfrak{B}$	$c\mathfrak{C}$	$b\mathfrak{D}$	$a\mathfrak{E}$	$c.1234$	$b.1235$	$a.1245$	$a.1236$
a^3df		+ 3			− 90			− 6	+ 6
a^2e^2	+ 3				− 195			+16	
a^2bef			+ 33	+ 10	+ 360		− 4	+ 6	−22
a^2bde	−12	+ 21		− 390	−1500		+24	−26	− 6
a^2c^2e	−16		−102		+ 900	− 16		−96	+16
a^2cd^2		−144	−216		+1800	+ 36		+96	
ab^3f				+ 155	+ 225		+ 4		+16
ab^2ce	+50		+135	+ 100		+ 16	−84	+90	−10
ab^2d^2		+ 30		− 600	−1500		−24	−80	
abc^2d		+240	+120	+1600		−152	+64		
ac^4			+480			+ 96			
b^4e	−25			+ 125			+60		
b^3cd		−150	−150	−1000		+ 80	−40		
b^3c^3			−300			− 60			
	+266478575	−617359490	+144200810	+9656911	+9090785	−721004050	+90914175	−160758675	+11559295

Fourth Table.

	$f\mathfrak{A}$	$e\mathfrak{B}$	$d\mathfrak{C}$	$c\mathfrak{D}$	$b\mathfrak{E}$	$a\mathfrak{F}$	$d.1234$	$c.1235$	$b.1236$	$b.1245$	$a.1246$	$a.1345$
a^2ef	+ 3	+ 3				− 114					+ 4	
a^2bdf	−12		+ 33		− 90	− 264			+ 6	− 6	− 4	− 24
a^2be^2		+ 21			− 195	− 990				+16	− 4	+ 64
a^2c^2f	−16			+ 10		+ 468		− 4			−24	+ 24
a^2cde		−144	−102	− 390		+1320	− 16	+24			+24	−208
a^2d^3			−216			+1080	+ 36					+144
ab^2cf	+50			+ 155	+ 360	+ 900		+ 4	−22	+ 6	+24	
ab^2de		+ 30	+135		−1500	−2700	+ 16		− 6	−26	−20	− 40
abc^2e		+240		+ 100	+ 900	+ 900		− 84	+16	− 96		+ 60
$abcd^2$			+120	− 600	+1800	− 600	−152	− 24		+96		− 40
ac^2d			+480	+1600			+ 96	+64				
b^4f	−24				+ 225				+16			
b^3ce		−150		+ 125				+60	−10	+90		
b^2d^2			−150		−1500		+ 80			−80		
b^2c^2d			−300	−1000			− 60	−40				

Fifth Table.

	$f\mathfrak{B}$	$e\mathfrak{C}$	$d\mathfrak{D}$	$c\mathfrak{E}$	$b\mathfrak{F}$	$a\mathfrak{G}$	$e.1234$	$d.1235$	$c.1236$	$c.1245$	$b.1246$	$b.1345$	$a.1256$	$a.2345$	$a.1346$
a^3f^2	+ 3					− 19							+ 1		
a^2bef	+ 21	+ 33			− 114	− 608					+ 4		− 2		+16
a^2edf	−144		+ 10	− 90		+ 537		− 4	+ 6	− 6			−16	+ 20	−36
a^2ce^2		−102		− 195		− 245	− 16			+16			+16	− 80	−16
a^2d^2e		−216	− 390			+1740	+ 36	+24					+16	+ 60	+36
ab^2df	+ 30		+ 155		− 264	− 245		+ 4			− 4	− 24	−15	− 80	−16
ab^2e^2		+135			− 990	−1700	+ 16				− 4	+ 64		+240	
abc^2f	+240			+ 360	+ 468	+1740			−22	+ 6	−24	+ 24		+ 60	+36
$abcde$		+120	+ 100	−1500	+1320	−2000	−152	− 84	− 6	−26	+24	−208		−860	− 20
abd^3			− 600		+1080	+ 600		−24				+144		+960	
ac^3e		+480		+ 900		+ 600	+ 96		+16	−96				+960	
ac^2d^2			+1600	+1800		− 400		+64		+96				−320	
b^3ef	−150			+ 225	+ 900				+16		+24				
b^3de		−150	+ 125		−2700		+ 80	+60			−20	− 40			
b^2c^2e		−300			+ 900		− 60		−10	+90		+ 60			
b^2cd^2			−1000	−1500	− 600			−40		−80		− 40			

Sixth Table.

	*f*𝔈	*e*𝔇	*d*𝔈	*c*𝔉	*b*𝔊	*a*.𝔄	*f*.1234	*e*.1235	*d*.1236	*d*.1245	*c*.1246	*c*.1345	*b*.1256	*b*.2345	*b*.1346	*a*.2346	*a*.1356
a^2bf^2	+ 33				− 19	− 114							+ 1				+ 4
a^2cef	−102	+ 10		− 114		− 264	− 16	− 4			+ 4					− 24	− 4
a^2d^2f	−216		− 90			+ 468	+ 36		+ 6	− 6						+ 24	−24
a^2de^2		− 390	− 195			+ 900		+24		+16							+24
ab^2ef	+135	+ 155			− 698	− 990	+ 16	+ 4					− 2		+16	+ 64	− 4
$abcdf$	+120		+ 360	− 264	+ 537	+1320	−152		−22	+ 6	− 4	− 24	−16	+ 20	−36	−208	+24
$abce^2$		+ 100		− 990	− 245	−2700		−84			− 4	+ 64	+16	− 80	−16	− 40	−20
abd^2e		− 600	−1500		+1740	+ 900		−24	− 6	−26			+16	+ 60	+36	+ 60	
ac^2f	+480			+ 468		+1080	+ 96				−24	+ 24				+144	
ac^2de		+1600	+ 900	+1320		− 600		+64	+16	−96	+24	−208				− 40	
acd^2			+1800	+1080						+96		+144					
b^3df	−150		+ 225		− 245		+ 80		+16				−80	− 80	− 16		
b^3e^2		+ 125			−1700			+60						+240			
b^2c^2f	−300			+ 900	+1740		− 60				+24			+ 60	+36		
b^2cde		−1000		−2700	−2000			−40	−10	+90	−20	− 40		−860	−20		
b^2d^3			−1500		+ 600					−80				+960			
bc^3e				+ 900	+ 600							+ 60		+960			
bc^2d^2				− 600	− 400							− 40		−320			

Seventh Table.

	*f*𝔇	*e*𝔈	*d*𝔉	*c*𝔈	*b*𝔄	*a*𝔦	*f*.1235	*e*.1236	*e*.1245	*d*.1246	*d*.1345	*c*.1256	*c*.2345	*c*.1346	*b*.2346	*b*.1356	*a*.2356	*a*.1456
a^3ef^2	+ 10			− 19		− 90	− 4					+ 1					− 6	+ 6
a^2def	− 390	− 90	− 114			+ 360	+24	+ 6	− 6	+ 4							+ 6	−22
a^2e^3		− 195				+ 225			+16									+16
ab^2f^2	+ 155				− 114	− 195	+ 4									+ 4	+16	
$abcef$	+ 100	+ 360		− 608	− 264	−1500	−84	−22	+ 6			− 2		+16	− 24	− 4	−26	− 6
abd^2f	− 600		− 264		+ 468	+ 900	−24			− 4	− 24				+ 24	−24	−96	+16
$abde^2$		−1500	− 990		+ 900			− 6	−26	− 4	+ 64					+24	+90	−10
ac^3df	+1600		+ 468	+ 537		+1800	+64			−24	+ 24	−16	+ 20	− 36			+96	
ac^2e^2		+ 900		− 245		−1500		+16	−96			+16	− 80	− 16			−80	
acd^2e		+1800	+1320	+1740					+96	+24	−208	+16	+ 60	+ 36				
ad^4			+1080								+144							
b^2ef	+ 125	+ 225			− 990		+60	+16							+ 64	− 4		
b^2cdf	−1000		+ 900	− 245	+1320		−40			+24		−15	− 80	− 16	−208	+24		
b^2ce^2				−1700	−2700			−10	+90				+240		− 40	−20		
b^2d^2e		−1500	−2700		+ 900				−80	−20	− 40				+ 60			
bc^3f				+1740	+1080								+ 60	+ 36	+144			
bc^2de			+ 900	−2000	− 600						+ 60		−860	− 20	− 40			
bcd^3			− 600	+ 600							− 40		+960					
c^4e				+ 600									+960					
c^3d^2				− 400									−320					

And the remaining seven Tables might of course be deduced from these by writing (f, e, d, c, b, a) instead of (a, b, c, d, e, f), and making the corresponding alterations in the top line of each Table.

18. The equations $\mathfrak{A}=0, \mathfrak{B}=0, \ldots, \mathfrak{M}=0$ consequently establish between the fifteen functions 1234, 1235, ... 3456 a system of fourteen equations, viz. the first and last three of these are

$$\begin{aligned} 1234&=0,\\ 1235&=0,\\ -160758675\,.\,1245&\\ +\ 11559295\,.\,1236&=0,\\ &\cdots\\ +\ 11559295\,.\,1456&\\ -160758675\,.\,2356&=0,\\ 2456&=0,\\ 3456&=0. \end{aligned}$$

To complete the proof that in virtue of the equations $\mathfrak{A}=0$, $\mathfrak{B}=0, \ldots, \mathfrak{M}=0$ all the fifteen functions 1234, 1235, ... 3456 vanish, it is necessary to make use of the identical relations subsisting between these quantities 1234, &c.; thus we have

$$a\,.\,1345+4b\,.\,1245+6c\,.\,1235+4d\,.\,1234=0,$$
$$b\,.\,1345+4c\,.\,1245+6d\,.\,1235+4e\,.\,1234=0,$$

which, in virtue of the above equations $1234=0$ and $1235=0$, become

$$a\,.\,1345+4b\,.\,1245=0,$$
$$b\,.\,1345+4c\,.\,1245=0,$$

giving (unless indeed $ac-b^2=0$) $1245=0$, $1345=0$; the equation $1245=0$ then reduces the third of the above equations to $1236=0$, and so on until it is shown that the fifteen quantities all vanish.

XXIV. *Researches on the Blood.—On the Action of Nitrites on Blood.* By Arthur Gamgee, *M.D., F.R.S.E., Assistant to the Professor of Medical Jurisprudence in the University of Edinburgh. Communicated by Professor* Frankland, *F.R.S.*

Received April 1,—Read May 7, 1868.

Introductory Propositions[1].

In the following propositions the leading facts which have been ascertained by previous observers with regard to the colouring-matter of the blood and its relation to gases have been condensed.

1. The colouring-matter of blood, to which the names of Cruorine[2] or Hæmoglobin[3] have been given, occurs in solution in the blood-corpuscles, but may by suitable treatment be obtained from them in the form of crystals, which when seen individually are of a yellow, and when seen collectively are of a reddish colour. Their solution possesses the property of absorbing light so as to yield a remarkable spectrum characterized by two very well-defined absorption-bands situated in the yellow and green portions of the spectrum, and by the cutting off of the greater part of the more refrangible rays[4].

These crystals are identical with those noticed by Leidig[5], Reichert[6], and Kölliker[7], and afterwards more fully described by Funke[8], Kunde[9], and Lehmann[10].

2. The composition of these crystals appears to be perfectly definite. The following are the results of the analyses of two independent observers, C. Schmidt[11] and Hoppe-Seyler[12]:—

[1] The author has thought it right to introduce these propositions in order to remove any doubts which might exist in the mind of the reader with regard to the identity of Cruorine or Hæmoglobin with the blood-crystals of Funke and Lehmann, and to place in as clear a light as possible the leading facts which have been made out on this subject.

[2] "On the Reduction and Oxidation of the Colouring-matter of the Blood, by Professor Stokes, F.R.S.," Proceedings of the Royal Society of London, vol. xiii. p. 357, paragraph 8.

[3] Berzelius applied the term Hæmatoglobulin to the colouring-matter of blood. Hoppe-Seyler and other German physiologists have adopted the term Hæmoglobin.

[4] Hoppe, Virchow's Archiv. vol. xxiii. p. 446 (1862).

[5] Zeitschrift f. wiss. Zoologie. 1849, Bd. I. p. 116.

[6] Müller's Archiv, 1849, p. 197.

[7] Zeitschrift f. wiss. Zoologie, 1849, Bd. I. p. 261.

[8] Zeitschrift f. rat. Med. Bd. I. p. 172 (1851).

[9] Zeitschrift f. rat. Med. Bd. II. p. 271.

[10] Qualis sit Hæmocrystallines natura chemica. Programm. Jena, 1856 (quoted by Hoppe). Lehmann's 'Physiological Chemistry,' translated by Dr. Day, Cavendish Society, vol. iii. pp. 485–495 (1854).

[11] Ueber Blutkrystalle. Dorpat, 1862, p. 33.

[12] Medicinisch-chemische Untersuchungen, zweites Heft, p. 187.

H. S.	C. S.
C = 53·64	C = 53·85
H = 7·11	H = 7·32
N = 16·19	N = 16·17
S = 0·66	S = 0·39
Fe = 0·43	Fe = 0·43
O = 21·02	O = 21·84

3. The colouring-matter exists in blood in combination with oxygen; this oxygen cannot be discovered by pyrogallic acid[1], nor is its absorption by the colouring-matter regulated by Dalton and Henry's law of absorption[2]. The combination is, however, of a loose character, and is broken up by the addition of reducing-solutions to blood[3], as well as by boiling blood, or merely exposing it in a Toricellian vacuum[4]. The blood-colouring-matter freed from its loose oxygen differs from the oxidized substance, (*a*) in colour[5], (*b*) in its absorption-spectrum[6]. To this substance the names of reduced or purple cruorine[7] and reduced hæmoglobin[8] have been given.

4. By passing hydrogen, carbonic acid, or laughing-gas through blood the oxygen is expelled, *i. e.* the colouring-matter is reduced[9]. Blood which has been reduced by hydrogen or carbonic acid may be made to furnish crystals of reduced hæmoglobin, which are isomorphous with those of the oxidized body, but which differ from it in colour and in optical characters when examined with the spectroscope[10].

5. "We may infer from the facts above mentioned that the colouring-matter of blood, like indigo, is capable of existing in two states of oxidation, distinguishable by a difference of colour and a fundamental difference in the action on the spectrum. It may be made to pass from the more to the less oxidized state by the action of suitable reducing agents, and recovers its oxygen by absorption from the air" (Stokes)[11].

6. The loosely combined oxygen of the colouring-matter may be expelled by carbonic oxide gas[12]; one volume of carbonic oxide takes the place of one volume of oxygen [13]. The compound thus formed differs from the O-compound in not yielding a less oxygenized substance when treated with suitable reducing agents[14]. It possesses an absorp-

[1] Bernard, Propriétés des liquides de l'organisme, t. i. p. 337 (1859).

[2] Lothar Meyer, "Die gase des Blutes," Zeitschrift f. rat. Med. Bd. VIII. p. 257 (1857).

[3] Stokes, *op. cit.*

[4] Hoppe-Seyler, Med.-chemisch. Untersuchungen, zweites Heft, p. 191.

[5] Stokes, Proceedings of Royal Society, *loc. cit.* [6] Ibid. [7] Ibid.

[8] Kühne, Lehrbuch der Physiologischen Chemie, 1866, zweites Lieferung, p. 218.

[9] Kühne, *op. cit.*, zw. Lief. p. 210. L. Hermann, "Ueber die wirkungen des Stickoxydgases auf das Blut," Müller's Archiv, 1865, p. 470. [10] Kühne, *op. cit.* p. 218.

[11] Stokes, Proceedings of the Royal Society, vol. xiii. p. 357, paragraph 8.

[12] Claude Bernard, Leçons sur les effets des substances toxiques et médicamenteuses. Paris, 1857, p. 158. Propriétés des liq. de l'organisme, vol. i. p. 365.

[13] De sanguine oxydo carbonico infecto. Auctor Lothar Meyer, Diss. Inaug. Chym. Vratislaviæ 1858. "Ueber die Einwirkung des Kohlenoxydgas auf Blut," Zeitschrift f. rat. Med. Dritte Reihe. V. Band. 1859, p. 82.

[14] Hoppe-Seyler, Zeitschrift für Anal. Chemie, vol. iii. p. 439.

tion spectrum almost identical with that of the O-compound[1]. It may be obtained in well-defined crystals which are isomorphous with those of the O-compound[2].

7. Similarly the CO-compound may be decomposed by nitric oxide, one volume of CO being apparently replaced by one volume of N_2O_2. The body thus formed may be obtained in crystals isomorphous with those of the O- and the CO-compounds. Like the latter it is unacted upon by reducing solutions[3].

8. Hydrocyanic acid when added to blood appears to combine with the colouring-matter; the nature of the compound formed has not, however, been ascertained. It may, however, be obtained in crystals identical with those of the O- and CO-compound. Its spectrum is identical with the former and, like it, is capable of reduction[4].

9. When heated, or when treated with strong acids and alkalies, hæmoglobin furnishes amongst other products of decomposition, an insoluble body called hæmatin[5], which was until recently supposed to be the colouring-matter of blood[6]. This body possesses no power of combining with oxygen. The absorption-coefficient of oxygen for a solution of hæmatin is nearly the same as for water[7].

Note.—The author has purposely avoided entering upon the description of the interesting physical properties and chemical reactions and relations of hæmatin, as these do not immediately concern the object of this memoir.

On the Action of Nitrites on Blood.

No one has hitherto investigated the action which the nitrites exert upon the blood and upon its colouring-matter. Two of the oxides of nitrogen have been studied in their relation to blood by Dr. Ludimar Hermann, viz. laughing-gas[8] and nitric oxide[9]. With regard to the first of these, it has been shown that it acts upon blood and hæmoglobin very much as hydrogen, carbonic acid, and nitrogen, whilst in reference to the second, Hermann has shown that it actually combines with the colouring-matter.

My attention was directed to the peculiar action of nitrites upon the blood-colouring-matter by observing that the blood of mice poisoned by exposure to an atmosphere impregnated with the vapour of nitrite of amyl presented a chocolate-colour. I was thus

[1] Hoppe-Seyler, Medicinisch-chemische Untersuchungen, zweites Heft, p. 203.

[2] Hoppe-Seyler, Med.-chem. Unters. p. 201.

[3] Dr. Ludimar Hermann, "Ueber die wirkungen des Stickstoffoxydgas auf das Blut," Müller's Archiv, 1865, pp. 469–481.

[4] Hoppe-Seyler, Med.-chem. Untersuchungen, 11 Heft, p. 206. Preyer, Virchow's Archiv, 1867, Sept. p. 125.

[5] Hoppe, Virchow's Archiv, Bd. XXIII. p. 446. Stokes, *op. cit.* p. 355 *et seq.*

[6] The reader who wishes to become acquainted with the different methods employed formerly for separating hæmatin are referred to Gorup-Besanez, Lehrbuch der Phys. Chemie, Braunschweig, 1862, p. 168 *et seq.* Berzelius clearly distinguished the difference between the unaltered colouring-matter and the coagulated colouring-matter.—Traité de Chimie, par J. T. Berzelius, Paris, 1833. t. vii. pp. 48–65.

[7] Lothar Meyer, De sanguine oxydo carbonico infecto, pp. 9–12.

[8] "Ueber die physiologischen wirkungen des Stickstoffoxydulgases," Müller's Archiv, 1864, p. 521.

[9] "Ueber die wirkungen des Stickstoffoxydgas auf das Blut," Müller's Archiv, 1865, p. 469.

led to find that the optical properties of blood were considerably modified by the action of nitrites. I embodied the results of my first observations in a preliminary note which I read before the Royal Society of Edinburgh on the 4th of March, 1867[1]. At that time I was acquainted with little more than the changes in the optical characters induced by nitrites.

The nature of the changes and the influence which they exert upon the relation of the blood to gases are chiefly to be discussed in the present paper.

I. *On the Changes in the Colour of the Blood induced by Nitrites.*

When defibrinated and well-arterialized blood is mixed with a solution of nitrite of potassium or nitrite of sodium, its colour becomes almost immediately altered; from its beautiful florid-red it changes to a more or less chocolate-brown colour. The rapidity with which this change takes place appears to vary considerably in the case of different samples of blood. Just as great differences are found to exist in the relative rate with which the colour of blood (and its spectrum) is affected by standard solutions of alkalies and acids, so considerable variations occur in the rapidity of the action of nitrites. In the course of this investigation the nitrites whose action on blood has been chiefly examined have been those of potassium, sodium, silver, and amyl. The alkaline nitrites were always prepared by decomposing repeatedly crystallized nitrite of silver by means of an exactly equivalent quantity of the alkaline chloride. The nitrite of amyl was prepared by the action of nitrous acid upon repeatedly fractionated amylic alcohol. The product was also repeatedly fractionated, and only those portions which boiled between 95° and 100° C. collected as pure.

The differences in the rapidity with which nitrites and other active substances act upon blood, probably depend much upon physical conditions affecting the blood-corpuscles. The freshly-drawn blood of the Dog is almost instantaneously affected in colour when mixed with solutions of nitrites, whereas in the case of the blood of the Ox and Sheep occasionally twenty minutes, or even longer, will elapse before a solution of nitrite of potassium or sodium effects the characteristic change. The blood of the Dog, it must be remarked, is used in preference to that of the Ox and Sheep in the preparation of hæmoglobin, because its blood-corpuscles when treated with water burst so much more readily than those of the Ox and Sheep.

When it is desired to act upon blood with nitrite of amyl, the latter should be dissolved in alcohol, and a few drops of the alcoholic solution added to the blood.

Solutions of hæmoglobin are immediately affected by the addition of nitrites.

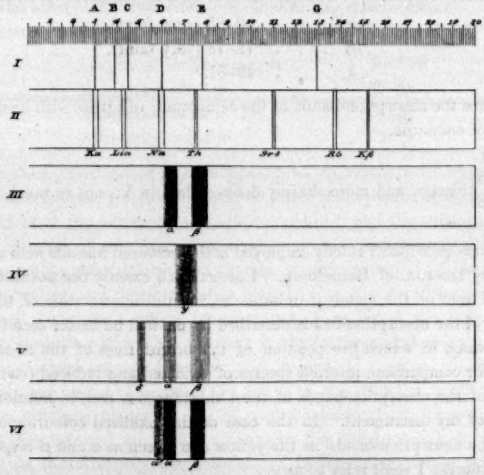
When blood has, through the addition of nitrites, assumed a chocolate-colour, the latter is immediately changed to a red on the addition of ammonia.

The change in colour induced by nitrites in no way depends upon an alteration in the shape of the blood-corpuscles, as I have ascertained by numerous experiments.

[1] "Note on the Action of Nitric Oxide, Nitrous Acid and Nitrites on Hæmoglobin," Proceedings of the Royal Society of Edinburgh, 1867, No. 73, p. 108.

II. *On the Spectrum of Blood acted upon by Nitrites.*

If normal arterialized blood be suitably diluted so as to admit of an accurate examination of its absorption-spectrum, and if after the position of the two characteristic absorption-bands has been noted, a solution of any nitrite be added to it, it will be noticed that when the solution begins to change from a red to a brown tint, the absorption-bands undergo notable changes. The two sharply-defined absorption-bands of the oxidized colouring-matter become fainter and fainter, and are, indeed, only visible when a comparatively thick layer of the fluid is examined. At the same time, if the layer be sufficiently thick, it will be noticed that an additional, though comparatively faint, absorption-band appears in the red: this band appears absolutely to coincide with that of acid hæmatin; it is seen to greatest advantage when so thick a layer of solution is examined that all but the red rays are cut off. The complete change, as seen with the spectroscope, is always coincident with the complete change of colour. If, now, the solution be made alkaline by the addition of ammonia, the colour changes from the chocolate-brown to blood-red again; simultaneously the absorption-band in the red disappears, and the two absorption-bands between D and E become more distinct again. In addition, however, it is noticed that the portion of the spectrum occupying the confines of the yellow and orange has become shaded by a less well-defined absorption-band.



I. indicates approximately the position of FRAUNHOFER'S lines A, B, C, D, E, and G in relation to the millimetre scale of the spectroscope used in this research.

II. indicates the position occupied by several of the bright lines of the metals in relation to the millimetre scale of the spectroscope used.

Kα	30
Liα	43
Na	60
Th	75
Irδ	110
Rbβ	136
Rbα	137
Rβ	152

III. indicates the absorption-spectrum of oxidized hæmoglobin as seen in relation to the millimetre scale of my instrument.

α	60–65
β	72–78.

IV. indicates the absorption-band of reduced hæmoglobin (purple cruorine).

γ	65–74.

V. indicates the spectrum of blood which has been treated with nitrites. The crystals of the nitrite compound of hæmoglobin also exhibit the same spectrum.

α	60–65 (very faint).
β	72–78 (very faint).
δ	49–51.

VI. indicates the absorption-bands of the compounds of nitrite with hæmoglobin after the addition of ammonia.

α′ 56–59.

β 60–65, α 72–78 } blacker and more sharply defined than in V., not so much so as in III.

The spectroscope which I chiefly employed in this research was one with a single prism, constructed by Desaga, of Heidelberg. I ascertained exactly the positions of the most characteristic lines of the metals in relation to the millimetre scale of the instrument. The position of the absorption-bands described by me will be better seen by comparison with the drawing, in which the position of the bright lines of the metals is marked. I have also, for comparison, inserted spectra of oxidized and reduced cruorine, in which the position of the absorption-bands of these substances is seen in relation to the millimetre scale of my instrument. In the case of the oxidized colouring-matter, I shall distinguish the absorption-bands in the yellow and green as α and β respectively; that of reduced cruorine I shall refer to as γ.

Obs. I. A solution of defibrinated Ox's blood was made by diluting 5 cub. centims. to the volume of one litre with distilled water. The solution was placed for examination

in a hollow glass prism, which permitted me to examine definite and varying thicknesses of fluid. The prism had a capacity of 200 cub. centims.

With a layer of solution 2 centimetres thick, the following observations were made:—

Extent of spectrum visible . . .	30–110
Absorption-band in yellow (α) .	60–65
Absorption-band in green (β) . .	71–76

Five drops of nitrite of amyl were now added; after ten minutes a slight fading of the band in the green and a brown tinge in the colour of the fluid was noticed. After fifteen minutes the two absorption-bands α and β had become so exceedingly faint and ill-defined as not to be capable of accurate measurement. There was also a faint absorption-band in the red, having its centre at 49. A stratum of the fluid 3·8 centimetres broad was now examined with the following results:—

Extent of spectrum visible . . .	35–110
Absorption-band α	60–65 (very faint)
Absorption-band β	71–76 (very faint)
Absorption-band δ (*in red*) . . .	48–51

The fluid was now made faintly ammoniacal. The band in the red instantly disappeared; the hæmoglobin bands became much darker; the orange was, however, shaded partly by a faint absorption-band which appeared to run into the band α. The following observations were made:—

Length of visible spectrum . . .	30–110
Absorption-band α	60–65
Absorption-band α'	56–59
Absorption-band β	72–78

The observation just detailed is merely taken as an example of what always occurs when a nitrite acts upon blood, and when the latter is afterwards treated with ammonia.

Obs. II. If after the action of a nitrite upon blood and the subsequent addition of ammonia, a solution of sulphide of ammonium or a reducing iron-solution be added, there is a very rapid and extraordinary change, viz. the spectrum of the original blood is first re-established, and thereafter gives place to that of reduced hæmoglobin.

The reduced blood then differs in no respect from blood that has been reduced without the previous addition of nitrites; it yields, when shaken with air, the spectrum of the oxidized colouring-matter, and instead of a brown chocolate-colour, it possesses again the colour of blood. The action of the reducing-solution in first of all changing the nitrite spectrum to that of oxidized hæmoglobin takes place quite independently of the external air. If, for example, the blood-solution in which a nitrite had induced its characteristic action be placed in the prism bottle, and then a long pipette full of sulphide of ammonium be plunged to the bottom of it, the two sharply-defined absorption-bands of O-Hb at once appear at the point where the fluids have

come in contact; whereas it is only after some time that the further reducing-action of the sulphide makes itself manifest.

Obs. III. Five cub. centims. of freshly defibrinated blood of the Ox were diluted to the volume of one litre, and the solution was then examined in a hollow prism. A layer of solution two centimetres thick was examined. The spectrum of this solution was as follows:—

Band α	60–65
Band β	72–76
Extent of spectrum . . .	30–110

One cub. centim. of a solution of nitrite of potassium of unknown strength was added to 200 cub. centims. of the above blood-solution. Almost instantaneously the bands became much fainter. About two minutes after the addition of the nitrite, on examining a stratum two centimetres broad, the two absorption-bands had become almost invisible, appearing as mere shadings of the spectrum. There was a very faint band with its centre at 50. On examining thicker strata of fluid the original bands could be seen more distinctly, and the band in the red was very well marked. A solution of ammonia was added to the fluid. The red band disappeared at once, and the two bands became more intense. The orange was observed to be shaded, a faint absorption-band appearing to overshadow it and to join the band α; as in other observations to determine this point it was impossible for me to decide whether the band in the orange was separated from the band α by any unshaded portion, *i. e.* whether it was an absolutely separate band. On bringing a soda-salt into the flame of the lamp whose light was passed through the blood-solution and examined, the yellow line was seen to be bounded above by a very marked absorption-band extending from 60 to 65, and below by the shadowy but distinct band in the orange (α'). The following observations were now made:—

α . .	60–65
α' . .	56–59
β . .	71–78

On now passing a long pipette charged with sulphide of ammonium to the bottom of the fluid, the sharply defined absorption-band of O-Hb appeared.

α . .	60–65
β . .	70–78.

The bands were very much blacker than before, and the absorption-band in the orange had disappeared. After some time the fluid had assumed a purple tint, indicating reduction. On then examining it a single broad band, that of reduced hæmoglobin, was seen. This band extended from 65–74. The fluid in the prism was now poured into a beaker and thoroughly shaken with air, then reintroduced into the prism bottle. The colour had now changed again from purple to red, and the following reading obtained:—

α . .	61–65
β . .	70–78

The two observations which I have adduced suffice to show the action which the solution of any nitrite exerts upon the colouring-matter. In my observations I have most frequently treated blood with solutions of nitrite of potassium and sodium. The action of a solution of nitrite of silver is, however, similar. The latter solution should be freshly prepared, and should not be much exposed to light. The observations should also, in this case, be made within a short time after the addition of the nitrite to the blood-solution, as otherwise the solution often becomes turbid.

The fact that ammonia altered the colour of blood which had been treated with nitrites from a chocolate-colour to red, might be explained on the supposition that some change had occurred in the constitution of the body formed under their influence, although it was quite possible that the spectrum of the body in a decidedly alkaline fluid might be different from that of the same substance in a nearly neutral solution. In order to determine whether the difference induced in the colour and spectrum was merely due to the alkalinity of the fluid, or to the presence of free ammonia, I prepared a dilute solution of ammonia. This solution contained 10 grms. of ammonia in 1 litre. I then made an exactly equivalent solution of phosphoric acid. The solutions were so exactly made that when equal quantities were mixed the fluid had no action on litmus. A weak solution of hæmoglobin was made, and a solution of nitrite of sodium added to it. As soon as the bands α and β had almost disappeared and the band in the red had appeared, I added 1 cub. centim. of the solution of ammonia. Immediately thereafter the absorption-band in the red disappeared, and the spectrum previously described as developed by the action of ammonia was seen, *i. e.* the orange was shaded over, and the bands α and β became more distinct.

One cub. centim. of the dilute solution of phosphoric acid was now added. Instantly the spectrum returned to the condition in which it had been before the addition of ammonia. On again adding another cub. centim. of ammonia, the band in the red disappeared and the orange became shaded over. By repeated observations I found that if care were taken to measure out exactly the same volume of the dilute acid and ammonia, the change from one spectrum to the other might be almost indefinitely repeated.

Without wishing at this stage to enter into a discussion concerning the cause of the changes which nitrites induce, I shall confine myself to remarking that the observations which I have described appeared to me to show—

First, that nitrites exert a marked influence both on the colour and the spectrum of blood, due obviously to a chemical change exerted on the colouring-matter.

Secondly, that, whatever the nature of the change, it is obviously not one which materially alters the composition of the colouring-matter, as by mere action of reducing-agents all effects of the change disappear; and

Thirdly, that the fact of reducing-agents developing, without the intervention of the atmospheric oxygen, the spectrum of oxidized hæmoglobin before exerting their reducing-action, proves that by the action of nitrites the loose oxygen of the blood is neither expelled nor removed.

III. *On the Influence of Nitrites in modifying the Respiratory Functions of the Blood.*

Under the term respiratory functions of the blood, I mean to include those physical and chemical processes which are concerned in the absorption of oxygen by blood, in its combination with the blood-colouring-matter, and its retention thereby so as to be readily available for purposes of oxidation, as well as those functions and processes which are concerned in the formation and evolution of the carbonic acid of the blood.

If the action which any substance exerts upon any or all of these various physical and chemical processes had to be ascertained, the methods of inquiry ought, in my opinion, to be equally varied.

Hitherto the method which has been almost exclusively employed to furnish evidence upon the influence which individual medicinal and poisonous substances exert upon blood, has consisted in finding out whether their addition to blood influenced the absorption of oxygen or the evolution of carbonic acid. For this purpose two samples of blood derived from the same source (one of which has been subjected to the action of the poison under examination) are brought in contact with the same quantity of oxygen or atmospheric air, and after the fluid and gas have been allowed to remain in contact for some time, the amount of oxygen absorbed and carbonic acid evolved is ascertained. This is done by determining, first, the contraction which the fluid has undergone, and, secondly, the amount of carbonic acid, nitrogen, and oxygen present in the gas after contact. Were it easy to place the blood under precisely similar circumstances in the two cases, the evidence yielded by the method would be very much greater than it really is. In order to ensure a very marked action of the blood upon the air, they must be brought together by agitation; the violence and the length of time during which this is continued are circumstances having the most marked influence upon the rapidity and amount of gaseous interchanges which take place. The difficulty of securing the same degree of agitation is very considerable, and probably introduces one of the chief errors attaching to this method of experimenting.

In such experiments I believe it to be most essential that the total volume of gas remaining after the experiment, and any contraction or dilatation which may have occurred, should be determined with the greatest accuracy. Some experimenters have not considered this essential; for in experiments to test the influence of poisonous agents on blood, they have merely made percentage analyses of the air after contact.

A second method which suggests itself as most valuable in investigating the action of poisonous agents upon the respiratory functions, is based upon the property which carbonic oxide gas possesses of displacing the oxygen of blood. The normal condition of the colouring-matter will obviously be tested in a valuable manner if we agitate it thoroughly with air and oxygen, and then bring it in contact with carbonic oxide gas. If the gas displaces as much oxygen from the poisoned blood as from normal blood which has been under exactly the same circumstances, we shall have evidence to show that the addition of the poison has, first, not prevented the normal absorption of oxygen; secondly, has not removed the oxygen from the blood by a special chemical action of its

own; and thirdly, has not so altered the compound of oxygen with blood-colouring-matter as to render the oxygen irremovable by carbonic oxide gas.

A third method consists in adding the poisonous agent to blood, arterializing it as completely as possible, and then boiling out the gases of the blood *in vacuo*, determining their amount and ascertaining their composition.

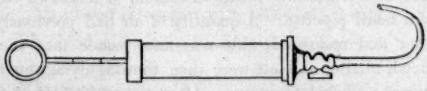
It is well known that by merely exposing the blood to the temperature of the body in a Torricellian vacuum, the hæmoglobin which it contains is reduced as completely as by contact with reducing-agents. The oxygen which has been in combination with it can therefore be determined with the greatest accuracy; and in addition we can determine the amount and rate of evolution of the more or less loosely combined carbonic acid existing in the blood. This method, taken by itself, possesses perhaps more value than either of the others to which I have alluded.

The fourth method to which I must allude has already, in some cases, afforded information upon the action of certain poisons on blood which could scarcely have been observed in any other way, viz. spectrum analysis. If used in connexion with the three other methods of research which I have thought it proper to allude to in detail, we may expect that our knowledge of the mode of action of poisons on blood will become both accurate and complete. Having shown what light it appears to throw upon the action of nitrites upon blood, I shall detail the experiments which have been performed according to the three remaining methods.

Experiments in which Blood acted upon by Nitrites was agitated with Air.

All the gas-analyses made in the course of this investigation have been conducted with the admirable instrument of Dr. Frankland and Mr. Ward; its value for such a research as the present one cannot be overestimated; for, besides permitting of the most accurate analysis of the gases (which have been in contact with blood) being made, it enables the experimenter to separate perfectly the gas from the blood, and to determine, with great ease and remarkable accuracy, the volume of the gas after contact with blood.

The air or gas whose action upon blood has to be studied is first measured in the eudiometer, and then passed into the laboratory-tube; into the latter is then introduced, by means of a very perfect metallic syringe of the form shown in the annexed cut, an



accurately measured quantity of the blood to be acted upon. The stopcocks of the eudiometer and laboratory-tube having been closed, the clamp which unites these is taken off; the laboratory-tube is then closed with the thumb, and being taken out of the mercurial trough, is agitated as long and as violently as is required. It is then

allowed to remain at rest for a considerable number of hours, so as to allow the froth to subside, and (unless this has been done immediately after the agitation of the tube had been completed) it is again clamped to the eudiometer. When the time has elapsed during which it is desired to keep the gas and blood in contact, the gas can be decanted in the most perfect manner from the blood which remains in the laboratory-tube; it is then measured and analyzed. The accuracy with which this separation of gas from the blood can be effected constitutes one of the greatest merits of the instrument for such experiments. Where gases are placed in contact with blood in ordinary absorption-tubes, it is impossible to determine with sufficient accuracy the amount of gas remaining after the experiment. It is true that the gas may be decanted with a gas-pipette. It is, however, impossible in this way to decant the whole of the gas without carrying with it some of the fluid from which it is desired to separate it. The small gasometers of BUNSEN[1] which were used by Dr. HARLEY[2] in his experiments on blood, although they allow of a transference of the gas which they contain to be easily effected, do not permit the gas which they contain to be sufficiently accurately read off to enable the exact change of volume which has occurred to be ascertained.

An admirable manner of conducting these experiments consists in measuring out successively two, as nearly as possible equal, quantities of gas, which are transferred to two laboratory-tubes of the same size, each of which fits accurately to the eudiometer. Into one tube is introduced the normal blood, and into the other the same quantity of blood to which has been added the substance whose action it is desired to study. Both tubes are then agitated in the same manner and for a like time; having been allowed to remain at rest for eighteen or twenty hours, one of the laboratory-tubes is connected to the eudiometer, and its gaseous contents are decanted and measured. This laboratory-tube having been emptied of blood and decanted, the gas in the eudiometer is analyzed. The other laboratory-tube is then connected with the eudiometer, and the same series of operations gone through. If one sample of blood have been placed in contact with its measured volume of gas an hour later than the other, it will on the following day also be decanted, measured, and analyzed one hour later; during the preceding hour there has then been ample time to analyze the gas from the first tube.

I. One cubic inch of freshly defibrinated blood of the Sheep, which had been arterialized by shaking with air, was introduced in the laboratory-tube of FRANKLAND'S apparatus. Then 2 cub. centims. of a solution of nitrite of sodium of unknown strength were introduced by means of a small pipettte. A quantity of air had previously been introduced into the eudiometer and measured; this was now passed into the laboratory-tube. The mixture of blood, nitrite, and air were then thoroughly agitated by shaking; the blood readily acquired a chocolate-colour. After twenty-three hours the gas was decanted from the blood, measured, and analyzed.

[1] Gasometry, translated by Dr. ROSCOE, 1857, p. 20.

[2] "On the Influence of Physical and Chemical Agents upon Blood," Philosophical Transactions, 1865, p. 687.

Blood employed	16·38 cub. centims.
Air placed in contact with blood	225·7 vols.[1]
After contact with blood for twenty-three hours	225·8 ,,
After absorption of carbonic acid	225·8 vols.
After addition of hydrogen	343·17 ,,
After explosion	203·30 ,,

In 100 parts of gas.

Oxygen	20·65
Nitrogen	79·35

The volume of gas before and after contact with blood has remained the same.

Vol. of gas before contact =225·7 vols. =25·14 cub. centims. at 0° C. and 0^{m}·76.
Oxygen present in air before contact (calculated)=47·3 vols.
Oxygen present in air after contact (found) . =46·62 vols.
The amount of oxygen absorbed by the blood used= 0·68 vol.=0·075 c.c. at 0°C.& 0^{m}·76.

The blood and nitrite which had been used in this experiment was at its conclusion examined with the spectroscope. The spectrum exhibited a dark absorption-band in the red, with very great faintness of the normal bands.

In the experiment just detailed, in which the blood brought in contact with air had been fully acted upon by nitrite of sodium, no marked absorption of oxygen had taken place. A cubic inch of blood had not absorbed one-tenth of a cubic centimetre of oxygen.

II. This observation is intended also to illustrate the action of nitrite of sodium upon blood.

The blood used was defibrinated blood of the Sheep, well arterialized by agitation with air. It was found by direct experiment that 1 cubic inch of this blood was fully and readily acted upon by 1·5 cub. centim. of a solution of nitrite of sodium of known strength. A sufficient quantity of air having then been introduced in the eudiometer and measured, not quite 16 cub. centims. of blood were thrown into the laboratory tube, then 2 cub. centims. of the solution of nitrite of sodium added. This quantity of solution contained 0·016 grm. of $Na\,NO_2$. The gas and blood were then agitated.

Vol. of blood about 16 cub. centims.
Air placed in contact with blood 271·2 vols.

After contact with blood and nitrite for twenty-four hours	271·5 vols.
After absorption of carbonic acid	268·6 ,,
After addition of hydrogen	394·03 ,,
After explosion	223·36 ,,

[1] I have not thought it advisable to give in the body of the paper the uncorrected readings made in the course of the gas-analyses. I have reduced all these to corrected volumes:
1 volume =0·1114 cub. centim. at 0° C. and 0^{m}·76.

In 100 parts of gas.	
Carbonic acid	1·06
Oxygen	20·97
Nitrogen	77·97
	100·00

Vol. of air before contact . =271·2 vols. =30·211 cub. centims. at 0° C. and 0m·76.
„ after contact . =271·5 „ =30·245 „ „ „
Vol. of oxygen in gas before exp. (calculated)=56·84
„ after experiment (found) . =56·89

In this case spectrum analysis proved that the blood had been fully acted upon by the nitrite. There had been no absorption of oxygen by the blood; the only changes had consisted in the diffusion of a little carbonic acid out of, and nitrogen into the blood.

III. The jugular vein of a dog was opened and the blood allowed to flow into a beaker. It was stirred with a glass rod so as to separate the fibrin, care being taken to shake it as little as possible, so as not to favour its arterialization. When the fibrin had been separated, the blood still preserved a very venous colour.

A syringeful, viz. 20 cub. centims., of this blood was introduced into a laboratory-tube, then 5 cub. centims. of a solution of nitrite of sodium were added to it. The blood and nitrite were agitated together, and allowed to remain in contact for two hours. At the end of that time air, which had been measured in the eudiometer, was transferred to the laboratory-tube and agitated with the mixture of blood and nitrite.

Immediately after the agitation the colour of the blood became much darker. After twenty hours' contact the gas was decanted, measured, and analyzed.

Volume of blood taken	20 cub. centims.
Vol. of air before contact	238·5 vols.
„ after contact	240·5 „
After absorption of carbonic acid . .	240·5 „
After addition of hydrogen	363·8 „
After explosion	224·6 „

In 100 parts of gas.	
Oxygen	19·28
Nitrogen	80·72
	100·00

Vol. of air before contact 238·5 vols. =43·16 cub. centims. at 0° C. and 0m·76.
Oxygen present in the gas before contact (calc.) 49·98 vols.
„ „ after contact . . 46·40 „
3·58 vols. =0·66 cub. centim. at 0° C.[1].

[1] In this case as the gas was measured at a different division of the eudiometer, the *absolute* volume as compared with the *arbitrary* volume is different from that mentioned in the note to Observation I.

From this observation it would appear that although nitrites diminish to a remarkable extent the capability which venous blood possesses of absorbing oxygen, yet they do not abolish it entirely. The action was such, however, that highly venous blood did not absorb more oxygen than average samples of thoroughly arterialized blood would have done.

IV. In this experiment, before acting upon the blood with nitrite of sodium, it was treated for half an hour with a rapid stream of carbonic acid. The object of this was partially to reduce the blood before acting upon it. The blood used was that of the Sheep, defibrinated. After passing the gas through it for half an hour, the blood, when examined in thin layers, presented a decidedly venous hue. It was then treated with nitrite-of-sodium solution. Two cub. centims. of the same solution as was used in the last experiment were added to every 25 cub. centims. of blood. Carbonic acid was then again passed through the blood.

Twenty cub. centims. were then introduced into the laboratory-tube, and a previously measured quantity of atmospheric air was passed from the eudiometer into the laboratory tube. On shaking the laboratory tube a considerable evolution of gas took place, so that when replaced into the mercurial trough a considerable quantity of mercury was expelled, and the quantity of gas in it appeared very sensibly increased.

After a contact of twenty hours the gas was decanted from the blood, measured, and analyzed.

Volume of blood taken	20 cub. centims.
Atmospheric air brought in contact with the blood	255·4 vols.
After contact with the blood for twenty hours .	322·2 „
After absorption of carbonic acid	254·3 „
After addition of hydrogen	388·96 „
After explosion	235·20 „

In 100 parts of gas.

Oxygen	15·9
Carbonic acid . . .	21·07
Nitrogen	63·03
	100·00

Volume of atmospheric air taken 255·4 vols. =47·37 cub. centims. at 0° and 0^{m}·76
Volume of O in air taken (calculated) 53·53 vols.
Volume of O remaining . . . . 51·25 „
Amount of oxygen absorbed by blood= 2·28 vols. =0·421 cub. centim. at 0° C. and 0^{m}·76.

The above experiment confirms the results of those previously described, as the amount of oxygen absorbed by blood which had been for a considerable time treated with carbonic acid was not as great as that absorbed by the same amount of the most arterialized blood. It is interesting, moreover, as furnishing an example (although an exag-

gerated one) of the fallacious conclusions to which mere percentage analyses of the gases which have been in contact with blood may lead. The determination of the composition of the gas would, by itself, have led to the opinion that a large quantity of oxygen had been absorbed by this very venous-looking blood. An accurate knowledge of the total amount of gas before and after contact shows that this apparent diminution of oxygen was merely relative, and due to the dilution of the gases by the large amount of carbonic acid which had, on agitation, diffused out of the fully saturated blood.

V. In the experiments now to be described, pure defibrinated and strongly arterialized blood was placed in contact with a measured volume of atmospheric air. Shortly after a like quantity of the same blood, to which nitrite of potassium had been added, was placed in contact with nearly the same volume of atmospheric air in another tube.

(*a*)

20 cub. centims. of thoroughly arterialized defibrinated blood of the Sheep brought in contact with a measured volume for twenty-two hours.

Vol. of air taken	337·6 vols.
After contact for twenty-two hours	331·08 „
After absorption of carbonic acid	324·5 „
After addition of hydrogen	483·9 „
After explosion	295·4 „
Carbonic acid	1·98
Oxygen	18·98
Nitrogen	79·04
	100·00
Vol. of air taken	= 37·6 c. c. at 0° and 0^m·76.
O in air taken (calculated)	70·76 vols.
O remaining	62·84 „
	7·92 „
O absorbed by 20 cub. centims. of arterialized blood	0·9 c. c. at 0° C. and 0^m·76.

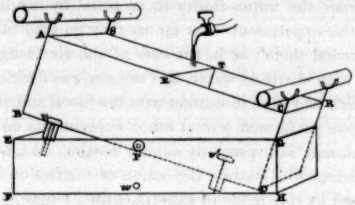
(*b*)

20 cub. centims. of the same blood as that used in (*a*) had 0·02 grm. of pure KNO_3 added to them, and were then brought in contact with air for twenty hours.

Vol. of air taken	348·2 vols.
After contact for twenty-two hours	349·3 „
After absorption of carbonic acid	346·8 „
After addition of hydrogen	527·3 „
After explosion	319·21 „
Carbonic acid	0·71
Oxygen	19·85
Nitrogen	79·44
	100·00
Vol. of air taken	= 38·8 c. c. at 0° C. and 0^m·76.
O in air taken (calculated)	72·98 vols.
O remaining	69·36 „
	3·62 „
O absorbed by 20 c. c. of arterialized blood treated with KNO_3	0·4 c.c. at 0° C. and 0^m·76.

VI. In the two experiments now to be described it was determined to take two portions of the same blood, to one of which a certain quantity of distilled water was added, and to the other the same quantity of water holding nitrite of potassium in solution, and to place these in tubes of the same capacity in contact with the same volume of atmospheric air, and then, in order to give the greatest possible facilities to the gas to act upon the blood, to subject them to prolonged and thorough shaking. For obvious reasons a mechanical contrivance must be devised if it be desired either to shake two tubes to the same extent or to continue the process of shaking for many hours. The instrument which I have employed for this purpose is merely a modification of a very ingenious piece of apparatus which was suggested by Mr. C. Hanbury, Jun., for washing photographic prints. It consists of a tin box, A B C D, well balanced and swinging freely

in gudgeons fixed in the sides of the tin box E F, G H. The box A B C D is divided by



a central partition, K T, into two equal compartments. Into the bottom of the box are fitted, one in each side, the valves v, v', which are so constructed that when either half of the box is dependent, the valve of that side opens. The apparatus is brought beneath a stream of water so that the water can enter one half of the box a little to the side of the central partition; when a certain amount of water has accumulated, that side of the box more than balances the other, and swings over with a powerful jerk; this movement, however, brings the opposite side of the box under the tap, and this commences to fill. In the meantime the valve in the side first filled is opened and the water is escaping. Thus the box swings alternately from side to side. Supports are attached to either end of the box for supporting the absorption-tubes containing the blood and gas. If the apparatus be well constructed, the two tubes are placed in exactly the same circumstances, each receiving the most thorough shake several times in a minute.

(*a*)

25 cub. centims. of defibrinated and well-arterialized blood of the Sheep were diluted with 10 cub. centims. of distilled water. 25 cub. centims. of this mixture of blood and water was placed in a tube of the capacity of 55 cub. centims., which was immediately corked.

The tube was tied to one of the supports of the shaking machine at 4.30 P.M. of one day, and the process of agitation continued until 1.15 on the following day. During this time the tubes received 1944 shakes.

After the air had been in contact (for 22 hours and 45 minutes) with the blood, it was decanted by means of Professor MILLER's gas-pipette and analyzed.

Vol. of gas taken . . .	154 vols.
After absorption of CO_2 .	152 „
After addition of H . .	246·8 „
After explosion . . .	155·5 „

Composition of gas in 100 parts.

CO_2	1·29
O	19·76
N	78·95
	100·00

(*b*)

The same quantity of blood as was used in (*a*) was mixed with 10 cub. centims. of distilled water containing 0·1 grm. of KNO_2 in solution. 25 cub. centims. of this mixture of blood and solution of nitrite of potassium was placed in a tube of the capacity of 55 cub. centims., which was immediately corked.

The tube *b* was treated in all respects like the tube *a*.

After the air had been in contact with the blood for 22 hours and 45 minutes, it was decanted by means of Professor MILLER's gas-pipette and analyzed.

Vol. of gas taken . . .	159 vols.
After absorption of CO_2 .	159 „
After addition of H . .	245·3 „
After explosion . . .	145·2 „

Composition of gas in 100 parts.

CO_2	0·00
O	20·99
N	79·01
	100·00

In these experiments I was only able to make percentage analyses of the gases after contact with blood, from the impossibility to measure, by the method used, the exact volume of gas after the experiment. As far as they go they fully confirm the experiments which had preceded them; as in the case of the air brought in contact with the normal blood mixed with distilled water, the oxygen was decidedly diminished, whilst in the case of that which had been in contact with the blood and nitrite, the air remained quite unaltered. I have performed several other experiments on exactly the same plan as those just described, and with precisely similar results. As, however, those which I have adduced are sufficient to illustrate the action of nitrites on blood, in so far as that action can be discovered by this mode of experimenting, I have preferred to omit giving further details, more especially as I myself attach little importance to these percentage analyses of air left in contact with blood[1].

Experiments in which Blood was acted upon by Nitrites and then treated with Carbonic Oxide.

In the experiments now to be described I have made use of carbonic oxide as a reagent for the detection and expulsion of the loosely combined oxygen existing in blood. The plan of the experiments has generally consisted in taking defibrinated blood of the Sheep and agitating it with air so as effectually to arterialize it, then treating it with a solution of some nitrite, and bringing it in contact with carbonic oxide gas. The carbonic oxide gas used was always prepared by the action of pure sulphuric acid on pure formiate of magnesium; in several of the first experiments I analyzed the gas before bringing it in contact with blood. I invariably found that the gas which I prepared in this way was absolutely pure.

As I wished in these experiments merely to find out the amount of oxygen which the carbonic oxide was capable of expelling from the blood, I confined myself to determining

[1] Since the experiments just described were performed, I have made others with the view of determining whether the power which the blood-colouring-matter appears to possess of ozonizing the atmospheric oxygen which comes in contact with it, is destroyed by the addition of nitrites.

Alexander Schmidt has pointed out that when a drop of diluted blood is placed upon bibulous paper which has been soaked in tincture of guaiacum and dried, at the edges of the drop the paper assumes a blue colour such as is produced by the action of ozone. I find that the reaction can be obtained with blood which has been fully acted upon by nitrites. In order to observe this reaction, the blood should be diluted with twenty times its volume of water. I may mention that all specimens of tincture of guaiacum do not give the reaction; of three samples which I purchased, only one was found to render the paper sufficiently sensitive to show the reaction, although the other two possessed the colour, smell, and taste of the genuine tincture.

It has been shown by Schönbein that when peroxide of hydrogen is added to blood, a copious evolution of oxygen occurs, as when the same reagent acts upon certain peroxides.

Blood which has been acted upon by nitrites effervesces on the addition of peroxide of hydrogen, just as normal blood does.

In so far, then, as we can judge by the two experiments which bear most directly on this point, we must conclude that nitrites do not destroy the ozonizing properties which the blood-colouring-matter appears to possess.

the amount of carbonic acid by absorption with potash and of oxygen by absorption with pyrogallic acid. The residual gas, consisting of carbonic oxide and nitrogen, was not further analyzed, but merely measured.

Obs. I. 25 cub. centims. of defibrinated and well-arterialized blood of the Sheep were mixed with 3 cub. centims. of a solution of KNO_2. The 3 cub. centims. contained 0·03 grm. of pure nitrite of potassium. The blood was allowed to remain in contact with the nitrite for 24 hours; at the end of that time it had acquired the characteristic chocolate-colour and spectrum which I have already described.

Pure carbonic oxide was prepared by the action of pure sulphuric acid upon chemically pure formiate of magnesium. A measured quantity of the gas was brought into the laboratory-tube, into which had already been introduced 20 cub. centims. of the mixture of blood and nitrite of potassium.

The blood and gas were agitated together and then left in contact for 24 hours. At the end of that time the gas was transferred, measured, and analyzed.

Quantity of blood used	17·8 cub. centims.
Carbonic oxide taken	273·4 vols.
After 24 hours' contact gas measured	273·0 „
After absorption of carbonic acid	271·5 „
After absorption of oxygen	270·5 „

Composition in 100 parts.

Carbonic acid	0·54
Oxygen	0·36
Carbonic oxide and nitrogen . . .	99·10
	100·00

Vol. of carbonic oxide before contact = 273·4 vols. = 30·45 cub. centims. at 0° and $0^m·76$
Vol. of carbonic acid exhaled = 1·57 vol. = 0·167 cub. centim. „
Vol. of oxygen exhaled = 1·0 vol. = 0·1114 cub. centim. „

The mixture of blood and nitrite was examined after the completion of the experiment, and it was found that the optical characters remained unaltered. The colour was still brown, and the blood exhibited the spectrum of blood which has been acted upon with nitrites.

In the above experiment the agitation of 20 cub. centims. of a mixture containing 17·8 cub. centims. of blood (diluted to 20 cub. centims. with a solution of nitrite of potassium) with 30·45 cub. centims. of carbonic oxide gas only resulted in the evolution of 0·11 cub. centim. of oxygen gas—a quantity which is quite insignificant when compared to that invariably yielded by either venous or arterial blood. In so far as an opinion can be formed from the results of one experiment, it would then appear either that under the influence of nitrites the loose oxygen of blood-colouring-matter had been removed, or that its constitution had so altered that its oxygen was no longer capable of expulsion by carbonic oxide gas.

Obs. II. 25 cub. centims. of well-arterialized Sheep's blood were treated with 3 cub. centims. of distilled water containing 0·03 grm. of KNO_2 in solution, and after being mixed were allowed to remain exposed to the air; then after an interval of two hours 20 cub. centims. of the mixture were brought in contact with a measured volume of carbonic oxide gas.

The carbonic oxide gas, which was prepared by the action of sulphuric acid upon formiate of magnesium, was found, on analysis, to be perfectly pure.

The blood was allowed to remain in contact with the gas for 21 hours, then decanted, measured, and analyzed.

Carbonic oxide taken measured	315·9 vols.
After contact with blood for 21 hours	316·2 ,,
After absorption with potash	314·8 ,,
After absorption with pyrogallic acid	314·8 ,,

Composition in 100 parts.

Carbonic acid	0·44
Carbonic oxide and nitrogen	99·56
	100·00

Carbonic oxide taken = 315·9 vols. = 35·19 cub. centims. at 0° C. and $0^m·76$
Carbonic acid exhaled = 1·4 vol. = 0·156 ,, ,, ,,

In the above experiment a mixture of blood and nitrite containing 17·8 cub. centims. of blood furnished on agitation with 35·19 cub. centims. of CO, no trace of oxygen gas!

Obs. III. 50 cub. centims. of defibrinated and well-arterialized blood of the Sheep were treated with a few drops of an alcoholic solution of nitrite of amyl. In a few minutes the chocolate coloration was very marked, and the spectrum had the character of that of blood treated with nitrites.

Carbonic oxide was prepared by the action of sulphuric acid on formiate of magnesium. The gas was analyzed and found to be perfectly pure. 20 cub. centims. of the blood acted upon by nitrite of amyl were brought in contact with a measured volume of it, and the gas and blood having been mixed by agitation, were allowed to remain in contact for twenty hours.

Volume of blood taken	20 cub. centims.
Carbonic oxide measured	387·9 vols.
After contact with blood and nitrite	398·4 ,,
After absorption of carbonic acid	391·1 ,,
After absorption of oxygen	390·57 ,,

Composition of gas in 100 parts.

Oxygen	0·38
Carbonic acid	1·58
Carbonic oxide and nitrogen	98·04
	100·00

Gas taken (CO) measured		44·359 cub. centims. at 0° C. and 0^{m}·76	
Carbonic acid exhaled	=6·3 vols.	=0·7 cub. centim.	,,
Oxygen exhaled	=0·5 vol.	=0·05 cub. centim.	,,

The apparent increase in the volume of gas after contact with the mixture of blood and nitrite of amyl is due to the tension of the vapour of the nitrite of amyl. The amount of oxygen exhaled by the blood is quite insignificant.

Exp. IV. In the last experiment it was pointed out that the volume of gas appeared to have increased after contact with the blood; this was, however, due to the tension of the vapour of nitrite of amyl. It is a curious fact that when carbonic oxide acts upon blood, although very considerable changes take place in the composition of the gas, its volume is almost imperceptibly affected. I shall in a separate paper, devoted to the consideration of the relation which the volume of carbonic oxide absorbed by blood bears to that of the gases exhaled, point out how this insignificant change in volume can be explained. At present it will suffice to remark that, since the volume of gas does not change materially, we may bring it in contact with blood, agitate it thoroughly, and then take a fraction of the gas and analyze it without having to wait for many hours to have elapsed, as is the case in the first experiments on the action of atmospheric air upon blood. In the latter, as the gas is very liable to change in volume, we are obliged, after agitating the blood with air, to wait for a considerable number of hours so as to allow of such a subsidence of froth as will enable us perfectly to separate the gas from the fluid. In these experiments, as the volume of the gas in contact with the blood alters considerably, mere percentage analyses do not suffice to show to what extent the gas has been altered in composition, whereas in those in which atmospheric air is replaced by carbonic oxide, we may dispense with a measurement of the gas after action, and rest satisfied that, by a determination of the percentage composition, we shall obtain a full insight into the changes which have occurred. The ground upon which this assertion is based will be seen by reference to the analyses and experiments which I quote in a paper "On the Relation which the volume of CO absorbed by Blood bears to that of the O displaced."

In the experiments now to be described, recently defibrinated blood, which had been arterialized by shaking with air, was brought in contact with a measured volume of pure carbonic oxide. Having been agitated for the space of half a minute, and then left in contact with the blood for one hour and twenty minutes, the gas was decanted and the amount of oxygen which had exhaled was determined. Then the same quantity of the blood mixed with nitrite was treated in exactly the same manner, and the changes which occurred in the composition of the gas which had been added to it determined.

(*a*) Normal blood.

Vol. of blood used 20 cub. centims.
Vol. of CO taken 468·5 vols. =41·03 cub. centims.
Vol of gas analyzed . . 315 vols.
After absorption of CO_2 . 313 ,,
After absorption of O . . 305 ,,

(*b*) Blood and Nitrite.

25 cub. centims. of the same blood as was used in (*a*) were treated with 5 cub. centims. of water, containing 0·05 grm. of KNO_2 in solution.

20 cub. centims. of the mixture brought in contact with the gas and left in contact for one hour and twenty minutes.

Composition of gas in 100 parts.

Oxygen	2·53
Carbonic acid	0·63
Carbonic oxide and nitrogen	96·84
	100·00

Assuming that the volume of gas in contact with the blood did not alter, *i. e.* still continued after contact for one hour and twenty minutes to be 368·5 vols., or 41·63 cub. centims., the amount of oxygen exhaled by the blood would amount to 9·32 vols., or to 1·03 cub. centims. at 0° and 0^{m}·76.

Vol. of CO taken 362·8 vols.=40·4 cub. centims.

Vol. of gas analyzed	355·8 vols.
After absorption of CO_2	355·0 ,,
After absorption of O	354·8 ,,

Composition of gas in 100 parts.

Oxygen	0·00
Carbonic acid	0·22
Carbonic oxide and nitrogen	99·78
	100·00

After the conclusion of this experiment the blood was examined with the spectroscope. The hæmoglobin bands were very faint and a marked band in the red. NH_3 caused this to disappear and shaded the orange. Sulphide of ammonium was then added. It readily reduced the blood; on blowing through it, the bands of oxidized hæmoglobin came out brilliantly. On now passing a stream of CO through the blood thus acted upon, it readily acquired the CO colour and became irreducible.

Exp. V. In the experiments now to be described, the conditions were as nearly as possible the same as in Exp. IV., with the exception that the blood used was treated with a stream of pure oxygen gas before being brought in contact with the carbonic oxide gas, or before having the nitrite of potassium added to it.

a (pure blood).

The carbonic oxide was prepared by the action of pure sulphuric acid upon pure formiate of magnesium.

Vol. of blood	20 cub. centims.
Vol. of CO taken	373·98 vols.
Period of contact with blood	60 minutes.
Gas analyzed	340·0 vols.
After absorption of CO_2	340·0 ,,
After absorption of O	326·0 ,,

Composition of gas in 100 parts.

Oxygen	4·11
Carbonic acid	0·00
Carbonic oxide and nitrogen	95·89
	100·00

Assuming the volume of gas not to have changed during the experiment, but to have remained 373·9 vols. or 41·66 cub. centims., the O exhaled would amount to 15·37 vols.=1·71 cub. centim. at 0° C. and 0^{m}·76.

b (blood + KNO_3).

The gas used was prepared as in *a*. 20 cub. centims. of a mixture of blood and solution of nitrite of potassium used. It contained 16·6 cub. centims. of blood.

Vol. of CO taken	388·5 vols.
Period of contact with blood	85 minutes.
Gas analyzed	373·0 vols.
After absorption of CO_2	373·0 ,,
After absorption of O	370·0 ,,

Composition of gas in 100 parts.

Oxygen	0·80
Carbonic acid	0·00
Carbonic oxide and nitrogen	99·20
	100·00

Assuming the volume of gas not to have changed during the experiment, but to have remained 388·5 vols. or 43·2 cub. centims., the amount of O exhaled would be 3·10 vols.=0·345 cub. centim. at 0° C. and 0^{m}·76.

In the experiments which have preceded it has been shown that, after the action of nitrites has been exerted upon blood, carbonic oxide is incapable of thrusting out the

loose oxygen; this gas, whose action on normal blood is so powerful, has for the time lost all power of action. It has also been shown that although CO loses its property of acting upon blood treated with nitrites, it acquires it again if the blood be, subsequently to the action of nitrites, acted upon by sulphide of ammonium.

It now remains for me to consider, before passing to the next series of experiments, whether nitrites have any action upon blood treated with carbonic oxide. Without detailing all the experiments which I have performed to obtain information on this point, I may state that nitrites appear to have no power of acting upon CO-blood. Whenever blood had been well agitated with carbonic oxide so as to acquire the well-known florid character and to become irreducible, it was found to be totally unaffected by the addition of solution of nitrites.

Experiments in which the Gases of the Blood are boiled out in vacuo, *their amount determined, and their composition ascertained.*

Magnus was the first experimenter who made use of a mercurial pump in order to obtain the gases of the blood[1]. Before his time several inquirers had attempted to discover the presence of dissolved gases in the blood, but their results were so contradictory that little reliance could be placed upon them. Thus whilst Sir Everard Home[2] had asserted that the blood evolves gases when placed *in vacuo*, Stevens[3] and Hoffmann[4] in this country, and Gmelin, Mitscherlich, and Tiedemann[5] in Germany had stated that they had arrived at a different result. The failure of these experimenters was undoubtedly due to the imperfection of their air-pumps; for, as Magnus discovered, it is essential, in order to remove the gases which exist in a state of solution and loose chemical combination in blood, to place it in a very perfectly exhausted space. Working with his very ingenious arrangement, Magnus found that blood yielded *in vacuo* from about 7 to 10 volumes per cent. of mixed gases, which consisted of a mixture of carbonic acid, oxygen, and nitrogen. These numbers are extremely low when compared with those obtained by experimenters who have worked according to more recent and improved methods. After Magnus, Lothar Meyer determined the gases existing in the blood[6]. Instead of a mercurial pump, he made use of the well-known method devised by Bunsen for boiling out the gases of water[7]. His plan of working enabled him to secure as good a vacuum as Magnus obtained, and to boil the blood in addition; it is probably due to this that Meyer obtained much larger quantities of gases than Magnus.

[1] Poggendorff's 'Annalen,' 1837, p. 583, "Ueber die im Blute enthaltenen Gase, Sauerstoff, Stickstoff, und Kohlensaüre."

[2] The Croonian Lecture for 1817, Philosophical Transactions, 1818, p. 181.

[3] Observations on the Blood, by W. Stevens, London, 1832.

[4] London Medical Gazette, 1833.

[5] Poggendorff's 'Annalen,' xxxi. t. 289,

[6] Die Gase des Blutes, "Zeitschrift, f. rat. Med." Bd. VIII. p. 257 (1857).

[7] Gasometry, &c., by Robert Bunsen. Edited by Dr. Roscoe, 1857, p. 16.

From 100 volumes of arterial blood of the Dog he obtained from about 49·5 to 54 volumes of total gases at 0° C. and 760 millims. pressure, containing from 12·43 to 18·42 volumes of oxygen, and from 26·25 to 34·75 volumes of carbonic acid, besides from 2·83 to 5·04 volumes of nitrogen. By his method MEYER was, however, only able to get a small proportion of the carbonic acid in a free condition, by far the larger proportion only being evolved on the addition of an acid; tartaric acid was, therefore, in his experiments, always added to the blood after the free carbonic acid had been boiled off, and then the more firmly combined carbonic acid was obtained.

The very small amount of carbonic acid which could be obtained by MEYER's method, without the addition of acid, was probably due to the blood having been mixed with from ten to twenty times its volume of water before being boiled out; for all more recent experimenters who have used the mercurial pumps for exhausting blood, and who have boiled the undiluted blood *in vacuo*, have succeeded in boiling off the greatest portion of the carbonic acid of the blood without adding any acid.

LUDWIG, and his pupils SETSCHENOW and SCHÖFFER, were the first to use the modern mercurial pump (which, as modified by GEISSLER, has proved so invaluable in researches on the blood) for the purpose of extracting the gases of the blood.

Professor PFLÜGER, with the aid of GEISSLER, of Bonn, has succeeded of late years in constructing the most perfect mercurial pumps. This physiologist has not only insisted upon the necessity of exposing the blood to a true Torricellian vacuum, as LUDWIG and his pupils had done, but contended that an arrangement whereby the gases of the blood were freed from all watery vapour was essential in order to obtain, without the addition of acids, all the carbonic acid of blood. His determinations made with the dry vacuum (das trockne vacuum) are probably the most correct which we yet possess[1]. Recently PFLÜGER has recommended that, in addition to having a drying chamber (containing sulphuric acid) in connexion with the blood-receptacle, as large a vacuum as possible should be employed, *i. e.* the vessel containing the blood to be exhausted should be connected with vessels of very large capacity, in which a Torricellian vacuum has been obtained; by this method he finds that the gases may be obtained from blood in as many minutes as the process formerly occupied hours, and that the amount of oxygen obtained is indeed greater when a large than when a small vacuum is employed.

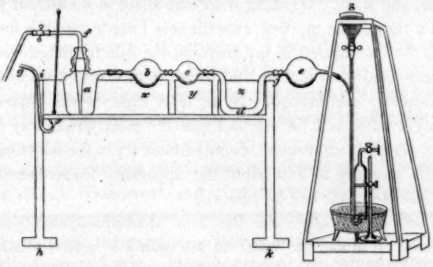
The following Table exhibits the mean of the most recently published experiments of PFLÜGER on the gases of arterial blood.

	At 0° C. and 0^{m}·76 pressure.	At 0° C. and 1 metre pressure.
Total gases.....	58·3 vols.	44·9 vols.
Oxygen	22·2 „	16·9 „
Carbonic acid	34·3 „	26·6 „
Nitrogen.....	1·8 „	1·4 „

[1] PFLÜGER, "Die Normalen Gasmengen des arteriellen Blutes nach verbesserten Methoden." Centralblatt für die Med. Wissenschaften, 26 October 1868.

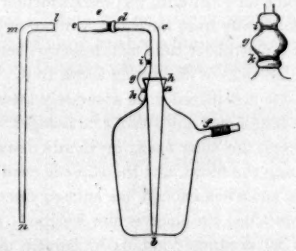
In my experiments on the gases of the blood, I have used SPRENGEL'S mercurial aspirator for exhausting my apparatus and obtaining the gases of blood. The results which I have obtained with this instrument have been remarkably satisfactory, and of such a nature as to lead me to think that it may supersede the very expensive and more complicated mercurial blood-pump of GEISSLER. In the annexed diagram I have exhibited the apparatus which I employed.

J h i k exhibits a section of a wooden stand which is 4 feet high, and which supports three tin boxes, *x*, *y*, and *z*. The box *x* is intended to be used as a water-bath, and



is heated by a gas-burner placed below it. *y* and *z* are tin boxes to be filled with cold water.

a exhibits the glass bottle into which is received the blood to be exhausted, and of which a larger diagram is given below. The neck *a* of this bottle is very accurately



ground, and into it fits the ground tube *d c b*, which dips to the bottom of the bottle, and which is bent at *c*. The bottle has an exit-tube, *f*.

The open end of *b c d* is closed by means of a narrow tube of black india-rubber, which is wired (after having been smeared on the inside with melted india-rubber) and furnished with a very perfect steel clip. Besides having the tube *d c b* very perfectly ground, the tightness of the apparatus is further secured by pouring melted shellac into the hollow

space remaining between the rim *h* and the tube, and thereafter slipping the caoutchouc cap *g* over the neck, and wiring it at *i* and *k*.

The exit-tube *f* is connected to a bulb *b* of the capacity of 100 cub. centims.; this bulb is supported in the water-bath *x* along with the bottle *a*; it is connected with the bulb *c*, which rests in a separate tin vessel *y* containing cold water; *c* is united to the U-tube *d*, and this is surrounded by cold water in the tin vessel *z*. The U-tube *d* is joined to the bulb *e*, which is connected to a very perfect Sprengel tube fixed in a firm wooden stand. The various tubes and bulbs are united with the aid of black tubing, melted india-rubber, and wire. Working with care there is no difficulty in getting this apparatus admirably tight. In my first experiments I made use of a more complicated Sprengel tube than the one figured in my diagram, *i. e.* one furnished with a gauge-tube on which a millimetre scale was etched, and which dipped into a vessel in which a very perfectly boiled barometer also dipped. In my later experiments I have used the simplest form of Sprengel tube, as I have found that the most satisfactory and perfect test consists in getting a perfectly unbroken column of mercury in the fall-tube, and observing that on allowing the mercury to flow after the apparatus has remained exhausted for some hours, not a particle of air can be removed.

When the tightness of the apparatus has been ascertained, and it is desired to use it to extract the gases of blood, the water in the bath *x* is heated to about 100° Fahr., and the temperature kept as constant as possible by regulating the supply of gas to the burner which heats it. A small glass tube (see diagram representing the blood-receptacle, page 613), *l m n*, having an internal diameter of a millimetre, bent at right angles at *m*, is filled with the blood to be analyzed. The end *n* is closed with the fore finger of the right hand, and with the aid of the other hand the end *l* of the tube is inserted into the end *o* of the india-rubber tube on *b c d*, until its passage further is resisted by a clip at *e*. The finger may now be taken away from the end *n*, without any risk of the blood which it contains escaping. The india-rubber tube had, however, better be wired to the glass tube *l m n*, so as to remove all risks of air getting access to it.

The blood to be analyzed is now placed in an accurately measured flask or bottle, and after its volume has been determined the vessel containing it is weighed. When this operation has been completed, the tube *l m n*, previously described, is plunged to the bottom of the flask containing the blood, and the clip *e* is cautiously opened; the blood is sucked into the vacuum, and when enough has entered the clip is rapidly and firmly closed. The flask which contained the blood is now weighed; after this, distilled water is allowed to flow out of an accurately graduated burette until the level of fluid in the flask is the same as it was before the blood was taken for analysis. The amount of distilled water poured out of the burette indicates the volume of the blood used.

The blood which has thus been allowed to flow into the receptacle enters into violent ebullition; a tube filled with mercury has, of course, previously been placed in the pneumatic trough connected with the Sprengel tube, and now mercury is made to flow through the latter incessantly. In about six minutes from the time when the blood

was admitted into the heated blood-receptacle the evolution of gases is most rapid. Very soon, however, the bells of gas become fewer and fewer, and in about twenty-five minutes after the pumping has been commenced the process is virtually at an end. For the sake of greater accuracy I have in my experiments generally exhausted the blood for one hour.

At the end of this time the blood-receptacle and the first bulb contain a dry red mass; the second bulb contains a little fluid blood which has spirted over, whilst the U-tube contains the greater part of the water of the blood which has condensed there. Some of this is also deposited in the large bulb situated between the U-tube and the Sprengel aspirator.

After describing separately my experiments, I shall draw attention to some interesting facts connected with the use of the Sprengel tube as a blood-pump, and show that the success of my experiments throws doubt upon statements which have lately been made in reference to the conditions which are most favourable for the extraction of gases from the blood.

In order that the observations which have been made upon the gases of blood treated with nitrites should be understood, and to prove the accuracy and value of the methods used, I must quote some analyses of blood to which no nitrite was added.

I. In this experiment the gases were determined in venous blood obtained from the right side of the heart of a dog. Having exposed the external jugular vein, I passed a catheter into the right auricle, and when this had been completely filled with blood, its free extremity was inserted into the india-rubber tube attached to the glass tube leading into the blood-receiver. On opening the clip the blood flowed rapidly into the vacuous receiver.

The quantity of blood admitted was 21·52 grms. The pumping was carried on for three quarters of an hour. There was no trace of gas then coming off. At the end of that time the 21·52 grms. of blood had been, through evaporation, reduced to 8·648 grms.

The following Table exhibits the amount and composition of the gases obtained.

Gases evolved by 100 volumes of the blood of the right side of the heart of a Dog[1].

	At 0° C. and 0^{m}·76 pressure.	At 0° C. and 1 metre pressure.
Total gases.....	66·03 vols.	47·59 vols.
Oxygen	12·76 „	9·19 „
Carbonic acid.....	49·95 „	36·00 „
Nitrogen.....	3·32 „	2·40 „

II. In this experiment defibrinated and well-arterialized blood of the Sheep was analyzed.

[1] I have thought it well to state the volume of gases obtained at 1 metre pressure, as well as at 0^{m}·76, as the majority of the German experimenters who have lately made determinations of the gases of the blood have calculated the volumes at 0° C. and 1 metre.

The blood was allowed to flow into the receiver at one o'clock, and the exhaustion was carried on exactly for one hour.

Weight of blood used	31 grammes.
Volume of blood used	29·6 cub. centims.
Volumes of gas obtained	167·8 vols.
After absorption of CO_2	70·19 ,,
After absorption of O	6·62 ,,

100 parts of gas contain—

Oxygen	37·88
Carbonic acid	58·17
Nitrogen	3·95
	100·00

167·8 vols.	of total gases	=18·69	cub. centims.	at 0° C.	and 0·m76.
6·62 ,,	Nitrogen	= 0·737	,,	,,	
97·61 ,,	Carbonic acid	=10·873	,,	,,	
63·57 ,,	Oxygen	= 7·073	,,	,,	

Gases evolved by 100 volumes of defibrinated blood of the Sheep arterialized by agitation with atmospheric air.

	At 0° C. and 0m·760 pressure.	0 nd 1 metre pressure.
Total gases.....	61·12 vols.	46·97 vols.
Oxygen	23·88 ,,	18·14 ,,
Carbonic acid	36·72 ,,	27·90 ,,
Nitrogen.....	2·48 ,,	1·88 ,,

III. In this experiment the same blood was used as in II., with the exception that 0·195 of pure nitrite of potassium were added to 100 parts of blood.

Weight of blood taken . . .	43·2 grammes.
Volume of blood taken . . .	41·2 cub. centims.

The bath into which the blood-receiver was plunged had a temperature of 98° Fahr. From the very moment that the blood entered the receiver the mercury was allowed to flow freely through the aspirator. The gas was collected in a laboratory-tube of FRANKLAND'S apparatus.

The evolution of gas appeared to be most brisk at 3.13, *i. e.* eight minutes after the blood had been admitted into the receiver. The temperature of the latter was gradually brought up to 110° Fahr. At 4.5 (after fifty-two minutes) the laboratory-tube was removed to FRANKLAND'S apparatus and the gas analyzed. Still another tube having been substituted, the pumping was carried on until 4.50, the temperature of the water being taken up to 130° Fahr. Only a tiny bubble (which it would have been vain to attempt to measure) was collected during the forty-five minutes which followed the first hour's pumping.

Volume of blood exhausted 41·2 cub. centims.
Total gases obtained =145·8 vols.
After absorption of CO_2 = 11·65 „
After absorption of O = 9·58 „

Total gases obtained	=145·8 vols.
Carbonic acid	134·15
Oxygen	2·07
Nitrogen	9·58

Composition of gas in 100 parts.

Oxygen	1·41
Carbonic acid	92·00
Nitrogen	6·59

100 volumes of arterialized blood of the Sheep mixed with 0·195 grm. of pure nitrite of potassium yield—

	At 0° C. and 0^{m}·76 pressure.	At 0° C. and 1 metre pressure.
Total gases	39·91 vols.	30·30 vols.
Oxygen	0·56 „	0·42 „
Carbonic acid	36·38 „	27·64 „
Nitrogen	2·97 „	2·25 „

IV. In this experiment 100 cub. centims. of recently defibrinated and thoroughly arterialized blood of the Sheep were treated with four minims of nitrite of amyl diluted with sixteen minims of rectified spirit. After having been mixed with the nitrite for one hour and forty-five minutes, a portion of the blood was admitted into the exhausted receiver. The exhaustion was carried on for one hour. After twenty-five minutes the vacuum appeared perfect, and scarcely any more bells of gas could be removed.

Weight of blood used	40·5 grammes.
Volume of blood used	38·27 cub. centims.
Total gases obtained	178·12 vols.
After absorption of CO_2	21·64 „
After absorption of O	8·64 „

Composition of gas in 100 parts.

Oxygen	7·29
Carbonic acid	87·85
Nitrogen	4·86
	100·00

Total gases obtained	=178·12 vols.	=19·84 cub. centims.	at 0° C. and	0^{m}·76.
Oxygen	= 13·0 „	= 1·44 „	„	„
Carbonic acid	=156·48 „	=17·43 „	„	„
Nitrogen	= 8·64 „	= 0·96 „	„	„

100 volumes of arterialized blood of the Sheep treated with nitrite of amyl yielded—

	At 0° C. and 0m·760 pressure.	At 0° C. and 1 metre pressure.
Total gases.....	51·84 vols.	39·39 vols.
Oxygen	3·78 „	2·87 „
Carbonic acid.....	45·54 „	34·61 „
Nitrogen.....	2·52 „	1·91 „

V. In this experiment the same blood was used as in the last, but without the addition of nitrite of amyl.

Unfortunately the blood was allowed to spurt into the U-tube, and the contents of the latter beginning to froth, I discontinued the pumping before the gas had been entirely removed. As by far the larger part of the gas had been obtained, I considered that it would be amply sufficient for the purpose of comparison with IV.

Weight of blood used . . .	37 grammes.
Volume of blood used . . .	34·97 cub. centims.
Total gases obtained	147·4 vols.
After absorption of CO_2 . . .	71·62 „
After absorption of O . . .	6·94 „

Composition of gas in 100 parts.	
Oxygen	43·87
Carbonic acid	51·41
Nitrogen	4·72
	100·00

Total gases obtained 147·4	vols.	= 16·42	cub. centims. at 0° and 0m·760.	
Oxygen . . . . 64·68	„	= 7·20	„	„
Carbonic acid . . 75·78	„	= 8·44	„	„
Nitrogen . . . . 6·94	„	= 0·77	„	„

100 volumes of arterialized blood of the Sheep evolved. (Exhaustion not completed.)

	At 0° C. and 0m·76 pressure.	At 0° C. and 1 metre pressure.
Total gases.....	46·95 vols.	35·68 vols.
Oxygen	20·58 „	15·64 „
Carbonic acid.....	24·13 „	18·33 „
Nitrogen.....	2·24 „	1·71 „

VI. Five drops of nitrite of amyl were added to 50 cub. centims. of blood of the Sheep. Before the addition of the nitrite the blood had been thoroughly arterialized.

Weight of blood used . . .	28·7 grammes.
Volume of blood	27·3 cub. centims.

Exhaustion was carried on for one hour. The temperature of the bath in which the blood-receptacle was plunged was at first 100° Fahr., and was raised gradually to 120°.

The great bulk of the gas had been collected within fourteen minutes from the commencement of the exhaustion.

Volumes of gas obtained	83·44
After absorption of CO_2	22·2
After absorption of O	6·67 vols.

Composition of gas in 100 parts.

Oxygen	18·61
Carbonic acid	73·39
Nitrogen	8·00
	100·00

83·44 vols. of total gases = 9·29 cub. centims at 0° C. and 0m·76.
15·53 „ oxygen . = 1·73 „ „
61·24 „ carbonic acid = 6·82 „ „
6·67 „ nitrogen . = 0·743 „ „

100 volumes of blood treated with nitrite of amyl yielded—

	At 0° C. and 0m·760 pressure.	At 0° C. and 1 metre pressure.
Total gases.....	34·02 vols.	25·85 vols.
Oxygen	6·33 „	4·81 „
Carbonic acid.....	24·90 „	18·92 „
Nitrogen.....	2·79 „	2·12 „

Having described the experiments in which the gases of normal blood and of blood treated with nitrites were extracted by ebullition *in vacuo*, I shall point out the facts which were either confirmed or made out in these experiments.

Three of the analyses illustrate the composition and quantity of the gases obtained from healthy blood; from these it will be seen that venous blood taken directly from the right side of the heart yielded 9·19 cub. centims. (at 0° C. and 1 metre pressure) of oxygen for every 100 cub. centims., that the amount obtained from two samples of arterialized blood was 18·14 and 15·64 cub. centims. (at 0° C. and 1 metre) for every 100 cub. centims. of blood. The addition of nitrites to well-arterialized blood was shown to lead to an enormous diminution in the amount of oxygen which could be removed by the pump; the amount of oxygen obtained being lowest in the case where the nitrite had been in contact with blood during the longest time, and highest when the nitrite was added to the blood only a short time before its exhaustion. A sample of blood which yielded 18·148 vols. of oxygen per 100 vols. of blood, after the addition of nitrite of potassium, gave up only 0·425 cub. centim. of oxygen.

In another case, where the amount and composition of the gases were determined in normal blood, and in the same blood after the addition of nitrite of amyl, it was found that 100 cub. centims. of the pure blood yielded 15·64 cub. centims. of O (at 0° C. and

1 metre pressure), whilst 100 cub. centims. of the same blood after the addition of nitrite of amyl yielded only 2·87 cub. centims.

These experiments, therefore, bear out, in a striking and satisfactory manner the results of those in which carbonic oxide was made to act upon blood which had been treated with nitrites.

The latter experiments showed that carbonic oxide had lost its power of thrusting out oxygen, although from certain facts which had been previously described, it appeared that the oxygen of the blood is neither expelled nor permanently taken possession of by nitrites.

The fact that the loose oxygen which can be thrust out by CO is identical with the oxygen which is given up by blood *in vacuo*, led me to suppose that bodies which, as nitrites, have the power of locking up the loose oxygen of hæmoglobin and preventing its expulsion by CO, would likewise prevent its removal by the air-pump. The hypothesis has proved to be a correct one.

I must now make a few remarks upon the use of SPRENGEL'S mercurial aspirator in the extraction of the gases of the blood.

In the detailed account of the individual experiments I have shown that, using the arrangement which I have employed, the removal of gases may be completed in a very short time (twenty-five or thirty minutes), and that the amount of gases obtained agrees admirably with the most successful of PFLÜGER'S analyses. Hitherto it has only been by PFLÜGER'S method (with the so-called dry vacuum) that the gases could be rapidly and completely removed from the blood; and as the excellence of the method has been supposed to depend upon the removal of the vapour of water by the acid in the drying chamber, it appears to be worth while examining how it is that as good results can be obtained with the arrangement which I have employed. The object of the drying chamber containing sulphuric acid, in PFLÜGER'S method, is to make the vacuum as perfect as possible by removing the steam as well as the air. It is thus intended to obviate the influence which the tension of the vapour of water would have in causing the gases to be retained. It appears to me, however, that whilst the blood is still in ebullition it would be quite hopeless to try to obtain *a dry vacuum*, as simultaneously with the drying of the air by the action of the sulphuric acid, it would again become saturated with the vapour of water given off by the blood, so that whilst the sulphuric acid probably hastened the evaporation it would scarcely influence the tension of the steam.

The sulphuric acid would, however, tend to establish a continued current of steam from the receptacle in which the blood is boiled to the chamber containing sulphuric acid, and probably this current aids very much in sweeping out the gases from the blood. When SPRENGEL'S apparatus is used, all the apparatus connected with the blood-receiver is thoroughly and rapidly swept by a continual current of steam, and to this probably some of its efficacy in removing the gases of the blood may be referred. Probably it is this current which is the cause of the superiority of PFLÜGER'S arrangement, and of my adaptation of SPRENGEL'S apparatus, over the other methods which have been employed.

On the Nature of the Action which Nitrites exert on the Colouring-matter of Blood.

Whatever might be the nature of the change induced by nitrites in hæmoglobin, it resulted from my experiments that it could not be one which deeply altered the constitution of the substance, seeing that the addition of certain reagents at once caused all the effects of the action to disappear and revealed the continued existence of oxidized hæmoglobin in the blood. Nitrites had, by my experiments, been shown to resemble in no way those agents which thrust oxygen out of the blood; on the other hand, it had been proved that the action of nitrites resulted in the locking up of the oxygen of the blood so as to render it irremoveable by CO or by a vacuum. A consideration of all the facts which I had observed led me to believe that probably nitrites might actually link themselves to oxidized hæmoglobin—a supposition which has been verified in the most ample manner.

In the experiments now to be described I have always made use of hæmoglobin prepared from the blood of the Dog, in the following manner. The dog whose blood was to be used for the preparation of hæmoglobin was kept for a considerable time under the influence of chloroform, and then bled to death.

The blood was allowed to coagulate, and the serum separated as completely as possible.

After twenty-four hours the blood-clot was broken up and firmly squeezed in linen or calico. The red fluid thus expressed was mixed with one and a half times its volume of distilled water, and set aside for three or four hours. At the end of that time the fluid was filtered through Swedish paper and mixed with one-fourth of its volume of eighty per cent. spirit. It was then placed in a vessel surrounded by ice and salt, and set aside until the following morning. The fluid was usually found to have become semi-solid from the separation of magnificent microscopic crystals of hæmoglobin. These were collected on a filter, washed with distilled water, and then dissolved in water at about 38° C. The clear red solution was filtered, treated with one-fourth of its volume of alcohol (eighty per cent.), and frozen. The hæmoglobin had at the end of twenty-four hours again crystallized out.

1. *Action of Nitrite of Potassium on Hæmoglobin.*

When solutions of hæmoglobin are treated with solutions of nitrite of potassium, the colour changes, as in the case of blood similarly acted upon, to a dark brown. Simultaneously the spectrum assumes the characters observed when nitrites act upon blood.

If a saturated solution of hæmoglobin be treated with a solution of nitrite of potassium until its colour is thoroughly changed, and if one-fourth of its volume of alcohol be then added, and the fluid set aside in a freezing-mixture, a brown deposit separates after some hours, having very much the colour of chocolate. This deposit consists of magnificent microscopic crystals, quite undistinguishable in form from those of oxidized hæmoglobin, but differing from these in appearing much less coloured, under the microscope, than the crystals of O-Hb, and in possessing only a faint yellow, but no red colour.

When a layer of these crystals is examined with the micro-spectroscope of Mr. SORBY, or with an ordinary spectroscope, the spectrum of nitrite-blood is seen to perfection.

The chocolate-coloured deposit may be dissolved in water, with the aid of a little heat, and the solution having been filtered may be treated with alcohol and frozen, when crystals identical in shape, colour, and optical properties with those precipitated the first time will again separate. The crystals when agitated with water give to it a dirty-brown colour; sometimes I have obtained them of such a size that, when shaken with water, they could be seen to float about as minute needles. If to this dirty-brown fluid a drop of ammonia be added the change is most marked and beautiful; for the fluid assumes the magnificent red colour of hæmoglobin.

If the crystals are drained by being placed on filtering paper laid on porous slabs, and then dried as rapidly as possible *in vacuo* over sulphuric acid, a reddish-brown mass is obtained which readily crumbles to powder. The crystalline form of the substance is lost in the process of drying. The dry body, when powdered, is soluble in pure water at the temperature of the body; it is much more readily soluble in water containing a trace of free ammonia; in this case a red, instead of a brown solution is obtained.

When the dry body is ignited an ash is left which has an alkaline reaction, and which contains only oxide of iron and potash.

3·379 grammes of the nitrite-of-potassium compound of hæmoglobin yielded on ignition a red ash, which was treated with water and evaporated, and then after the addition of hydrochloric acid ignited.

The chloride of potassium obtained weighed 0·014 grm.

100 grammes would therefore yield 0·47 of KCl.

5·2 grammes of the nitrite-of-potassium compound prepared at a different time yielded 0·026 grm. of KCl.

100 grammes would therefore yield 0·57 grm. of KCl.

The amount of potassium in the nitrite compound was by these analyses shown to be so small as to render very accurate results scarcely attainable. The results agree very nearly with the view that a molecule of hæmoglobin combines with a molecule of nitrite of potassium.

2. *Action of Nitrite of Sodium on Hæmoglobin.*

If a solution of nitrite of sodium be substituted for a solution of nitrite of potassium, the changes which have been described under 1 all occur, and crystals are obtained which only differ from the first by containing Na instead of K.

3. *Action of Nitrite of Silver on Hæmoglobin.*

When a solution of hæmoglobin is treated with a solution of freshly prepared and several times recrystallized nitrite of silver, it immediately becomes chocolate-coloured, and presents the spectrum of blood treated with nitrites. The solution reacts exactly as solutions of Hb which have been treated with the alkaline nitrites when ammonia

and sulphide of ammonium are added. When a concentrated solution of hæmoglobin is treated with a solution of nitrite of silver and then with alcohol, and placed in a freezing-mixture, a precipitate separates, which is usually amorphous, and has a chocolate colour. Sometimes it is very readily soluble in water, holding a trace of ammonia in solution; at other times it is soluble only with difficulty.

This precipitate, when washed and ignited, leaves a residue consisting entirely of silver and oxide of iron.

When solutions of hæmoglobin which have been treated with nitrite of silver are exposed, or set aside for some time, a precipitate separates, which is scarcely soluble in ammonia; the solution obtained has, however, all the properties of ammoniacal solutions of the nitrite compounds of hæmoglobin.

a. 4·279 grms. of the nitrite of silver compound yielded 0·0517 grm. of Ag Cl. 1·042 grm. of the same sample yielded 0·0067 grm. of $Fe_2 O_3$.

b. 3·591 grms. of the nitrite of silver compound prepared on another occasion and from different blood, yielded 0·0177 grm. of Ag Cl.

The first sample, therefore, contained in 100 parts 0·759 of silver, corresponding to 1·083 of $Ag NO_2$; whilst the second contained almost exactly half the quantity, or ·346 of silver, corresponding to 0·494 grm. of $Ag NO_2$ per 100 parts of the compound.

These two analyses clearly indicate that, although nitrite of silver enters into actual chemical combination with hæmoglobin, the amount which is taken up varies remarkably.

Preyer has calculated the molecular weight of hæmoglobin on the assumption that each molecule of this substance yields a molecule of hæmatine. The molecular weight of hæmatine being 626 ($C_{32} H_{34} N_4 Fe O_6$), that of hæmoglobin would on the above assumption amount to 13280.

On the assumption that one molecule of hæmoglobin has the power of combining with and retaining loosely one molecule of oxygen, 13280 grammes of hæmoglobin would combine with and retain 32 grammes of oxygen, or 1 gramme of hæmoglobin would have the power of retaining 1·3 cub. centim. of oxygen at 0° C. and 1 metre pressure; this number agrees, as perfectly as can be expected, with direct observations made to determine this point.

If one molecule of oxygenized hæmoglobin combined with one molecule of a nitrite, 13312 grammes of oxidized hæmoglobin would combine with 154 grammes of $Ag NO_2$, and 100 parts of the compound should contain 1·14 gramme of $Ag NO_2$, or 0·79 of Ag. Comparing these numbers with those which I found, we have

In 100 parts.	Calculated.	Found. (1)	Found. (2)
Silver	0·79	0·759	0·346
Iron	0·419	0·44	

4. *Action of Nitrite of Amyl on Hæmoglobin.*

If, instead of the solutions of the nitrites already mentioned, we add an alcoholic solu-

tion of nitrite of amyl to a solution of hæmoglobin, then treat the mixture with one-fourth of its volume of alcohol and expose it to cold, after some hours a brown deposit separates having very much the appearance of chocolate, and forming on microscopic examination the characters which have been already described as possessed by the bodies formed when hæmoglobin is treated with other nitrites. These crystals can, like the others, be recrystallized without undergoing any change. On one occasion (20th December 1867) I placed in a test-tube a magma of these crystals along with their mother-liquor, and on examining them now, after an interval of nearly three months, I find them still perfectly preserved and possessing all the characteristic features. This is probably due to the preservative influence of the nitrite, as I have hitherto failed in all attempts to preserve hæmoglobin crystals. I may remark that when hæmoglobin is dried, even *in vacuo*, the crystals always crumble to pieces.

In order to show that the chocolate-coloured crystals of the nitrite compounds of hæmoglobin are identical in form with those of O-hæmoglobin, I shall place two microscopic photographs at the end of this paper[1]. The first represents the compound of nitrite of potassium with hæmoglobin; the second the body formed under the influence of nitrite of amyl, and which we may, reasoning by analogy, conclude to be a compound of nitrite of amyl with hæmoglobin. These microscopic photographs I owe to the kindness and skill of Mr. Nicol, a very able photographer in Edinburgh.

In now drawing the account of my experiments to a close, I shall state the conclusions which in my opinion may be legitimately drawn from them, and then add certain observations on the relation of the nitrite compounds of hæmoglobin to the O-, CO-, and N_2O_2-compounds.

Conclusions.

1. When a solution of any nitrite acts upon the blood, peculiar changes occur in the colour, and simultaneously in the absorption-spectrum.

2. These changes in the optical properties of blood are due to the formation of compounds presenting the same crystalline form, colour, and spectrum, whatever the nitrite which has been employed in their preparation.

3. These bodies appear to be compounds of the nitrite used with oxidized hæmoglobin.

4. The substances formed by this process of chemical addition, although isomorphous with hæmoglobin, differ from it in many of those remarkable properties upon which its functions in the economy of the body depend. By this process of addition the blood-colouring-matter appears to have lost its power of absorbing oxygen.

5. The addition of nitrites to hæmoglobin appears to result in the locking up of the loosely combined oxygen, so as to make it irremoveable by CO, or by a vacuum.

Observations.

We have hitherto been acquainted with hæmoglobin itself as well as with its O-, CO-, and N_2O_2-compounds. These compounds are all isomorphous, and possess almost the

[1] These have not been reproduced, but are preserved in the Archives of the Royal Society.

same physical characters; in all the O free hæmoglobin has apparently linked itself to a molecule of O, CO, and N_2O_2 respectively, the stability of the compound being least in the case of the O- and greatest in the case of the N_2O_2-compound.

All these bodies, and preeminently the O-compound, appear to be examples of a class of bodies which stand, as it were, on the boundary line which separates chemical from physical combination—to be, in fact, examples of the class of so-called molecular compounds. Like other molecular compounds their composition varies greatly within certain limits, and is influenced by circumstances and conditions which have no action on chemical compounds proper[1].

That a body possessing such a very complicated molecular structure as hæmoglobin should present numerous points of attachment, as it were, for the linking-on of such active, condensed bodies as the nitrites, is more than probable, and it is not remarkable that, as in the case of other combinations of a molecular kind, such as the union of salts with their water of crystallization, of bases with sugar, of albumen with metallic oxides, of iodine with the compound ammonias, the amount of the simpler body added to the more complex, should vary within wide limits.

Simultaneously with the observations which I have conducted, and which have shown the power of nitrites to combine with hæmoglobin, those which have lately been made by Hoppe-Seyler and Preyer, although discrepant in many particulars, seem to agree in proving that hydrocyanic acid possesses the property of linking itself to hæmoglobin, forming a body which is isomorphous with it, but which physiologically is an inert body, having lost the power which, normally, hæmoglobin seems to possess of ozonizing atmospheric oxygen.

[1] My friend Mr. James Dewar, Assistant to the Professor of Chemistry in the University of Edinburgh, first suggested to me the idea of the molecular nature of the compounds of hæmoglobin.

XXV. *On the Mysteries of Numbers alluded to by* FERMAT.—Second Communication.

By the Right Hon. Sir FREDERICK POLLOCK, *Bart., M.A., F.R.S., F.S.A., F.G.S., &c.*

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IN the last paper I described the mode of constructing a square (which, for brevity, I shall call *The Square*) which would necessarily be made up of three different series; the indices in the margin and in the small squares were explained, and I must refer to that paper for the explanations.

I propose in this paper to show (from *that square* and a *supplemental one*) that all the odd numbers possess the properties that I have ascribed to them. *The Square* also proves the first theorem of FERMAT, viz. that every number is composed of three triangular numbers or less from the 2nd theorem (relating to the squares), which I believe has not hitherto been done.

If any one will take the trouble to examine any odd number, he will find that it may be divided into four squares (generally in several ways or forms) in some set of four roots of the squares composing the number; two of the roots will be equal, two of the same, or another set of four will differ by 1; two will differ by 2, two by 3, and so on, as far as the number is large enough to have roots of sufficient magnitude to furnish such differences. This, of course, cannot be done by one set of roots, except by a few of the low numbers*. If the four roots do not furnish all the differences, then he will discover that there is another or more forms that will furnish the whole of them.

The algebraic sum of the four roots will also, in some or other of the forms, be equal to 1, 3, 5, 7, &c., that is, to every odd number within the compass of the given number. These properties, which may be discovered in any odd number, I propose to show belong to all odd numbers.

It may be well, in order to make this statement quite clear, to take an example. 73 is one of the terms in the series 1, 3, 7, 13, 21, &c., a series increasing by 2, 4, 6, 8, &c.; every term of the series has roots of the form n, n, n, $n \pm 1$.

73 is composed of four squares in four different forms; the four sets of roots are

	Differences of roots.
4, 4, 4, 5	0, 1, 8, 9
2, 2, 4, 7	2, 3, 4, 5, 6, 11
0, 1, 6, 6	5, 7, 12, &c.
1, 2, 2, 8	10, &c.

* 3, 5, 7, 11, 15, and 23 have each but one form, and the last (23) is the highest odd number that has only one form of roots.

These forms give the differences of roots set opposite to them; 12 is the limit, as the lowest number which can give a difference of 13 is 85 (0, 0, 6, 7). 6.7 have a difference of 13.

roots.

So with reference to the algebraic sum of the roots:

4, 4, 4, 5 gives 1, 7, 9, and 17
2, 2, 4, 7 gives 1, 3, 7, 11, 15
0, 1, 6, 6 gives 1, 11, 13
1, 2, 2, 8 gives 3, 5, 7, 9, 11, 13.

It is remarkable that the sum of the roots being equal to every odd number is immediately connected with FERMAT'S first proposition of the triangular numbers; it is also remarkable that the second proposition of the differences of the roots being equal to every number, odd or even, is immediately connected with FERMAT'S second proposition of every number consisting of four squares or less. The connexion between the sum of the roots and FERMAT'S first proposition was observed by me in the year 1854, and is mentioned in a paper which the Royal Society did me the honour to publish in the Philosophical Transactions for that year, vol. cxliv. p. 315, as Theorem C: see also p. 318.

Before I proceed to show the other properties of *The Square*, I think it right to call attention to the manner in which a change in the roots of the four squares alters the sum of the squares themselves. This subject was touched upon by me in a paper published in the Philosophical Transactions for 1859, vol. cxlix. p. 49, in which it is stated, and a sort of proof (not a satisfactory one) given, that in some form of division into four squares the roots will be equal, will differ by 1, by 2, by 3, &c.

I propose now to call attention to the effect, or result, of altering the roots so as to increase or decrease the sum of the squares. If the roots of two of the four squares that compose any odd number differ by n, and the larger of the two be increased by 1, and the smaller be decreased by 1, the sum of the squares will be increased by $2n+2$. Let p and $p+n$ be the roots that differ by n, the sum of their squares will be

$$2p^2+2pn+n^2.$$

If $p-1$ and $(p+n)+1$ be squared, the sum of their squares will be

$$p^2-2p+1+p^2+2pn+n^2+2p+2n+1,$$

or

$$2p^2+2pn+n^2+2n+2;$$

the increase is $2n+2$.

If $n=0$ the increase is 2,
if $n=1$ the increase is 4,
if $n=2$ the increase is 6,

and so on, always *twice the difference* $+2$.

There is a similar theorem for reducing the sum of the squares; if p and $p+n$ become

$p+1$ and $p+n-1$, the sum of the roots will then be diminished by $2n-2$. The algebraic sum of the roots is not altered by this increase of one and decrease of another of them by the same number.

For, if

$$\pm a \pm b \pm c \pm d = 1, \text{ or } 2n+1,$$

then

$$\pm a \pm (b \mp n) \pm (c \pm n) \pm d = 1, \text{ or } 2n+1.$$

All these changes reciprocate; thus, if a number has two roots equal, the number which is greater by 2 will have two roots differing by 2; and *vice versâ*, if a number has two roots differing by 2, the number which is less by 2 will have two roots equal; thus, if 13 has two roots differing by 2 as 0, 2, then 11, which is less by 2, will have two roots equal, viz. 1, 1.

If an odd number has roots differing by 0, 1, 2, 3, 4, 5, &c., the four roots of the odd numbers, next in succession, one after another, will be discovered till the differences are exhausted; and if *every* number has this property, the succession will continue through the whole series of odd numbers, and every odd number will be composed of four squares, and therefore every number will be so likewise; for every even number is ultimately the double of an odd number, and the double of four squares is itself composed of four squares.

There is another mode of altering the roots, substantially the same, but occasionally applicable when the other is not. It is mentioned as Theorem A in page 313 of the Philosophical Transactions for 1854; but I was not *then* aware of its full effect, and did not pursue the subject.

If the difference between the sum of two of the four roots, and the sum of the other two be n, and each of the larger be decreased by n, and each of the smaller e bincreased by n, the increase in the sum of the squares will be $2n$, if $n=1$ the increase is 2.

This is a proper place to mention a property of the series mentioned above, 1, 3, 7, 13, &c. In a former paper (see Philosophical Transactions, 1854, page 315) I called this a *gradation series*, but I was not then familiar with its properties, and did not pursue the subject as I propose now to do. Every term in the series has roots of the form $n, n, n, n \pm 1$. If any term, as 13 (which is composed of the squares of the four roots 1, 2, 2, 2), be increased by the methods above mentioned, taking care to keep the sum of the roots always equal to the number 7 (the sum of the roots of 13), it will be found that, from 13 to 21 inclusive, every odd number will be composed of four squares, the sum of whose roots is 7. In this case the process cannot be carried beyond 21; for 23 being of the form $8n+7$, cannot be composed of less than four squares: 5 is too large to be one of the roots, and four is unavoidable, since $23-16$ leaves a remainder of 7, which cannot be expressed by less than four squares. The only form, therefore, of roots for 23 is 1, 2, 3, 3, and their sum is 9. But the sum of the roots from the number 21 to 31 will be 9; and from 21 to 31 the odd numbers can be made of four roots whose sum shall in every case be 9; and this goes two steps beyond 31; and as the number

increases which may belong to the sum of the roots, the number of odd numbers in succession which may be formed increases faster than the interval between the terms of the *gradation series*. This will appear by an examination of the numbers themselves; but the interval may be filled up more readily in the manner above mentioned; and if any two terms in the *gradation series* be taken, the odd numbers from the one to the other, both inclusive, may be made up of the squares of roots whose sum shall be the number indicated by the sum of the roots of the first of the two terms.

Thus 3, 3, 3, 4 are the roots whose squares make the number 43; and from 43 to 57, which is composed of 3, 4, 4, 4, every odd number can be found, the sum of whose roots shall equal 13. Thus

From 43 to 57 is but 7 steps; but the series of odd numbers consisting of squares whose roots equal 13 goes on 6 steps further.

$$
\begin{cases}
\overset{0}{3},\overset{0}{3},\overset{1}{3},4 = 43 \\
\overset{1}{2},\overset{1}{3},\overset{0}{4},4 = 45 \\
\overset{1}{2},\overset{0}{3},\overset{2}{3},5 = 47 \\
\overset{0}{2},\overset{2}{2},\overset{1}{4},5 = 49 \\
\overset{2}{1},\overset{1}{3},\overset{1}{4},5 = 51 \\
\overset{0}{2},\overset{1}{2},\overset{3}{3},6 = 53 \\
\overset{2}{1},\overset{0}{3},\overset{3}{3},6 = 55 \\
\overset{1}{1},\overset{2}{2},\overset{2}{4},6 = 57 (=3, 4, 4, 4)
\end{cases}
$$

$$
\begin{aligned}
\overset{3}{0},\overset{2}{3},\overset{0}{5},5 &= 59 \\
\overset{3}{0},\overset{1}{3},\overset{2}{4},6 &= 61 \\
\overset{1}{1},\overset{1}{2},\overset{4}{3},7 &= 63 \\
\overset{2}{0},\overset{3}{2},\overset{1}{5},6 &= 65 \\
\overset{0}{1},\overset{3}{1},\overset{3}{4},7 &= 67 \\
\overset{2}{0},\overset{2}{2},\overset{3}{4},7 &= 69
\end{aligned}
$$

The differences between the roots are marked in order to compare them with those in the next number

So

$$
\begin{cases}
\overset{1}{3},\overset{0}{4},\overset{0}{4},4 = 57 \\
\overset{0}{3},\overset{1}{3},\overset{1}{4},5 = 59 \\
\overset{2}{2},\overset{0}{4},\overset{1}{4},5 = 61 \\
\overset{1}{2},\overset{2}{3},\overset{0}{5},5 = 63 \\
\overset{1}{2},\overset{1}{3},\overset{2}{4},6 = 65 \\
\overset{3}{1},\overset{1}{4},\overset{0}{5},5 = 67 \\
\overset{3}{1},\overset{0}{4},\overset{2}{4},6 = 69 \\
\overset{2}{1},\overset{2}{3},\overset{1}{5},6 = 71 \\
\overset{0}{2},\overset{2}{2},\overset{3}{4},7 = 73 = 4,4,4,5
\end{cases}
$$

From 57 to 73 is 8 steps (one more than the last); but the series of odd numbers goes on 10 steps further.

$$
\begin{aligned}
&\overset{2}{1},\overset{1}{3},\overset{3}{4},7 = 75 \\
&\overset{4}{0},\overset{1}{4},\overset{1}{5},6 = 77 \\
&\overset{1}{1},\overset{3}{2},\overset{2}{5},7 = 79 \\
&\overset{3}{0},\overset{3}{3},\overset{0}{6},6 = 81 \\
&\overset{3}{0},\overset{2}{3},\overset{2}{5},7 = 83 \\
&\overset{1}{1},\overset{2}{2},\overset{4}{4},8 = 85 \\
&\overset{0}{1},\overset{5}{1},\overset{1}{6},7 = 87 \\
&\overset{2}{0},\overset{4}{2},\overset{1}{6},7 = 89 \\
&\overset{0}{1},\overset{4}{1},\overset{3}{5},8 = 91 \\
&\overset{2}{0},\overset{3}{2},\overset{3}{5},8 = 93
\end{aligned}
$$

The differences are the same, but are reversed, and this occurs alternately.

Again:

Here the next term in the gradation series (91) is reached by 9 steps; but the series of odd numbers goes on 15 steps further.

4, 4, 4, 5 = 73
3, 4, 5, 5 = 75
3, 4, 4, 6 = 77
3, 3, 5, 6 = 79
2, 4, 5, 6 = 81
3, 3, 4, 7 = 83
2, 4, 4, 7 = 85
2, 3, 5, 7 = 87
1, 4, 6, 6 = 89
1, 4, 5, 7 = 91 = 4, 5, 5, 5
2, 3, 4, 8 = 93
1, 3, 6, 7 = 95
2, 2, 5, 8 = 97
1, 3, 5, 8 = 99
0, 4, 6, 7 = 101
1, 2, 7, 7 = 103
1, 2, 6, 8 = 105
0, 3, 7, 7 = 107
$0, \overset{3}{3}, \overset{3}{6}, \overset{2}{8} = 109$
$1, \overset{1}{2}, \overset{3}{5}, \overset{4}{9} = 111$
$0, \overset{4}{4}, \overset{0}{4}, \overset{5}{9} = 113$
$0, \overset{3}{3}, \overset{2}{5}, \overset{4}{9} = 115$
$0, \overset{2}{2}, \overset{5}{7}, \overset{3}{8} = 117$
$1, \overset{0}{1}, \overset{5}{6}, \overset{3}{9} = 119$
$0, \overset{2}{2}, \overset{4}{6}, \overset{3}{9} = 121$

4, 5, 5, 5 = 91
4, 4, 5, 6 = 93
3, 5, 5, 6 = 95
3, 4, 6, 6 = 97
3, 4, 5, 7 = 99
2, 5, 6, 6 = 101
2, 5, 5, 7 = 103
2, 4, 6, 7 = 105
3, 3, 5, 8 = 107
2, 4, 5, 8 = 109
1, 5, 6, 7 = 111=5, 5, 5, 6
2, 3, 6, 8 = 113
1, 4, 7, 7 = 115
1, 4, 6, 8 = 117
2, 3, 5, 9 = 119
2, 2, 7, 8 = 121
1, 3, 7, 8 = 123
2, 2, 6, 9 = 125
1, 3, 6, 9 = 127
0, 4, 7, 8 = 129
0, 5, 5, 9 = 131
0, 4, 6, 9 = 133
1, 2, 7, 9 = 135
0, 3, 8, 8 = 137
0, 3, 7, 9 = 139
1, 2, 6, 10= 141.

So much for the series 1, 3, 7, 13, 21, &c.; but *The Square* has three series. The other two (when the first term is 1) are 1, 5, 13, 25, 41, 61, &c., and 1, 3, 9, 19, 33, 51, 71, &c.; and each of these affords a similar facility of passing from one term to the next; but these last series assist each other, and the condition is, not the sum of all the roots being the same, but the difference of two of them, and any term of the second being halfway between two of the first; thus $25+8=33$, and $33+8=41$, which is true throughout both series.

Roots.

25 = 3, 0, 0, 4
27 = 3, 1, 1, 4
29 = 3, 0, 2, 4
31 = 2, 1, 1, 5
33 = 2, 0, 2, 5

But 33 is in the other series, and is represented by

4, 0, 1, 4 = 33
3, 0, 1, 5 = 35
4, 1, 2, 4 = 37
3, 1, 2, 5 = 39
4, 0, 3, 4 = 41

The first five terms have each two roots whose difference is 7; the second five terms have each two roots whose difference is 8; and this process may be continued throughout the two series. 51 is the middle term between 41 and 61; 73 is the middle term between 61 and 85; and each of these admits of the same treatment as 25, 33, and 41.

If a *Supplemental Square* be appended to *The Square*, and be constructed on the reverse principle of diminishing towards the *left* as the other increases towards the right, beginning with precisely the same odd number (see Diagram No. 1), it is manifest that a series of numbers may be obtained less than the original number, and which will finally terminate, giving all the numbers from which the odd number in the beginning of *The Square* may proceed, if each be placed in succession in the first position of *The Square*; for it is clear that if the given odd number be lessened by 4, 8, 12, &c., and each of these numbers be again lessened by 2, 6, 10, &c., till the operation can be carried no further, the process which takes place in the formation of *The Square* itself will be reversed, and therefore if any number so obtained in *The Supplemental Square* be placed in the first position in *The Square*, and 4, 8, 12, &c. be added, and then to each of these 2, 6, 10, &c., at length the given odd number will be reached; and in this way it will be seen that the odd number will be found in any position whatever that its magnitude (as a number) would enable it to fill and properly occupy. And now, if all the numbers less than the given number have the properties, which it is alleged belong to all odd numbers, viz. of differences of two roots and sum of all the roots, then the given odd number will be accompanied by numbers less than itself, having such properties that they properly (that is, according *to the laws of The Square*) occupy their place in *The Square*; and it must follow that their composition will indicate what roots the given odd number ought to have in order to comply with the exigencies of the place in *The Square* that it occupies; for, in respect to any one of the series that compose *The Square*, if one

term in the series be correct, that is, have the proper sum of two roots, or the proper sum of all the roots, then every other term in that series will also be correct (see last paper) and may be derived from that term, and it will therefore have all the qualities that belong to the different portions of *The Square*, and in every one it will be divided into four squares, and the roots will appear. It may be said the first does not appear to have two roots equal, or two differing by 1, but (see Diagram No. 1) where the top row is called A, the second B, the third C, the fourth D, and the terms in each row are distinguished by 1, 2, 3, 4, and because 25 in A 1 has two roots differing by 1, 29 in A 2 will have two roots differing by 3; and because 17 in A 1 has two roots differing by 1, 29 in A 3 will have two roots differing by 5; and because 5 has two roots differing by 1, 29 in A 4 will have two roots differing by 7. But in A 2, 29 (which is derived from 27 the sum of whose roots is 1) will have the sum of its roots 3; and for a similar reason in A 3 it will have the sum of its roots 5 (derived from 23 the sum of whose roots is 1 in B 2), and in A 4 it will have the sum of its roots 7.

But if the sum of the four roots $=2n+1$ and the difference between two of them $=2n+1$, the other two must be equal to one another, or they would prevent the sum of the roots being $2n+1$.

A similar mode of reasoning applies to show that 29 must have two of its roots differing by 1. For, because 27 has two roots equal, therefore 29 in B 1 (derived from 27) has two roots differing by 2, and in C 1 (derived from 21) has two roots differing by 4, and in D 1 (derived from 11) has two roots differing by 6. But if any number has the sum of its roots equal to $2n+1$ and to $2n+3$, and two of the roots differ by $2n+2$, the other two must differ by 1, or the sum of all the roots could not be both $2n+1$ and $2n+3$.

I propose now to point out some of the results of what has been already stated.

There can be no doubt that every number, odd or even, is composed of four squares or less. And whether this be proved by the *gradation series*, or by the combination of *The Square* and the *Supplemental Square* (if these furnish a proof), or by LAGRANGE's method from the prime numbers, or by any other, the result is the same, the *proposition is true*; and it follows that a number of the form $4n+2$ must be composed of two odd squares, or of two odd squares and one even one, or two odd squares and two even ones.

In the first case $4n+2=4a^2+4a+1+4b^2+4b+1$; deducting 2 from each side and dividing by 4, $n=a^2+a+b^2+b$; that is, four trigonal numbers, of which two are equal and the other two are equal. In the second case $4n+2=4a^2+4a+1+4b^2+4b+1+4c^2$, $n=a^2+a+b^2+b+c^2$; but a^2+a+c^2 equals 2 trigonal numbers, therefore n equals four and trigonal numbers, of which two are equal. In the third case $n=a^2+a+b^2+b+c^2+d^2$; and as a^2+a+c^2 and b^2+b+d^2 are each equal to 2 trigonal numbers, n in every case is equal to four trigonal numbers, that is, to $a^2+a+b^2+c^2+c+d^2$.

In EULER's 'Opuscula Analytica,' vol. ii. p. 4, he says, "LAGRANGE's demonstration as to the square numbers is deduced from principles of such a kind that clearly no assistance can be expected from *that* to prove the rest of FERMAT's theorems*.

* Quæ autem ex ejusmodi principiis est deducta, ut inde nullum plane subsidium ad reliqua demonstranda expectari possit.

Now the above observation on a number of the form $4n+2$ shows that LAGRANGE'S method may be used to prove that every number is composed of four trigonal numbers as well as of four squares, and thus is brought into immediate connexion with *The Square*, and enables it to divide the first term (whatever odd number it may be) into 4 squares the algebraic sum of whose roots shall be 1, and the consequence of that is, that the $\frac{\text{1st term}-1}{2}$ will be a number composed of 3 trigonal numbers or less; it may be as well here to insert the proof of this.

If the sum of two of the roots differ from the sum of the other two by 1, the 2 sums of the roots must be of the form $2a+1$ and $2a$, and the four roots will be of the form $a+p+1$, $a-p$, $a+q$, $a-q$, and the sum of the roots squared will be

$$4a^2+2p^2+2q^2+2a+2p+1;$$

deducting 1, and dividing by 2, the number will be $2a^2+a+p^2+p+q^2$; $2a^2+a$ is a trigonal number, and p^2+p+q^2 is an expression for 2 trigonal numbers. Now if 1 be the first number in *The Square*, every number in it will be of the form $1+2a^2+2a+2b^2$, which is 4 squares, and expressed by their roots is $a+1$, a, b, b; and the sum of any two terms added together will be an even number; and as every possible value of a and b is to be found in *The Square*, every even number may be obtained by adding together some two of the numbers in *The Square*.

Another result is that, as every number is of the form $a^2+a+b^2+c^2+c+d^2$, every even number may be composed of 4 of the terms of the series 0, 2, 4, 8, 12, 18, 24, &c., the terms of which are alternately $2b^2$ and $2a^2+2a$.

The law of the series is obvious enough; beginning with 0, the differences are 2, 2, 4, 4, 6, 6, ... $2n$, $2n$. It is a convenient mode of using this series to place the terms in two columns, putting all the $2b^2$ in one column and all the $2a^2+2a$ in another, as below.

2	4
8	12
18	24
32	40
50	60
72	84
&c.	&c.

Every even number may be made by some 2 terms of each column, and any 2 squares, if equal, may be increased by 2, 8, 18, 32, &c., that is, by $2b^2$, by changing the roots—

	Increase.
n, n into $n-1$, $n+1$	2
$n-2$, $n+2$	8
$n-3$, $n+3$	18
&c. &c.	$2b^2$

In like manner any 2 roots that differ by 1 may be increased by 4, 12, 24, 40, &c. $(2a^2+2a)$ by changing

	Increase.
$n, n+1$ into $n-1, n+2$	4
$n-2, n+3$	12
$n-3, n+4$	24
&c. &c.	$(2a^2+2a)$

Another conclusion is, that if *The Square* have as the first number in it all the odd numbers *in succession* which are to be found in *The Square* when 1 is the first number, then every other odd number will be obtained as some one of the numbers thus formed. For every such *Square* will begin with a number of the form of $1+2a^2+2a+2b^2$; and in forming *The Square* from that, $2m^2+2m+2n^2$ will be added, therefore all the possible combinations of $2a^2+2a+2b^2$ and $2m^2+2m+2n^2$ will be found, with every value of a and b, m and n; that is, *every odd number will be found.*

But if any odd number whatever be made the first number in *The Square*, and a *Supplemental Square* be formed, and the numbers in the *Supplemental Square* be successively put in the first place in *The Square*, the assumed number will be found in some of the terms at the top, and also in some of those at the side. And the necessary consequence is, if we are allowed to notice the numbers of *The Square* when 1 is the first number, that any odd number is not only equal to $1+2a^2+2a+2b^2+2c^2+2c+2d^2$, but also to $1+2a^2+2a+2b^2+2c^2+2c$, or $1+2a^2+2a+2b^2+2d^2$; that is, the four squares may have two equal roots, or two roots differing by 1.

I now propose to show in what manner *The Square* can obtain a division of its first term into four squares, the algebraic sum of whose roots ± 1; the result of which is that the first term less 1, divided by 2, will be composed of 3 trigonal numbers (see Theorem C in Philosophical Transactions, vol. cxliv. p. 315).

Every odd number is of the form $4n+1$, or $4n+3$. If it be of the form $4n+3$, then in the $(2n+1)$th term of the series which increases by 2, 4, 6, 8, 10, &c., the roots will be $n, n+1, n+1, n+1$.

For $4n+3+2n$ terms of 2, 4, 6, 8, &c.

$$=4n+3+(2+4n)\times\frac{2n}{2}$$

$$=4n^2+6n+3=\begin{cases} n^2 \\ (n+1)^2 \\ (n+1)^2 \\ (n+1)^2 \end{cases}$$

the roots of which are $n, n+1, n+1, n+1$. The index of this term is $4n+1$ (since it is the $(2n+1)$th term of 1, 3, 5, 7, &c.), that is, 2 less than the sum of the roots.

In like manner, if the number be of the form $4n+1$, then the 2nth term in the series already mentioned will have as roots $n, n, n, n+1$.

For $4n+1+(2n-1)$ terms of 2, 4, 6, 8, &c.

$$=4n^2+2n+1=\begin{cases} n^2 \\ n^2 \\ n^2 \\ (n+1)^2 \end{cases}$$

which, when expressed in roots, is n, n, n, $n+1$. The index of the term is $4n-1$ (being the $2n$th term of 1, 3, 5, 7, &c.).

It will be seen that in each case the sum of the roots exceeds the index of the term by 2; it is therefore obvious that if the sum of the roots could be diminished by 2 (without altering the sum of the squares), the sum of the roots would be equal to the index, and the roots of every term in the diagonal would be obtained, with the sum of roots equal to the index of the term; and as the first number in *The Square* is in this diagonal, the roots of the given number would be obtained, whose sum would equal 1.

The difference between the sums of the squares of the roots of any two terms in the *gradation series* is the sum of the roots of the larger, minus 1. The difference between the sums of their roots is 2; therefore, if the sum of the roots of the larger, minus 1, can be added to the roots of the smaller, a set of roots will be obtained whose sum will be 2 less than that of the larger, but the sum of their squares will be equal to it. (See Diagram No. 2.)

Take the case of 35 as the first term of *The Square*. In the $(n+1)$th term of the diagonal, 8, 9, 9, 9 will be the roots of the squares which compose that term. The next term in the *gradation series* will have the roots 9, 9, 9, 10, but will be two more than the position requires, as will appear from calculation. The difference between $8^2+9^2+9^2+9^2$ and $9^2+9^2+9^2+10^2$ will be $9+9+9+10-1$, that is, 36, but that would be 2 too much. If, therefore, the squares of 8, 9, 9, 9 be increased by 34, the number will be what that place in *The Square* requires, and the sum of the roots will be 35, which is the index of that term.

Now the squares of the roots 8, 9, 9, 9 may be increased by 34 by adding $12+18+4$, which are three terms in the two columns above mentioned; 12 may be added by changing 8, 9 into 6, 11; and 18 may be added by changing 9, 9 into 6, 12. The roots then are 6, 11, 6, 12, and to these 4 is to be added. This may be done by changing 11, 12 into 10, 13, and the result is $\overset{0}{6}, \overset{4}{6}, \overset{3}{10}, 13$, whose sum is 35, and is therefore equal to the index of the term.

To reduce roots whose sum is 35 to roots having the same differences but whose sum will equal 1, deduct 9 from each root, and 9×4 will be deducted, and the sum will then be -1, which by changing all the signs will become $+1$. From each of the roots 6, 6, 10, 13 deduct 9, and they will then be -3, -3, 1, 4, and the sum of their squares $=35$.

If 1 be deducted from 35, and the remainder be divided by 2, the quotient (17) will be composed of not exceeding three trigonal numbers.

There remains one other matter to be mentioned, viz. a certain remarkable relation which all the polygonal numbers bear to each other, and which forms a connexion that runs through them all; from which it would seem to follow that a solution of the theorem as to one, would be a solution as to all the rest (except the first).

This relation arises in the square numbers by a property of the gradation series, already in part alluded to, viz. as to the odd numbers, by which the interval between any two terms can be filled up, all the terms having, as to the odd numbers, the sum of the roots of the squares that compose them equal to the sum of the roots of the first term; but the intervals, as to the *even* numbers, may be also filled up by making the sum of the roots 1 less than that of the roots of the odd numbers (see the Table in Diagram No. 3), which is thus constructed: a term in the gradation series is assumed (in this case 73); its roots are 4, 4, 4, 5, and the roots of all the odd numbers between that and the next term are found by the processes mentioned in the former part of this paper. The roots of the even numbers are obtained by an analogous process, and these are used as bases or roots of the polygonal numbers, which are placed in columns, with their sums, as appears in the Table. See Diagram No. 4 for the mode in which the polygonal numbers are formed.

It will be observed that the sum of the roots or bases is 17, but if they be used to form trigonal numbers, the increment of the sum of the resulting trigonal numbers, above the sum of the roots or bases, is 28; and so on of the rest, each successive column increasing by the same number, viz. 28. If the roots or bases be n, n, n, $n \mp 1$, that is, a term in the gradation series, the increment of the sums of the successive columns will be $2n^2 \mp n$, a trigonal number.

Again, in the trigonal numbers the difference between the sums of the first and second term is 0; in the square numbers it is 1, in the pentagonal numbers 2, in the hexagonal numbers 3, in the heptagonal numbers 4; but in all of them the difference between the second and third terms is 1, and this continues throughout. The difference between the 3rd and 4th, the 5th and 6th, the 7th and 8th, &c. increases by 1 in each column, but the difference between the 2nd and 3rd, the 4th and 5th, the 6th and 7th, &c. is always 1 in each column; and the result is, that by adding 1 in the pentagonal column, by adding 1, or 1, 1 in the hexagonal, by adding 1, or 1, 1, or 1, 1, 1 in the heptagonal, every number, odd or even, can be made by not exceeding four square numbers, or five pentagonal numbers, or &c., as clearly appears by the Table.

This corresponds with what was discovered by CAUCHY, published at the end of LEGENDRE'S "Théorie des Nombres," viz. that four only of each class of numbers is necessary, the rest may be supplied by 1, repeated as often as necessary. But I must not omit to say that, although all the odd numbers are sufficiently obedient, there is one class of even numbers quite refractory, viz. the powers of 2. They may easily be expressed in squares, pentagonal numbers, &c., but they cannot be brought within the rule that otherwise prevails.

DIAGRAM No. 1.

					1	3	5	7	9
5	17	25	29	0 A	1 29	3 2	5 3	7 4	9
3	15	23	27	2 B	1 3 1	1 5 2	3 7 3	5 9 4	7 11
	9	17	21	4 C	3 5 1	1 7 2	1 9 3	3 11 4	5 13
		7	11	6 D	5 7 1	3 9 2	1 11	1 13	3 15
				8	7 9	5 11	3 13	1 15	1 17

Roots or Bases.				Sums.	Trigonal Numbers.				Sums.	Square Numbers.				Sums.
4	4	4	5	= 17	10	10	10	15	= 45	16	16	16	25	= 73
2	3	5	6	= 16	3	6	15	21	= 45	4	9	25	36	= 74
3	4	5	5	= 17	6	10	15	15	= 46	9	16	25	25	= 75
3	3	3	7	= 16	6	6	6	28	= 46	9	9	9	49	= 76
3	4	4	6	= 17	6	10	10	21	= 47	9	16	16	36	= 77
2	3	4	7	= 16	3	6	10	28	= 47	4	9	16	49	= 78
3	3	5	6	= 17	6	6	15	21	= 48	9	9	25	36	= 79
2	2	6	6	= 16	3	3	21	21	= 48	4	4	36	36	= 80
2	4	5	6	= 17	3	10	15	21	= 49	4	16	25	36	= 81
2	2	5	7	= 16	3	3	15	28	= 49	4	4	25	49	= 82
3	3	4	7	= 17	6	6	10	28	= 50	9	9	16	49	= 83
1	3	5	7	= 16	1	6	15	28	= 50	1	9	25	49	= 84
2	4	4	7	= 17	3	10	10	28	= 51	4	16	16	49	= 85
2	3	3	8	= 16	3	6	6	36	= 51	4	9	9	64	= 86
2	3	5	7	= 17	3	6	15	28	= 52	4	9	25	49	= 87
2	2	4	8	= 16	3	3	10	36	= 52	4	4	16	64	= 88
1	4	6	6	= 17	1	10	21	21	= 53	1	16	36	36	= 89
1	3	4	8	= 16	1	6	10	36	= 53	1	9	16	64	= 90
1	4	5	7	= 17	1	10	15	28	= 54	1	16	25	49	= 91
4	5	5	5	= 19	10	15	15	15	= 55	16	25	25	25	= 91
3	3	5	7	= 18	6	6	15	28	= 55	9	9	25	49	= 92

Pentagonal Numbers.			Sums.
22	22	22	35 = 101
5	12	35	51 = 103
12	22	35	35 = 104
12	12	12	70 = 106
12	22	22	51 = 107
5	12	22	70 = 109
12	12	35	51 = 110
5	5	51	51 = 112
5	22	35	51 = 113
5	5	35	70 = 115
12	12	22	70 = 116
1	12	35	70 = 118
5	22	22	70 = 119
5	12	12	92 = 121
5	12	35	70 = 122
5	5	22	92 = 124
1	22	51	51 = 125
1	12	22	92 = 127
1	22	35	70 = 128
22	35	35	35 = 127
12	12	35	70 = 129

Hexagonal Numbers.			Sums.
28	28	28	45 = 129
6	15	45	66 = 132
15	28	45	45 = 133
15	15	15	91 = 136
15	28	28	66 = 137
6	15	28	91 = 140
15	15	45	66 = 141
6	6	66	66 = 144
6	28	45	66 = 145
6	6	45	91 = 148
15	15	28	91 = 149
1	15	45	91 = 152
6	28	28	91 = 153
6	15	15	120 = 156
6	15	45	91 = 157
6	6	28	120 = 160
1	28	66	66 = 161
1	15	28	120 = 164
1	28	45	91 = 165
28	45	45	45 = 163
15	15	45	91 = 166

Heptagonal Numbers.			Sums.
34	34	34	55 = 157
7	18	55	81 = 161
18	34	55	55 = 162
18	18	18	112 = 166
18	34	34	81 = 167
7	18	34	112 = 171
18	18	55	81 = 172
7	7	81	81 = 176
7	34	55	81 = 177
7	7	55	112 = 181
18	18	34	112 = 182
1	18	55	112 = 186
7	34	34	112 = 187
7	18	18	148 = 191
7	18	55	112 = 192
7	7	34	148 = 196
1	34	81	81 = 197
1	18	34	148 = 201
1	34	55	112 = 202
34	55	55	55 = 199
18	18	55	112 = 203

DIAGRAM No. 4.

Natural numbers .	1	2	3	4	5	6	7	8	9	10	&c.
Trigonal numbers .	1	3	6	10	15	21	28	36	45	55	&c.
Squares	1	4	9	16	25	36	49	64	81	100	&c.
Pentagonal . . .	1	5	12	22	35	51	70	92	117	145	&c.
Hexagonal . . .	1	6	15	28	45	66	91	120	153	190	&c.
Heptagonal . . .	1	7	18	34	55	81	112	148	189	235	&c.
Octagonal , . .	1	8	21	40	65	96	133	176	225	280	&c.

Explanatory Remarks on Diagram No. 2.

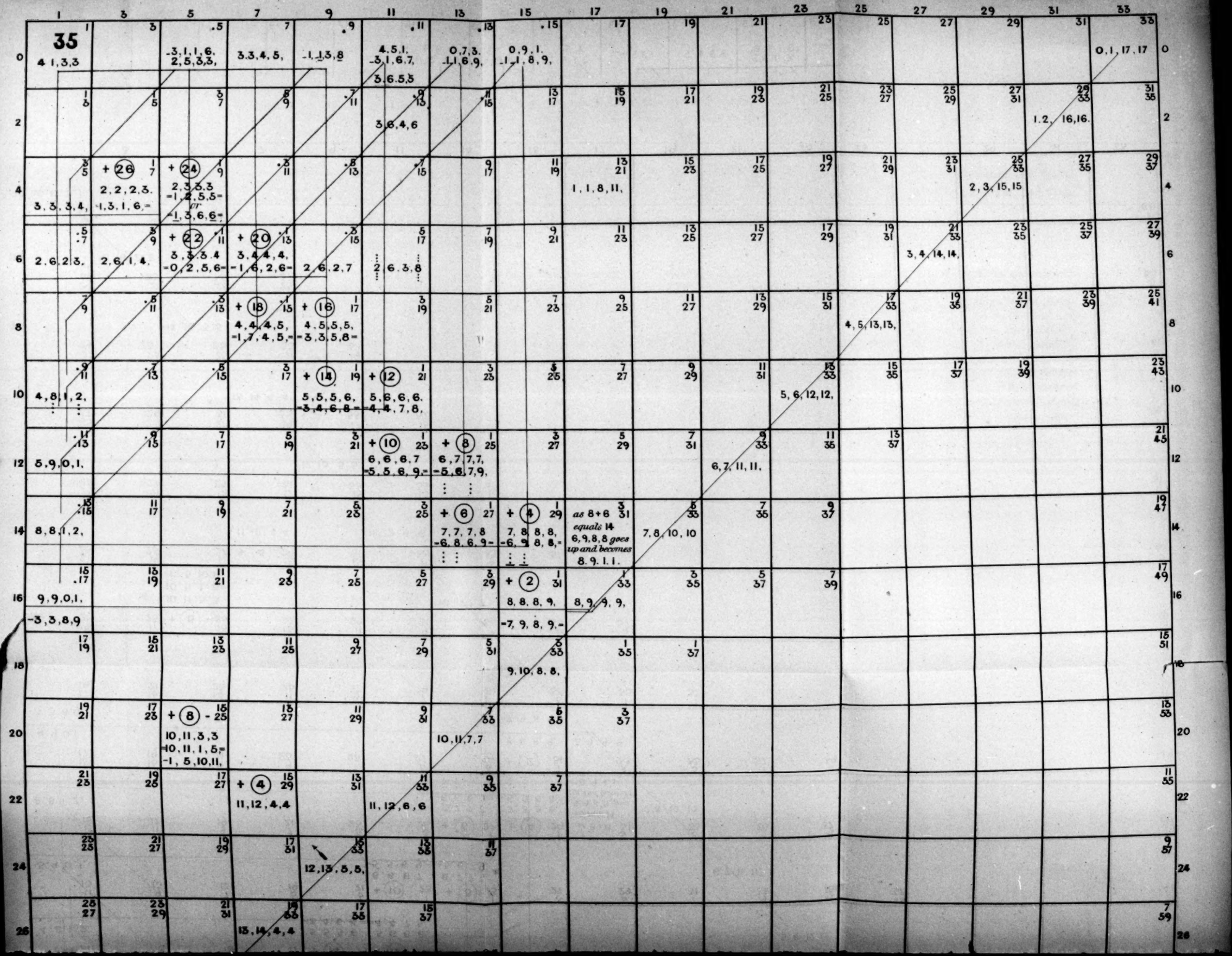
The number in the first square is 35, a number of the form $4n+3$; the nearest division in whole numbers of 35 is 17, 18, and the nearest division of these is 8, 9, 9, 9, which are the roots of the $(n+1)$th term down the diagonal (in this case the 9th); these roots, when 8 and 9 are negative, give a difference of 1, and therefore the roots traverse from the place they occupy to the top corner, becoming there 0, 1, 17, 17, and also to the opposite corner, becoming there 1, 1, 16, 17. If the roots ascend in the diagonal towards 35, diminishing each root every step by one, they will require a numeral to be added, viz. 4, 6, 12, 16, &c., to make them equal to the number that should be found in the square they occupy. This may be easily done by the assistance of the 2 columns of $2b^2$ and $2a^2+2a$; a few of them are corrected and marked thus, =0, 2, 5, 6= by way of example. The term in the gradation series next below 8, 9, 9, 9 has the roots 8, 8, 8, 9, but it requires 2 to be added, and when corrected is 7, 9, 8, 9. If the roots 8, 8, 8, 9 (diminished by 1 each) at every step be taken up the 2nd diagonal, they will require a numeral to be added, viz. 2, 6, 10, 14, &c., to make them equal to the required number; this also may be done in the same way as the other, whenever the correction produces 1 as a root; the roots of the 1st number may be obtained; thus in the 4th square of the diagonal the corrected roots are

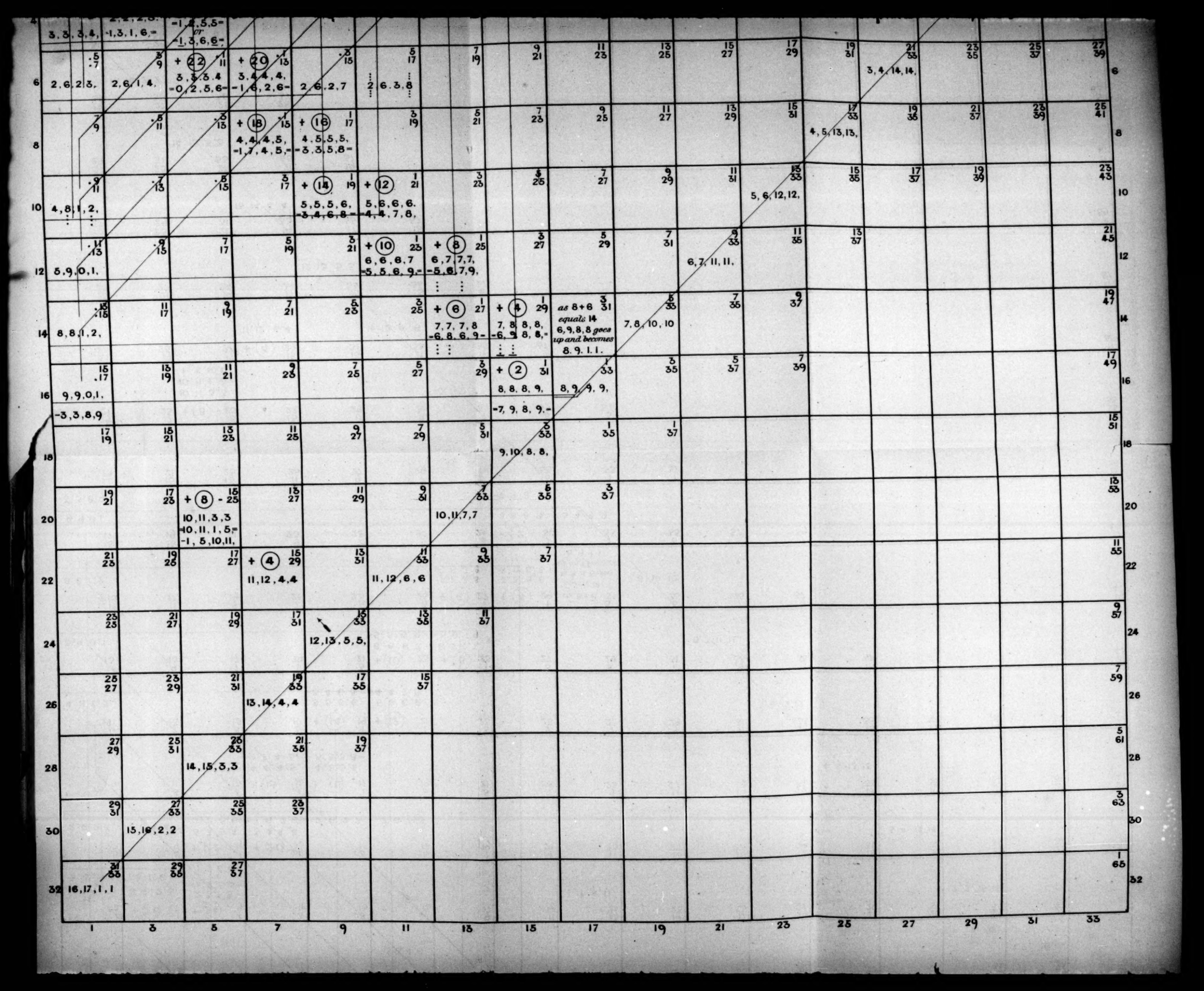
$$1, 6, 2, 6, \text{the sum of which is } 15;$$

but by making 1 negative, $\overset{3}{-1}, \overset{4}{2}, 6, \overset{9}{6}=13$, and lessening the roots by 3 each, they become $-4-133$, the sum of whose squares is 35, and the algebraic sum of the roots 1; the number $\frac{35-1}{2}$, that is 17, is composed of 3 trigonal numbers, viz. 10, 6, 1. All the squares in the line have a similar property, 12, 13, 5, 5; the 4th below to the left from 8, 9, 9, 9 may be carried up, diminishing the roots by 1 each every step, but adding as a numeral 4, 8, 12, &c., the roots will always (one or more) have 1 among them; but it may not always be easy to find out which; the following mode cannot fail: 8, 8, 8, 9 is 34 less than 8, 9, 9, 9, each term in the Gradation Series is less than the next above it by the sum of the roots of the next, minus 1; 34 will be added by adding 18, 12, and 4; 18 will be added by changing 8, 8 into 5, 11; 12 may be added by changing 8, 9 into 6, 11; the roots will then be 5, 6, 11, 11, and to these 4 will be added by changing 5, 6 into 4, 7; 4, 7, 11, 11, when squared (each of them), =8, 9, 9, 9 when squared and added together; but the sum of the roots of 4, 7, 11, 11 is 33, the index of the square in which 8, 9, 9, 9 is; it will therefore traverse upwards to the left and diminish each root by 8; the roots will be $-4-133$ as before.

Note.—Every square that is crossed by a red line, whether perpendicular, horizontal, or transverse, will have one or more forms of roots, the sum of the squares of which will be the number belonging to that square.

DIAGRAM N° 2.


35
4 1,3,3
-3,1,1,6,
2,5,3,3,
3.3,4,5,
-1,1,3,8
4,5,1,
-3,1,6,7,
3,6,5,5
0,7,3,
-1,1,6,9,
0,9,1,
-1,-1,8,9,
0,1,17,17
3,6,4,6
1,2, 16,16.
3,3,3,4,
2,2,2,3,
-1,3,1,6,=
2,3,3,3
=1,2,5,5=
or
=1,3,6,6=
1,1,8,11,
2,3,15,15
2,6,2,3,
2,6,1,4,
3,3,3,4
=0,2,5,6=
3,4,4,4,
-1,6,2,6=
2,6,2,7
2,6,3,8
3,4,14,14,
4,4,4,5,
=1,7,4,5,=
4,5,5,5,
=3,3,5,8=
4,5,13,13,
4,8,1,2,
5,5,5,6,
=3,4,6,8=
4,4,7,8,
5,6,12,12,
5,9,0,1,
6,6,6,7
=5,5,6,9=
6,7,7,7,
=5,6,7,9,
6,7,11,11,
8,8,1,2,
7,7,7,8
=6,8,6,9=
7,8,8,8,
=6,9,8,8,=
as 8+6 equals 14
6,9,8,8 goes up and becomes
8.9.1.1.
7,8,10,10
9,9,0,1,
-3,3,8,9
8,8,8,9,
-7,9,8,9,-
8,9,9,9,
9,10,8,8,
10,11,3,3
=10,11,1,5=
-1,5,10,11,
10,11,7,7
11,12,4,4
11,12,6,6
12,13,5,5,
13,14,4,4


3,3,3,4,
2,2,2,3,
-1,3,1,6,=
-1,3,5,5=
or
-1,3,6,6=
2,6,2,3,
2,6,1,4,
+ 22
3,3,3,4
=0,2,5,6=
+ 20
3,4,4,4,
-1,1,2,6=
2,6,2,7
6,3,8
3,4,14,14,
4,4,4,5,
-1,1,4,5,=
4,5,5,5,
-3,3,5,8=
4,5,13,13,
4,8,1,2,
+ 14
5,5,5,6,
-3,4,6,8=
+ 12
5,6,6,6,
-4,4,7,8,
5,6,12,12,
5,9,0,1,
+ 10
6,6,6,7
-5,5,6,9=
+ 8
6,7,7,7,
-5,6,7,9,
6,7,11,11,
8,8,1,2,
+ 6
7,7,7,8
-6,6,6,9=
+ 4
7,8,8,8,
-6,7,8,8,=
as 8+6
equals 14
6,9,8,8 goes
up and becomes
8.9.1.1.
7,8,10,10
9,9,0,1,
-3,3,8,9
+ 2
8,8,8,9,
-7,9,8,9,=
8,9,9,9,
9,10,8,8,
+ 8
- 25
10,11,3,3
=10,11,1,5=
-1,5,10,11,
10,11,7,7
+ 4
11,12,4,4
11,12,6,6
12,13,5,5,
13,14,4,4
14,15,3,3
15,16,2,2
16,17,1,1

XXVI. *On a Method of making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light.* By J. Clerk Maxwell, *F.R.SS. L. & E.*

Received June 10,—Read June 18, 1868.

There are two distinct and independent methods of measuring electrical quantities with reference to received standards of length, time, and mass.

The electrostatic method is founded on the attractions and repulsions between electrified bodies separated by a fluid dielectric medium, such as air; and the electrical units are determined so that the repulsion between two small electrified bodies at a considerable distance may be represented numerically by the product of the quantities of electricity, divided by the square of the distance.

The electromagnetic method is founded on the attractions and repulsions observed between conductors carrying electric currents, and separated by air; and the electrical units are determined so that if two equal straight conductors are placed parallel to each other, and at a very small distance compared with their length, the attraction between them may be represented numerically by the product of the currents multiplied by the sum of the lengths of the conductors, and divided by the distance between them.

These two methods lead to two different units by which the quantity of electricity is to be measured. The ratio of the two units is an important physical quantity, which we propose to measure. Let us consider the relation of these units to those of space, time, and force (that of force being a function of space, time, and mass).

In the electrostatic system we have a force equal to the product of two quantities of electricity divided by the square of the distance. The unit of electricity will therefore vary directly as the unit of length, and as the square root of the unit of force.

In the electromagnetic system we have a force equal to the product of two currents multiplied by the ratio of two lines. The unit of current in this system therefore varies as the square root of the unit of force; and the unit of electrical quantity, which is that which is transmitted by the unit current in unit of time, varies as the unit of time and as the square root of the unit of force.

The ratio of the electromagnetic unit to the electrostatic unit is therefore that of a certain distance to a certain time, or, in other words, this ratio is a *velocity*; and this velocity will be of the same absolute magnitude, whatever standards of length, time, and mass we adopt.

The electromagnetic value of the resistance of a conductor is also a quantity of the nature of a velocity, and therefore we may express the ratio of the two electrical units

in terms of the resistance of a known standard coil; and this expression will be independent of the magnitude of our standards of length, time, and mass.

In the experiments here described no absolute measurements were made, either of length, time, or mass, the ratios only of these quantities being involved; and the velocity determined is expressed in terms of the British Association Unit of Resistance, so that whatever corrections may be discovered to be applicable to the absolute value of that unit must be also applied to the velocity here determined.

A resistance-coil whose resistance is equal to about 28·8 B. A. units would represent the velocity derived from the present experiments in a manner independent of all particular standards of measure.

The importance of the determination of this ratio in all cases in which electrostatic and electromagnetic actions are combined is obvious. Such cases occur in the ordinary working of all submarine telegraph-cables, in induction-coils, and in many other artificial arrangements. But a knowledge of this ratio is, I think, of still greater scientific importance when we consider that the velocity of propagation of electromagnetic disturbances through a dielectric medium depends on this ratio, and, according to my calculations*, is expressed by the very same number.

The first numerical determination of this quantity is that of Weber and Kohlrausch†, who measured the capacity of a condenser electrostatically by comparison with the capacity of a sphere of known radius, and electromagnetically by passing the discharge from the condenser through a galvanometer.

The Electrical Committee of the British Association have turned their attention to the means of obtaining an accurate measurement of this velocity, and for this purpose have devised new forms of condensers and contact-breakers; and Sir William Thomson has obtained numerical values of continually increasing accuracy by the constant improvement of his own methods.

A velocity which is so great compared with our ordinary units of space and time is probably most easily measured by steps, and by the use of several different instruments; but as it seemed probable that the time occupied in the construction and improvement of these instruments would be considerable, I determined to employ a more direct method of comparing electrostatic with electromagnetic effects.

I should not, however, have been able to do this, had not Mr. Gassiot, with his usual liberality, placed at my disposal his magnificent battery of 2600 cells charged with corrosive sublimate, with the use of his laboratory to work in.

To Mr. Willoughby Smith I am indebted for the use of his resistance-coils, giving a resistance of more than a million B. A. units, and to Messrs. Forde and Fleeming Jenkin for the use of a galvanometer and resistance-coils, a bridge and a key for double contacts.

Mr. C. Hockin, who has greatly assisted me with suggestions since I first devised the

* "A Dynamical Theory of the Electromagnetic Field," Philosophical Transactions, 1865.

† Pogg. Ann. Aug. 1856, Bd. xcix. p. 10.

experiment, undertook the whole work of the comparison of the currents by means of the galvanometer and shunts. He has also tested the resistances, and in fact done everything except the actual observation of equilibrium, which I undertook myself.

The electrical balance itself was made for me by Mr. BECKER.

The electrostatic force observed was that between two parallel disks, of which one, 6 inches diameter, was insulated and maintained at a high potential, while the other, 4 inches diameter, was at the same potential as the case of the instrument.

In order to insure a known quantity of electricity on the surface of this disk, it was surrounded by the "guard-ring" introduced by Sir W. THOMSON, so that the surface of the disk when in position and that of the guard-ring were in one plane, at the same potential, and separated by a very narrow space. In this way the electrical action on the small disk was equal to that due to a uniform distribution over its front surface, while no electrical action could exist at its sides or back, as these were at the same potential with the surrounding surfaces.

The large disk was mounted on a slide worked by a micrometer-screw. The small disk was suspended on one arm of a torsion-balance so that in its position of equilibrium its surface and that of the guard-ring were in one plane.

If E is the difference of potential between the two disks in electromagnetic measure, the attraction between them is

$$\frac{E^2 a^2}{8v^2 b^2}, \quad \ldots \ldots \ldots \ldots \ldots \quad (1)$$

where a is the radius of the small disk, b its distance from the large one, and v is the velocity representing the ratio of the electromagnetic to the electrostatic unit of electricity.

The electromagnetic force observed was the repulsion between two circular coils, of which one was attached to the back of the suspended disk, and the other was placed behind the large disk, being separated from it by a plate of glass and a layer of HOOPER'S compound. A current was made to pass through these coils in opposite directions, so as to produce a repulsion

$$= 2\pi nn' \frac{2A}{B} y^2, \quad \ldots \ldots \ldots \ldots \quad (2)$$

where n and n' are the number of windings of each coil, y is the current, and

$$\frac{2A}{B} = \{E_c \tan^2\gamma - 2(F_c - E_c)\} \frac{b' \sin\gamma}{2\sqrt{a_1 a_2}}, \quad \ldots \ldots \quad (3)$$

where

$$c = \sin\gamma = \frac{2\sqrt{a_1 a_2}}{\sqrt{(a_1+a_2)^2 + b'^2}}; \quad \ldots \ldots \quad (4)$$

a_1 and a_2 are the mean radii of the coils, and b' the mean distance of their planes, and E_c and F_c are the complete elliptic functions for modulus $c = \sin\gamma$.

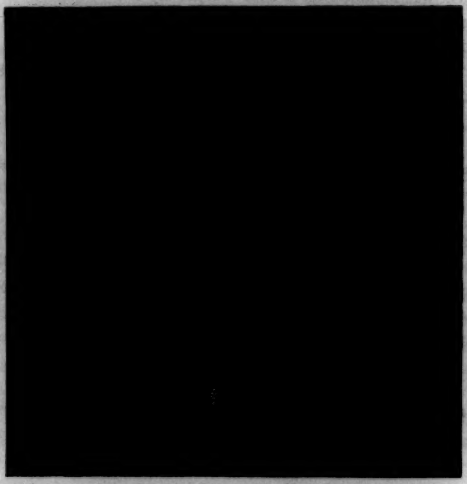
When b' is small compared with a', $\frac{2A}{B}$ becomes very nearly $\frac{2a'}{b'}$.

If we take into account the fact that the section of each coil is of sensible area, this formula would require correction; but in these coils the depth was made equal to the breadth of the section, whence it follows, by the differential equation of the potential of two coils, given at p. 508 of my paper on the Electromagnetic Field,

$$\frac{d^2M}{da^2}+\frac{d^2M}{db^2}+\frac{1}{a}\frac{dM}{da}=0, \quad \ldots\ldots\ldots\ldots \quad (5)$$

that the correction is a factor of the form $\left(1-\frac{1}{12}\frac{\alpha'^2}{a^2}\right)$, where α' is the depth of the coil —a correction which is in this case about 1—·000926.

The suspended coil, besides the repulsion due to the fixed coil, experiences a couple due to the action of terrestrial magnetism. To balance this couple, a coil exactly similar was attached to the other arm of the torsion-balance, and the current in the second coil was made to flow in the opposite direction to that in the first. When the current was made to flow through both coils, no effect of terrestrial magnetism could be observed.



A. Suspended disk and coil.
A'. Counterpoise disk and coil.
C. Fixed disk and coil.
B_1. Great battery. B_2. Small battery.
G_1. Primary coil of galvanometer. G_2. Secondary coil.
R. Great resistance. S. Shunt.
K. Double key. *g*. Graduated glass scale.
C. Electrode of fixed disk.
x. Current through R.
x'. Current through G_1. $x-x'$. Current through S.
y. Current through the three coils and G_2.
M. Mercury cup. T. Torsion head and tangent screw.

One quarter of the micrometer-box, disks, and coils is cut away to show the interior. The case of the instrument is not shown. The galvanometer and shunts were 10 feet from the Electric Balance.

The torsion-balance consisted of a light brass frame, to which the suspended coils and disks were attached so that the centre of each coil was about eight inches from the vertical axis of suspension. This frame was suspended by a copper wire (No. 20), the upper end of which was attached to the centre of a torsion head, graduated, and provided with a tangent screw for small angular adjustments. The torsion head was supported by a hollow pillar, the base of which was clamped to the lid of the instrument so as to admit of small adjustments in every direction.

The fixed disk and coil were mounted on a slide worked by a micrometer-screw, and were protected by a cylindrical brass box, the front of which, forming the guard-ring, 7 inches in diameter, had a circular aperture 4·26 inches diameter, within which the suspended disk, 4·13 inches diameter, was free to move, leaving an interval of ·065 of an inch between the disk and the aperture. A glass scale with divisions of $\frac{1}{100}$ of an inch was attached to the suspended disk on the side which was not electrified, and this was viewed by a microscope attached to the side of the instrument and provided with cross wires at the focus.

The disk worked by the micrometer was carefully adjusted by the maker, so as to be parallel to the inner surface of the guard-ring, or front face of the micrometer-box. This front face of the micrometer-box, when in position in the instrument, was made vertical by means of three adjusting screws. The suspended disk was then pressed against the fixed disk by means of a slight spring, and the fixed disk was gradually moved forward by the micrometer-screw, while at the same time the graduated scale was observed through the microscope. In this way the graduations on the scale were compared with the readings of the micrometer. This was continued till the large disk came into contact with the guard-ring at one point, when the regularity of the motion was interrupted. A very small motion was then sufficient to bring the whole circumference of the disk into contact with the guard-ring, when the motion ceased altogether. This motion was not much more than one-thousandth of an inch.

This disk was then brought to the position of first contact, and the microscope was adjusted so that a known division of the glass scale was bisected by the cross wires. A small piece of silvered glass was fastened to the outside of the guard-ring, and another to the back of the suspended disk; and these were adjusted so as to be in one plane, and to give a continuous image of reflected objects when the disks were in contact and the surface of the suspended disk was therefore in the plane of the surface of the guard-ring. The fixed disk was then screwed back, and the torsion-balance was adjusted so that the suspended disk when in equilibrium was in precisely the same position as before. This was tested by observing the coincidence of the zero division of the glass scale with the cross wires of the microscope, and by examining the reflections from the two pieces of silvered glass. The torsion-balance could be moved bodily in any horizontal direction by adjusting the base of the pillar; it could be raised or lowered by a winch, and it could be turned about any horizonal axis by sliding weights, and round the vertical axis by a tangent screw of the torsion head. In this way the position of equilibrium

of the suspended disk could be made to coincide with the plane of the guard-ring to the thousandth of an inch; and the adjustment when made continued very good from day to day, soft copper wire, stretched straight, not having the tendency to untwist gradually which I have observed in steel wire. The weight of the torsion piece was about 1 lb. 3 oz., and the time of a double oscillation about fourteen seconds. The oscillations of the suspended disk, when near its sighted position, were found to subside very rapidly, the energy of the motion being expended in pumping the air through the narrow aperture between the guard-plate and the suspended disk.

The electrical arrangements were as follows:—

One electrode of Mr. GASSIOT'S great battery was connected with a key. When the key was pressed connexion was made to the fixed disk, and thence, through Mr. WILLOUGHBY SMITH'S resistance-coils, to a point where the current was divided between the principal coil of the galvanometer and a shunt, S, consisting of Mr. JENKIN'S resistance-coils. These partial currents reunited at a point where they were put in connexion with the other electrode of the battery, with the case of the instrument, and with the earth.

Another battery was employed to send a current through the coils. One electrode of this battery was connected with a second contact piece of the key, so that, when the key was pressed, the current went first through the secondary coil of the galvanometer, consisting of thirty windings of thick wire, then through the fixed coil, then to the suspension wire, and so through the two suspended coils to the brass frame of the torsion-balance and the suspended disk. A stout copper wire, well amalgamated, hanging from the centre of the torsion-balance into a cup of mercury, made metallic communication to the case, to earth, and to the other electrode of the battery.

When these arrangements had been made, the observer at the microscope, when the suspended disk was stationary at zero, made simultaneous contact with both batteries by means of the key. If the disk was attracted, the great battery was the more powerful, and the micrometer was worked so as to increase the distance of the disk. If the disk was repelled, the fixed disk had to be moved nearer to the suspended disk, till a distance was found at which, when the scale was at rest and at zero, no effect was produced by the simultaneous action of the batteries. With the forces actually employed the equilibrium of the scale at zero was unstable; so that when the adjustment was nearly perfect the force was always directed from zero, and contacts had to be made as the scale was approaching zero, in such a way as to bring it to rest, if possible, at zero.

In the meantime the other observer at the galvanometer was taking advantage of these contacts to alter the shunt S, till the effects of the two currents on the galvanometer-needle balanced each other.

When a satisfactory case of equilibrium had been observed simultaneously at the galvanometer and at the torsion-balance, the micrometer-reading and the resistance of the shunt were set down as the results of the experiment.

The chief difficulties experienced arose from the want of constancy in the batteries, the ratio of the currents varying very rapidly after first making contact. I think that

by increasing considerably the resistance of the great battery-circuit, the current could be made more uniform.

When a sufficient number of experiments on equilibrium had been made, a current was made to pass through the secondary coil of the galvanometer, and was then divided between a shunt of 31 units B. A. and the primary coil of the galvanometer with a resistance S′ added. S′ was then varied till the needle was in equilibrium. In this way the magnetic effects of the two coils were compared.

The resistance of the galvanometer and of all the coils were tested by Mr. Hockin, who also made all the observations with the galvanometer and its adjusting shunts.

To determine v from these experiments, we have first, since the attraction is equal to the repulsion,

$$\frac{E^2}{8v^2}\frac{a^2}{b^2}=2\pi nn'\frac{2A}{B}y^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If x is the current of the great battery passing through the great resistance R, and if x' of this passes through the galvanometer whose resistance is G, and $x-x'$ through the shunt S to earth, then

$$E=Rx+Gx' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$Gx'=S(x-x'). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Also if g_1 is the magnetic effect of the principal coil of the galvanometer, and g_2 that of the secondary coil, then when the needle is in equilibrium

$$g_1x'=g_2y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In the comparison of the coils of the galvanometer, if x_1 and y_1 are the currents through each, we have

$$g_1x_1=g_2y_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

But y_1 is divided into two parts, of which x_1 passes through the galvanometer G and the shunt S′, and the other, y_1-x', passes through the shunt of 31 Ohms. Hence

$$x_1(G+S')=(y_1-x_1)31. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From these equations we obtain as the value of v,

$$v=\frac{1}{4\sqrt{nn'\pi}}\frac{a}{b}\sqrt{\frac{B}{2A}}\left(\frac{RG}{S}+R+G\right)\frac{31}{G+S'+31}, \quad . \quad . \quad . \quad . \quad . \quad (12)$$

an equation containing only known quantities on the right-hand side. Of these, n and n' are the numbers of windings on the two coils, a is the mean of the radii of the suspended disk and the aperture, b is the distance between the fixed disk and the suspended disk, $\frac{2A}{B}$ is found from a_1 and a_2, the mean radii of the coils, and b' their mean distance by equation (3).

R is the great resistance, G that of the galvanometer, S that of the shunt in the principal experiment, and S′ that of the additional resistance in the comparison of galvanometer-coils.

In this expression the only quantities which must be determined in absolute measure are the resistances. The other quantities which must be measured are the ratios of the radius of the disk to its distance from the fixed disk, and the ratio of the radius of the coils to the distance between them. These ratios and the number of windings in the coils are of course abstract numbers.

In the experiments,

$n=144$ $\qquad$ $n'=121$

$a=2{\cdot}0977$ inches. $\qquad$ $a'=1{\cdot}934$ inch.

To determine a', the circumference of every layer of the coils was measured with watch-spring, the thickness of which was ·008 inch.

One turn of the micrometer-screw was found by Mr. HOCKIN to be equal to ·0202 inch. If m is the micrometer-reading in terms of the screw,

$$b=m-12{\cdot}70, \qquad b'=m+26{\cdot}31.$$

In terms of the micrometer measure we have for a and a',

$$a=103{\cdot}85 \text{ turns}, \qquad a'=95{\cdot}75 \text{ turns}.$$

The resistances were determined by Mr. HOCKIN as follows:

R=1 102 000 Ohms.

G= 46 220 „

The experiments were made for two days, using a small battery charged with bichromate of potash. The current due to this battery was found to diminish so rapidly that a set of GROVE'S cells was used on the third day, which was found to be more constant than the great battery. A proper combination of the two batteries would perhaps produce a current which would diminish according to the same law as that of the great battery. Another difficulty arose from the fact that when the connexions were made, but before the key was pressed, if the micrometer was touched by the hand the disk was attracted. This I have not been able satisfactorily to account for, except by leakage of electricity from the great battery through the floor. When the micrometer was not touched, the disk remained at its proper zero. In certain experiments I kept my hand always on the micrometer in order to be able to adjust it more accurately. These experiments gave a value of v much too small, on account of the additional attraction. When I discovered the attraction, I took care to make the observations without touching the micrometer, and took advantage of the attraction to check the oscillations of the disk. The experiments in which these precautions were taken agree together as well as I could expect, and lead me to think that, with the experience I have acquired, still better results might be obtained by the same method. It must be borne in mind that none of the results were calculated till after the conclusion of all the experiments, and that the rejected experiments were condemned on account of errors observed while they were being made.

Any leakage arising from want of insulation of the fixed disk would introduce no

error, as the difference of potentials between the two disks is measured by the current in the galvanometer, through a known resistance, independently of any leakage.

All that is essential to accuracy is that the position of equilibrium before making contact should be at true zero, the same as when there is no electrical action, and that this equilibrium should not be disturbed when simultaneous contact is made with both batteries.

Experiments on May 8. $S'=1710$ Ohms.

Number of experiment.	Great battery-cells.	Small battery-cells.	Distance of disks by micrometer.	Resistance of shunt S.	Value of v in Ohms.
1	1000	6	12·41	6870	28·591
2	1000	6	12·36	6940	28·430
8*	1800	8	16·99	5074	28·886
9	1800	8	17·02	5110	28·686
10	1800	8	19·91	4430	28·910
11	1800	7	20·07	4410	28·850
12	2600	9	25·08	3700	28·762
13	2600	9	25·12	3690	28·795
14	2600	9	25·29	3680	28·735
15	2600	9	25·18	3690	28·752
16	2600	9	25·19	3695	28·709
17	1800	7	19·69	4435	29·474

Mean value of $v=28·798$ Ohms, or B. A. units,
or 288 000 000 metres per second,
or 179 000 statute miles per second.

The "probable error" is about one-sixth per cent.

Experiments 3, 4, 5, 6, 7 were rejected on account of the micrometer being touched during the observation of equilibrium. These experiments gave an average value of $v=27·39$.

The value of v derived from these experiments is considerably smaller than that which was obtained by MM. Weber and Kohlrausch, which was 31·074 Ohms, or 310 740 000 metres per second.

Their method involved the determination of the electrostatic capacity of a condenser, the electrostatic determination of its potential when charged, and the electromagnetic determination of the quantity of electricity discharged through a galvanometer.

The capacity of the condenser was measured by dividing its charge repeatedly with a sphere of known radius. Now, since all condensers made with solid dielectrics exhibit the phenomena of "electric absorption," this method would give too large a value for the capacity, as the condenser would become recharged to a certain extent after each discharge, so that the repeated division of the charge would have too small an effect on the potential. The capacity being overestimated, the number of electrostatic units in the discharge would be overestimated, and the value of v would be too great.

In pointing out this as a probable source of error in the experiments of MM. Weber and Kohlrausch, I mean to indicate that I have such confidence in the ability and fidelity

* In experiment 8 Mr. Hockin and I changed places.

with which their investigation was conducted, that I am obliged to attribute the difference of their result from mine to a phenomenon the nature of which is now much better understood than when their experiments were made.

On the other hand, the result of present experiments depends on the accuracy of the experiments of the Committee of the British Association on Electric Resistance. The B. A. unit is about 8·8 per cent. larger than that determined by WEBER in 1862, and about 1·2 per cent. less than that derived by Dr. JOULE from his experiments on the dynamical equivalent of heat by comparing the heating effects of direct mechanical agitation with those of electric currents.

I believe that Sir WILLIAM THOMSON's experiments, not yet published, give a value of v not very different from mine. His method, I believe, also depends on the value of the B. A. unit.

The lowest estimate of the velocity of light, that of the late M. FOUCAULT, is

298 000 000 metres per second.

Note on the Electromagnetic Theory of Light.

In a paper on the Electromagnetic Field* some years ago, I laid before the Royal Society the reasons which led me to believe that light is an electromagnetic phenomenon, the laws of which can be deduced from those of electricity and magnetism, on the theory that all these phenomena are affections of one and the same medium. Two papers appeared in POGGENDORFF's 'Annalen' for 1867 bearing on the same subject. The first, by the late eminent mathematician BERNHARD RIEMANN, was presented in 1858 to the Royal Society of Göttingen, but was withdrawn before publication, and remained unknown till last year. RIEMANN shows that if for LAPLACE's equation we substitute

$$\frac{d^2V}{dt^2}-\alpha^2\Delta^2V+\alpha^2 4\pi\rho=0, \quad \ldots\ldots\ldots\ldots \quad (13)$$

V being the electrostatic potential, and α a velocity, the results will agree with known phenomena in all parts of electrical science. This equation is equivalent to a statement that the potential V is propagated through space with a certain velocity. The author, however, seems to avoid making explicit mention of any medium through which the propagation takes place, but he shows that this velocity is nearly, if not absolutely, equal to the known velocity of light.

The second paper, by M. LORENZ, shows that, on WEBER's theory, periodic electric disturbances would be propagated with a velocity equal to that of light. The propagation of attraction through space forms part of this hypothesis also, though the medium is not explicitly recognized.

From the assumptions of both these papers we may draw the conclusions, first, that action and reaction are not always equal and opposite, and second, that apparatus may be constructed to generate any amount of work from its own resources.

* Philosophical Transactions, 1865, p. 459.

For let two oppositely electrified bodies A and B travel along the line joining them with equal velocities in the direction AB, then if either the potential or the attraction of the bodies at a given time is that due to their position at some former time (as these authors suppose), B, the foremost body, will attract A forwards more than A attracts B backwards.

Now let A and B be kept asunder by a rigid rod.

The combined system, if set in motion in the direction AB, will pull in that direction with a force which may either continually augment the velocity, or may be used as an inexhaustible source of energy.

I think that these remarkable deductions from the latest developments of WEBER and NEUMANN'S theory can only be avoided by recognizing the action of a medium in electrical phenomena.

The statement of the electromagnetic theory of light in my former paper was connected with several other electromagnetic investigations, and was therefore not easily understood when taken by itself. I propose, therefore, to state it in what I think the simplest form, deducing it from admitted facts, and showing the connexion between the experiments already described and those which determine the velocity of light.

The connexion of electromagnetic phenomena may be stated in the following manner.

Theorem A.—If a closed curve be drawn embracing an electric current, then the integral of the magnetic intensity taken round the closed curve is equal to the current multiplied by 4π.

The integral of the magnetic intensity may be otherwise defined as the work done on a unit magnetic pole carried completely round the closed curve.

This well-known theorem gives us the means of discovering the position and magnitude of electric currents, when we can ascertain the distribution of magnetic force in the field. It follows directly from the discovery of ŒRSTED.

Theorem B.—If a conducting circuit embraces a number of lines of magnetic force, and if, from any cause whatever, the number of these lines is diminished, an electromotive force will act round the circuit, the total amount of which will be equal to the decrement of the number of lines of magnetic force in unit of time.

The number of lines of magnetic force may be otherwise defined as the integral of the magnetic intensity resolved perpendicular to a surface, multiplied by the element of surface, and by the coefficient of magnetic induction, the integration being extended over any surface bounded by the conducting circuit.

This theorem is due to FARADAY, as the discoverer both of the facts and of this mode of expressing them, which I think the simplest and most comprehensive.

Theorem C.—When a dielectric is acted on by electromotive force it experiences what we may call electric polarization. If the direction of the electromotive force is called positive, and if we suppose the dielectric bounded by two conductors, A on the negative, and B on the positive side, then the surface of the conductor A is positively electrified, and that of B negatively. If we admit that the energy of the system so electrified resides

in the polarized dielectric, we must also admit that within the dielectric there is a displacement of electricity in the direction of the electromotive force, the amount of this displacement being proportional to the electromotive force at each point, and depending also on the nature of the dielectric.

The energy stored up in any portion of the dielectric is half the product of the electromotive force and the electric displacement, multiplied by the volume of that portion.

It may also be shown that at every point of the dielectric there is a mechanical tension along the lines of electric force, combined with an equal pressure in all directions at right angles to these lines, the amount of this tension on unit of area being equal to the amount of energy in unit of volume.

I think that these statements are an accurate rendering of the ideas of FARADAY, as developed in various parts of his 'Experimental Researches.'

Theorem D.—When the electric displacement increases or diminishes, the effect is equivalent to that of an electric current in the positive or negative direction.

Thus, if the two conductors in the last case are now joined by a wire, there will be a current in the wire from A to B.

At the same time, since the electric displacement in the dielectric is diminishing, there will be an action electromagnetically equivalent to that of an electric current from B to A through the dielectric.

According to this view, the current produced in discharging a condenser is a complete circuit, and might be traced within the dielectric itself by a galvanometer properly constructed. I am not aware that this has been done, so that this part of the theory, though apparently a natural consequence of the former, has not been verified by direct experiment. The experiment would certainly be a very delicate and difficult one.

Let us now apply these four principles to the electromagnetic theory of light, considered as a disturbance propagated in plane waves.

Let the direction of propagation be taken as the axis of z, and let all the quantities be functions of z and of t the time; that is, let every portion of any plane perpendicular to z be in the same condition at the same instant.

Let us also suppose that the magnetic force is in the direction of the axis of y, and let β be the magnetic intensity in that direction at any point.

Let the closed curve of Theorem A consist of a parallelogram in the plane $y\,z$, two of whose sides are b along the axis of y, and z along the axis of z. The integral of the magnetic intensity taken round this parallelogram is $b(\beta_0-\beta)$, where β_0 is the value of β at the origin.

Now let p be the quantity of electric current in the direction of x per unit of area taken at any point, then the whole current through the parallelogram will be

$$\int_0^z bpdz,$$

and we have by (A),

$$b(\beta_0-\beta)=4\pi\int_0^z bpdz.$$

If we divide by b and differentiate with respect to z, we find

$$\frac{d\beta}{dz}=-4\pi p. \quad \ldots \quad (14)$$

Let us next consider a parallelogram in the plane of $x\,z$, two of whose sides are a along the axis of x, and z along the axis of z.

If P is the electromotive force per unit of length in the direction of x, then the total electromotive force round this parallelogram is $a(\mathrm{P}-\mathrm{P}_0)$.

If μ is the coefficient of magnetic induction, then the number of lines of force embraced by this parallelogram will be

$$\int_0^z a\mu\beta dz,$$

and since by (B) the total electromotive force is equal to the rate of diminution of the number of lines in unit of time,

$$a(\mathrm{P}-\mathrm{P}_0)=-\frac{d}{dt}\int_0^z a\mu\beta dz.$$

Dividing by a and differentiating with respect to z, we find

$$\frac{d\mathrm{P}}{dz}=-\mu\frac{d\beta}{dt}. \quad \ldots \quad (15)$$

Let the nature of the dielectric be such that an electric displacement f is produced by an electromotive force P,

$$\mathrm{P}=kf, \quad \ldots \quad (16)$$

where k is a quantity depending on the particular dielectric, which may be called its "electric elasticity."

Finally, let the current p, already considered, be supposed entirely due to the variation of f, the electric displacement, then

$$p=\frac{df}{dt}. \quad \ldots \quad (17)$$

We have now four equations, (14), (15), (16), (17), between the four quantities β, p, P, and f. If we eliminate p, P, and f, we find

$$\frac{d^2\beta}{dt^2}=\frac{k}{4\pi\mu}\frac{d^2\beta}{dz^2}. \quad \ldots \quad (18)$$

If we put

$$\frac{k}{4\pi\mu}=\mathrm{V}^2, \quad \ldots \quad (19)$$

the well-known solution of this equation is

$$\beta=\varphi_1(z-\mathrm{V}t)+\varphi_2(z+\mathrm{V}t), \quad \ldots \quad (20)$$

showing that the disturbance is propagated with the velocity V.

The other quantities p, P, and f can be deduced from β.

Thus, if

$$\left.\begin{aligned} \beta &= c\cos\frac{2\pi}{\lambda}(z-\mathrm{V}t), \\ p &= \frac{c}{2\lambda}\sin\frac{2\pi}{\lambda}(z-\mathrm{V}t), \\ \mathrm{P} &= e\mu\mathrm{V}\cos\frac{2\pi}{\lambda}(z-\mathrm{V}t), \\ f &= \frac{c}{4\pi\mathrm{V}}\cos\frac{2\pi}{\lambda}(z-\mathrm{V}t). \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

I have in the next place to show that the velocity V is the same quantity as that found from the experiments on electricity.

For this purpose let us consider a stratum of air of thickness b bounded by two parallel plane conducting surfaces of indefinite extent, the difference of whose potentials is E.

The electromotive force per unit of length between the surfaces is $\mathrm{P}=\frac{1}{b}\mathrm{E}$.

The electric displacement is $f=\frac{1}{k}\mathrm{P}$.

The energy in unit of volume and the tension along the lines of force per unit of area is $\frac{1}{2}\mathrm{P}f$.

The attraction X on an area πa^2 of either surface is

$$\left.\begin{aligned} \mathrm{X} &= \tfrac{1}{2}\pi a^2\mathrm{P}f \\ &= \tfrac{1}{2}\frac{\pi}{k}\frac{a^2}{b^2}\mathrm{E}^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

If this area is separated by a small interval from the rest of the plane surface, as in the experiment, and if this interval is small compared with the radius of the disk, the lines of force belonging to the disk will be separated from those belonging to the rest of the surface by a surface of revolution, the section of which, at any sensible distance from the surface, will be a circle whose radius is a mean between those of the disk and the aperture. This radius must be taken for a in the equation (22)*.

Let us next consider the magnetic force near a long straight conductor carrying a current y. The magnetic force will be in the direction of a tangent to a circle whose axis is the current; and the intensity will be uniform round this circle. If the radius is b, and the magnetic intensity β, the integral round the circle will be $2\pi b\beta=4\pi y$ by (A).

* [Note added Dec. 28, 1868.—I have since found that if a_1 is the radius of the disk, and a_2 that of the aperture of the guard-ring, and b the distance from the large fixed disk, then we must substitute for $\frac{a^2}{b^2}$ the more approximate expression $\frac{a_1^2}{b^2}+\frac{a_2^2-a_1^2}{2b(b+\alpha)}$, where α is a quantity which cannot exceed $\frac{\log_e 2}{\pi}(a_2-a_1)$.—J. C. M.]

Hence

$$\beta = 2\frac{y}{\delta}. \quad \ldots \quad (23)$$

Let a wire carrying a current y' be placed parallel to the first at a distance b, and let us consider a portion of this wire of length l. This portion will be urged across the lines of magnetic force, and the electromagnetic force Y will be equal to the product of the length of the portion, multiplied by the current and by the number of lines which it crosses per unit of distance through which it moves, or, in symbols,

$$\left.\begin{aligned} \mathrm{Y} &= ly'\mu\beta \\ &= 2\mu\frac{l}{b}yy' \end{aligned}\right\} \quad \ldots \quad (24)$$

If the two wires instead of being straight are circular, of radius a', and if b' the distance between them is very small compared with the radius, the attraction will be the same as if they were straight, and will be

$$\mathrm{Y} = 2\mu\frac{2\pi a'}{b'}yy'. \quad \ldots \quad (25)$$

When b' is not very small compared with a', we must use the equation (3) to calculate the value of $\frac{2\mathrm{A}}{\mathrm{B}}$ by elliptic integrals.

Making X = Y and comparing with equation (6), we find

$$v^2 = \frac{\mu k}{4\pi}; \quad \ldots \quad (26)$$

but, by (19),

$$\mathrm{V}^2 = \frac{k}{4\pi\mu}.$$

Hence

$$v = \mu\mathrm{V}, \quad \ldots \quad (27)$$

where v is the electromagnetic ratio and V is the velocity of light.

But since all the experiments are made in air, for which μ is *assumed* equal to unity, as the standard medium with which all others are compared, we have finally

$$v = \mathrm{V}, \quad \ldots \quad (28)$$

or the number of electrostatic units in one electromagnetic unit of electricity is numerically equal to the velocity of light.

XXVII. *On Supersaturated Saline Solutions.* By CHARLES TOMLINSON, *F.R.S.*

Received April 21,—Read May 28, 1868*.

I. *History.*

THE phenomena of supersaturation have been studied chiefly with reference to solutions of GLAUBER'S salt. In 1809, ZIZ of Mayence† showed that the sudden crystallization of these solutions is not due to agitation; that the vessels containing them need not be hermetically sealed, but if put under a bell-glass, or loosely covered as with a capsule, they can be preserved during a long time; that solids brought into contact with the solutions act as *nuclei* and produce instant crystallization, especially when dry; if wet or boiled up with the solution they become *inactive*. The best nucleus is a crystal of the salt itself. Air, if artificially dried, ceases to be a nucleus. Three varieties of the sodic sulphate are noticed, *i. e.* the *anhydrous*, the ordinary 10-*atom hydrate*, and also a modified salt formed when supersaturated solutions are left to cool down in closed vessels. This salt contains less water of crystallization than the ordinary salt, and is more soluble. If the vessel in which it is formed be suddenly opened, or a nucleus touch the mother-liquor, this instantly solidifies into the 10-atom hydrate, and the crystals of the more soluble salt become opaque, like the boiled white of egg.

In 1819 GAY-LUSSAC‡ referred the state of supersaturation to the inertia of the saline molecules, the molecular condition of the sides of the vessel, and other causes. He also showed that solutions of some other salts besides GLAUBER'S salt exhibit the phenomena of supersaturation. In 1832 the number of such salts was stated by Dr. OGDEN§ to be not less than 21.

In 1825 FARADAY‖ published some experiments on the supersaturated solutions of GLAUBER'S salt. GRAHAM¶, TURNER**, URE, and others also contributed new facts; but the most elaborate inquiry was made by M. LÖWEL between the years 1850 and 1857, the results of which are contained in six memoirs††. According to this writer, the modified salt is formed in consequence of a molecular change which takes place when a solution saturated at 218° FAHR. cools down to 64°. The modified salt is much more soluble at ordinary temperatures than the 10-atom hydrate, its maximum solubility being at 80°·6. The sodic sulphate has three maxima of solubility, viz. 93°·2 for the 10-atom hydrate, 78°·8 to 80°·6 for the 7-atom hydrate, and 62°·6 to 64°·4 for the anhydrous salt. At

* Rewritten by the author from the original paper in a more condensed form.

† Schweigger, Journal for 1815, Band XV.

‡ Annales de Chimie et de Physique, 2nd ser. vol. xi.

§ Edinburgh New Philosophical Journal.

‖ Quarterly Journal of Science, vol. xix.

¶ Transactions of the Royal Society of Edinburgh.

** Elements of Chemistry.

†† Annales de Chimie et de Physique, 3rd ser. vol. xxix., xxxiii., xxxvii., xliii., xliv., xlix.

these three temperatures the saturated solutions are about equally rich in salt. The 7-atom hydrate and the anhydrous salt can only maintain their molecular constitution when in contact with the mother-liquor in closed vessels, in which they are sheltered from the air and from other bodies that act upon them as nuclei. No sooner are they exposed to the air than they become opaque and warm, and assume the molecular constitution of the 10-atom hydrate as well as its solubility. Hence the conclusion is that supersaturated solutions of the sodic sulphate are not really so, since they hold a salt of much greater solubility at common temperatures than the ordinary salt. It is also stated that the solutions of the sodic carbonate and of the magnesic sulphate form salts of a lower degree of hydration, and of greater solubility than the normal salts. LÖWEL's general conclusion is that all cases of supersaturation are such in appearance only, and not in fact. As to the function of nuclei and the inner sides of the flasks in determining crystallization, he regards it as the effect of one of those mysterious contact actions known as *catalytic*, of which science has not yet been able to give a satisfactory explanation. Bodies that appear to be active in inducing crystallization are designated *catalytic* or *dynamic*; while bodies that are apparently inactive are termed *non-catalytic* or *adynamic*. "It appears certain," he says, "that but for the mysterious action which the air and other bodies exert on supersaturated solutions, we should obtain sulphate of soda only in the modified state; that is, crystallized with seven equivalents of water, and possessing at ordinary temperatures of the air a much greater solubility than that of the ordinary 10-atom salt."

Later inquirers have endeavoured to explain the action of nuclei. GERNEZ* tried 220 solids, and of these he selected thirty-nine that were active in inducing crystallization, of which eighteen were insoluble: these were carefully washed in distilled water, and dried out of contact with air. When dry they were found to be inactive. The twenty-one soluble substances were purified by recrystallization, and they all became inactive. Hence it is concluded that sulphate of soda is the only nucleus for solutions of the same salt. That is to say, whenever a glass rod or other body acts as a nucleus, it is contaminated with minute portions of the salt itself, which is supposed to exist in the air, not only of towns, but in the country. According to this view, the supersaturated solution of any other salt can only be crystallized by a saline nucleus of its own kind.

II. *Definitions.*

In the present memoir the terms *chemically clean* or *chemically unclean*, or simply *clean* and *unclean*, are to be taken with the following limitations:—

1. A chemically clean surface is one that has not a greater attraction for the dissolved matter than for the liquid which dissolves it. A supersaturated aqueous solution, for example, whether of a salt or of a gas, will adhere perfectly to such surface, and there will be no separation of salt or of gas.

2. But as there is little or no adhesion between water and a fatty, oily, or greasy

* Comptes Rendus, vol. lx. p. 833. See also Schiff, Ann. der Chem. und Pharm. vol. cxi. p. 68.

surface, while there is strong adhesion between a salt or a gas and such surface, the presence of such a surface in a supersaturated solution will cause a separation of the salt, or of the gas.

3. Hence a surface contaminated ever so slightly with oily, fatty, or greasy matter will act as a nucleus in such solutions.

4. A nucleus may be defined as a body that has a stronger adhesion for the salt or the gas than for the liquid which holds it in solution.

5. Bodies that have been exposed to the products of respiration or of ordinary combustion, or that have been handled or wiped with a cloth, are covered more or less with a greasy film, and act as nuclei in aqueous solutions. So also the dust of the room raised into the air is more or less contaminated with greasy or organic matter, and acts as a nucleus.

6. By washing such bodies in a solution of caustic alkali, in sulphuric acid, in ether, &c., such oily or greasy film is got rid of. In this way the sweepings of a room may be made inactive as nuclei. It is proposed to name bodies or surfaces that have been treated in this way *catharized*, from καθαρός.

III. *Action of Nuclei.*

This action may first be illustrated by taking care to ensure the absence of nuclei. Previous observers insist on the great sensitiveness to cold of sodic sulphate solutions; but if precautions be taken to ensure chemical purity, this sensitiveness disappears. For example:—Two ounces of large crystals of this salt (which had been purified by repeated crystallization) in a dry and slightly efflorescent state were put into a clean flask with one ounce of distilled water, and the flask was carefully heated so as to avoid the deposit of anhydrous salt, and then boiled. Two thin glass test-tubes, each of $1\frac{1}{2}$ oz. capacity, and about $\frac{3}{4}$ inch in diameter, were filled with strong sulphuric acid, emptied, and left to drain with their mouths under water. A glass funnel washed with spirits of wine, had a clean filter adapted to it, and the boiling solution was passed through the filter into the two tubes. These were immediately plugged with cotton-wool, and passed over the flame of a spirit-lamp, when one of the tubes gave a kick, and threw down a small quantity of anhydrous salt. Both tubes were now placed in a vertical position in a rack, and left to cool. At about 50° F. they were put into a freezing-mixture in which a thermometer rapidly sank to 10°; the tubes were moved about in the mixture, and then left in it during some hours, while the temperature slowly rose from 10° to 40°, when the solution in one tube was still liquid, bright, and transparent; and equally so in the other tube, only at the bottom there was a small group of 7-atom crystals, built, as it were, on the deposit of anhydrous salt already referred to. The cotton-wool was now taken out so as to admit a nucleus from the air, and crystallization instantly set in at the surface, spreading rapidly downwards, and carrying with it sufficient water to convert the whole solution into the 10-atom hydrate, and also to impart to the 7-atom hydrate three additional equivalents of water. There is a considerable rise of temperature

accompanying this action. By cautiously fusing the salt after each experiment, these phenomena may be repeated with the same solution a number of times. It is very possible, however, that the tube may cease to retain its condition of chemical purity perfect; in which case the ordinary phenomena consequent on reduction of temperature will take place. It is not easy to maintain a chemically clean surface. A short exposure to the air, or a mere touch, will often suffice to impart a greasy film, or motes or dust, which prevent or lessen adhesion, and render a so-called "inactive" solid "active." When a glass rod has been kept in water or passed through flame and dried, or cooled not in contact with the air, it is more or less chemically clean, and remains so while being sheltered. When Herr Ziz found that a knitting-needle was "active" in one solution, and that by passing it through the cork which confined another solution it became inactive, he simply made the wire chemically clean by the friction.

Some liquids, such as spirits of wine, act as nuclei by combining with a portion of the water of the solution, and liberating a little salt which acts as a nucleus.

My experiments lead me to the conclusion that air, in its pure state, is not a nucleus; for not only may a supersaturated solution be briskly shaken up in a bottle half-full of air*, but air if previously passed through water may be made to bubble through a supersaturated solution without producing any separation of the salt.

When air appears to act as a nucleus, I believe that it is simply performing the part of a carrier of some solid particle not chemically clean. If a boiling solution be poured into a bottle, which is then tightly corked, or the stopper inserted, the solution in cooling down will have a partial vacuum over it. If, now, the cork be loosened, air will rush in, and almost certainly drag in with it a solid nucleus. But if the flask be plugged with cotton-wool, motes &c. will be kept back while the air streams in as the solution cools, and the plug may be removed without always producing immediate crystallization. Thus a solution opened in a garden, on a fine clear still day, is less likely to crystallize than if opened in a room, where a step across the carpet raises a cloud of solid particles. So also a narrow-necked bottle is less likely to catch nuclei than a wide one. Löwel found that he could keep his solutions from crystallizing longest in tubes of narrow bore. Those in tubes of from $\frac{1}{2}$ to $\frac{3}{4}$-inch bore, crystallized on being opened; in those of less than half an inch the solutions remained longer in the liquid state, and they could be kept for weeks exposed to the air in tubes of $\frac{1}{4}$-inch bore. All this seems intelligible without recurring to any mysterious undiscovered force, such as M. Löwel so frequently refers to.

By using chemically clean vessels and thermometers, we get rid of various anomalous cases of crystallization referred to by previous observers. And not only so; but we reduce most of the phenomena of supersaturation to the case of nucleus or no nucleus. In chemically clean vessels there is no nucleus, and the modified salt, if formed at all,

* In such a case it is a long time before the air becomes disengaged and the solution clear—an effect which belongs to many saline solutions a familiar example of which is afforded by the long white foaming trail left behind a steamer in sea-water. This effect is not produced in river-water.

is the result of certain conditions which will be examined in the next section. But it by no means follows that supersaturated solutions in cooling down undergo any change in molecular condition, or produce a modified salt of a lower degree of hydration. Strong solutions of many salts may be cooled down below the freezing-point of water, and become viscous without crystallizing. For example:—Five parts of sodic acetate to one part of water were boiled, filtered, and boiled again at 240°, in a flask, with a thermometer in the solution, the neck being plugged with lamp-cotton. The solution was left in a cold room during about twenty hours, and then placed for some hours in a freezing-mixture. The solution remained some time at 14°, when the thermometer being gently lifted up several times, without being taken out of the flask, the viscous solution ran off from the bulb like a thickish syrup. The thermometer was restored to its place, and a wire passed down so as to touch the liquid. It immediately and rapidly crystallized into an amorphous mass, and became quite solid; the temperature rose to 104°, thus showing a sudden rise of 90°.

Now here is a solution which in an open evaporating-dish crystallizes at 150° and becomes solid a few degrees below this; yet it can be cooled down to 14° or lower, and be kept in that state for hours without crystallizing. It becomes viscid at this low temperature, and so far it undergoes a molecular change; but this solution has no tendency, so far as I am aware, to form a salt of modified character. It may be cooled at least 140° below its point of crystallization and yet not crystallize, simply because there is no nucleus to start the action. In the open evaporating-dish, on the contrary, no sooner is the boiling solution in contact with the air, than its surface becomes dotted with specks and filaments that are floating about. If these be carefully watched, one or more of them will be seen to increase in size by the aggregation of saline particles about it, and suddenly from this point fans of crystals begin to radiate. In the meantime the sides of the vessel are active in separating the saline molecules, and the more so in proportion to the absence of chemical purity.

The sodio-potassic tartrate also gives remarkable results. Two ounces and a half of newly formed crystals were fused in their water of crystallization, one ounce of water was then added, and the solution raised to the boiling-point. The solution cooled down to the temperature of the air without depositing crystals. The flask was then kept for some hours in ice and water, and afterwards in ice and salt until the temperature of the solution was 14°. The flask was repeatedly moved round and round in the freezing-mixture, but the solution remained perfectly fluid. The flask was now taken out, and the thermometer lifted up several times, then quite out of the flask; the solution was also touched with a wire, but crystallization did not set in until the side of the flask had been scratched with a wire not chemically clean. (If clean, it would have had no effect.) The scratches instantly became chalky white, and crystallization set in very gradually; each crystal as it was formed dragged a viscid line as it slowly descended. There was at first but little rise of temperature, but in the course of half an hour the mercury gradually rose to 64°, thus showing a rise of 50°.

The sodic arseniate may be taken as another example. Two parts of this salt to three of water were boiled at 223°, and filtered into a clean bottle with a thermometer passing through the plug. As the solution cooled it became viscid; but at 35° there was no sign of crystallization. The thermometer was taken out of the flask for an instant, and then reinserted. The effect was striking. The thermometer had caught up some chemically unclean particles from the air, and these acted as nuclei. The whole of the immersed portion of the thermometer became at once covered with crystalline tufts which propagated their action, and the temperature rose to 54°.

Cases like these may be multiplied; for it must not be supposed that I am dealing with exceptional instances with respect to these motes and particles. The air of every house is full of them (as a sunbeam often reveals); they come in contact with solutions and start crystallization, or they enter by the necks of flasks and bottles, and render their inner surfaces "active."

The cotton-wool used in plugging the vessels is not chemically clean; it induces crystallization if the solution touch it for an instant, as it may sometimes do on inclining or shaking the flask for the purpose of watching the effects of cooling. We can also induce crystallization in any part of the vessel by making that part chemically unclean. For example, a large tube was washed out with a caustic alkaline solution, rinsed first with water, then with spirit, and again with water, when a boiling solution of sodic phosphate was poured into it, and in another experiment sodic sulphate (5 salt to 1 water); but before plugging it, the upper part of the inside of the tube was touched with a finger slightly greased. When cold the solution was brought into contact with different parts of the tube that were clean, without producing crystallization; but the moment the solution came upon the edge of the finger-mark, crystallization set in, and the solution became solid.

I believe this experiment will explain the statement so often made as to the necessity of keeping supersaturated solutions quite still; for if stirred or agitated they suddenly deposit the excess of salt. In such cases, no care having been taken to produce chemically clean surfaces, the solution itself does something in that direction to the extent that it occupies in the vessel; but agitation brings a portion of the solution against the upper and more "active" part of the sides, or the rod used for stirring is equally "active," and either may perform the part of a nucleus accordingly.

The salt used for making a solution is likely to be unclean. During its formation in an open evaporating-dish, and its subsequent handling, packing, &c., it contracts various impurities, among which may often be found hairs and other greasy substances, which do not always lose their property of becoming nuclei during the boiling of the solution. They may be removed by filtration; but as they often contaminate the vessel in which the salt is boiled, it is desirable to filter the hot solution into a second clean flask, in which it may be boiled up again. In the case of the acetates, a few drops of acetic acid should be added to the second flask to make up for any loss during the boiling. But in such case the temperature may pass beyond the

boiling-point without giving any signal whatever, until the liquid bursts into vapour with explosive violence, or is simply turned out of the flask without any noise at all. Five ounces of sodic acetate were fused in its water of crystallization, boiled, and set aside to cool. The liquid contained a number of hairs or motes. At 150° it began to crystallize, and the whole solution became filled with crystalline needles. The salt was again raised to the boiling-point, and it boiled quickly and readily with the usual symptoms, both before and during the boiling. The solution was now filtered into a chemically clean flask wet with acetic acid, and again placed over the lamp. It rose to 240° without any sign, and a few degrees above this it suddenly burst into vapour. I have not repeated this experiment; but have no doubt that, instead of crystallizing at 150°, the clean solution might be reduced to a very low temperature without crystallizing, simply from the absence of the nuclei which were so abundant in the unfiltered solution.

*[It is said that the most powerful nucleus is a crystal of the salt itself. This statement must be received with reference to the condition of clean and unclean. When supersaturated solutions are lightly covered, so as to allow a portion of the water to escape, a crystalline crust often forms on the side of the flask. This crust does not act as a nucleus, because it is clean; and the solution, being supersaturated, does not dissolve it. I have kept such a solution of ammonium-phosphate in contact for days together with the crystalline crust that had formed above it with little or no corroding action. These crusts may be readily formed in the case of the magnesic sulphate solution (2 salt to $1\frac{1}{2}$ water) in clean tubes placed in strong sulphuric acid under an exhausted receiver. A crust will form on the surface of the cold solution in about twenty minutes, and this may be allowed to fall through the solution without acting as a nucleus. Or a solution of the magnesic sulphate (4 salt to 4 water) may be made in a flask, and filtered into a clean flask; and while this is boiling, perfectly clean crystals (made so by washing in sulphuric acid and rinsing in water), contained in a clean, short tube, are to be suspended by a clean wire in the neck of the flask while steam is pouring off from it. The cotton-wool plug may now be inserted, the lamp removed, and the flask left to cool during twelve hours or so. The tube may then be lowered into the solution, and the crystals will not act as nuclei in inducing crystallization.

In a similar experiment with sodic sulphate the salt in the short tube became, of course, converted into the anhydrous salt; but it was quite inactive as a nucleus on the cold solution. Exposed to the air for a few moments it acted powerfully as a nucleus.]

Such cases as the above lead me to the conclusion that the state of supersaturation is simply a case of no nucleus. There is nothing to start the action which produces change of state, and there is no reason why the state should change, unless indeed, as may sometimes happen, mere reduction of temperature is the efficient agent in inducing crystallization by bringing the particles so close together that cohesion can prevail over adhesion. But in such cases it more often happens that the solution completely solidi-

* Those paragraphs within [ ] formed the subject of a Note sent in to the Royal Society, August 13, 1868.

fies into the normal salt than partially into a salt of a modified character. The formation of the modified salt, which has given rise to much speculation, will next be considered.

IV. *On the Formation of the modified Salt.*

Two ounces of zincic sulphate and one ounce of water were heated in a clean flask until the solution boiled briskly. The solution was then filtered into chemically clean test-tubes, made so by washing them out with strong sulphuric acid. The tubes were plugged with cotton-wool and set aside to cool; they were bright and transparent; but on cooling to about 70° they became troubled by the formation of a monohydrated salt, which gradually subsided and formed a small deposit at the bottom of the tubes. Upon this deposit, after some hours, a fine crop of acicular crystals had sprung up, leaving the mother-liquor clear, but still supersaturated; for on taking out the cotton-wool from one tube, crystallization set in from the surface. On gradually raising the other tube to the temperature of 100°, or from that to 106°, the acicular crystals melted down and formed a dense lower stratum.

This experiment throws much light on the behaviour of those supersaturated saline solutions that form salts of a modified character, and of greater solubility than the normal salt. When the richly hydrated salts, which best display the phenomena of supersaturation, enter into solution at high temperatures, they give up all, or nearly all, their water of crystallization to the aqueous solvent. In the case of the sodic sulphate, for example, I quite agree with the accepted theory, that from the boiling-point of an aqueous solution down to about 60° it is the anhydrous salt that is held in solution; but I differ from that theory which supposes a molecular change to take place in the supersaturated solution at and below 60°, and the molecules to arrange themselves into the constitution of the 7-atom hydrate. There is no experimental proof that such is the case. We can get the 7-atom hydrate, it is true, when a strong solution is cooled down below 60°; but this is no proof that the solution passed through the molecular change which has been insisted on.

I have already given a case in which a strong solution of the sodic sulphate, contained in a tube made chemically clean with special precautions, was cooled down to 10° F. without any separation of the salt, in consequence of the absence of a nucleus, and the strong adhesion of the solution as a whole to the sides of the tube. If a strong solution of sodic sulphate be filtered into a tube washed without any special precautions to make it chemically clean, and the mouth be closed with cotton-wool, such a solution may be cooled down to 50° or less without depositing any salt. If we now place this tube in ice-cold water, and watch it carefully, a small number of octahedral crystals (the form of the anhydrous salt) will separate from the solution and subside, liberating heat-currents as they fall and when resting at the bottom. This experiment has been performed with supersaturated solutions of sodic sulphate of varying strengths and with the same result, unless crystallization set in from the surface by the dragging in of a nucleus con-

sequent on the cooling, or on the separation of a fragment of cotton-wool, and then the whole solution immediately became solidified into the 10-atom salt. The deposit of a few anhydrous crystals, as stated above, is, I think, a sufficient proof that the solution undergoes no molecular change at 60° or under, but continues to be a solution of the anhydrous salt until, by a proper reduction of temperature, it deposits the excess that it can no longer hold.

That which takes place in the case of the zincic sulphate solution applies equally here. The anhydrous sodic sulphate thus thrown down at 32° liberates heat, and the crystals partially enter again into solution. Or should the atmospheric temperature rise a few degrees, the operation is further assisted; the anhydrous crystals become wholly or partially dissolved, and a dense lower stratum is thus formed, containing not water enough for the formation of the 10-atom hydrate, but only sufficient for the 7-atom; and thus we have a comparatively small group of crystals of the 7-atom hydrate formed at the bottom of the solution.

In this way, as it appears to me, the formation of the 7-atom hydrate may be accounted for and rendered intelligible without having recourse to the doctrine of molecularity of which we really know so little. The mother-liquor resting on this 7-atom hydrate is still a supersaturated solution; so that if the flask be opened crystallization proceeds rapidly from the surface downwards, and carries down sufficient water to form the 10-atom salt throughout, and also to convert the 7-atom hydrate into the 10-atom; and in appropriating the three additional equivalents of water, the 7-atom hydrate becomes of an opaque white, from the breaking up of the mass into a multitude of minute 10-atom crystals. It is important to bear in mind the difference of locality in the formation of the two hydrates; the 7-atom hydrate originates at the bottom of the solution under circumstances which do not allow of sufficient water being present to form a salt of a higher degree of hydration than the 7-atom; whereas the 10-atom salt originates at the surface, and carries down in close well-packed crystalline lines sufficient water to form the 10-atom salt. These lines form so quickly, and are packed so closely together, that there is neither time nor space for the formation of the usual crystal; they pass to the very bottom of the vessel, and seem to penetrate while they render opaque the transparent crystals formed there.

M. Löwel states that supersaturated solutions of the sodic carbonate form two modified hydrates, viz. the 7HO *a* and the 7HO *b*, which, he says, not only differ in solubility from each other, but also from that of the 10-atom salt. He admits that the sodic carbonate differs in behaviour from the sodic sulphate, since the carbonate does not deposit crystals from the supersaturated solution merely by a reduction of temperature; "but such solutions remain saturated until, in consequence of lapse of time, and above all of low temperature, the inner surfaces of the flask have recovered their peculiar catalytic properties of determining crystallization which heat had deprived them of, the cause of which remains unknown." In chemically clean vessels I have not been able to reproduce M. Löwel's results. I have tried numerous experiments with solutions containing

5, 4, 3, and 2 parts salt to 1 of water: the solutions have been left to themselves, or they have been placed in freezing-mixtures at various temperatures; but the general result is a sudden crystallization from some point near the surface of the solution, rapidly converting the whole into a solid, with a rise of temperature of 35° or 40°.

M. Löwel also points out three varieties of the magnesic sulphate, the ordinary 7-atom salt, which he names 7HO *a*, a modified salt, 7HO *b*, and also a 6HO salt.

In the case of this salt also I have placed highly charged supersaturated solutions (2 salt to 1 water), contained in chemically clean tubes, in freezing-mixtures without any effect in producing crystallization. In one experiment a solution made with two ounces of salt and one of water was boiled and filtered into five tubes, four of which were chemically clean, while the fifth had been used for a solution of this salt and left lying on the table for two or three days. All five tubes on being filled were plugged with cotton-wool, and set aside to cool. When cold the four clean tubes were put into a freezing-mixture at 10°, and when this had slowly risen to between 30° and 40°, another freezing-mixture was prepared, to which the tubes were transferred; but still after another half hour the solutions remained perfectly bright and transparent. One tube was removed from the mixture and opened, when on touching the surface with a metal point, crystalline tufts, like thistle down, were propagated downwards, and soon the whole solution had solidified into the usual 7-atom salt. The fifth tube, which was not chemically clean, did not retain its solution in the liquid state: on examining it some hours after it had been filled, the contents had solidified into the ordinary salt. The other three tubes, on being removed from the freezing-mixture, showed no disposition to crystallize.

The magnesic sulphate parts readily with an atom of water and becomes milky white. A group of these milky crystals is sometimes deposited at the bottom of a long tube containing a strong and still warm solution of the salt, and partly immersed in cold water. A certain difference in density between the lower and upper portions of supersaturated solutions is favourable to this result. Indeed there is a tendency to stratification during the cooling of all these solutions, as may be seen on gently shaking the flask, when the effect is that of the mingling of layers of different densities, as when water is resting on syrup. I long thought that when a salt of a lower degree of hydration is produced, it was due to the formation of denser and more highly charged saline substrata. Gay-Lussac's experiment* teaches that saturated solutions contained in long tubes, and maintained at the same temperature during many months, are of the same density at every part of the column; and it seems probable that such is also the case with supersaturated solutions when cold and left undisturbed some time; but there are various difficulties in the way of determining this point experimentally. When the denser lower stratum is formed at all, it is, I believe, by the process already explained in the case of zincic sulphate and sodic sulphate. A salt of a lower degree of hydration, or an anhydrous salt, is thrown down on a reduction of temperature of the supersaturated solution, and the heat given out during a change of state, assisted by a rise of atmospheric

* Annales de Chimie et de Physique, vol. vii. p. 79.

temperature, enables the salt to go into solution, and thus a dense lower layer is formed from which the modified salt is produced.

The ammonium phosphate is another example of this action. A solution saturated at 212° was filtered into a clean flask wetted with liquor ammoniæ, and left some weeks as a supersaturated solution. It was then placed in a freezing-mixture, when it threw down a white powder, probably the anhydrous salt: the flask was now set aside, when after a few days the powder had partially disappeared, and a crop of transparent crystals had formed. But this result is not so readily produced if special means be taken to secure clean vessels. Five tubes were filled with sulphuric acid, rinsed with water, filled up with liquor ammoniæ, emptied, and a boiling saturated solution of the ammonium phosphate filtered into them. When cold, these tubes were put into a freezing-mixture composed of sodic sulphate and hydrochloric acid, and after half an hour transferred to a similar mixture, without producing any immediate effect. On examining the tubes next day a small quantity of white powder was found at the bottom of three of the tubes, with transparent crystals in the form of flat quadrangular prisms.

Strontic nitrate would appear to be favourable to the exhibition of this class of phenomena, since a strong solution on cooling down to about 62° deposits anhydrous octahedral crystals. But these are bright, hard, and transparent, and apparently not affected when the tube is put into water at 212°, or into a freezing-mixture at 10°, or when the crystals are touched with a wire. On removing the cotton-wool and scratching the side of the tube, the scratches became immediately filled up with minute crystals, and a copious shower of the hydrated salt took place with the liberation of heat-currents. After a time these formed a considerable mass, and on putting the tube into water at 70° the salt melted down, but more rapidly at 90°; at 200° the remaining part of the mass fell down into an opaque white powder, from which the original octahedra emerged as sharp and transparent as when first formed.

Many salts behave like the sodic carbonate in maintaining a state of supersaturation down to a certain temperature, and then suddenly becoming solid throughout. The rhombic sodic phosphate does this, as also the plumbic acetate. A boiling solution of plumbic acetate ($2\frac{1}{2}$ salt to 1 water) was left to cool slowly down to about 40° (the temperature of the room), when it suddenly crystallized with a considerable rise in temperature. The whole formed a solid mass, not amorphous, but consisting of needles of the normal salt, as is nearly always the case when crystallization commences at the surface. On gently warming the mass, these needles fell apart, and revealed the fibrous structure. This is a common case in salts that have considerable solubility. A boiling saturated solution of zincic acetate, on the contrary, not only cooled down to 40° without crystallizing, but continued liquid during some hours in a mixture of snow and water at 32°. On removing the thermometer crystallization did not set in until the inner side of the flask at the level of the solution had been scratched, when the marks became chalky white as usual, crystallization of the normal salt set in, and small well-shaped crystals continued to fall during some hours with but slight rise in temperature. The excess of salt beyond the point of saturation was

deposited, and the mother-liquor was simply a solution saturated at the existing temperature. The cupric sulphate, citric acid, and other salts are examples of this action. In some cases supersaturated solutions, if not highly concentrated, may be frozen and thawed again, without any separation of the salt; whereas if such solutions are much stronger, they cannot be frozen at temperatures varying from 10° to zero. The cupric sulphate is a good example of this. According to Poggiale, 100 parts water at 86° dissolve 48·81 parts of the salt; this is equivalent to 234¼ grains of salt to one ounce of water. A solution of this kind was put into a freezing-mixture and sank slowly to about 16°, when it froze, and the thermometer rose to about 25°. The solidified solution now appeared almost white, but on leaving the flask exposed to an atmosphere of 52° during some hours, the solution thawed, and recovered its former brightness. A much stronger solution was now made (468 grains salt to 1½ ounce water) and filtered into a tube that had been washed out with sulphuric acid; this was repeatedly kept in a freezing-mixture at 10° without any deposit of salt.

V. *Anhydrous Salts.*

The salts hitherto treated of in this memoir as presenting the phenomena of supersaturation are hydrated, and in general the phenomena are best exhibited in those salts which are richest in water of crystallization. Whether anhydrous salts display these phenomena in their solutions is a point by no means settled; for while some observers maintain that the solutions of a few anhydrous salts do become supersaturated, others deny that such is the case. In perfectly clean flasks, and with clean solutions, I have never been able to detect a case of supersaturation in an anhydrous salt. A few examples may be given. Two ounces of nitre crystals were added to two ounces of water in a clean flask, say 100 parts of nitre in 100 of water, and the solution was raised to the boiling-point, filtered into a chemically clean flask, made so by washing it out with strong nitric acid. In this flask the solution was boiled again; the neck was then plugged with cotton-wool with a thermometer passing through the plug. When the temperature of the solution had fallen to about 120°, crystals began to grow up from the bottom of the flask. These crystals were built up of short prisms, diminishing in width and becoming smaller as they rose into the hotter and weaker part of the solution above.

Now, according to Gay-Lussac's Table, 100 parts of water at 131° F. contain 97·05 of nitre. My solution contained 100 parts. Supposing that, owing to the absence of a nucleus, the solution were supersaturated between the temperatures 131° and 118°, it could not remain so in consequence of the cohesive force of the saline particles prevailing over the adhesive force of the solvent; it begins to deposit crystals, and then the action goes on rapidly; the temperature rises a few degrees, then declines, and when cooled down to the temperature of the air, say 50°, the solution holds about 22 per cent. of the salt.

But is there any reason for supposing the solution to be supersaturated? When crystallization begins, it is at the coldest part of the flask that the action commences.

The coldest part is the bottom, and the coldest part of the bottom is that nearest the window. If the flask be suspended in the air crystallization will first begin on the window side. If the flask be placed on metal or other good conductor of heat, a ring of crystals will be formed at the bottom, while the thermometer is 6° or 8° above the point at which crystallization ought to take place; the bottom of the flask and the ring of solution in contact with it are prematurely chilled, while the rest of the solution is above the point of crystallization. If the flask be left on a thick block of wood or other bad heat-conductor, the crystals at the bottom will be formed later. A solution containing 125 parts of nitre in 100 of water ought, according to the Table, to begin to deposit salt at 150°. The flask containing such a solution at boiling heat was placed on a block of wood; at about 149° heat-currents suddenly rose like a cloud from the bottom of the solution to the surface, and crystallization instantly set in. If, before crystallization begins, the flask be plunged into cold water, the sides become coated with a shell of crystals, while the rest of the solution is still hot.

It might reasonably be expected, from its great solubility, that nitre would form supersaturated solutions; but it behaves as, I believe, anhydrous salts in general behave; its solution cools down until it is exactly saturated, and then it begins to deposit crystals of the normal salt without any modification whatever.

The potassic bichromate is cited, on the authority of Ogden and others, as an example of an anhydrous salt that produces supersaturated solutions. My experiments do not confirm this view. According to Kremer's Table, 200 parts water at 140° F. dissolve 100 parts of the salt. Such a solution in cooling down from 212° ought to deposit salt at about 140°, unless it becomes supersaturated. One ounce of newly crystallized dry salt was boiled in two ounces of water, filtered into a clean flask and again boiled, and the flask was closed. At 138° an abundance of crystalline flakes was thrown down, heat-currents ascended, and the temperature continued stationary for a minute or so. The crystals formed a lining over about two-thirds of that part of the flask that was occupied by the solution, but the deposit was most abundant on the side nearest the window*.

Sal-ammoniac.—According to Karsten, 100 parts water at 65°·75 dissolve 37·02 of the salt, or, what is the same thing, one ounce water (480 grains) dissolves 177·6 grains of the salt. Such a solution in cooling from 212° began to deposit salt at about 66°, and just below this the salt separated abundantly.

Sodic nitrate was also tried. According to Poggiale's Table, 100 parts of water at 158° will dissolve 129·6 of the salt, or $\frac{129\cdot6}{100}$ of $\frac{480}{1}=622$ grains. In such a solution raised to the boiling-point and allowed to cool down, the salt began to separate at 160°.

* Saturated solutions of the potassic bichromate undergo a remarkable deepening in colour under the action of heat; they may even become black and opaque if boiled a second or a third time in the same flask; but they recover their usual colour on cooling. The salt deposited in an open evaporating-dish from a hot saturated solution is almost white. If this light-coloured portion be placed on filtering-paper and allowed to dry in the open air, it does not deepen in colour; but if put in a platinum crucible and gently heated over a spirit-lamp, it becomes very dark in colour, and in cooling assumes and retains the usual colour of the salt. The phenomena in question resemble those pointed out by Sir David Brewster in the case of nitric peroxide.

Potassic chlorate, potassic ferrocyanide, baric nitrate, and plumbic nitrate gave similar results.

Ammonium-nitrate solution gave apparently some signs of supersaturation; but the observation is liable to error from the fact that at high temperatures ammonia is given off. According to KARSTEN, 100 parts of water at 64°·5 F. dissolve 199·54 parts of the salt. A hot solution (2 salt to 1 water) was filtered into a clean flask moistened with a few drops of *liq. ammon.* Such a solution remained during an hour or so at 40°, when it suddenly crystallized. I may, however, remark that a number of authorities differ in their determination of the solubility of this salt. But if the observation be correct, the behaviour of this anhydrous salt resembles that of some salts that crystallize with only a small proportion of water of crystallization. Baric chloride ($BaCl+2Aq$) is a case in point. According to BRANDE's Table, 100 parts water at 99°·5 F. dissolve 51 parts of the salt. This is equal to 244 grains of salt to one ounce of water. A solution of this kind cooled from the boiling-point to 100° without depositing salt, and it was not until the temperature had fallen to about 66° that the salt separated, and then it did so abundantly.

VI. *Summary.*

My experiments lead me to the conclusions, first, that the state of supersaturation in saline solutions is a real and not an apparent one, and is due in the majority of cases, where the solutions are kept in chemically clean flasks, to the absence of nuclei or other predisposing cause to crystallization; 2nd, that in the few cases where salts of a lower degree of hydration are formed in such solutions, the effect is not due to a predisposing molecular change in the solution, but to the deposit of a portion of the anhydrous salt, or of a monohydrated salt, which, entering again into solution, forms a dense lower substratum from which the modified salt crystallizes out; 3rd, that anhydrous salts do not form supersaturated solutions.

The salts examined in this memoir may be arranged into five groups according to their behaviour:—

I. Salts of which the supersaturated solutions remain liquid at low temperatures.

Examples:—Sodic acetate.
Sodic sulphate.
Sodic arseniate.
Sodic succinate.
Sodic borate.
Sodio-potassic tartrate.
Potash alum.
Magnesic sulphate.
Baric acetate.
Calcic chloride.
Cupric sulphate.

II. Salts of which the supersaturated solutions suddenly solidify at low temperatures.

Examples:—Sodic carbonate.
Sodic phosphate.
Plumbic acetate.
Sodic hyposulphite.
Strontic chloride.

III. Salts of which the supersaturated solutions deposit their excess of salt at low temperatures, or under the action of a nucleus, leaving the mother-liquor saturated.

Examples:—Zincic acetate.
Cupric sulphate.
Baric chloride.
Potassic arseniate.
Antimonio-potassic tartrate.
Citric acid.

IV. Salts of which the supersaturated solutions form modified salts of a lower degree of hydration.

Examples:—Sodic sulphate.
Zincic sulphate.
Magnesic sulphate.
Ammonium phosphate.

It will be seen that the sodic sulphate and the magnesic sulphate also occupy a place in Class I.

V. Anhydrous salts that do not form supersaturated solutions:—

Potassic nitrate.
Potassic bichromate.
Sal-ammoniac.
Sodic nitrate.
Potassic chlorate.
Potassic ferrocyanide.
Baric nitrate.
Plumbic nitrate.
Ammonium nitrate.

XXVIII. *The Caudal Heart of the Eel a Lymphatic Heart.—Effect of the force with which the lymph-stream is propelled therefrom on the flow of the blood in the Vein into which the heart opens.—Explanation of the appearance of blood propelled in successive drops, as if from the heart, along the Caudal Vein.—Influence which the force of the lymph-stream from the heart exerts in accelerating and promoting the flow of blood in the Caudal Vein.* By THOMAS WHARTON JONES, *F.R.S., Professor of Ophthalmic Medicine and Surgery in University College, London, Ophthalmic Surgeon to the University College Hospital, &c.*

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Introduction.

THE remarkable pulsating organ in the tail of the eel, which forms the subject of this paper, was discovered by the late Dr. MARSHALL HALL. He viewed it as belonging to the blood-vascular system, and named it the "Caudal Heart."

His description of it was founded on observations made on small eels under the microscope. In large eels the heart may, as he also pointed out, be seen with the naked eye by spreading the tail on a plate of glass and viewing it against the light. Not only, however, are the pulsations of the organ itself thus visible, but also the very peculiar appearance of successive drops of blood propelled, as if from the heart, with great velocity along the caudal vein, which was observed by Dr. MARSHALL HALL in his microscopical examinations, though incorrectly interpreted by him.

To explain the true nature of the phenomenon here referred to,—to prove thereby that the caudal heart belongs, not to the blood-vascular system, but to the lymphatic system,—and to inquire into the influence which the force of the lymph-stream from the heart exerts in accelerating and promoting the flow of blood in the caudal vein, constitute the object of the communication here presented to the Royal Society.

My observations, like those of Dr. MARSHALL HALL, were made on small eels under the microscope.

A third of an inch Objective, by the Messrs. MERZ of Munich, with the lowest eyepiece was the microscopical power employed.

The small eel used for the purpose (one of the size of those commonly kept in aquaria) was secured by wrapping it up in a bit of wet rag, leaving only about one inch of the tail hanging out.

The glass slide on which the animal was placed was about double the usual length.

Being laid on the slide with the tail in proper position, the animal was fixed by tying a string loosely round the slide and its body where enveloped in the rag.

The part of the tail to be examined was covered with water and a thin plate of glass.

Matters being thus arranged and the microscope adjusted, rapid pulsations are immediately detected, on directing the eye to the region of the extreme end of the vertebral column—abdominal side*.

No very well-defined heart-like organ is observable at first sight, but the pulsations seem as if in the tissue of the part generally. The skin and substance of the tail surrounding the heart are drawn somewhat together at each systole, and fall back into their previous state at each diastole. From this it may be inferred that the heart does not lie free in a special cavity, nor even in loose cellular tissue, but that it is imbedded in the substance of the tail.

Dissection of the tail of a large eel shows that the heart is connected with the adjacent bones†.

The great vein of the tail‡ is formed by the junction of two trunks, a larger and a smaller§. The larger trunk receives the venous radicles returning the blood from the terminal and abdominal parts of the caudal fin. The smaller trunk receives the venous radicles returning the blood from the dorsal part.

It is into this smaller trunk, near its junction with the larger, that the caudal heart opens.

The great artery of the tail‖ runs close alongside the vein, and is smaller than it, and somewhat tortuous. It gives off branches to the abdominal and dorsal parts of the caudal fin, whilst the continuation of it subdivides into the small ramifications distributed to the terminal part.

This continuation of the artery and the larger trunk of the great caudal vein diverge from each other in their course towards the end of the tail, and then, after having become small by the giving off of branches, approach curvingly and cross each other at the place where the spine ends and the terminal part of the caudal fin commences.

The elongated space bounded by the artery and vein thus diverging from and again approaching each other, is that in which the lymphatic heart is situated¶.

Description and explanation of the phenomena attending the propulsion of the lymph from the caudal heart into the caudal vein.

At the opening of the caudal heart into the vein there is a valve which prevents regurgitation of the lymph back from the vein into the heart; but, owing to the thick-

* Plate XXXV. figs. 1 & 2, E.

† This point in the anatomy of the caudal heart is described in detail in Part III. of my paper entitled, "Microscopical characters of the rhythmically contractile muscular coat of the veins of the Bat's wing, of the lymphatic hearts of the Frog, and of the caudal heart of the Eel."

‡ Plate XXXV. figs. 1 & 2, B.

§ Plate XXXV. figs. 1 & 2, C, D.

‖ Plate XXXV. fig. 1, A.

¶ Plate XXXV. fig. 1, E.

ness of the substance of the tail, a well-defined view of the valve and its workings, such as we have of the valves of the veins of the thin web of the bat's wing and their workings, cannot be obtained.

When, by the contraction of the heart, the lymph is propelled into the vein*, the flow of blood from that vessel into the great caudal trunk is interrupted by the force of the lymph-stream. From the place where the heart opens into the vein to the junction of the latter with the caudal trunk, colourless lymph thus replaces red blood; whilst in the caudal trunk itself, the lymph, still under the influence of the heart's force, so far displaces the blood as to flow in a colourless stream on one side of the vessel (the side corresponding to that on which the heart opens) for some distance, distinct from and unmingled with the blood-stream from the lower part of the vein and from its lateral branches†.

During the diastole of the heart, the stream of lymph into the vein intermitting, the flow of blood from that vessel into the great trunk of the caudal vein again takes place. No sooner, however, has a small quantity of blood entered than systole of the heart ensuing, the stream of lymph thereby propelled into the vein drives the small quantity of blood before it into the great caudal venous trunk‡, whilst it at the same time arrests, as before, the flow of blood into the great caudal vein from its tributary vessel.

In consequence of the arrestment of the flow of red blood in the vein under notice, though for so short a time, the red corpuscles appear to become, under the influence of contact with the lymph, aggregated together at the place §, so that when, by the diastole of the heart, the flow of blood is again permitted, it is a small mass of aggregated red corpuscles which enters the great caudal vein, and is, during the ensuing systole of the heart, driven before the stream of lymph issuing therefrom.

In the observation now described, we have thus presented to our eye the remarkable phenomenon of small drops of red blood or agglomerations of red corpuscles propelled in rapid succession in the colourless stream of lymph above described, as seen within and to one side of the great caudal venous trunk ‖.

Such is the true nature of the phenomenon which has hitherto been erroneously supposed to be owing to the caudal heart transmitting, at each systole, a drop of blood into the caudal vein.

The intermittent propulsion of drops of red blood into and along the caudal vein gives the appearance, at first sight, as if the vein pulsated. But neither that vessel nor the adjacent arterial trunk does so.

The undulatory appearance of the stream in the caudal vein is seen to be continued to a considerable distance upwards. The blood and lymph-streams at last mingle, and the result is a blood paler and brighter even than that in the adjacent artery.

* Plate XXXV. fig. 1, D; fig. 2, D–F. † Plate XXXV. fig. 1. ‡ Plate XXXV. fig. 2, G.

§ In a manner analogous to that in which the red corpuscles of a drop of blood, newly drawn from the body become aggregated in cases in which the plasma contains an excess of fibrin.

‖ Plate XXXV. fig. 2, G, G, G.

When the circulation in the tail is active and the blood flows with corresponding force in the great caudal venous trunk and its tributary vein, the blood-stream does not yield so readily, as above described, to displacement by the lymph-stream from the heart, but the propulsion of the drops of red blood presents a remittent rather than an intermittent character.

Influence of the force with which the lymph-stream is propelled from the caudal heart in accelerating and promoting the flow of blood in the caudal vein.

The force with which the lymph is propelled by the heart into the caudal vein may be judged of comparatively by contrasting the force of the stream, as indicated by rapidity of flow, in the tributaries of the vein, with that of the stream in the vein itself whilst the lymph is being propelled into it. Within the vein, the force of the lymph-stream is evidenced by the fact above mentioned, that at the moment of the propulsion of the lymph into the vein by the contraction of the heart, the onward course of the blood in the tributary vein is arrested and the column of blood in the caudal trunk displaced to one side; and also by the manner in which the drops of red blood or agglomerations of red corpuscles, which enter the caudal trunk from the tributary vein during the diastole, are seen driven on in the lymph-stream at each systole of the heart.

Through the medium of the stream of lymph propelled into the great caudal vein at each stroke of the heart, an impetus is communicated to the column of blood in that vessel, which we can observe has the effect of accelerating and promoting its onward flow to the blood-heart of the animal.

We thus see that though the caudal heart of the eel is a lymphatic heart, its function being to receive lymph on the one hand and to propel it into the great vein of the tail on the other, it at the same time performs the secondary function of accelerating and promoting the flow of the blood in that vessel in its course back to the blood-heart.

Teleologically speaking the great length of the tail of the eel seems to render a reason for such a provision.

In respect to the secondary function of the caudal heart of the eel here mentioned, an analogy may thus be traced between it and the rhythmically contractile veins of the much elongated wing of the bat.

How does the lymph enter the caudal heart?

This question does not properly come within the scope of the present paper. It may, however, be remarked that I have observed under the microscope no defined lymphatic vessels opening into the heart, and only an indistinct appearance of afferent streams of lymph.

If there had been any lymphatic canals opening into the heart, such as those the late Professor JOHANNES MÜLLER (in his paper entitled "Beobachtungen zur Analyse der Lymphe des Blutes und des Chylus" in POGGENDORFF's Annalen der Physik und Chemie for 1832, page 519) states that he injected through a slit made into the organ, and if a

stream of any kind passed through them into the heart, they could not fail being seen under the microscope.

The arrangement of the canals described by MÜLLER is identical with that of the blood-vessels of the caudal fin; and I cannot help suspecting that it was those very vessels which that distinguished physiologist had injected. Indeed he seems to have subsequently tacitly admitted as much when, in his 'Physiology' (BALY's Translation, 2nd edition, vol. i. p. 245, 1839), he says, without any expression of dissent or comment:—"Dr. MARSHALL HALL has discovered in the eel a kind of auxiliary venous heart situated at each side of the last caudal vertebra, which pumps the blood out of the small veins of the extremity of the caudal fin into the vena caudalis."

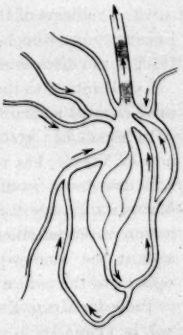
It may be proper to remark here that MÜLLER's injections of the alleged lymphatic canals appear to me, from his description (POGGENDORFF's Annalen, *loc. cit.*), to have been made, not from the caudal heart, nor even from the great caudal vein, but from the two tributary venous trunks which unite to form that vessel.

Historical retrospect regarding the discovery of the caudal heart of the eel, and the opinions entertained of its nature.

Dr. MARSHALL HALL made known his discovery of the caudal heart in the eel in his 'Critical and Experimental Essay on the Circulation of the Blood,' published in 1831.

He viewed it as an auxiliary heart belonging to the blood-vascular system, as previously stated, and erroneously supposed that the drops of red blood seen passing in rapid succession along the caudal vein were propelled from it.

His description of the phenomenon is in the following words:—"By a sudden contraction of this (the ventricle of the heart) it (*i. e.* the blood which had entered it in the manner he erroneously supposed) is gathered into a drop and propelled with great velocity, and at first with the peculiar appearance of successive drops, along a vessel which ascends along the inferior spinal canal, and which must, although it pursues a direction towards the heart, be considered an artery"*.



The anatomical relations between the caudal heart and blood-vessels, and between the blood-vessels themselves, as described and delineated by Dr. MARSHALL HALL, are altogether incorrect.

In his figure of the caudal heart, an outline copy of which is here annexed, vessels are represented as arising from the heart and returning, some to it and some to the caudal vein, after having formed a loop, an anatomical arrangement very different from the reality, as may be seen by comparing Plate XXXV. fig. 1 which accompanies this paper.

It need scarcely be remarked that an organ with such a structure as that described by

* *Op. cit.* p. 172.

Dr. MARSHALL HALL would have been inadequate for the performance of the function he attributed to the caudal heart of the eel.

Though the late Professor JOHANNES MÜLLER, in his paper in POGGENDORFF'S 'Annalen' for 1832 above quoted, remarks that "further observations must teach whether the function of this organ (the caudal heart in the eel, the discovery of which by Dr. MARSHALL HALL he had just learned from the 698th Number of FRORIEP'S 'Notizen') is to drive the lymph of the tail into the end of the caudal vein," he did not, in his paper on the lymphatic hearts of Amphibia, question Dr. MARSHALL HALL'S representation of it as a blood-heart.

In the paper referred to, which was published in the Philosophical Transactions for 1833, and is entitled, "On the Existence of four distinct Hearts, having regular pulsations, connected with the Lymphatic System in certain Amphibious Animals," Professor MÜLLER, then of Bonn, makes the following statement:—"In the vascular system of the blood, there certainly are particular places, besides the heart, which are capable of contractions; as, for example, the bulbus aortæ in fishes and batrachia, the venæ cavæ where they enter the atrium, and the pulsating organ discovered by Dr. MARSHALL HALL in the eel at the end of the vena caudalis, where that organ receives the branches of the extremity of the tail and conducts its blood into the vena caudalis. But organs of pulsation in the lymphatic system have hitherto been altogether unknown."

Again, Professor MÜLLER, in his 'Archiv für Anatomie und Physiologie' for 1842, p. 477, published a paper entitled "Bemerkungen über eigenthümliche Herzen des Arterien- und Venen-Systems," in which he says, "To the heart-like contractile parts of the blood-vascular system the following structures belong:—

"A. To the venous system:

"1. * * * * *

"2. The heart of the caudal vein of the eel. The pulsation at this place was seen by LEEUWENHOEK, who, however, did not examine into the nature of the thing. MARSHALL HALL is the discoverer of this heart."

I shall return to the claim set up for LEEUWENHOEK, as it has been reproduced in a more decided manner by Professor MILNE-EDWARDS of Paris.

In MÜLLER'S 'Archiv' for 1843, p. 224, Professor HYRTL, at that time of Prague, but now of Vienna, has a paper entitled "Ueber die Caudal- und Kopf-Sinuse der Fische, und das damit zusammenhängende Seitengefäss-System," in which he refers to Dr. MARSHALL HALL'S discovery of the caudal heart of the eel, and describes his own examination of the pulsations of the organ with the naked eye, in the tail of a large eel held against the window-pane, but says nothing in opposition to Dr. MARSHALL HALL'S opinion of the nature of the organ.

Professor MILNE-EDWARDS, in his "Leçons sur la Physiologie et l'Anatomie comparée," vol. iv. (Paris 1859) p. 476, correctly views the pulsating organ in the eel's tail as a lymphatic heart, but adduces no facts, either anatomical or physiological, to prove that the organ is of this nature. He merely refers to Professor MÜLLER as having recognized the organ to be a lymphatic heart.

That Professor MILNE-EDWARDS labours under a mistake in thus attributing to Professor MÜLLER the recognition of the true nature of the caudal heart of the eel is evident from the following facts:—

Professor MÜLLER, as we have seen, indeed remarks, in his paper in POGGENDORFF'S 'Annalen' above referred to, that "further observations must teach whether the function of this organ is to propel the lymph of the tail into the end of the caudal vein." But a reference to the quotations, also above given, from Professor MÜLLER'S paper in the Philosophical Transactions for 1833, and his paper in the 'Archiv' for 1842, p. 477, as well as from his 'Elements of Physiology,' by BALY, 2nd edition, 1839, pp. 216 and 245, shows that Professor MÜLLER did not in these more recent writings dissent from Dr. MARSHALL HALL in the opinion he entertained that the "caudal heart" in the eel belongs to the blood-vascular system.

So far as I have been able to ascertain, no one has hitherto given a correct explanation of the phenomenon of small drops of red blood propelled in rapid succession, *as if* from the caudal heart, along the caudal vein.

Without first showing that these drops of blood or agglomerations of red corpuscles are *not* propelled from the heart, and without showing that it is colourless lymph alone which is *really* propelled from the heart, no one could have been warranted in dissenting from Dr. MARSHALL HALL'S view as to the nature of the caudal heart of the eel, or in pronouncing, how correctly soever, that organ to be a lymphatic heart.

I now come, in conclusion, to the alleged anticipation of Dr. MARSHALL HALL'S discovery of the "caudal heart" by LEEUWENHOEK.

Professor MILNE-EDWARDS, adopting Professor MÜLLER'S reading of LEEUWENHOEK, as above quoted, more decidedly than Professor MÜLLER himself, says:—"Cet organe a été vaguement aperçu, il y a près de deux siècles, par LEEUWENHOEK, mais n'avait que peu attiré l'attention des physiologistes, lorsque, en 1831*, MARSHALL HALL publia à ce sujet des observations qu'il croyait être complétement nouvelles."

In regard to this claim for LEEUWENHOEK, I have not been able to satisfy myself that there is any foundation for it.

LEEUWENHOEK'S words in his 66th Epistle, which was addressed to "Nobilissimi Viri," the Fellows of the Royal Society of London, 'Opera Omnia,' tom. ii. p. 174, the reference given by Professor MILNE-EDWARDS, are:—

"Vidi quoque in omnibus arteriis, licet minutissimis, assidue, ac celeriter novam fieri accelerationem protrusionis in sanguine, ubi vero attenderem ad magnam arteriam in cauda jacentem, ibi protrusio sanguinis, a corde facta, multo erat vehementior."

Again, at page 175, LEEUWENHOEK says:—

"Imo etiam fere semper quam proxime ad pinnæ extrema in tenuissimis vasis sanguiferis, singulis momentis novæ protrusionis, quam sanguis a corde accipit accelerationem dignoscere poteram."

How far, with the knowledge of Dr. MARSHALL HALL'S discovery, the words "ubi vero

* 1836 is put down, but evidently by mistake.

attenderem ad magnam arteriam in cauda jacentem, ibi protrusio sanguinis, a corde facta, multo erat vehementior" might be admitted as having reference to the phenomenon of the transmission of the drops of red blood or agglomerations of red corpuscles in rapid succession along the caudal vein, opinions may perhaps differ, but certainly all will agree with me that no one could have been led by the words to suppose that LEEUWENHOEK was speaking of *a heart in the tail of the eel.*

Had it been the caudal heart to which LEEUWENHOEK refers, however, and not merely the branchial heart of the fish, he would, no doubt, have commented on the peculiarity of its position in the tail, and expressed as much admiration, at least, as he does at the general phenomena of the circulation in the fins of the eel, when he says, unwilling to reserve to himself so beautiful a sight, he invited several "Viri gravissimi" to come to see it—amongst others the celebrated CHRISTIAN HUYGHENS. These gentlemen, he tells us, "often exclaimed that they could not have believed anything more curious could have met their eye or been better demonstrated."

EXPLANATION OF THE PLATE.

PLATE XXXV.

Fig. 1. The caudal artery and its ramifications, the caudal vein and its two tributary trunks with their radicles, and the caudal heart, as seen in a small eel under the microscope.

No capillaries are represented.

A. The caudal artery, the ultimate ramifications of which are seen distributed in straight lines corresponding with the direction of the rays of the caudal fin.

B. The great caudal vein formed by the junction of the two tributary trunks C and D. The radicles of these trunks are seen parallel with the straight ultimate arterial ramifications.

E. The caudal heart communicating, by an opening provided with a valve, with the smaller tributary caudal venous trunk D, near where it joins the larger tributary caudal venous trunk C to form the great caudal vein B.

The smaller tributary venous trunk D, from the place where the caudal heart opens into it to its junction with the larger tributary trunk C, to form the great caudal vein B, is seen filled with the lymph-stream just propelled into it from the heart, whilst in the great caudal vein the lymph, still under the influence of the heart's force, has so far displaced the blood as to flow for some distance in a colourless stream on one side of the vessel.

Fig. 2. A diagrammatic view of the caudal heart, and the lymph-stream from it, with the drops of blood or agglomerations of red corpuscles which issue, one from

the smaller tributary caudal venous trunk (into which the heart opens) at each diastole, and are propelled along in the great caudal venous trunk at each systole of the heart*.

B. The caudal venous trunk.

C. The larger tributary caudal venous trunk.

D. The smaller tributary caudal venous trunk.

E. The caudal heart opening into the smaller tributary caudal venous trunk.

F F F. The lymph-stream from the heart, and its course along the great caudal vein.

G G G. The drops of blood or agglomerations of red corpuscles as propelled along in the lymph-stream.

* The phenomenon which it is here attempted to represent can properly be seen only in time and space.

XXIX. *On the Tides of Bombay and Kurrachee.* By WILLIAM PARKES, *M.Inst. C.E.* *Communicated by* G. B. AIRY, *Esq., Astronomer Royal, F.R.S.*

Received May 5,—Read May 28, 1868.

THE tides on the coasts of India present a marked difference from those on our own coasts in the large amount of diurnal inequality to which they are subject.

My attention was first directed to the subject in the course of an engineering survey of the Harbour of Kurrachee which I made in 1857–58, when I obtained between three and four months' continuous observations, a copy of which is deposited with the Royal Society. Subsequently I obtained from the Admiralty, through the kindness of Captain BURDWOOD, R.N., the loan of the records of three years' observations taken at Bombay in 1846, 1847, and 1848. Of these I plotted in a series of continuous curves the records for 1846, and deposited them, at the Astronomer Royal's request, at the Royal Observatory, Greenwich. These records, however, are not perfect. They were made by a self-acting machine, the adjustment of which does not appear to have been always accurately preserved; and I hope that they will be superseded as data for investigation by a better record for the year 1868. Taking them as they were, however, I discussed them to obtain the semimonthly curves of semidiurnal tide, and also formulæ for the approximate determination of diurnal tide.

In 1865 a careful series of observations was made at Kurrachee for the six months from March to August inclusive. They were made by a self-acting machine, the adjustment of which appears to have been fairly maintained, and the records are generally self-consistent. A register and diagram of High Water and Low Water formed from these observations is deposited with the Royal Society.

This series of observations was made the basis of a discussion for the curves of semidiurnal tide, and for a series of formulæ for diurnal tide.

In 1866, I received from the Secretary of State for India instructions to prepare tide-tables for Bombay and Kurrachee for the year 1867; and the tables made in accordance with these instructions have been compared with records of observations at Bombay and Kurrachee as under:

At Bombay, from January 28th to June 4th, by direct reading from a graduated staff at every ten minutes, under the superintendence of Mr. ORMISTON, C.E.

At Kurrachee, the machine made diagrams for the whole year.

The registers and diagrams of high and low water of these two records, and the continuous curves of the Bombay observations, are also deposited with the Society.

Plate XXXVI. is a specimen of the mode in which the times and heights of high and low water are exhibited in a graphic form.

The discussions of the records for semidiurnal tide were made in the ordinary manner, except that diurnal inequality was eliminated before grouping the heights and intervals. The curves of semimonthly inequality are appended (Plate XXXVII.), and the principal results are as follows:—

	Bombay.	Kurrachee.
	d h m	d h m
Time of highest tide of the springs after the transit nearest to noon or midnight	1 12 17	1 11 3
Mean of least lunitidal intervals . . .	1 11 32	1 10 10
Mean of greatest lunitidal intervals . .	1 13 6	1 11 42
Mean spring half-range	73 inches.	44 inches.
Mean neap half-range	33½ „	22 „

The effects of the variations of declination and parallax in semidiurnal tide have not been minutely investigated; but it was found that the corrections for moon's parallax adopted by the Admiralty, for tides of similar range on the English coast, were insufficient. The following were therefore adopted in the calculations of height of semidiurnal tide.

For all times of Moon's transit.

Parallax.	Correction.		Parallax.	Correction.	
	Bombay.	Kurrachee.		Bombay.	Kurrachee.
	inches.	inches.		inches.	inches.
54′	−10	−7	57½′	+ 2	+ 1
54½	− 8½	−5½	58	+ 3½	+ 2
55	− 7	−4	58½	+ 5½	+ 3½
55½	− 5	−3	59	+ 7½	+ 5
56	− 3½	−2	59½	+ 9	+ 6½
56½	− 2	−1	60	+11	+ 8
57	0	0	60½	+13	+ 9½
			61	+15	+11

I now proceed to describe those features which it is the special object of this communication to bring before the Royal Society, viz. those in which the tides on the coasts of India differ most remarkably from those on our own coasts.

It is well known that certain effects are produced by the attraction of the sun and moon upon the waters of the ocean, which ultimately result in a series of tide-waves having an average length or period of half a lunar day. The combination of the solar and lunar waves as the luminaries change their relative positions, produce the alternations of neap and spring tides. Without entering into any question of tidal theory, it may be assumed that the semidiurnal tide-wave is primarily due to the *tendency* of the waters to be drawn into two heaps, one on the side of the earth nearest to the attracting body, and the other on the side furthest from it. When the attracting body is in the plane of the equator, these two tendencies appear to be sensibly similar*, and the resulting

* A slight difference in the tides following the north and south transits has been detected by Mr. Bunt in the Bristol tides (Philosophical Transactions, 1867).

series of semidiurnal tide-waves present in their variations in height and time a nearly unbroken gradation.

When, however, either luminary is out of the plane of the equator, the tide formed on the nearer side is different from that formed on the further side of the earth; and the difference depends, not upon its origin being on the further or nearer side of the earth, but upon its being situated in the northern or southern hemisphere. Such of the variations in the tidal movements, therefore, as are due to the fact of their having taken their ultimate origin from the northern or southern hemisphere, recur every day instead of every half day; and hence we have a result which, if it could be separated from the semidiurnal features of the tide, would appear as a diurnal tide.

I have now to show how this supposition of an independent diurnal tide agrees with the observed tidal movements.

The diurnal tide being a combination of solar and lunar effects, is subject to the recurrence of neaps and springs like the semidiurnal tide, but beyond this it is subject to a recurrence of changes as the luminaries leave and reapproach the plane of the equator. The semidiurnal luni-solar tide is a combination of two tides of different periods, but each in itself of dimensions varying only slightly. The diurnal luni-solar tide, on the contrary, is a combination of two tides not only of different periods, but each in itself of ever-varying magnitude.

Again, it is well known that the time of semidiurnal tide varies to some extent during the half lunation, according as the solar portion of the tide precedes or follows the lunar portion; but as the lunar effect preponderates in the nearly constant ratio of about 3 to 1, a tide occurs once in every lunar half day. In the diurnal tide, however, the lunar influence does not always predominate. Twice in the course of a tropical month, when the moon is on the equator, her diurnal effect vanishes, and the period of diurnal tide then responds to the sun's influence alone. After the moon has crossed the equator, however, her influence rapidly increases, and the diurnal tide then again follows the lunar period. But the lunar effect, in thus recovering its preponderance, does so under changed conditions. The times of lunar diurnal high water and low water are now reversed. Thus there is an abrupt transition of twelve hours from the time of the vanishing tide to that of the reappearing one. The abruptness, however, is more or less counteracted by the solar diurnal tide, which maintains its regular periods. From these considerations it will be evident that, whether in respect of height or time, the diurnal features of the tide are of a far more complicated character than the semidiurnal features.

In considering the effect of the combination of the semidiurnal and diurnal tides, it will be obvious that if one diurnal *high water* takes place at the same time as a semidiurnal high water, the next diurnal *low water* will take place at the same time (or nearly so) as the next semidiurnal high water. Thus there will be a difference between the two successive semidiurnal high waters equal to the whole range of diurnal tide. Since, however, in the first semidiurnal tide the water ceases to rise at the same time as

the diurnal tide ceases to rise, and in the second it ceases to rise at the same time that diurnal tide ceases to fall, the times of actual high water (the combination of the two tides) will be the same as those of semidiurnal high water. Thus, though there is a diurnal inequality in *height* equal to the whole range of the diurnal wave, there is no inequality of *time*. If we now turn to the semidiurnal low waters, we find that the first, being between the two high waters in question, is coincident with diurnal *half ebb*, and the following one with diurnal *half flood*. Thus when the first semidiurnal tide ceases to fall, diurnal tide is still falling at its most rapid rate, and the actual combined tide continues to fall till the rate of fall of the diurnal tide is balanced by the rate of rise of the semidiurnal tide, and thus actual low water is *retarded*. When the second semidiurnal tide ceases to fall, diurnal tide is rising, and the actual tide has therefore been on the rise since the moment when the rate of semidiurnal fall was equal to the rate of diurnal rise. In this case the actual low water has been *accelerated*. Thus, one having been retarded and the other accelerated, a diurnal inequality in *time* is the result. The level of the diurnal tide, however, will have been the same at half flood and at half ebb, and will not therefore have affected the level of semidiurnal tide when at its lowest points. Hence in such case there is no diurnal inequality in *height* of low water.

It appears, then, that when there is no diurnal inequality in *high-water time*, there is none in *low-water height*, and when there is none in *high-water height* there is none in *low-water time*. This will be seen to be almost universally the case in the diagrams.

I now proceed to describe the mode adopted to compare the actual effects of diurnal tide with the changes in the positions of the sun and moon supposed to have produced them.

The diagrams (of which Plate XXXVI. is a specimen) are laid down in the manner usually adopted in tidal investigations, and from them the amount of diurnal inequality in height and time at any given high water or low water may be ascertained by simple measurement. By a similar process of measurement the time and range of any pure semidiurnal tide may be ascertained. From these two elements (the time and range of semidiurnal tide, and the amount of diurnal inequality in time and height) the time and range of the diurnal tide producing those diurnal inequalities may be ascertained by the following process:—

Let H be half range of semidiurnal tide.

h be half diurnal inequality in height.

t be half diurnal inequality in time.

c be interval of time between semidiurnal and diurnal tide.

D be half range of diurnal tide.

[In reference to the expressions (t and c) which denote values of time, 360 degrees is supposed to represent indifferently twenty-four hours, or the period of diurnal tide. Where the expression denotes a proportion of semidiurnal tide, double the value in degrees must be taken to represent the value in hours and minutes which it would denote if representing a proportion of diurnal tide.]

Then the general equation to the tidal curve being, Height at any time = Half range multiplied by cosine of interval between that time and high water, we have for total height of water at time t,

$$\mathrm{H}\cos 2t\,(\text{semidiurnal tide})+\mathrm{D}\cos\overline{c-t}\,(\text{diurnal tide}),$$

and this is a maximum when

$$0=-2\mathrm{H}\sin 2t+\mathrm{D}\sin\overline{c-t},$$

or

$$\frac{\mathrm{D}}{\mathrm{H}}=\frac{2\sin 2t}{\sin\overline{c-t}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathrm{A})$$

also the half diurnal inequality being

$$h=\mathrm{H}-(\mathrm{H}\cos 2t+\mathrm{D}\cos\overline{c-t}),$$

$$\frac{h}{\mathrm{H}}=1-\left(\cos 2t+\frac{\mathrm{D}}{\mathrm{H}}\cos\overline{c-t}\right) \quad \ldots\ldots\ldots \quad (\mathrm{B})$$

$$=1-\cos 2t-\frac{2\sin 2t}{\sin\overline{c-t}}\cos\overline{c-t}$$

$$\operatorname{cotan}\overline{c-t}=\frac{1-\frac{h}{\mathrm{H}}-\cos 2t}{2\sin 2t},$$

whence c, and by substitution in (A) D also may be found.

Then, by taking the successive corresponding values of H, h, and t, we may deduce the corresponding values of c and D, and thus obtain a series of heights and times of diurnal tides.

If we next apply the same system to the successive low waters, we obtain by independent means a second series which ought to correspond with the first. This process was tried, but it was found that there was a decided discrepancy between the two series of diurnal tides; and the values of diurnal inequality upon which the results were based were, moreover, not sufficiently sharply defined to give confidence in their correctness.

A process in some respects the reverse of this was therefore tried, viz. the assumption of a hypothetical diurnal tide, and its combination with the semidiurnal tide by means of an inversion of the formulæ given above. There were at the same time some data for determining the approximate values of the local constants which would enter into the expressions for diurnal tide. As has been before stated, whenever there is no diurnal inequality in time, the time of diurnal tide corresponds with that of actual tide, and the range is equal to the difference in height between two consecutive tides. Thus a simple inspection of the diagram gives four values for diurnal tide in each month, two when diurnal inequality in time vanishes at high water, and two when it vanishes at low water. But besides this, diurnal inequality in time of low water is sometimes for several consecutive days so small that a series of diurnal tides, in which the times are those of semidiurnal low water, and the ranges the total amount of diurnal inequality in height of low water, could not be far from the truth.

A series of trials based on these data led to the following results:—

1st. That the diurnal tide was referable to the transit of the moon immediately anterior to it, and not to one a day and a half anterior, as in the case of semidiurnal tide.

2nd. That the time of high or low water of solar or lunar diurnal tide was about three hours at Kurrachee, and two hours at Bombay after the transit of the sun or moon*.

3rd. That the amount of diurnal tide (being proportional to the declination and the cube of the parallax combined) was for the half-range at Kurrachee about $\frac{3}{10}$ of an inch for each degree of the sun's, and $\frac{9}{10}$ for each degree of the moon's declination when at the mean distance from the earth. At Bombay the range was taken to be $\frac{1}{3}$ more.

A series of diurnal tides was then calculated from the following expressions:—

Let M be moon's declination.

S be sun's declination.

P be moon's parallax (sun's parallax neglected).

$L=\frac{MP^3}{57^3}$ (57 being the mean value of moon's parallax).

T=moon's hour-angle.

C=proportion of half-range for each degree of sun's declination.

Then the height of diurnal tide at any time t (measured from the time of high water of solar-diurnal tide) is

$$(3L\cos\overline{T-t}+S\cos t)\times C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

This is a maximum when

$$0=3L\sin\overline{T-t}-S\sin t,$$

$$S\sin t=3L(\sin T\cos t-\sin t\cos T),$$

$$\frac{S}{3L}=\sin T\cot t-\cos T,$$

$$\cot t=\frac{\frac{S}{3L}+\cos T}{\sin T},$$

whence t may be found, and thence by substitution in (A) the range of the diurnal tide.

The application of the daily successive values of S, L, and T in these formulæ gave a series of hypothetical diurnal tides, and these were combined, by inverted forms of the expressions at p. 689, with the actual semidiurnal tides, so as to obtain the values of diurnal inequalities which they would produce.

These hypothetical diurnal inequalities were then compared with the actual ones. As had been anticipated, the assumed values for the constants which gave a coincidence in high-water time and low-water height did not give a coincidence in high-water height and low-water time. This result is indeed identical with that which was obtained when the series of diurnal tides, deduced from the high-water observations, was compared with those deduced from the low-water, but in its present form it offers greater facility for

* It has since been found that the times are nearly the same at both ports (see Postscript).

ascertaining the character of the correction required. We have now a series of observed heights of high water and low water, and a corresponding series of calculated heights. In order to ascertain whether the differences between these followed any regular law, they were set off as ordinates from a horizontal line, and a continuous line drawn through their extremities. This continuous line assumed a form obviously, though roughly, approximating to the curves of a series of diurnal lunar tides having a maximum half-range of about 12 inches at Bombay and 6 inches at Kurrachee*. It made its high water and low water nine hours at Bombay and ten hours at Kurrachee after the moon's transit, high water at full and change being at night in the summer, and in the morning in the winter. It had evident periods of disappearance twice in a month, two days at Bombay and three days at Kurrachee *before* the moon crossed the equator; and, like the first-described diurnal tide, it reappeared after vanishing with the times of high and low water reversed. There seemed to be some evidence of solar influence upon this tide, but of too uncertain a character to justify its being taken into account.

The incorporation of this empirical tide with the first diurnal tide effected a materially closer coincidence with the observed heights and times, but it cannot be taken as a final solution of the problem. A more perfect series of observations at Bombay may probably suggest some form of the correction more consistent with known physical causes.

The combination of the two diurnal tides may be effected by the following formulæ, similar to those given in page 690.

Let α be interval between the two tides.

D be half-range of first tide.

d be half-range of second tide.

θ be interval between the times of first and combined diurnal tides.

Then height of diurnal tide at time θ

$$= D \cos \theta + d \cos \overline{\alpha - \theta}\ ; \quad \ldots\ldots\ldots\ldots \quad (A)$$

and this is a maximum when

$$\operatorname{cotan} \theta = \frac{\frac{D}{d} + \cos \alpha}{\sin \alpha},$$

whence θ may be found, and the half-range by substitution in (A).

For the purposes of practical computation tables and diagrams were formed from the various formulæ above given, and under the authority of the Secretary of State for India, tide tables for Bombay and Kurrachee for the year 1867 were published. The times and heights given in these tables were compared with observation for four months at Bombay and eight months at Kurrachee. The diagrams give in a graphic form the result of the comparison, the small black dots showing the observed, and the red† ones the calculated times and heights. It should be stated that the calculated heights are

* This result is modified, so far as Bombay is concerned, by the process described in the Postscript.

† In Plate XXXVI. small circles.

plotted from the half-tide level of the day (shown on the diagrams), and not from a fixed level. Thus the disturbances of sea-level due to other than tidal causes are eliminated from the comparison.

The following are the analyses of the comparison:—

Bombay High-Water Heights.—242 cases.

31	or 13	per cent.	are correct.
94	„ 39	„	are within 1 inch.
140	„ 58	„	„ 2 inches.
175	„ 72	„	„ 3 „
198	„ 82	„	„ 4 „
221	„ 91	„	„ 5 „
231	„ 96	„	„ 6 „
237	„ 98	„	„ 7 „
240	„ 99	„	„ 8 „

The greatest error is 9 inches.

Bombay Low-Water Heights.—243 cases.

34	or 14	per cent.	are correct.
109	„ 45	„	are within 1 inch.
152	„ 62	„	„ 2 inches.
185	„ 76	„	„ 3 „
209	„ 86	„	„ 4 „
217	„ 89	„	„ 5 „
235	„ 97	„	„ 6 „
239	„ 98	„	„ 7 „
242	„ 99½	„	„ 8 „

The greatest error is 9 inches.

Bombay High-Water Times—246 cases.

75	or 30	per cent.	are within 5 minutes.
151	„ 61	„	„ 10 „
200	„ 81	„	„ 15 „
225	„ 91	„	„ 20 „
238	„ 97	„	„ 25 „

The greatest error is 30 minutes.

Bombay Low-Water Times.—246 cases.

68	or 28	per cent.	are within	5 minutes.
133	„ 54	„	„	10 „
178	„ 72	„	„	15 „
207	„ 84	„	„	20 „
226	„ 92	„	„	25 „
239	„ 97	„	„	30 „
244	„ 99	„	„	35 „

The greatest error is 37 minutes.

Kurrachee High-Water Heights.—479 cases.

51	or 11	per cent.	are correct.	
131	„ 27	„	are within	1 inch.
242	„ 50	„	„	2 inches.
335	„ 70	„	„	3 „
385	„ 80	„	„	4 „
435	„ 91	„	„	5 „
456	„ 95	„	„	6 „
467	„ 97	„	„	7 „
475	„ 99	„	„	8 „

The greatest error is 9 inches.

Kurrachee Low-Water Heights.—473 cases.

114	or 24	per cent.	are correct.	
259	„ 55	„	are within	1 inch.
369	„ 78	„	„	2 inches.
425	„ 90	„	„	3 „
458	„ 97	„	„	4 „
466	„ 98½	„	„	5 „
470	„ 99½	„	„	6 „

The greatest error is 8 inches.

The Kurrachee High-Water Times are rather more correct, and the Low-Water Times are rather less correct than those of Bombay; but the errors cannot be given in a tabular form, as there are obvious errors in the record owing to the imperfect adjustment of the machine, the limits of which cannot be exactly defined.

Postscript.

Since receiving the observations made at Bombay and Kurrachee in the year 1867 the author has subjected them to another process for obtaining the actual times and heights of diurnal tide, which has been more successful than that described in the paper.

The only data made use of were the diurnal inequalities in *height* at high and low water, the range of semidiurnal tide and the diurnal inequality in time, which were necessary to the previous process, being now altogether disregarded.

The diurnal inequalities in height were obtained from the diagrams by measuring the widths between the lines joining alternate tides where they were crossed by the vertical lines representing noon on successive days. The two daily values thus obtained are respectively the sine and cosine of an angle which represents the difference in time between semidiurnal and diurnal tide. Dividing the low-water by the high-water value gives the cotangent of that angle, and thence the angle itself. Thus the time of actual diurnal tide (first in relation to the time of semidiurnal low water, and then in relation to solar time) was obtained.

The actual range of diurnal tide was obtained by adding together the squares of the high-water and low-water values (sine and cosine), and taking the square root of the sum.

With these two series of results as ordinates, curves were drawn representing times and ranges of actual diurnal tide, which were thus presented in a convenient form for comparison with the diurnal tide which had been previously calculated.

The comparison confirmed the previous conclusion that the tide based on the simple declination theory was insufficient; and the empirical correction which had been adopted seemed to provide an approximation to the required addition to it, both in time and height. But it appeared that a better coincidence in time would have been obtained by assuming the diurnal tide at Kurrachee to be forty minutes earlier. This supposition was tested by treating the observations of 1865 in a similar manner, and also by recalculating a portion of the tides of 1867 with the earlier diurnal tide. In both cases the supposition was confirmed, a better agreement being obtained.

On treating the Bombay observations in the same manner, a fair general coincidence with the calculated diurnal tides was found to exist; but it was further found, on comparing together the Kurrachee and Bombay curves of actual diurnal tide (thus for the first time recorded for the same period), that the times were nearly identical at the two ports, and the range at Bombay about one-tenth greater than that at Kurrachee.

The tables for the four months over which the Bombay observations extend were recalculated with the diurnal tides which had been calculated for Kurrachee (but made forty minutes earlier, and increased in range by one-tenth), and the result was quite as good as that shown by the original tables. This fact would seem to point to the possibility that the diurnal tide is a vertical undulation, acting simultaneously, or nearly so, over a large area.

Appendix.

Explanation of the Diagrams.

PLATE XXXVI.

The four sets of waving lines running the whole length of the diagrams represent respectively High-Water Times, Low-Water Times, High-Water Heights, and Low-Water Heights.

The observed times are represented by small black circles, the day and approximate hour being given by their positions measured along a horizontal line, and their intervals after the moon's transit by their vertical positions according to the scale—the line A A representing thirty-three hours after the moon's transit throughout.

The observed heights are also marked by small black circles laid off by scale from the straight horizontal line B B, which represents mean sea-level. The diurnal inequality is shown by the widths of the spaces between the lines joining alternate black circles—coloured brown in the manuscript diagrams.

The blue* lines passing along the centres of these spaces represent the actual curves of semidiurnal tide. The uneven line B B is equidistant between the semidiurnal curves of High-Water and Low-Water heights, and represents the actual half-tide level.

Mean sea-level at Bombay is deduced from the four months' observations here given, and is 80 feet 4 inches above the "Town Hall Datum."

Mean sea-level at Kurrachee is deduced from the observations of 1857–58, and is 4 feet 9 inches above the "Harbour Works Datum."

The calculated times and heights are given by red† circles in the diagrams for 1867. The heights are plotted from the *actual half-tide level* (uneven line), and not from the *mean sea* (straight line).

The calculated semidiurnal curves of time and height are shown by red‡ lines.

The Bombay observations (1867) are believed to be sufficiently correct for most practical purposes. The same may be said of the Kurrachee observations of 1857–58.

The Kurrachee observations of 1865 and 1867 were taken by a self-registering machine at a spot a mile and a half higher up the harbour than that at which those of 1857–58 were taken; but numerous observations have been taken simultaneously at the two places, which prove that the tide-wave is scarcely appreciably affected in its passage up the harbour. The time of high water is, however, from five to ten minutes later at the upper station. The times of high and low water given on the diagrams of 1865 and 1867 are ten minutes earlier than those actually recorded, so as to make them comparable with those of 1857–58.

The working of the self-registering machine is not quite perfect, and its adjustment is

* Strong black lines. † Small open circles. ‡ Broken black lines.

doubtful in some respects; it will therefore be desirable not to place too much confidence on its records in some particulars.

They may be depended on for,—

1st. The forms of the mean semimonthly curves of height and time (very nearly).

2nd. The *range* of tide without reference to a fixed level.

3rd. Diurnal inequality in height or time.

But in the following the results should be taken with some reserve:—

1st. The actual time of any particular tide or short series of consecutive tides.

2nd. The height of any particular tide or short series of tides above a fixed level.

(N.B. The papers were generally renewed about once a week, and any error in placing a paper would be likely to be continued till it was removed.)

3rd. The comparative duration of the times of rising and falling water. The machine gives the average duration of flood tide a few minutes longer than that of ebb, a result which is not confirmed by direct observation.

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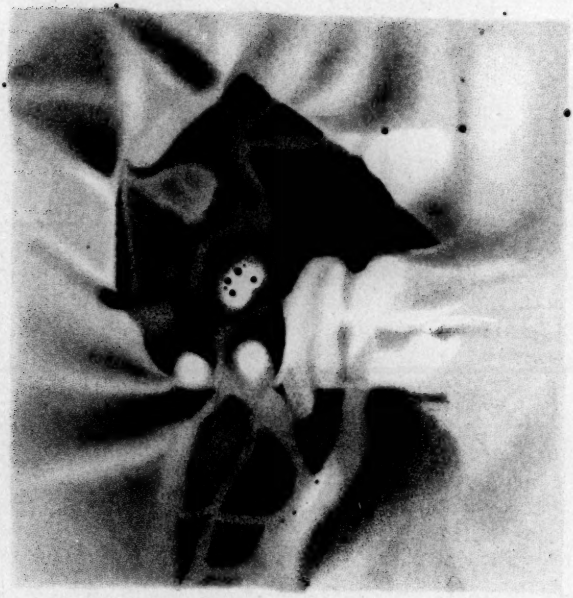

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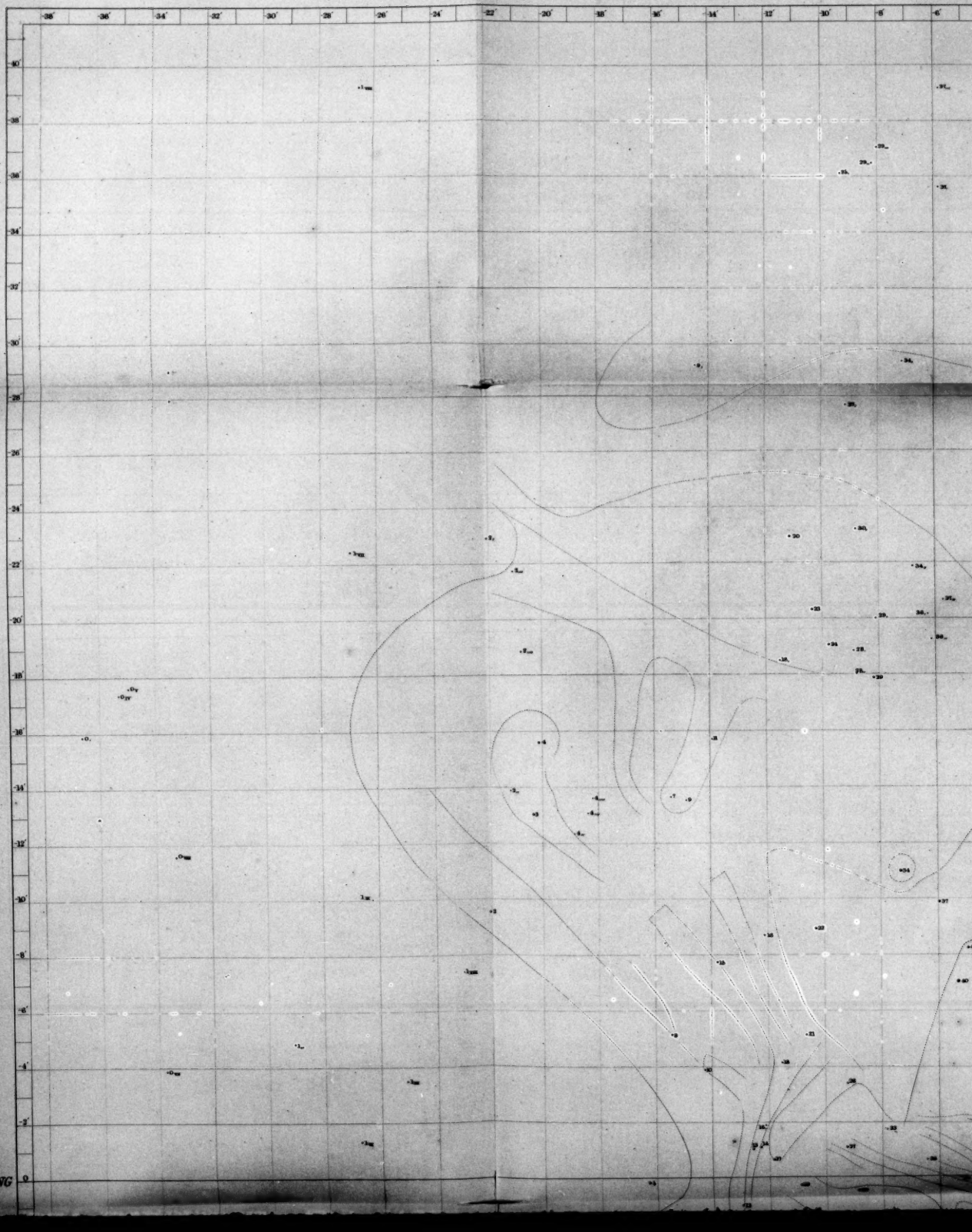
THE HUYGENIAN REGION OF THE GREAT NEBULA IN ORION.

Scale $\frac{1}{24}$ of 1 inch = 1 minute of space. J. Basire sc.

Given by Lord Rosse



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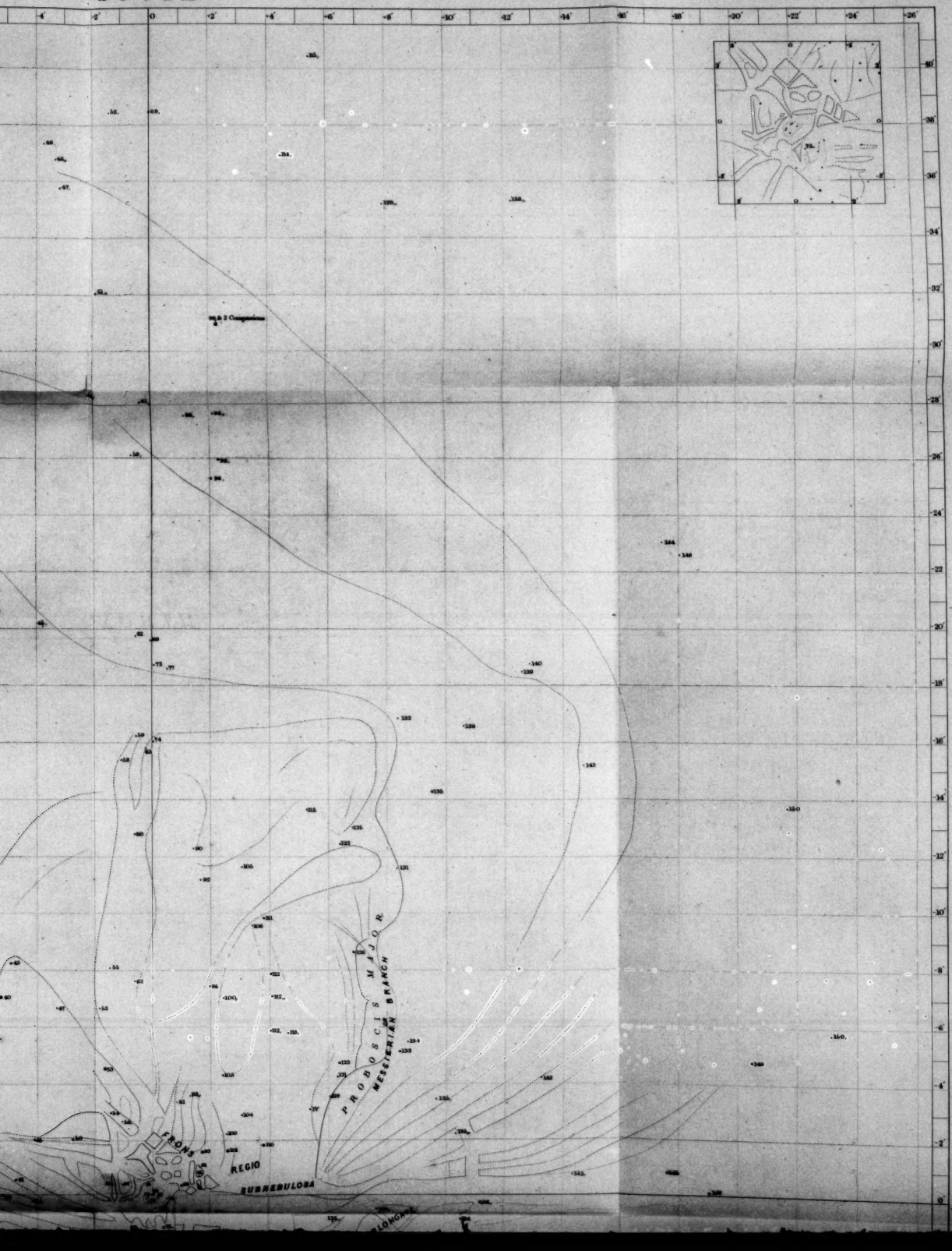
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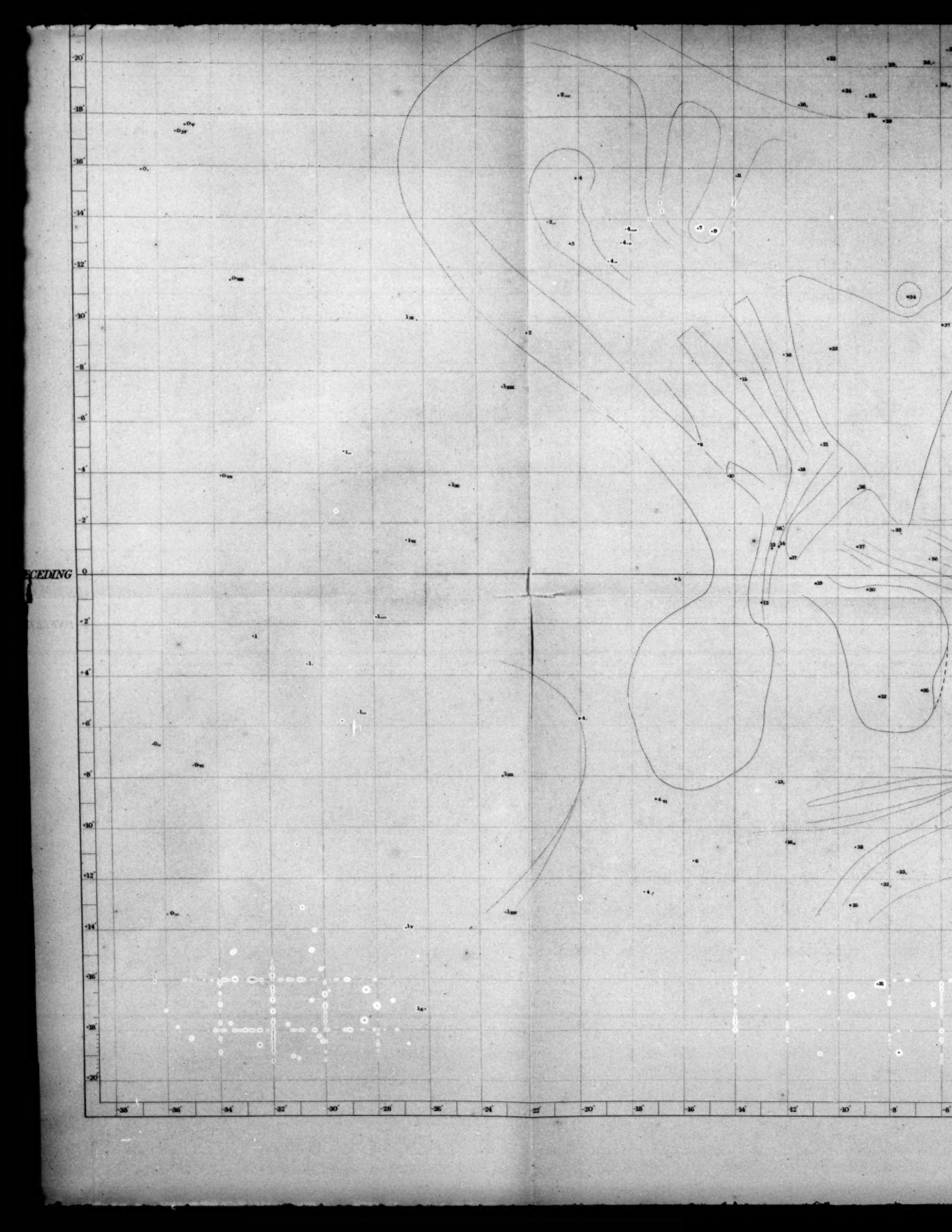

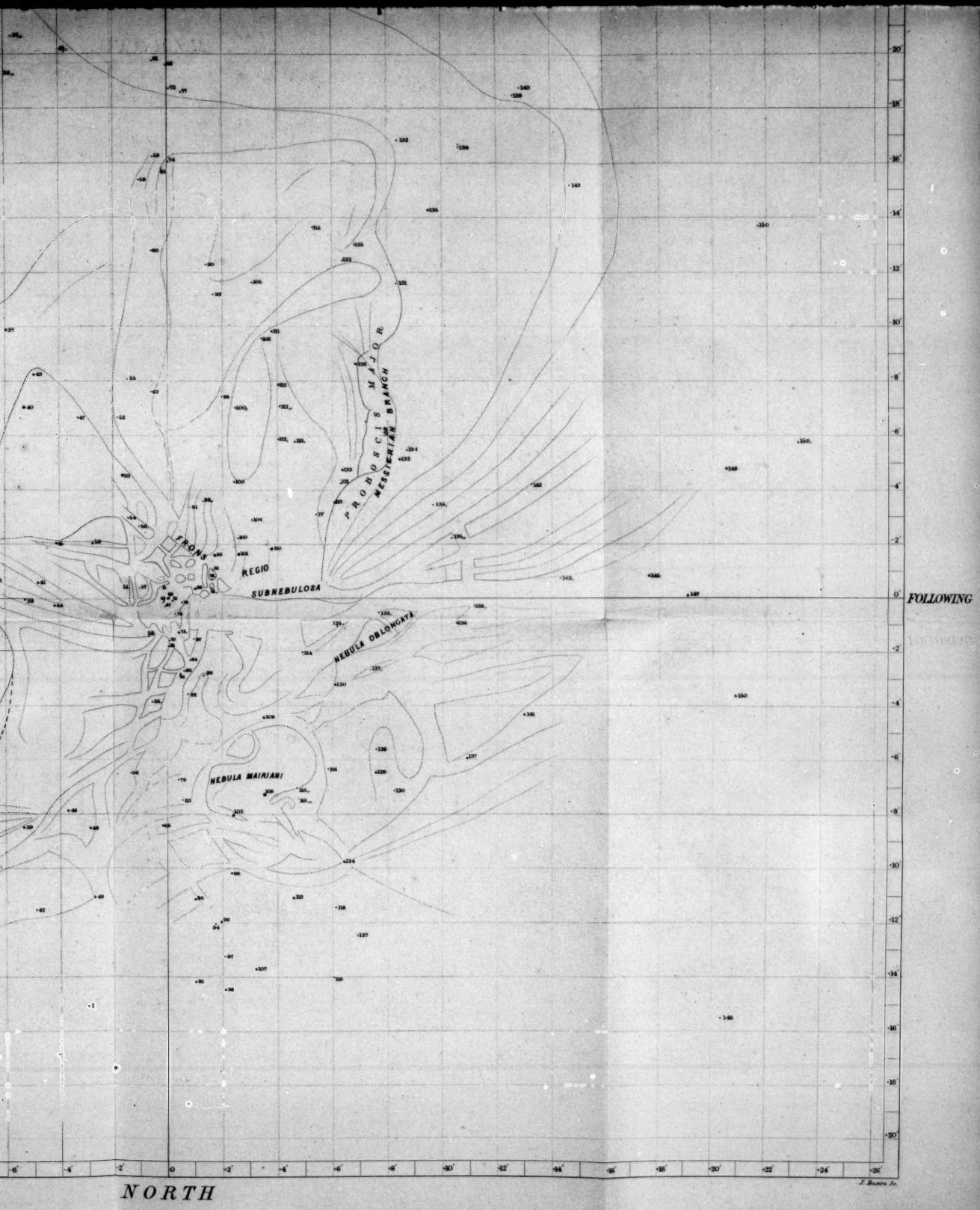
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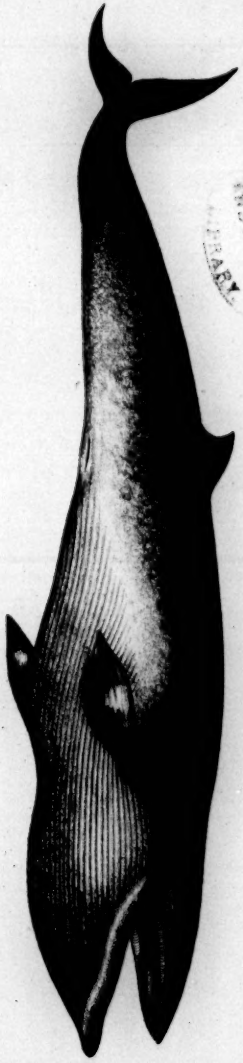

Fig. 1

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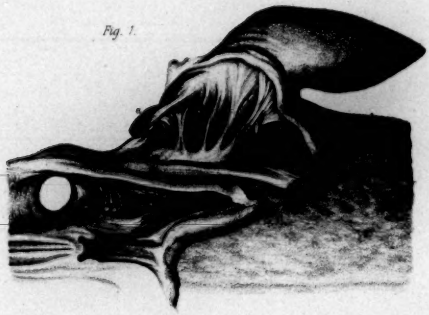
Fig. 1.



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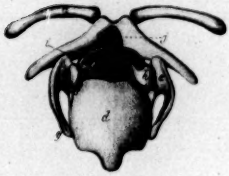

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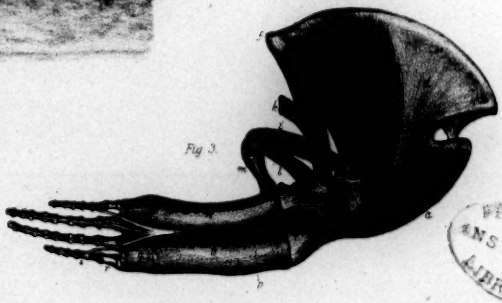

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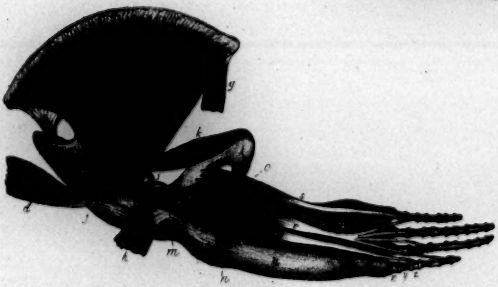

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Fig. 4.

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Fig. 3

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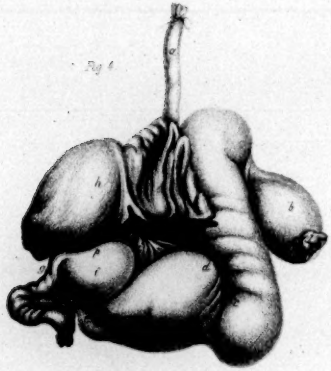

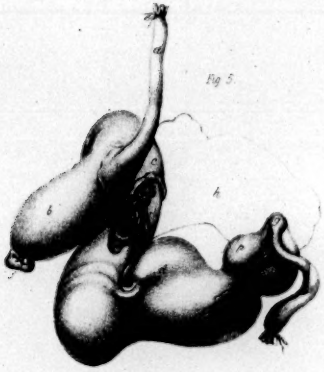

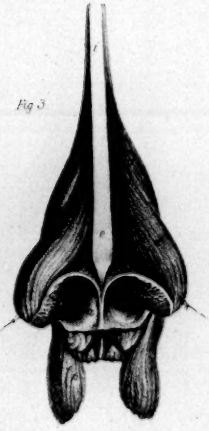

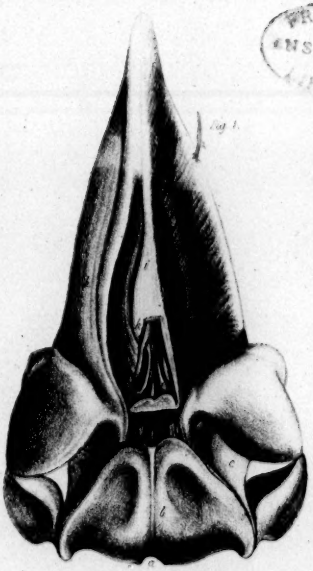

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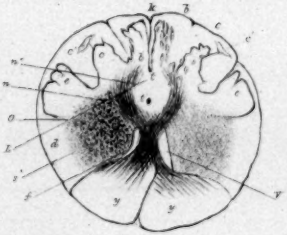

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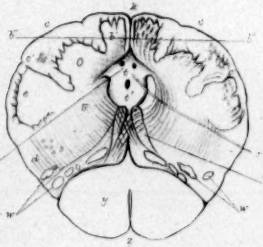

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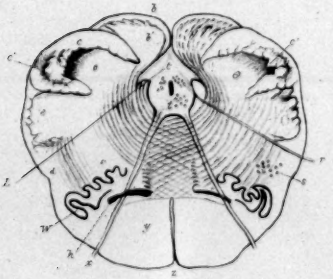

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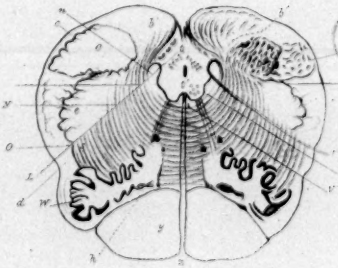

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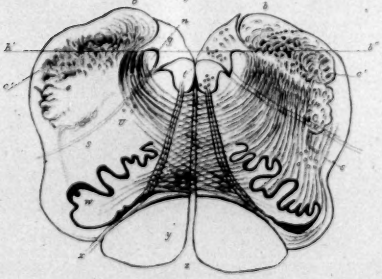

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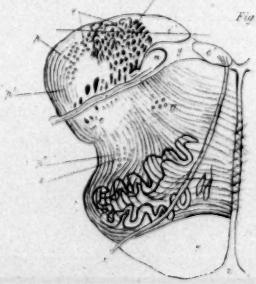

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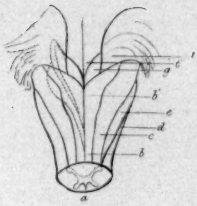

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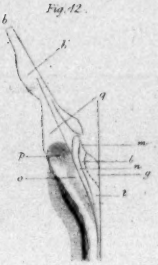

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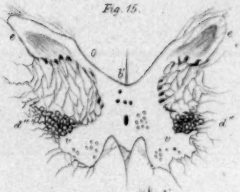

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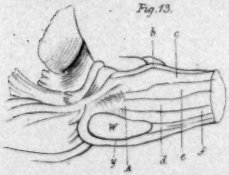

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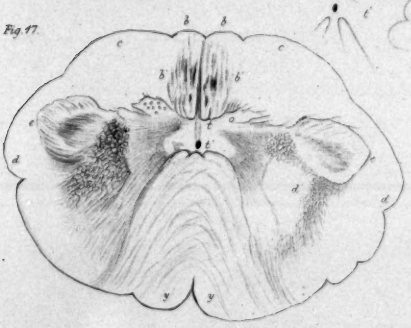

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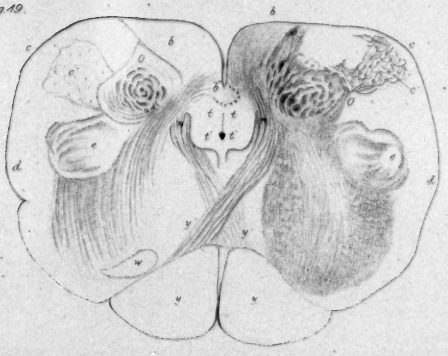

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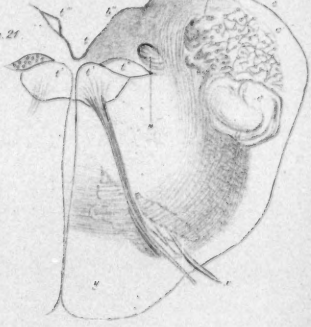

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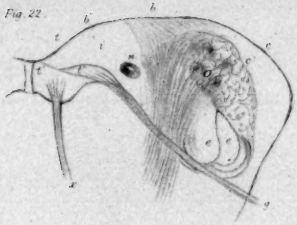

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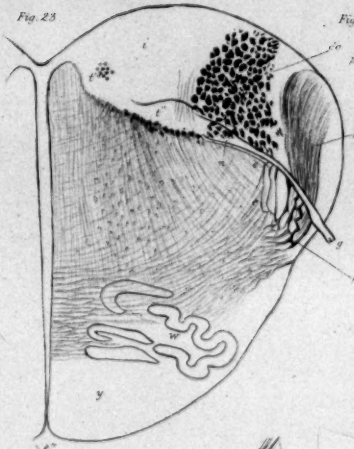

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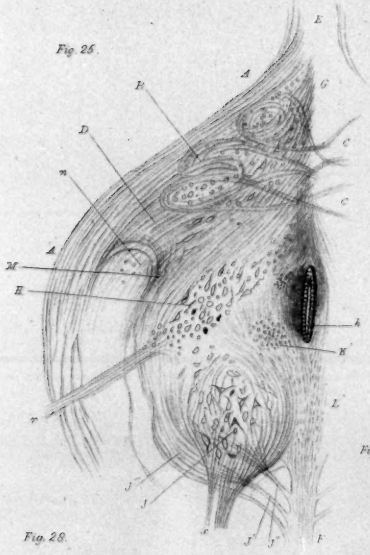

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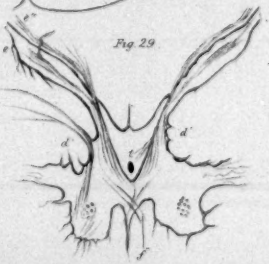

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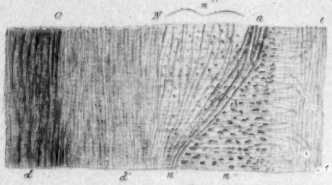

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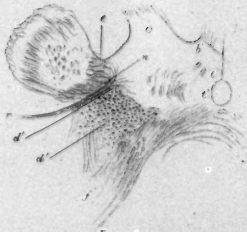

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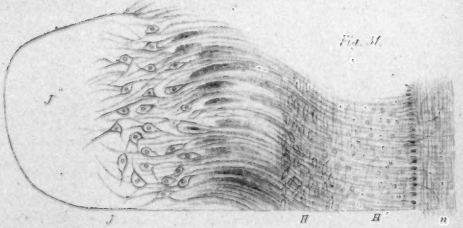

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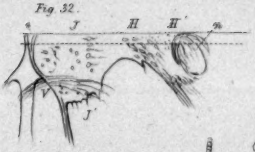

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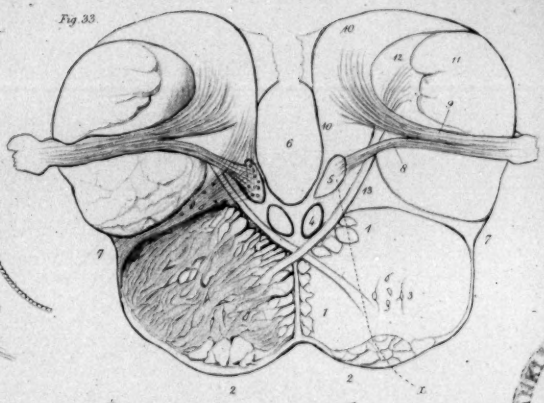

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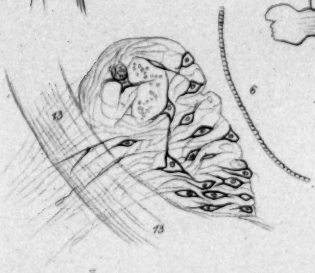

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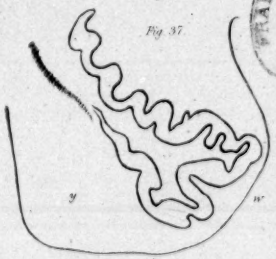

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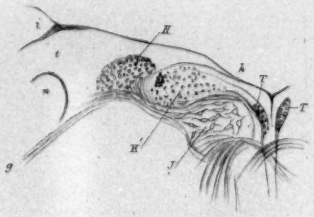

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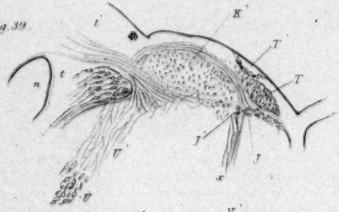

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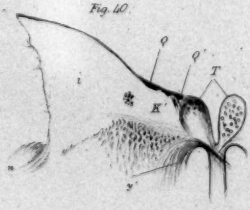

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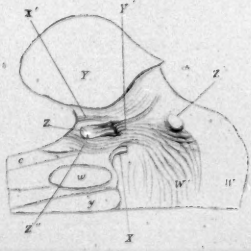

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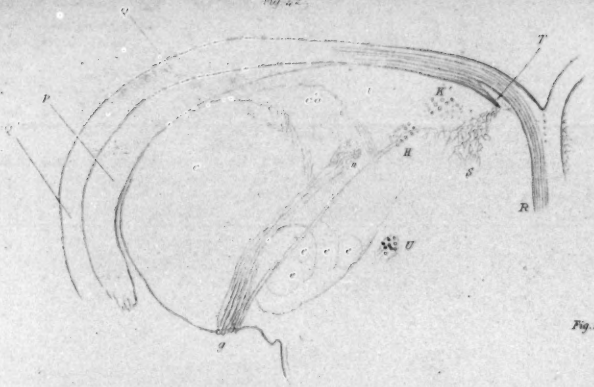

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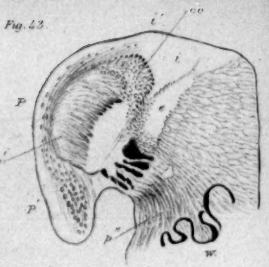

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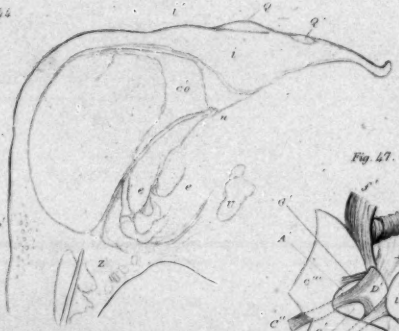

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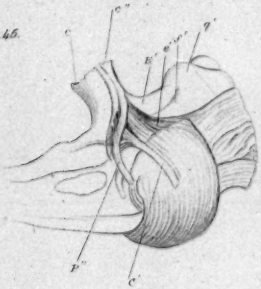

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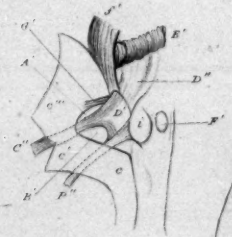

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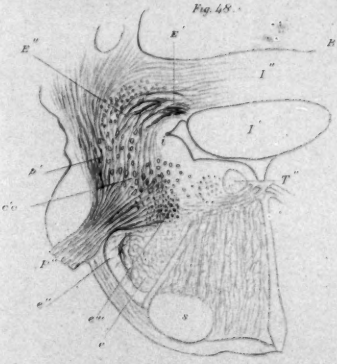
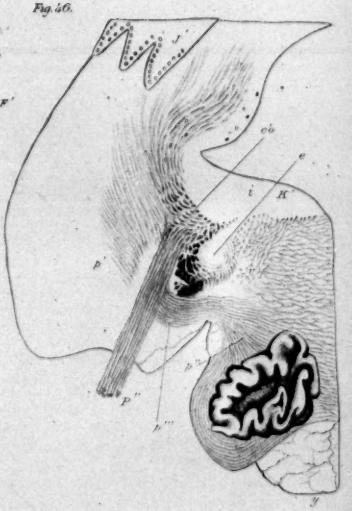
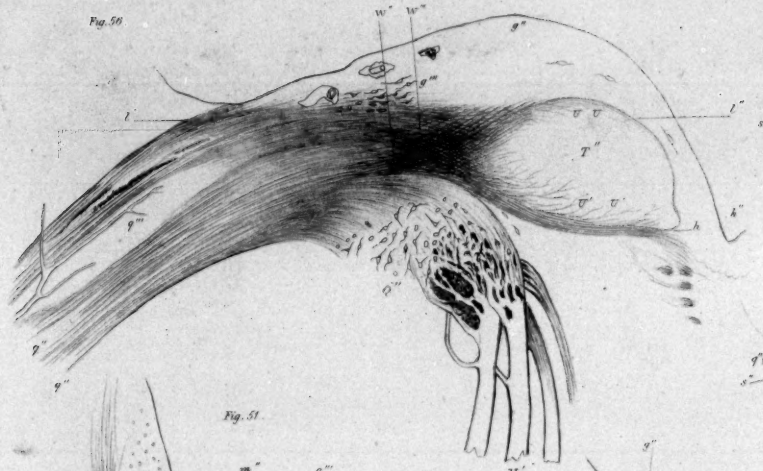


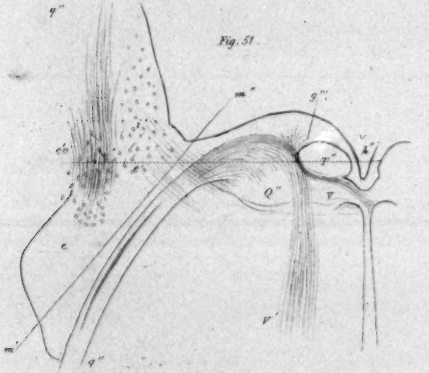
Fig. 46.

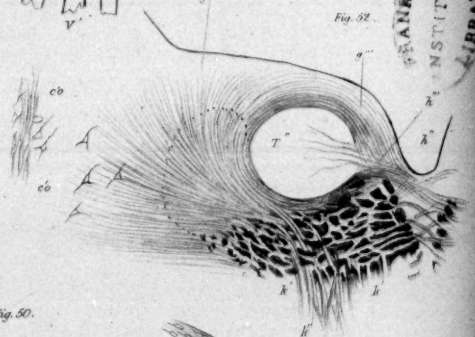


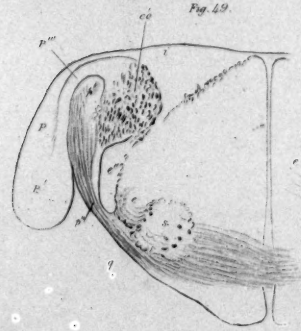
J. L. Clarke ad. nat. del. 1867.

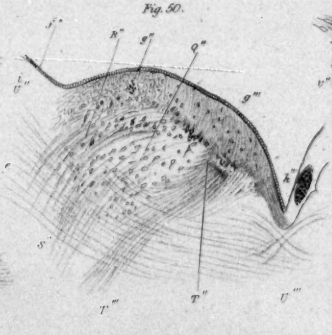
J. Basire

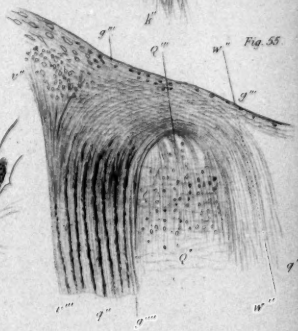












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J. L. Clarke ad nat. del. 1867.

J. Basire

Fig. 57.

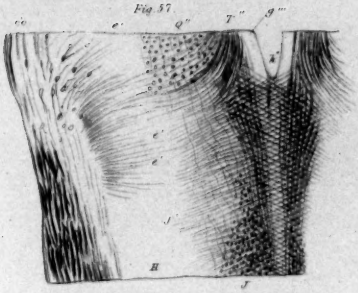


Fig. 58.

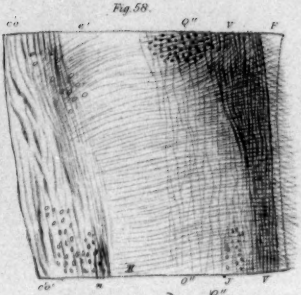


Fig. 59.

Fig. 60.

Fig. 64.

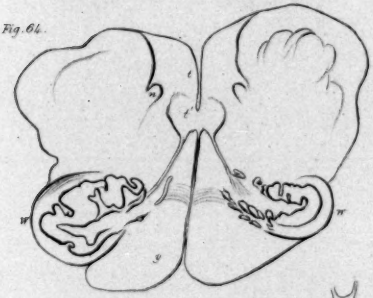


Fig. 64a.

Fig. 64b.

Fig. 62.

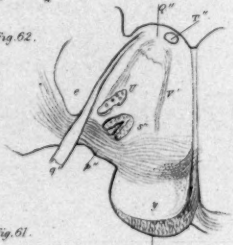
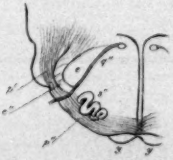


Fig. 61.



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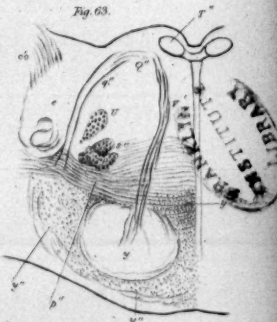

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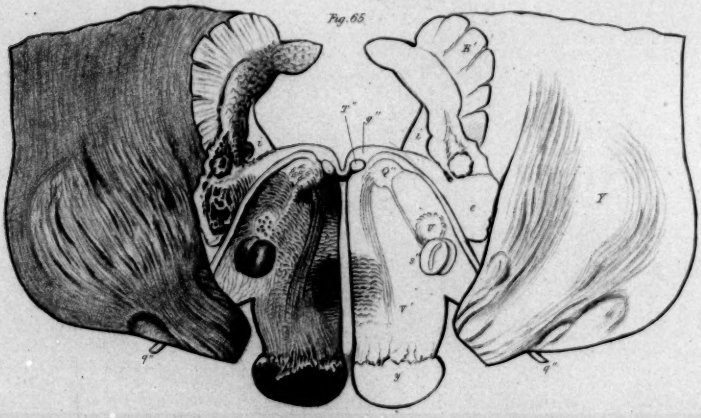


SOUTH

GASSENDI
after Sunrise
1862. 1863. 1865

Fig 1.

WEST

EAST

Scale of Geographical Miles

THEOPHILUS & part of CYRILLUS.
25 March 1863. 8 p.m. Power 200-400.
Moon's Decl. N. 22. 7 Sun's Decl. N. 1.44. Age 6.23 days

Fig 2

Scale of Geographical Miles

THEOPHILUS 24 April 1865.
Power 300-400.

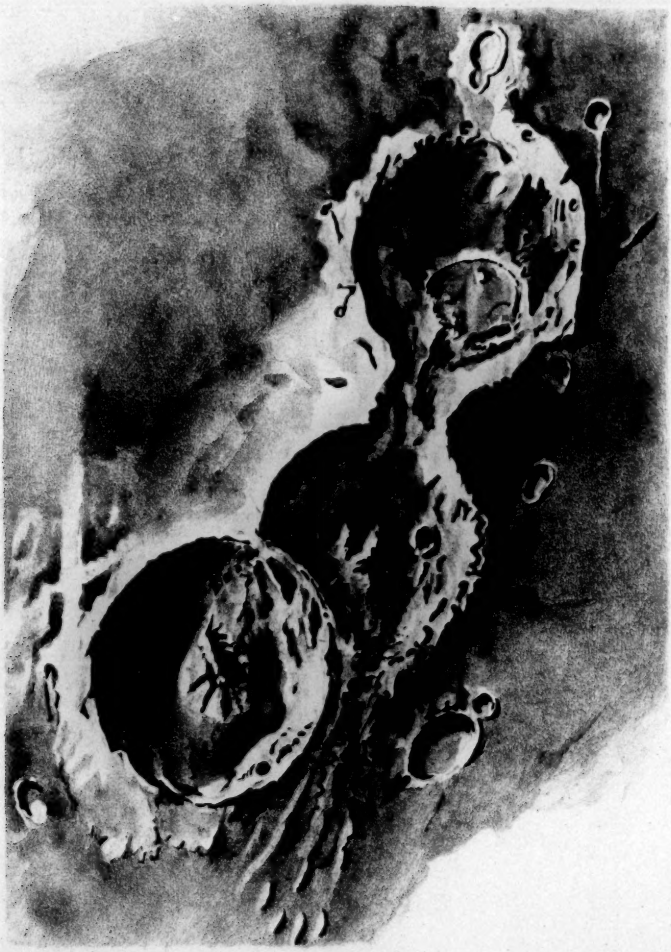
Fig 3.

NORTH

Engraved by J. Basire.

SOUTH

WEST

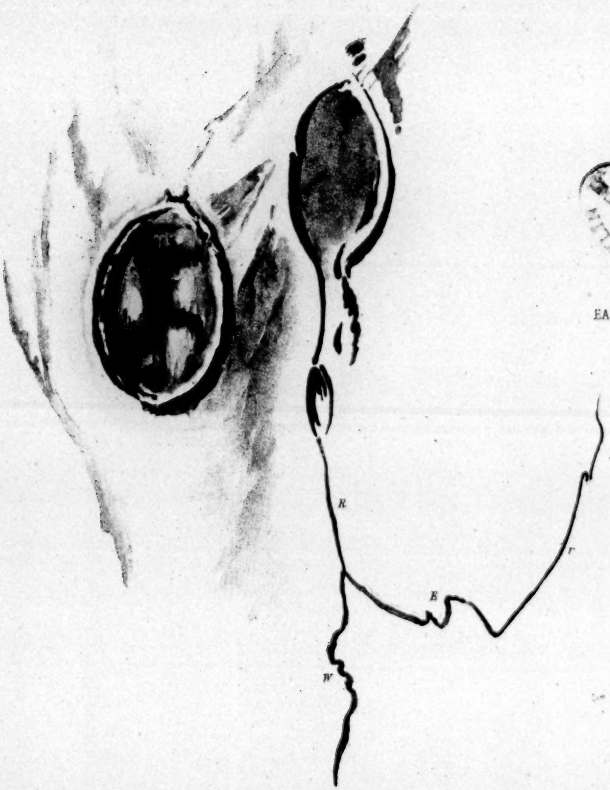


EAST

W. H. Wesley, Lith.

NORTH

J. Phillips, del.



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WEST

EAST

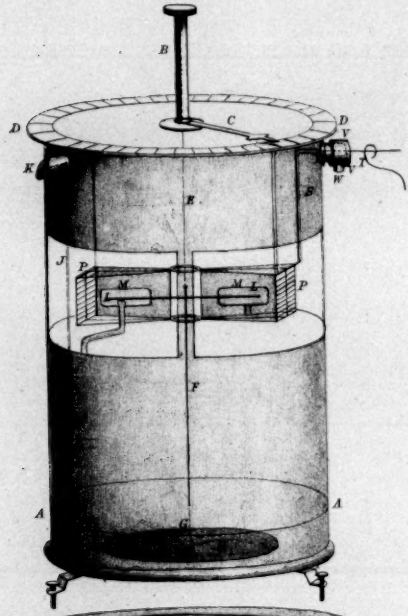

Fig. 1.

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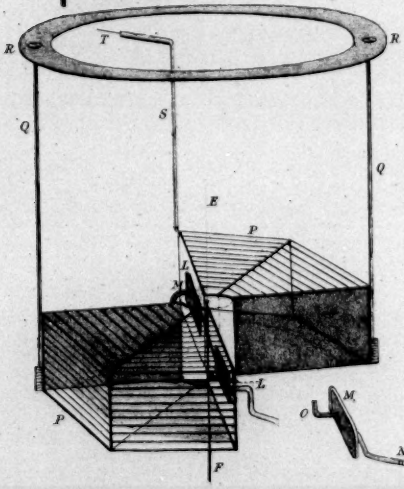

Fig. 2.

Fig. 3.

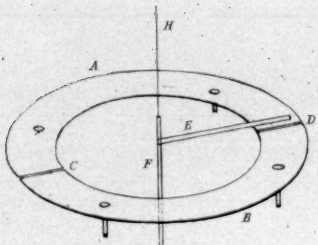

Fig. 4.

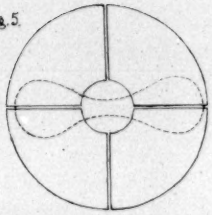

Fig. 5.

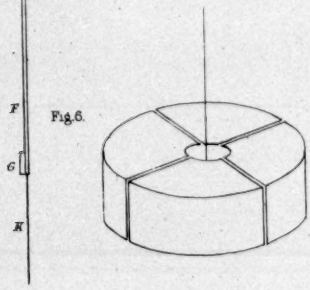

Fig. 6.

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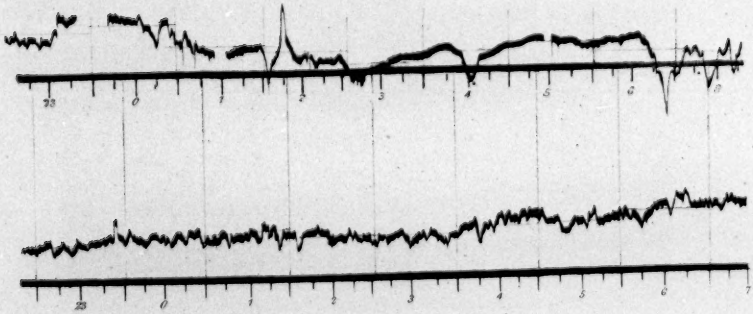
1864. June 3. 22. 35 guage at commt 254·9
4. 22. 14 " end 240·5
Ins. 3. 23. 25 good
Earth 4. 1. 0
Ins. 4. 1. 54 good

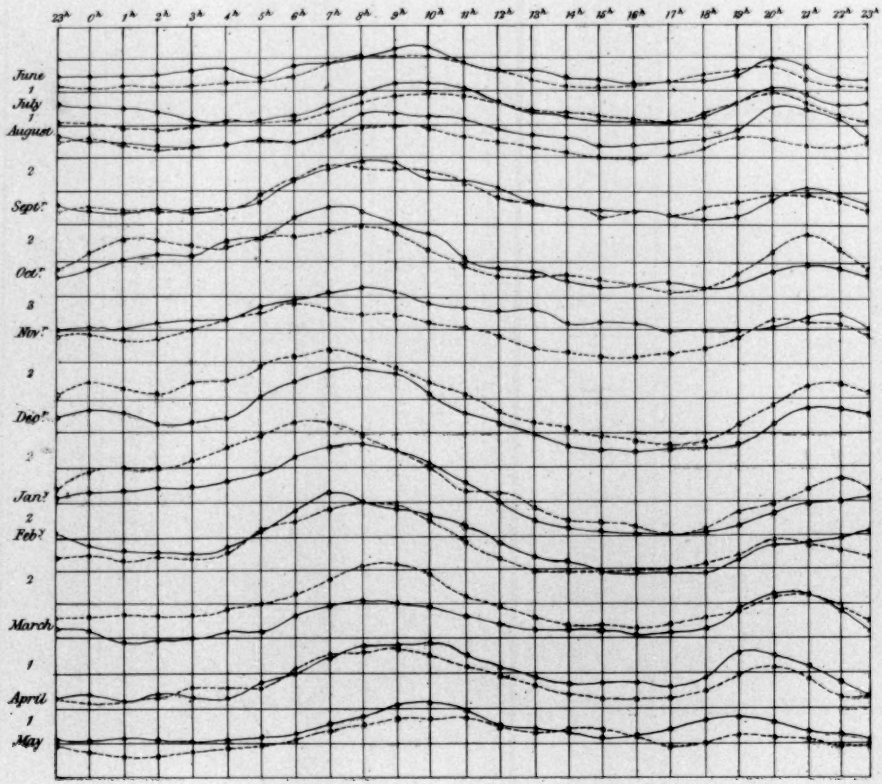


1864. June 4. 22. 15 guage at commt 240·5
5. 22. 24 end 240·0

Diurnal Curves of Atmospheric Electricity at Kew for each month from June 1862 to May 1864 inclusive.

The continuous lines belong to the first year, dotted lines to second year.

The numbers written between the names of the months indicate the amounts by which the origin of coordinates has been lowered on passing from each curve to the next below it.



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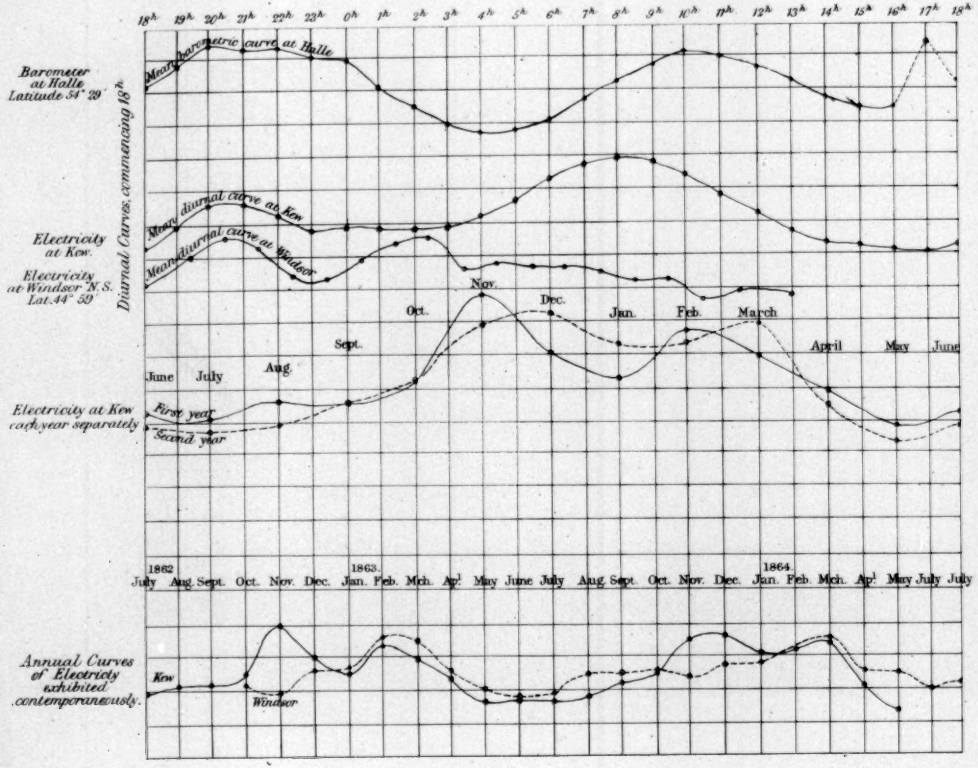
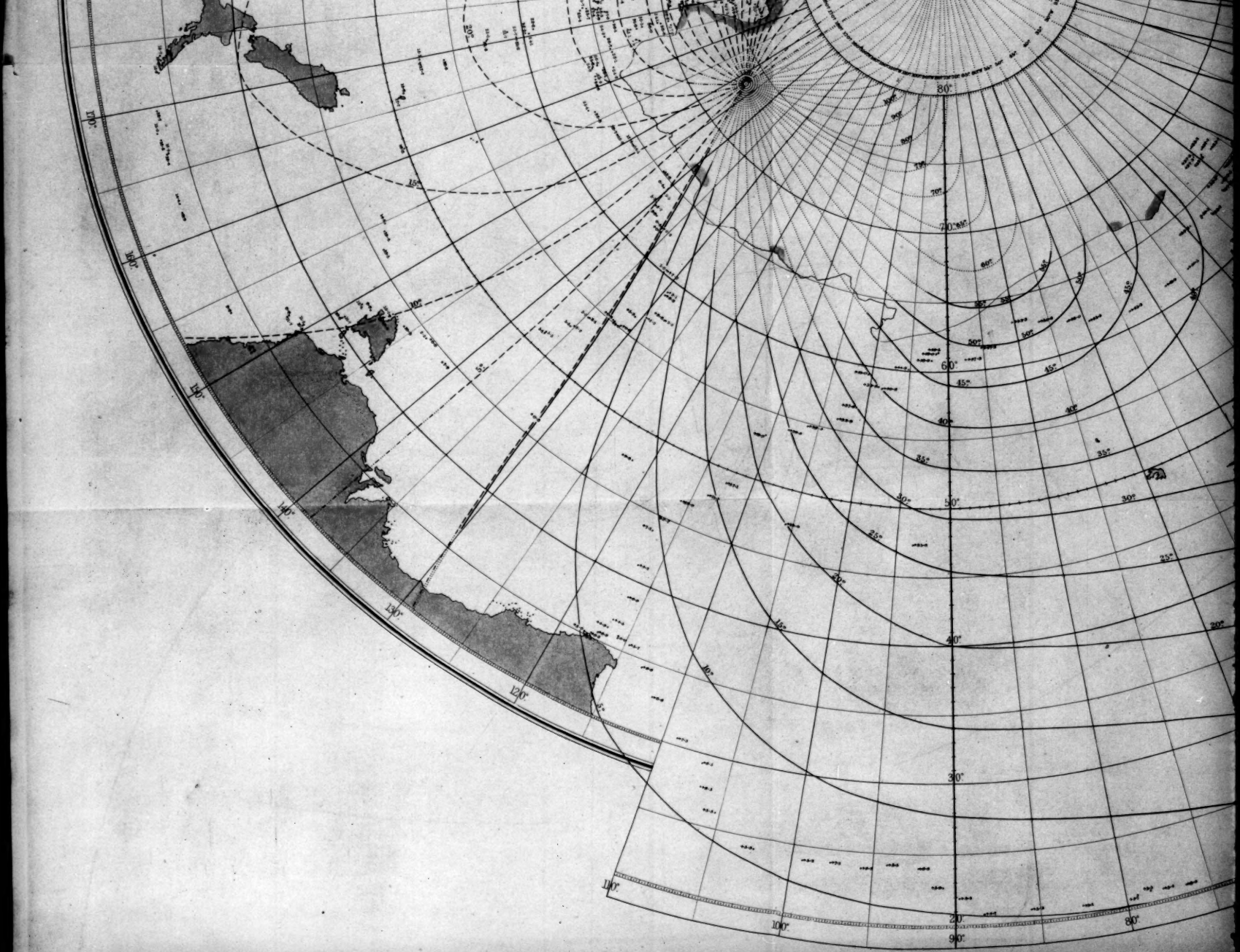
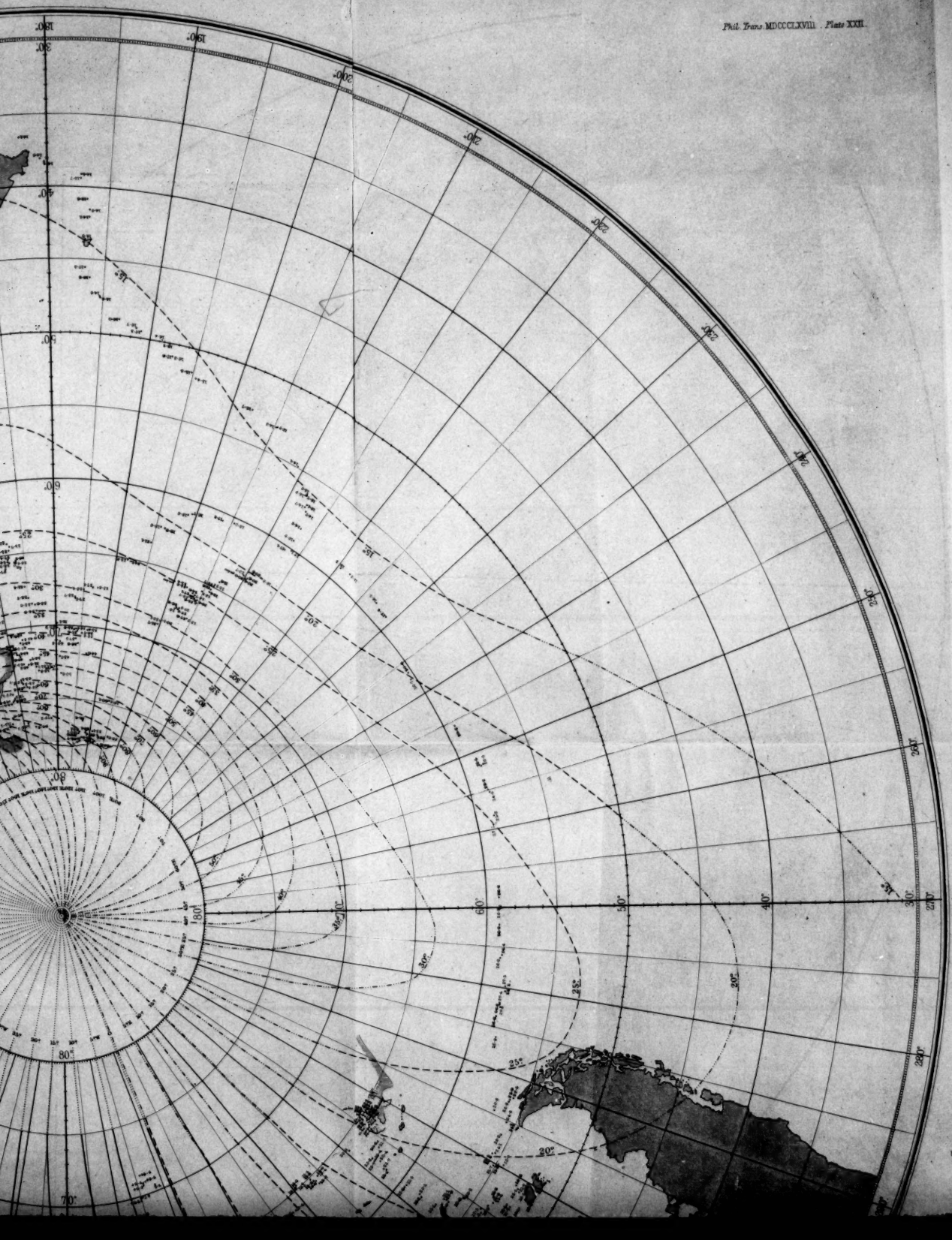
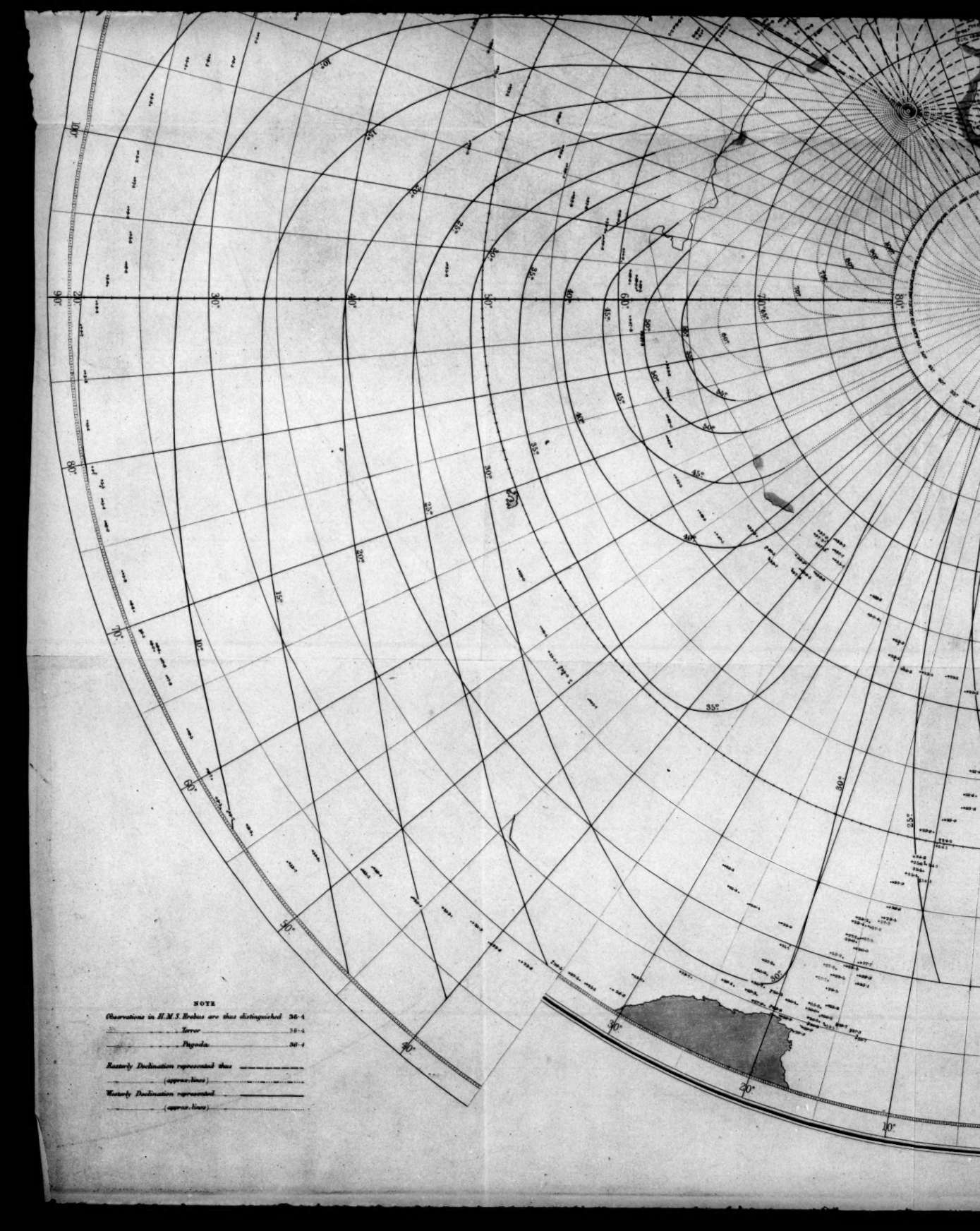
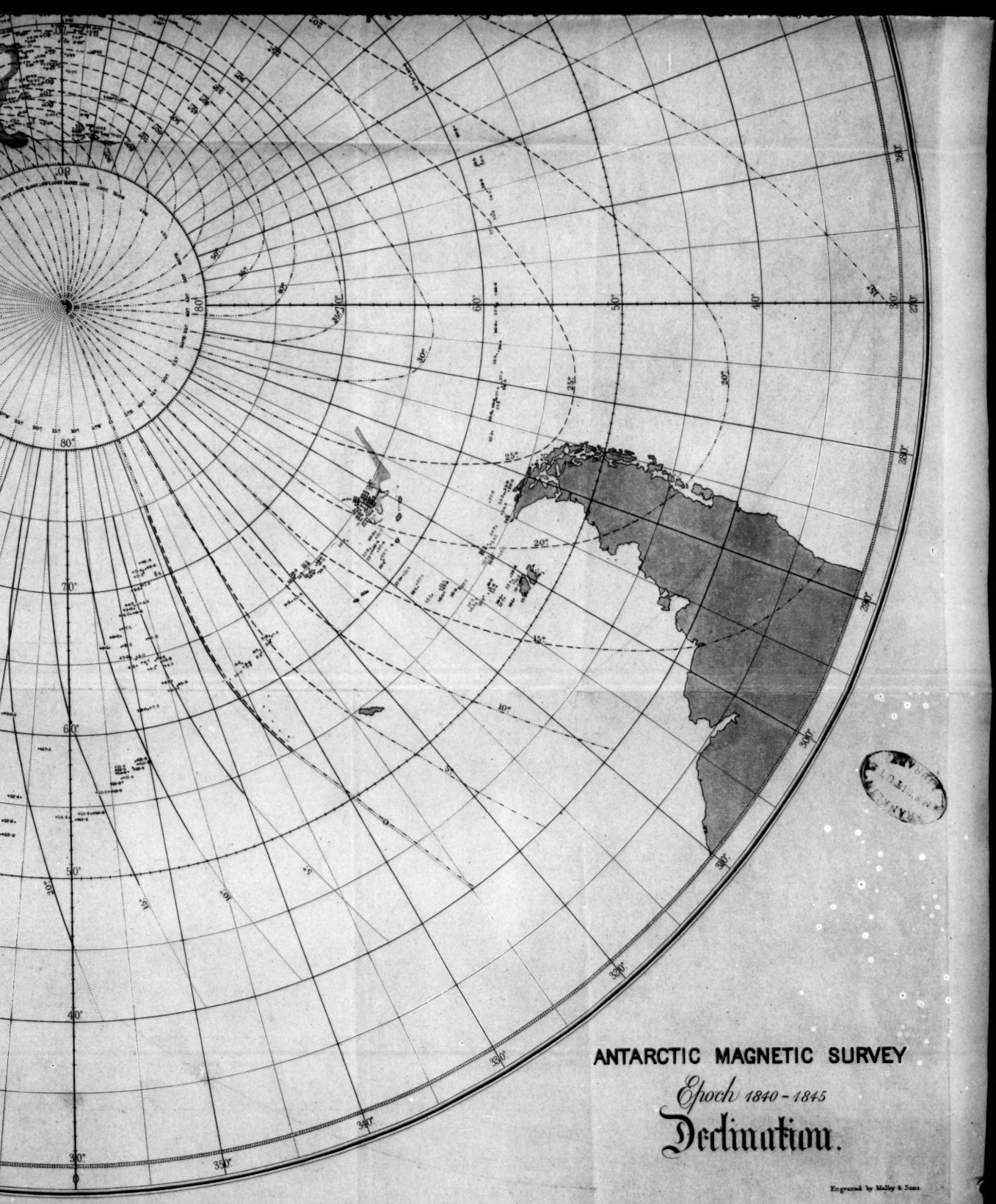
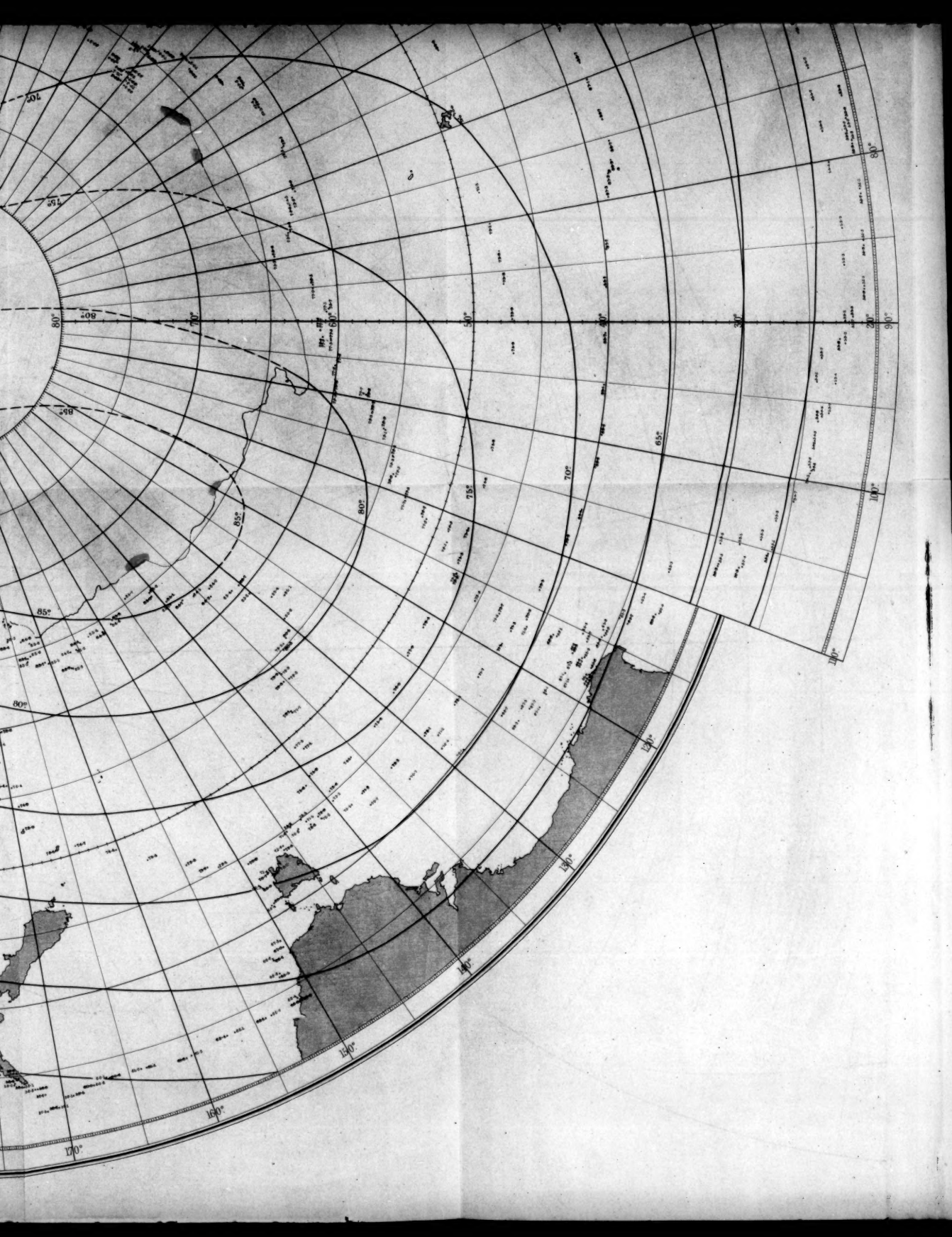


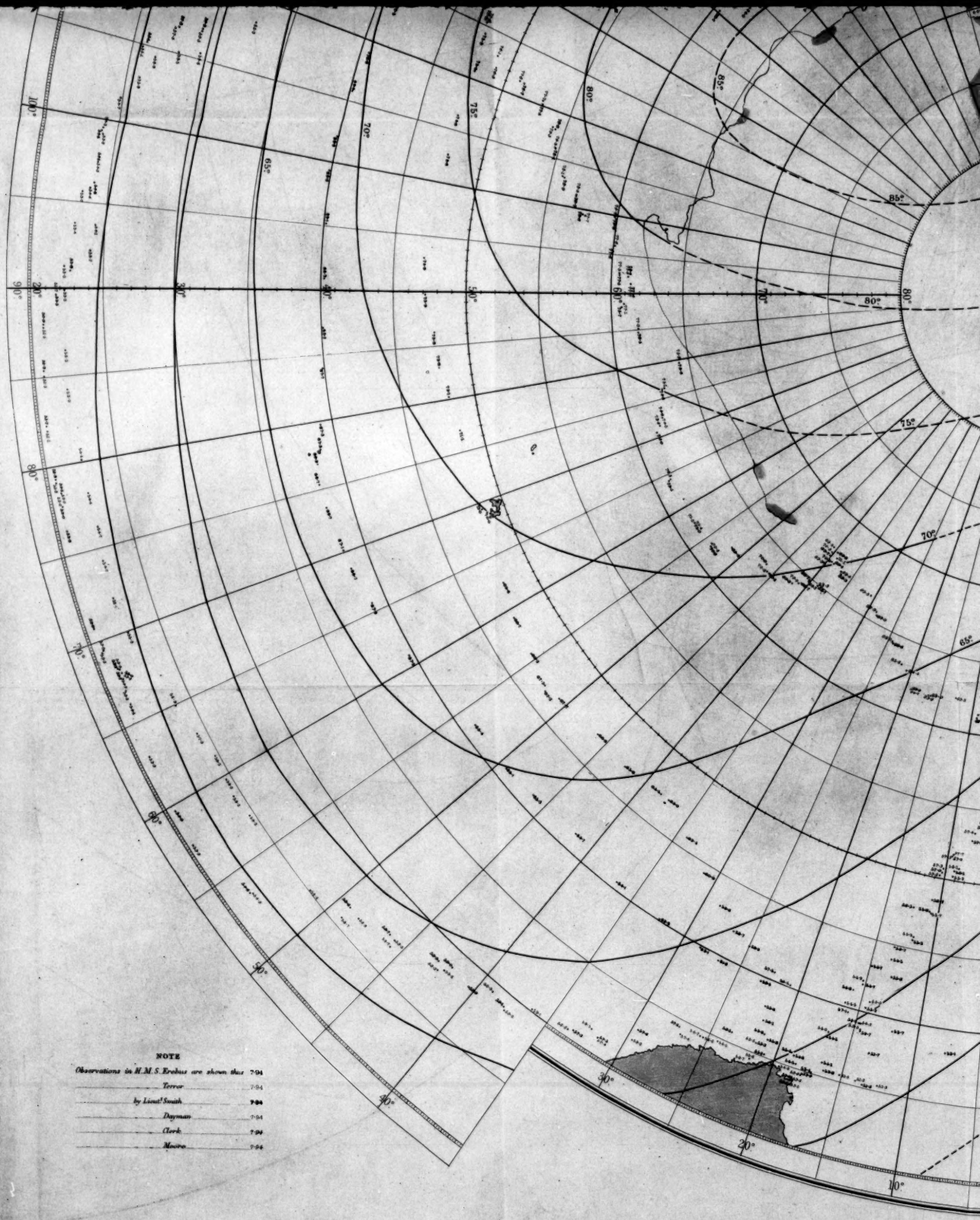


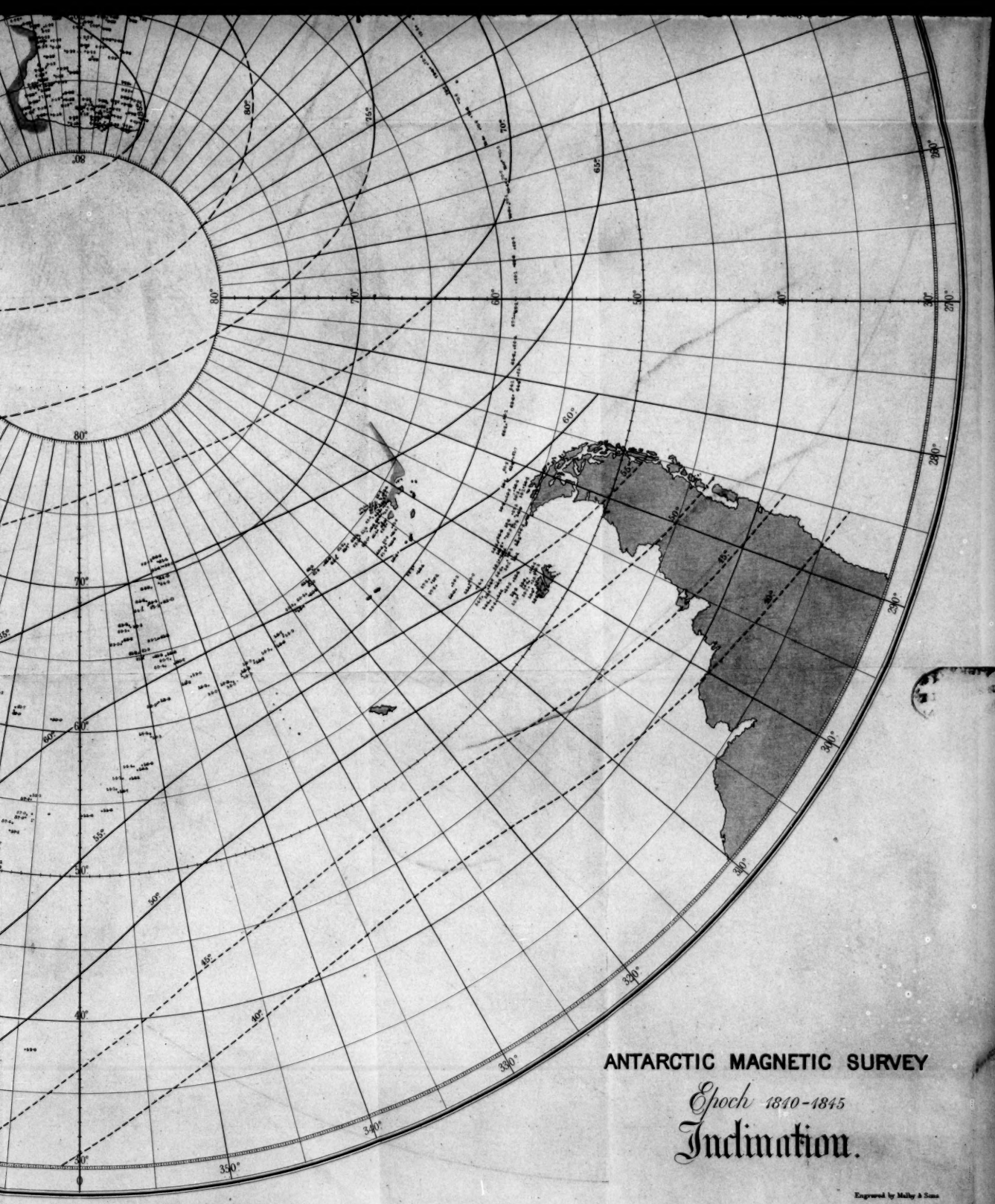

180°
190°
200°
210°
220°
230°
240°
250°
260°
270°
280°
290°
300°
310°
320°
330°
340°
350°
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°


NOTE
Observations in H.M.S. Erebus are thus distinguished 36·4
Terror 36·4
Pagoda 36·4
Easterly Declination represented thus
(approximate lines)
Westerly Declination represented
(approximate lines)


ANTARCTIC MAGNETIC SURVEY
Epoch 1840 - 1845
Declination.
Engraved by Malby & Sons


90°
80°
70°
60°
50°
40°
30°
20°
10°
85°
75°
65°
55°
45°
35°
25°
15°
5°
100°
110°
120°
130°
140°
150°
160°
170°


NOTE
Observations in H.M.S. Erebus are shown thus 7·94
Terror 7·94
by Lieut.t Smith 7·94
Dayman 7·94
Clerk 7·94
Moore 7·94



ANTARCTIC MAGNETIC SURVEY

Epoch 1840-1845

Inclination.

Engraved by Malby & Sons

15·0
14·0
13·0
12·0
11·0
10·0
15·0
14·0
13·0
14·0
15·0
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
180°
170°
160°
150°
140°
130°
120°
110°
100°

180°
170°
160°
150°
140°
130°
120°
110°
100°
90°
80°
70°
60°
50°
40°
30°
20°
10°
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350°
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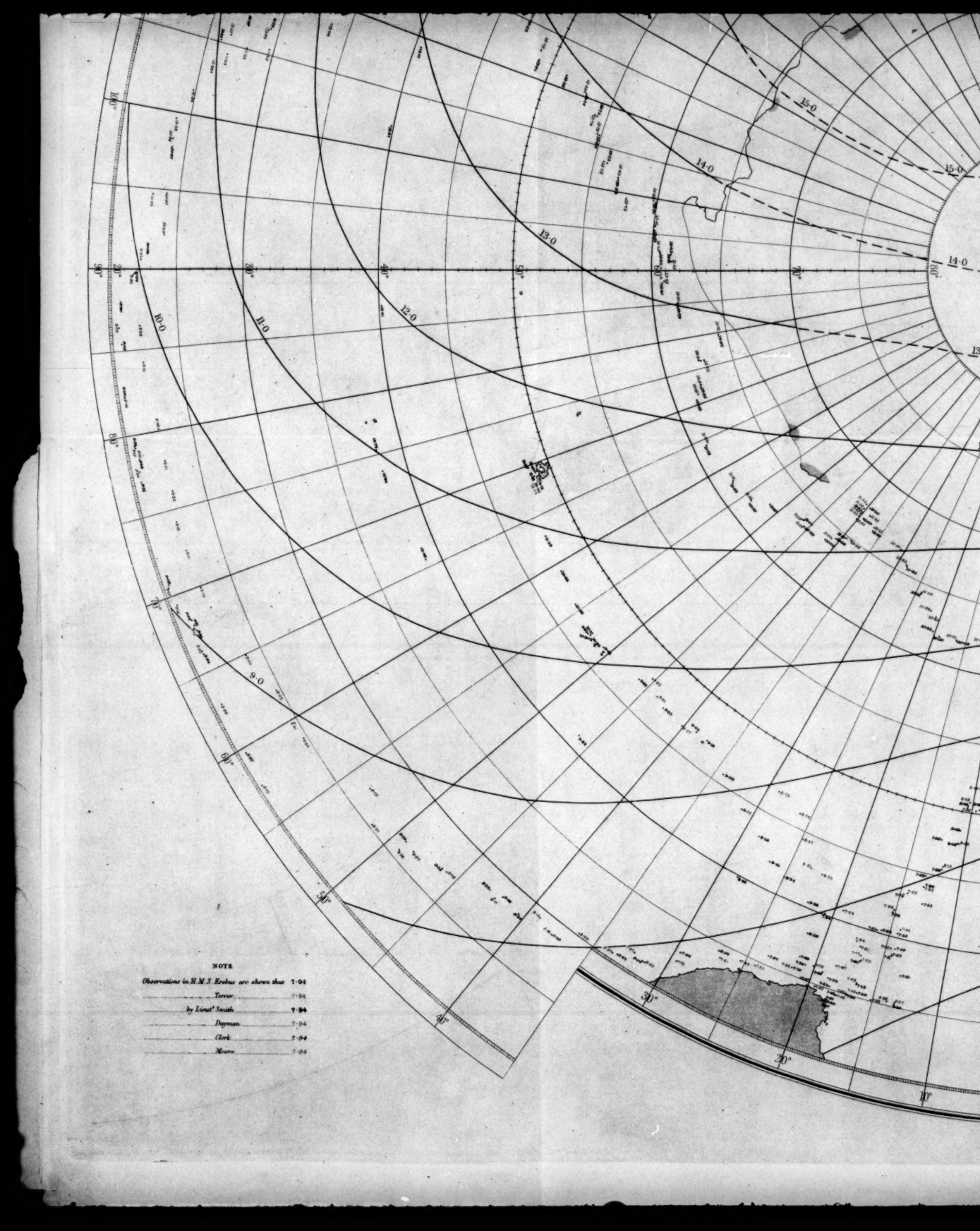
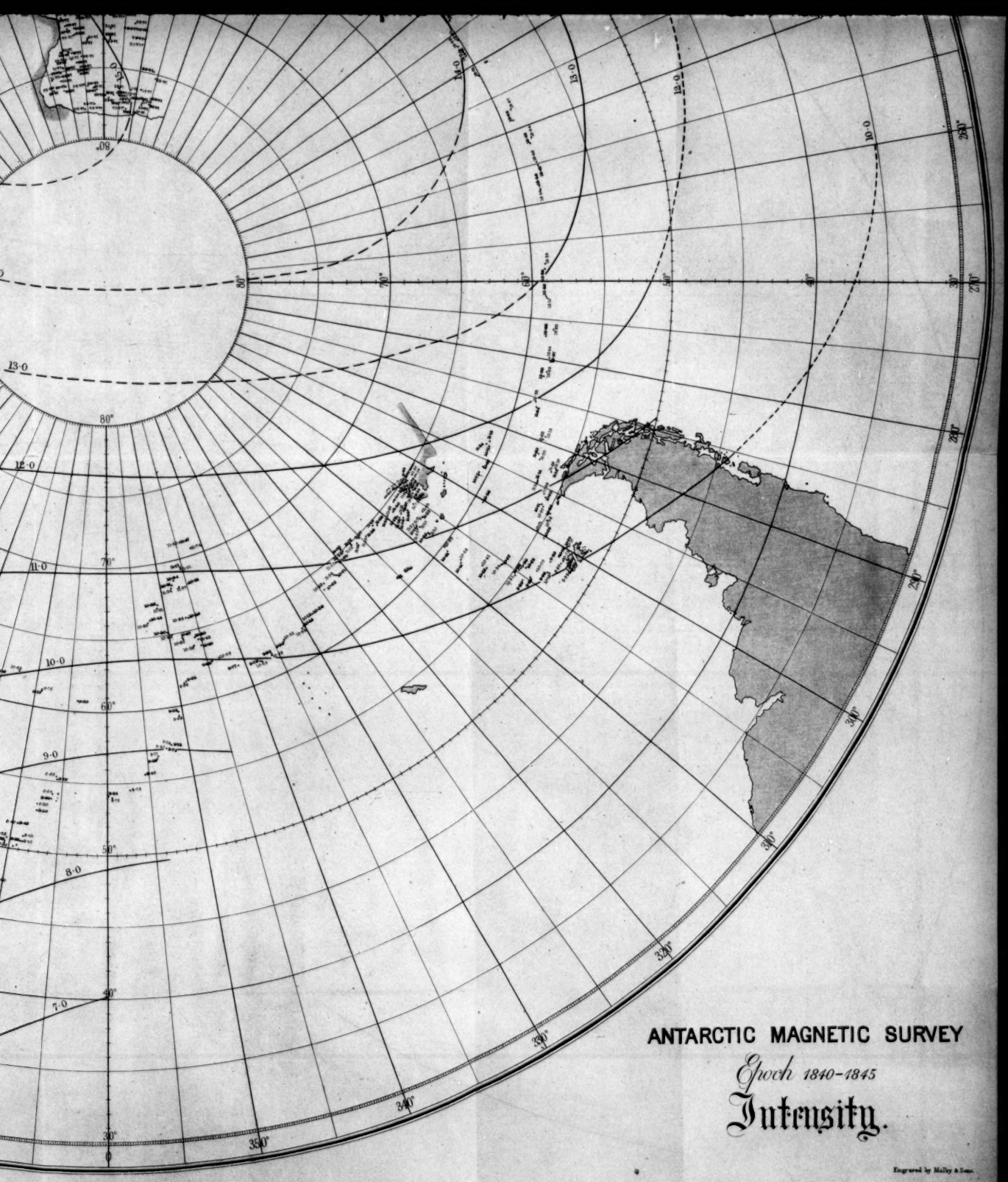
330°
320°
310°
300°
290°
280°
270°
260°
250°
240°
230°
220°
210°
200°
190°
11·0
12·0
13·0
14·0
15·0
16·0
80°
70°
60°
50°
40°
30°
20°
10°
0°


9·0
10·0
11·0
12·0
13·0
14·0
15·0
NOTE
Observations in H.M.S. Erebus are shown thus 7·94
Terror 7·94
by Lieut.t Smith 7·94
Dayman 7·94
Clark 7·94
Moore 7·94



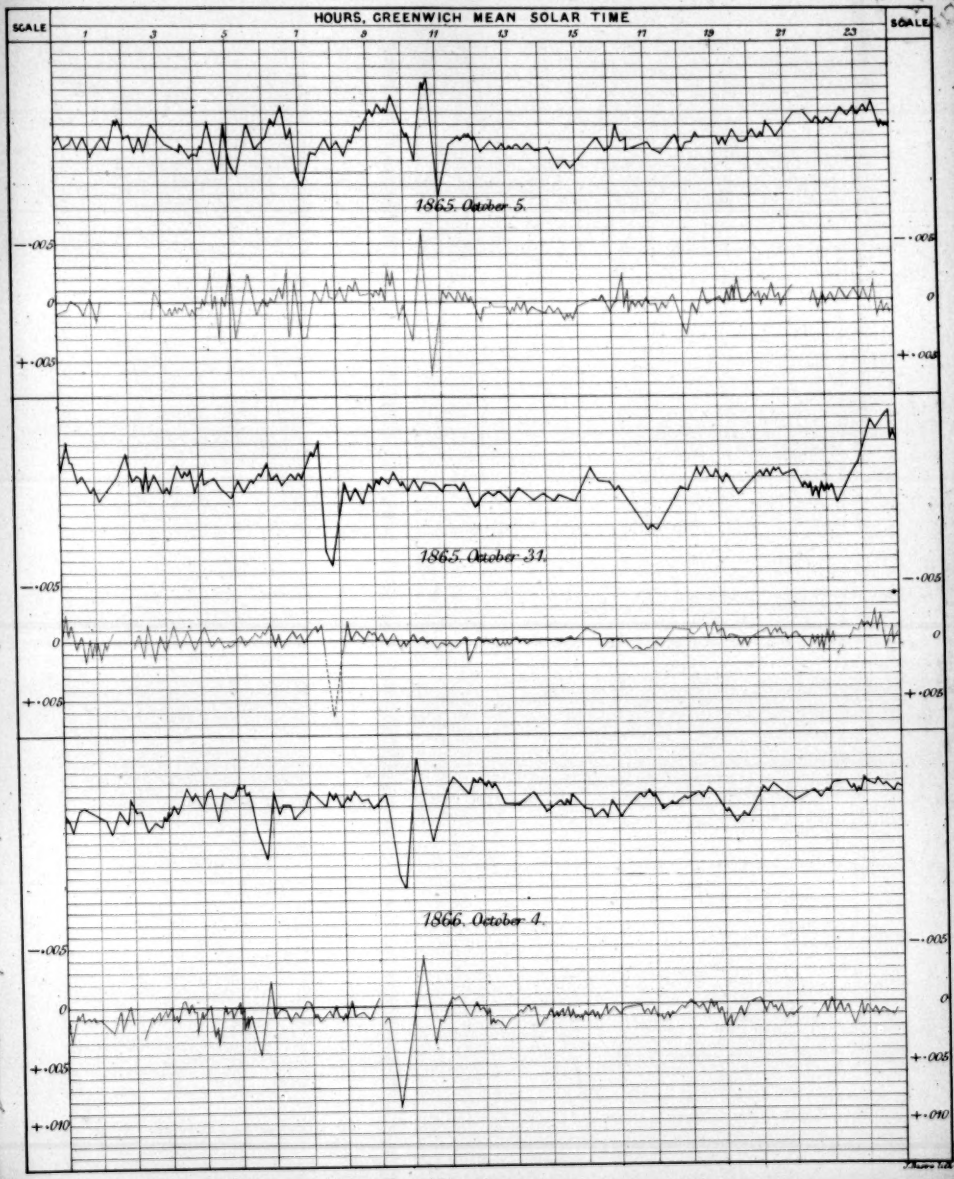
ANTARCTIC MAGNETIC SURVEY

Epoch 1840–1845

Intensity.

Engraved by Malby & Sons.

N. Force by Magnetometer.
N. Force by Earth Currents.



Phil. Trans. MDCCCLXVIII. Plate XXVI.

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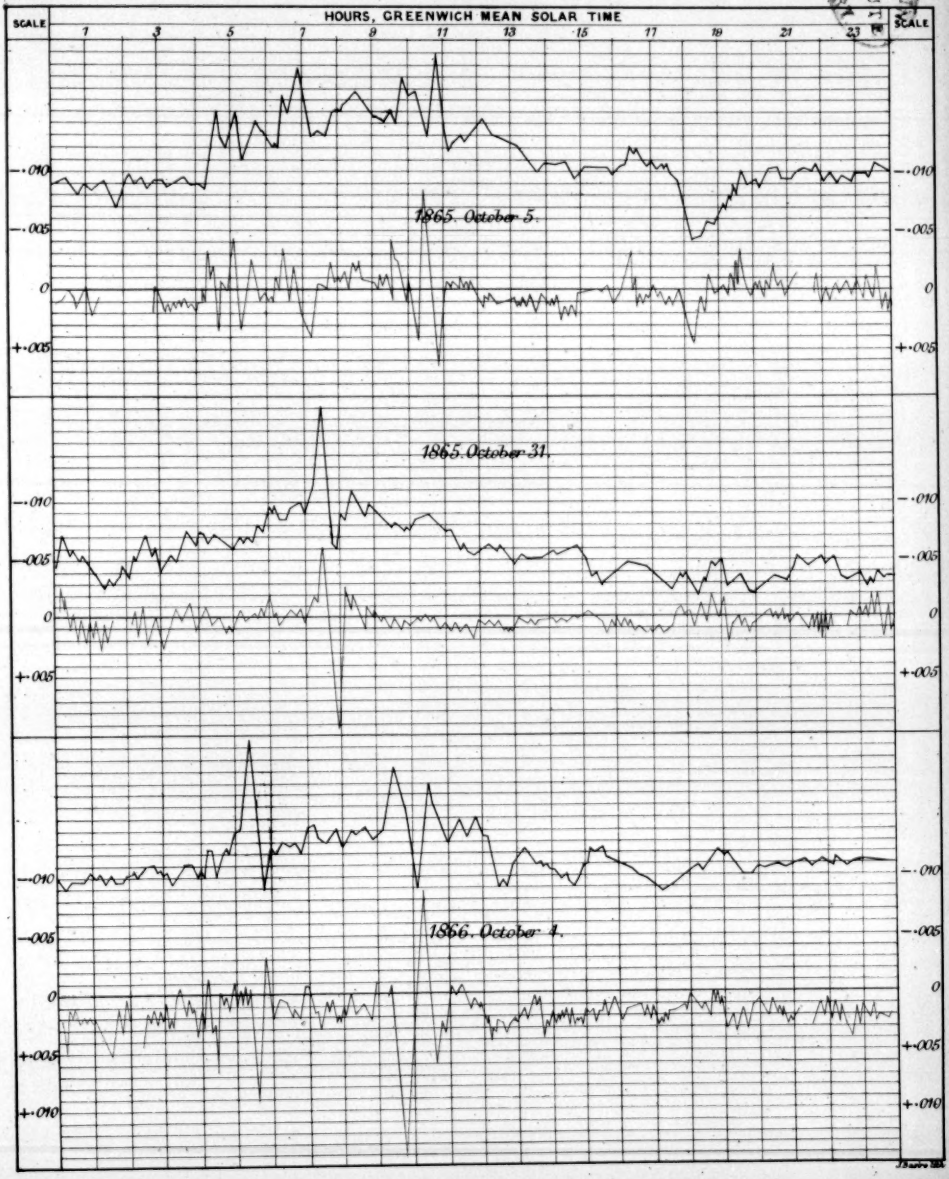
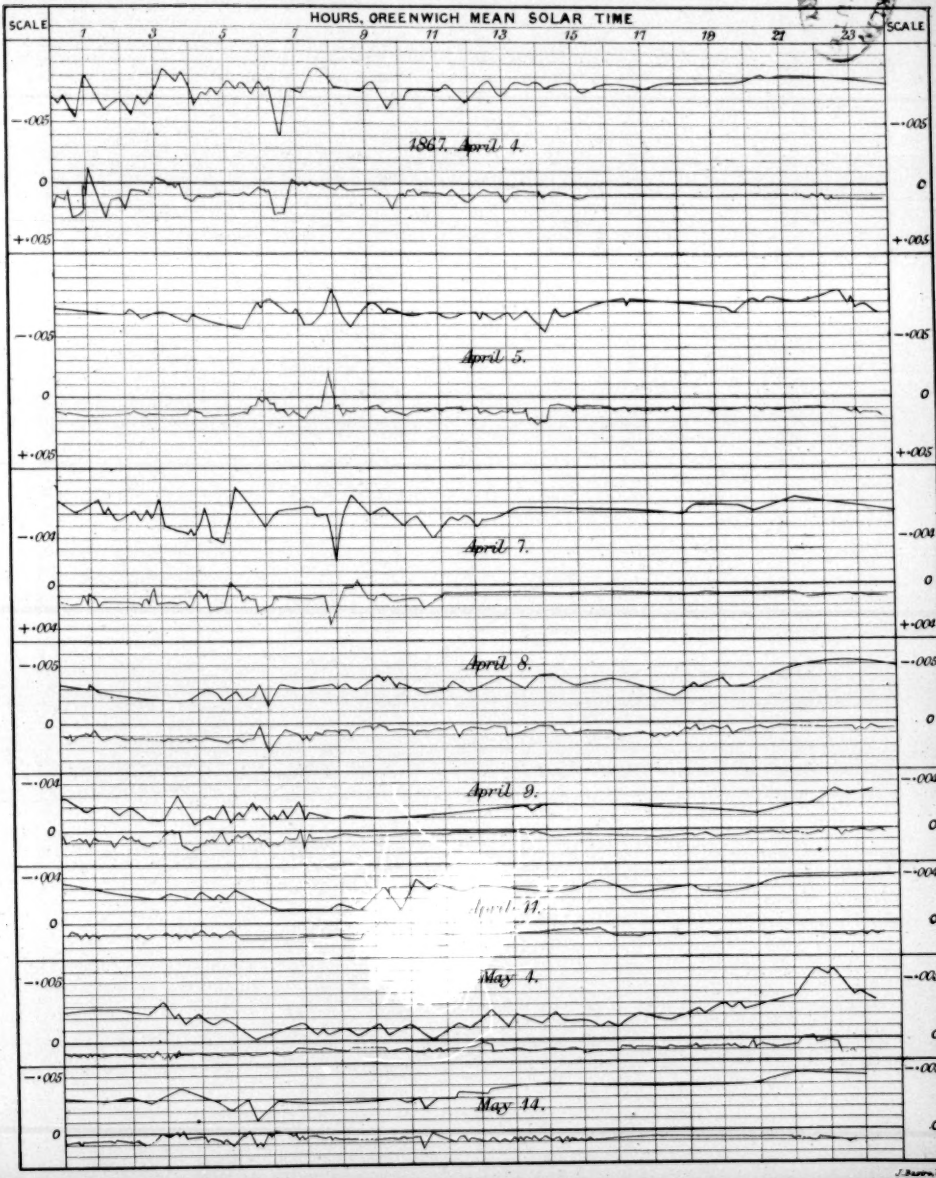
W. FORCE BY MAGNETOMETER.
W. FORCE BY EARTH CURRENTS.



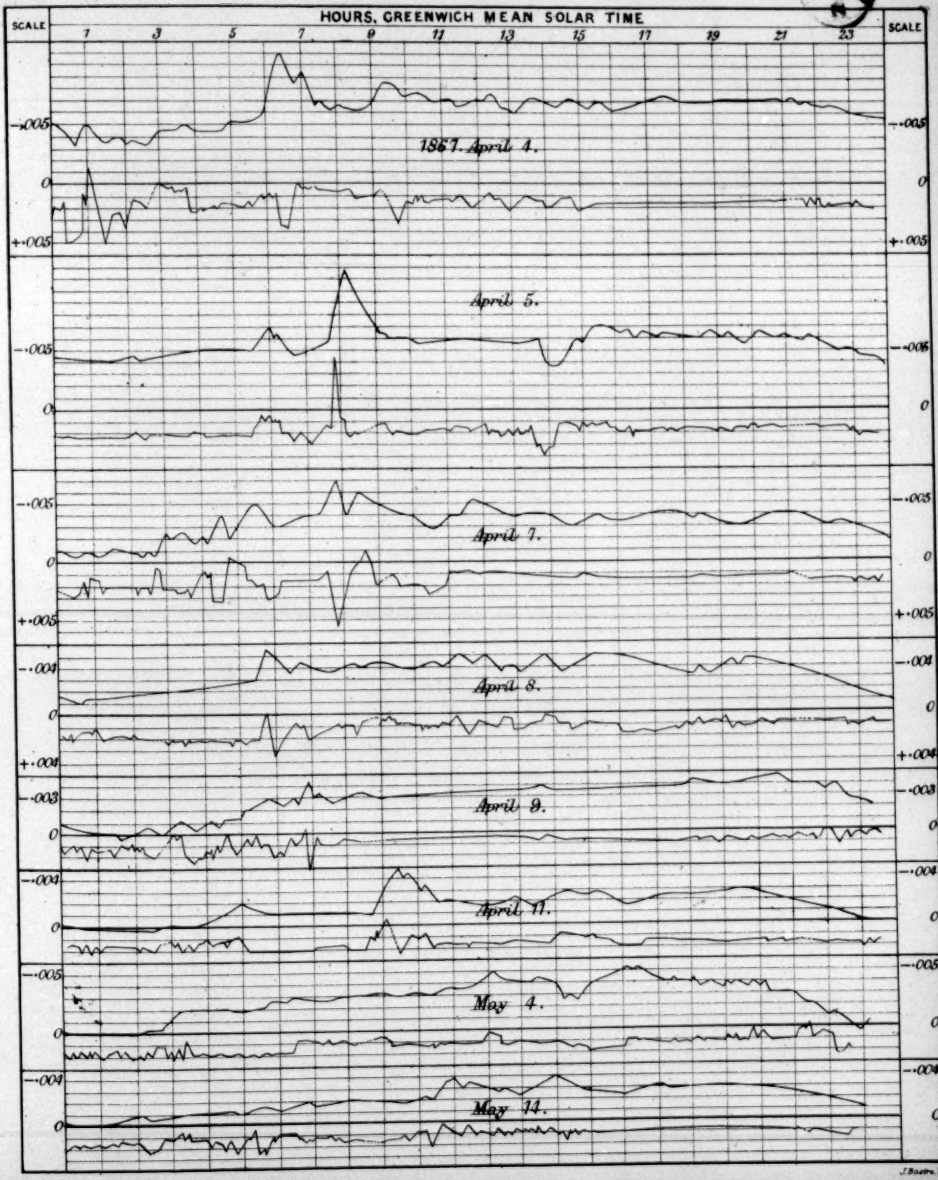
N. FORCE BY MAGNETOMETER.
N. FORCE BY EARTH CURRENTS.



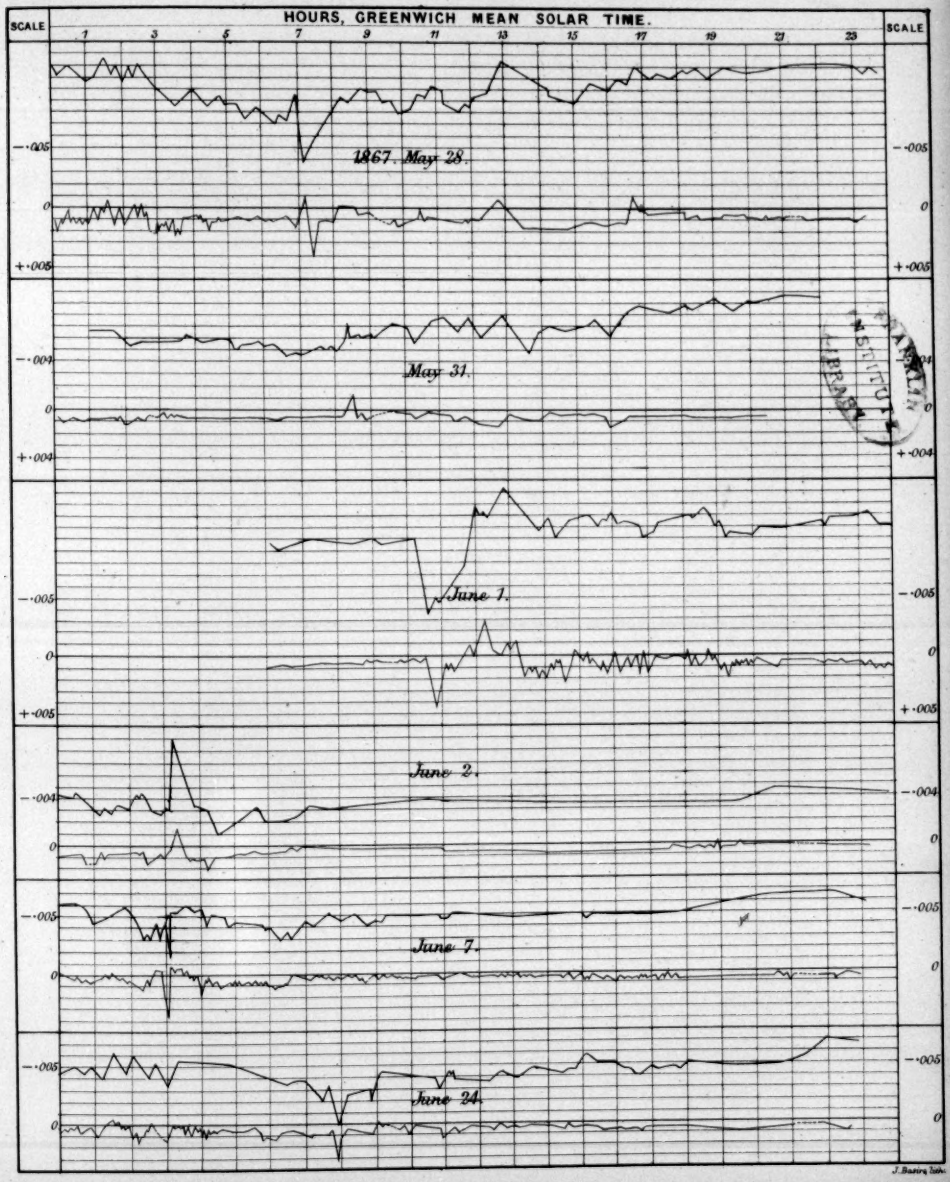
J. Basire lith.

ROYAL SOCIETY LIBRARY

W. Force by Magnetometer.
W. Force by Earth Currents.

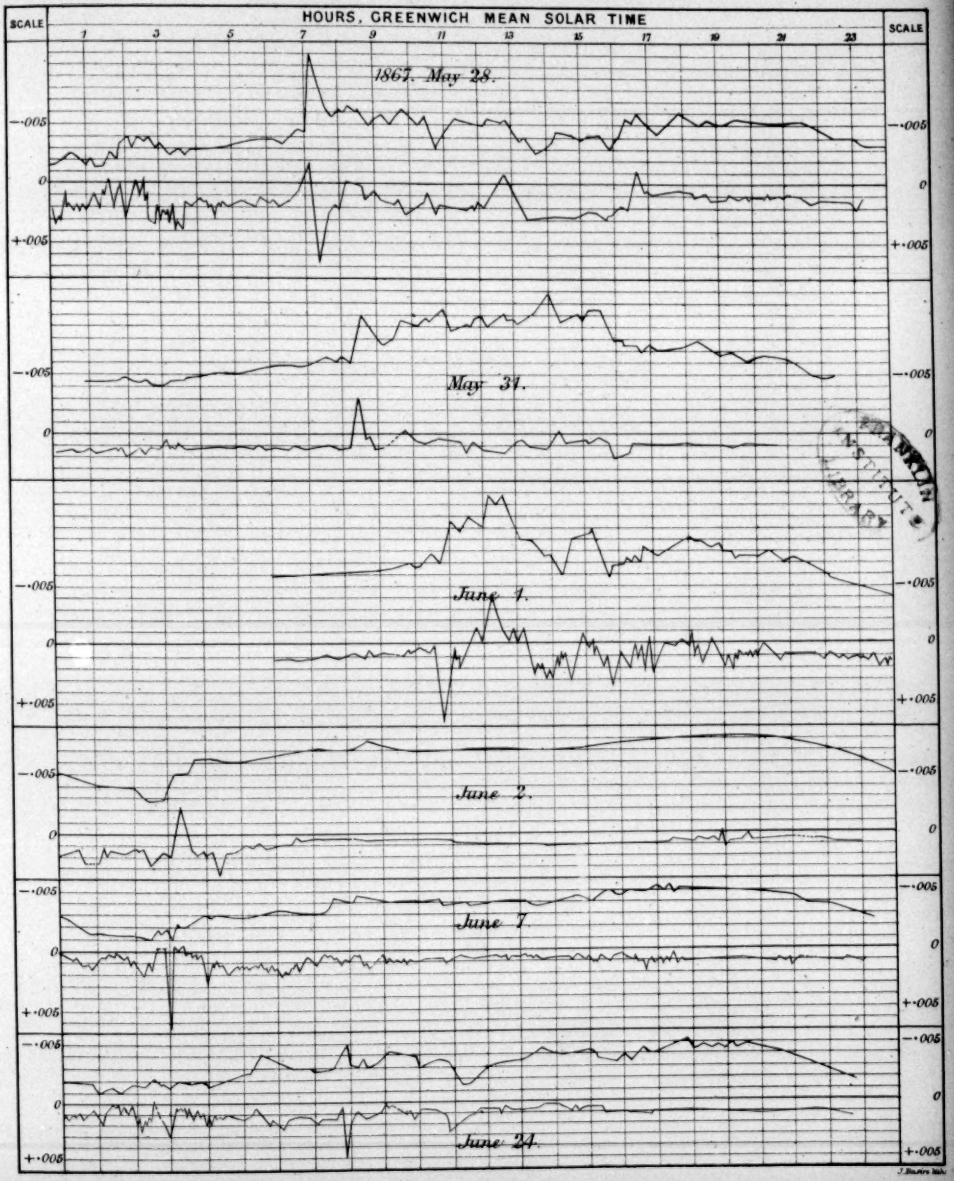


N. Force by Magnetometer.
N. Force by Earth Currents.



Phil. Trans. MDCCCLXVIII. Plate XXX.

W. Force by Magnetometer.
W. Force by Earth Currents.



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H.M.S. NORTHUMBERLAND.

Figure 1.

Plan.

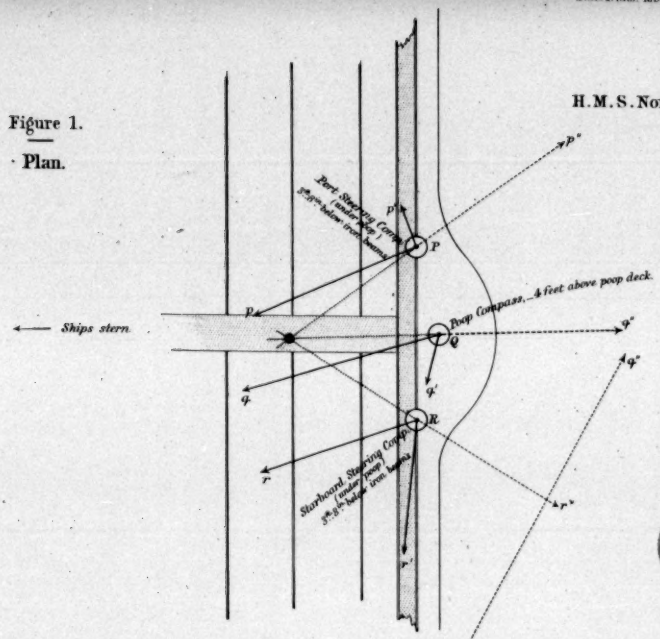
Ships stern.
Ships head.
Poop Compass. 4 feet above poop deck.
Port Steering Compass (under poop) 3 ft. 6 in. below iron beams.
Starboard Steering Compass (under poop) 3 ft. 6 in. below iron beams.

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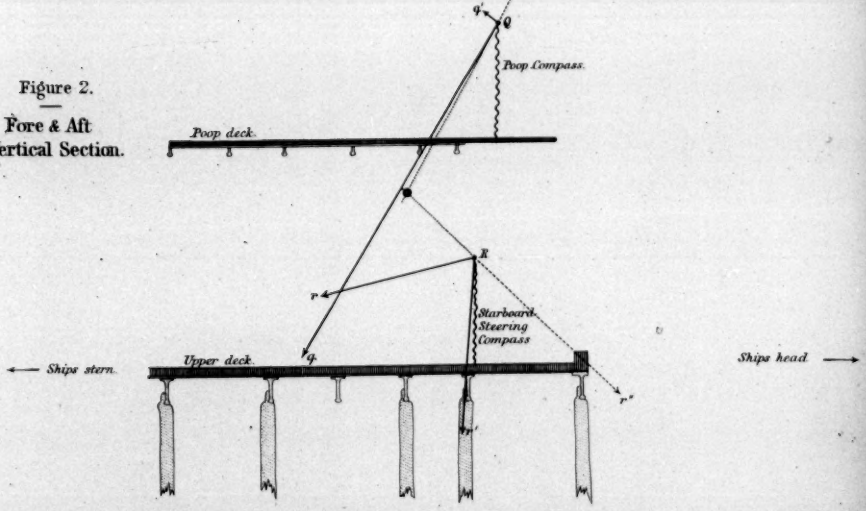
Figure 2.

Fore & Aft Vertical Section.

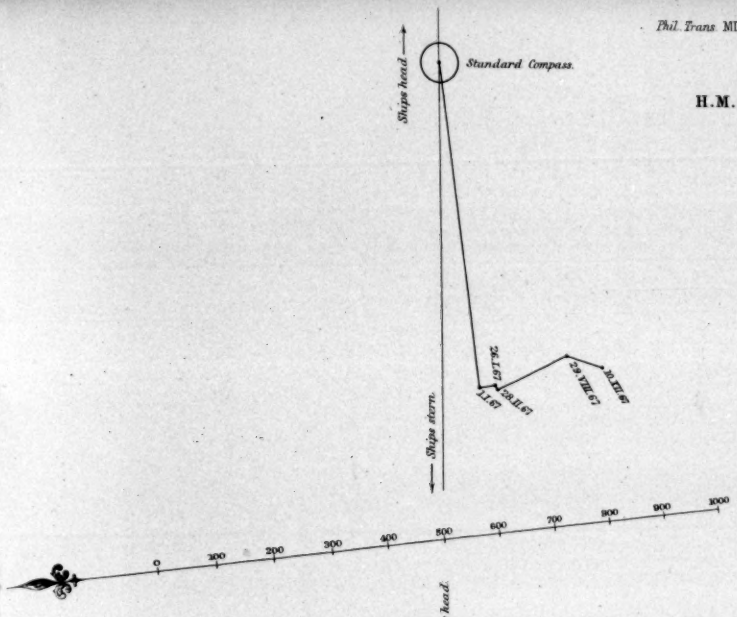


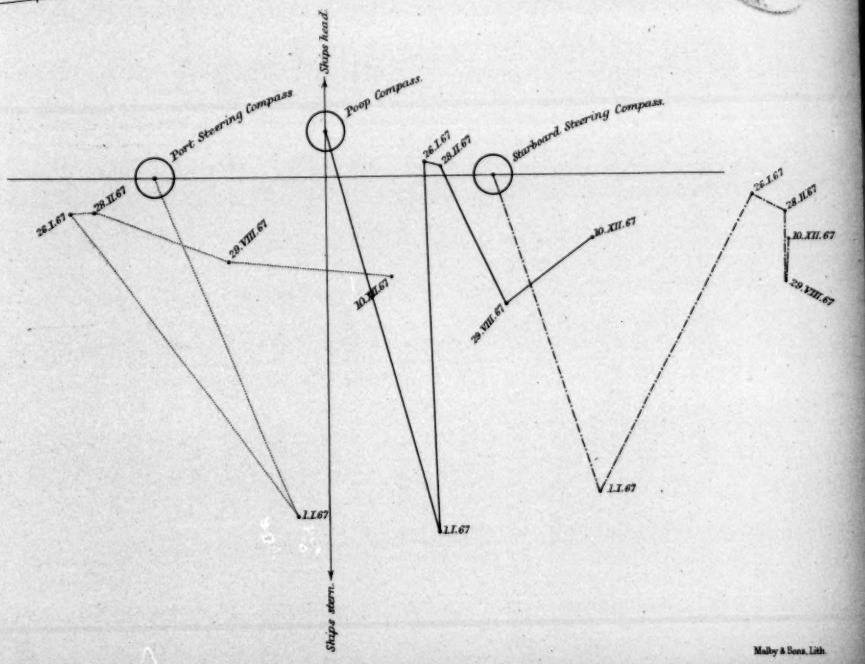
Scale of Feet, and also of $\frac{1}{10^{ths}}$ of Earth's horizontal force.

F. J. Evans, Del.

Malby & Sons, Lith.

H.M.S. Northumberland.





F.J.Evans, Del.

Malby & Sons, Lith.

Fig. 1.

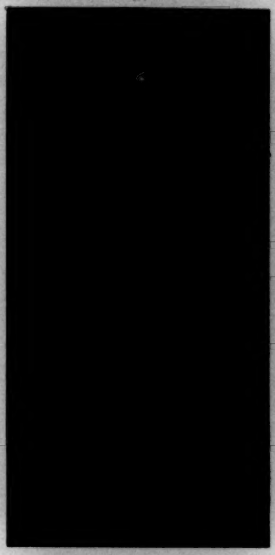


Fig 2.

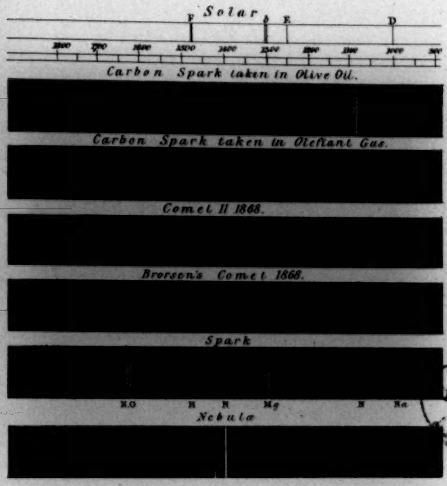


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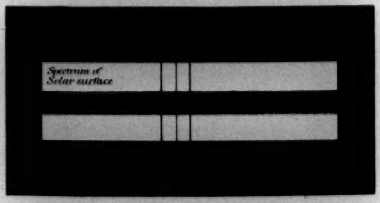
Fig. 3.



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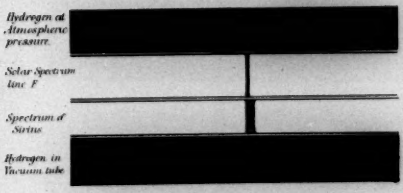

J. Basire, lith.

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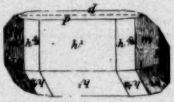


Fig. 2.

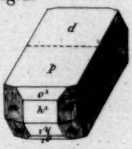
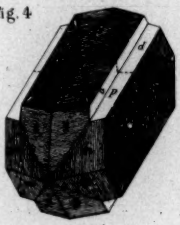


Fig. 3.

Fig. 4



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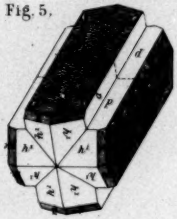
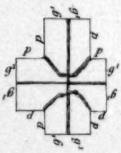


Fig. 6.

Fig. 7.



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WÖHLERITE.

Fig. 1.

Fig. 2.

Fig. 3.

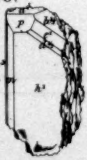


Fig 4.

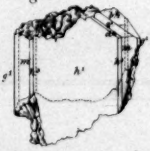
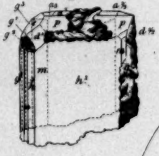


Fig. 5.

$a_3 - (b^1 b^{\frac{1}{2}} h^1)$

Fig. 6.

Fig. 7.



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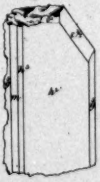

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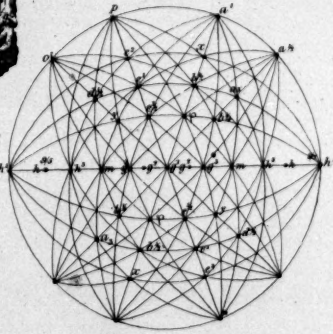
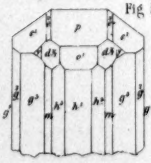


Fig. 9.

$\varphi - (b^1 d^{\frac{1}{2}} g^1)$

Fig. 10.



$y - (d^1 b^{\frac{1}{2}} g^1)$

$y - (d^{\frac{1}{2}} b^{\frac{1}{2}} g^1)$ $x - (b^1 b^{\frac{1}{2}} h^{\frac{1}{2}})$

$a_3 - (b^1 b^{\frac{1}{2}} h^1)$ $\varphi - b^1 d^{\frac{1}{2}} g^1)$

W.H. Weekley, Lith. Vincent, Brooks Day & Son, Imp.

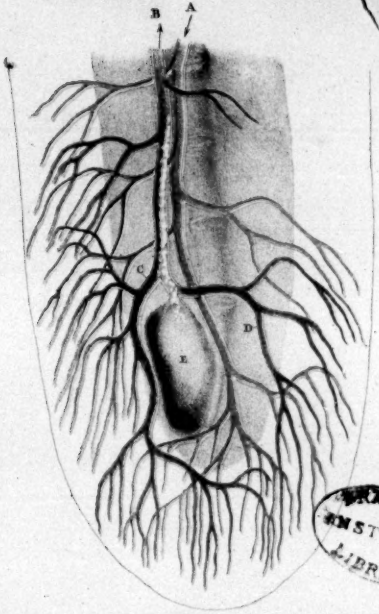


Fig 1

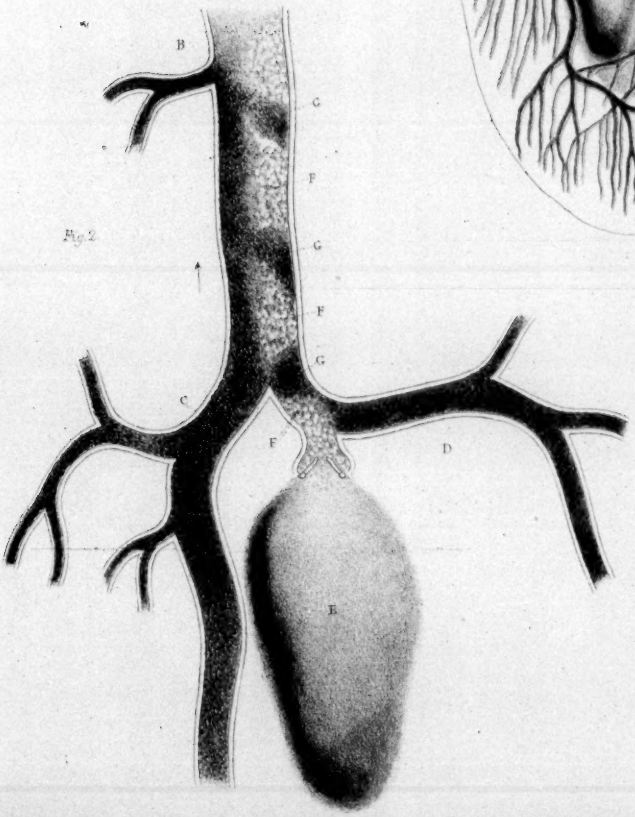


Fig 2

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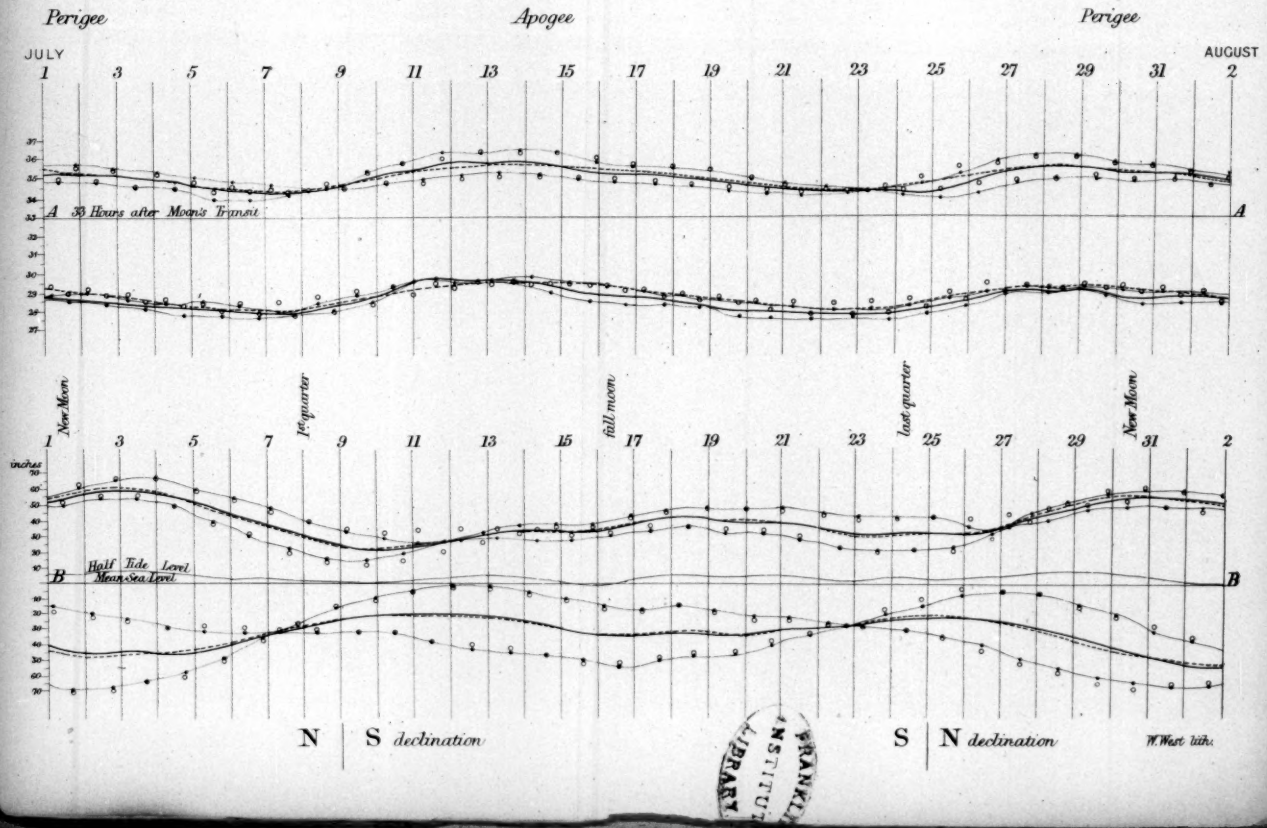
W.H. Hooley lith.

KURRACHEE TIDES. 1867.



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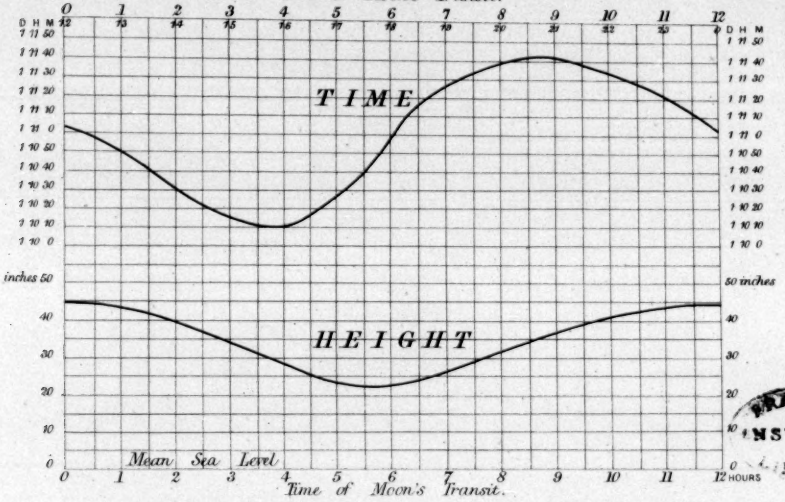
Kurrachee Tides. Mean Semimonthly Curves of Time and Height.



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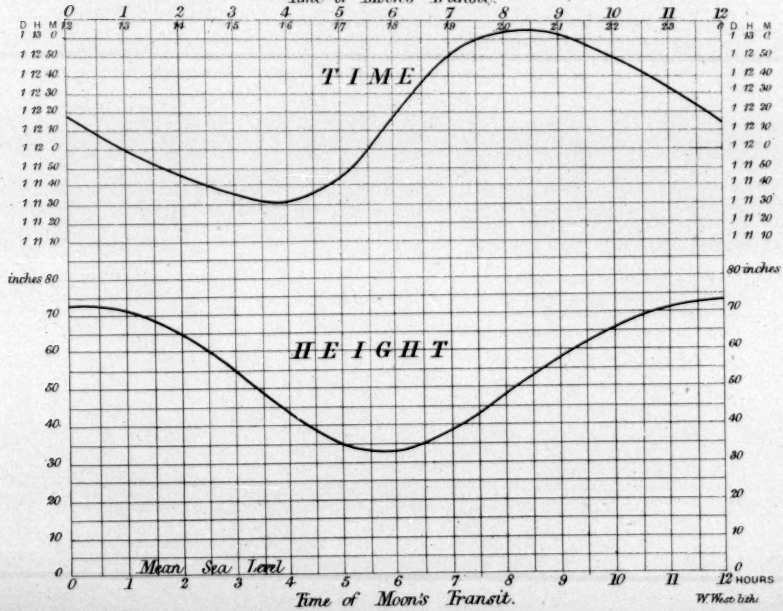
Bombay Tides. Mean Semimonthly Curves of Time and Height.